非牛顿流体渗透性井壁偏心环空二维流动

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内容提要 本文研究了适合带渗透性井壁的偏心环空非牛顿流体二维运动微分方程式，提出了牛顿流体与宾汉流体在该偏心环空中流动的流场解析解及幂律流体流场的数值解。文中还导出了计算牛顿流体与宾汉流体在实际井眼内流动的压降与流量计算公式。

主题词 非牛顿流体 渗透性井壁 偏心环空 二维流动

在石油钻井工程中，泥浆与水泥浆在井眼内的流动，通常可看作是宾汉流体或幂律流体在带渗透性井壁的偏心环空中的流动。由于这一渗透作用，使得偏心环空一维轴向流动变成二维流动，增加了理论上探讨的难度。因此，长期以来，有的学者假设井眼为非渗透的刚性井壁[1]。

在流体力学界，已有学者研究了牛顿流体在渗透性井壁同心环空中的二维流动[2]。文献〔2〕给出了这种流动的数值解。

用平板流模型研究偏心环空中的流动，已广为人知[1,3]。因此，仍然可用平板流模型研究带渗透性井壁的偏心环空中的流动。为讨论方便，本文假设：

①流动是稳定的；

②井壁渗透速度是均匀的；

③渗透速度与轴向速度相比很小。

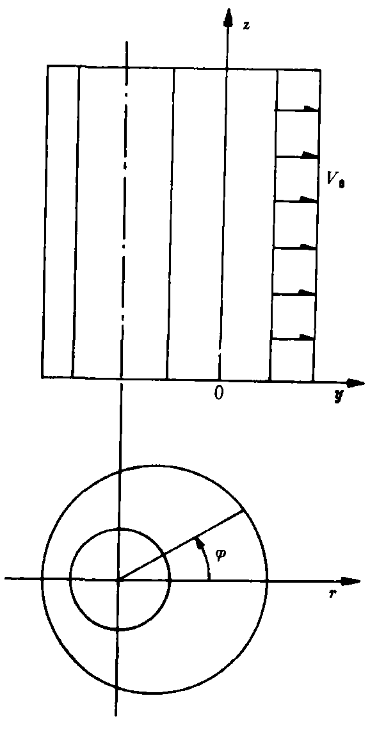

图1 偏心环空

流动微分方程式

由流体力学理论知[3]，偏心环空（图1）流动的平衡微分方程是：

$$\rho\left(\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\varphi}{r}\frac{\partial u_z}{\partial \varphi}+u_z\frac{\partial u_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz})+\frac{1}{r}\frac{\partial \tau_{\varphi z}}{\partial \varphi}+\frac{\partial \tau_{zz}}{\partial z}\right]-\rho g \qquad (1)$$

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考虑二维稳定流动且径向流速很小，有：

$$u_{\varphi}=0,\quad \frac{\partial u_z}{\partial t}=0,\quad \frac{\partial u_z}{\partial z}=0,\quad \frac{\partial \tau_{zz}}{\partial z}=0$$

对平板流，令 $r\to\infty$，并以$\frac{\partial}{\partial y}$、$u_y$、$\tau_y$ 分别代替$\frac{\partial}{\partial r}$、$u_r$、$\tau_{rz}$，(1) 式成为：

$$u_y\frac{\partial u_z}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{1}{\rho}\frac{\partial \tau_y}{\partial y}-g \tag{2}$$

(2)式即为非牛顿流体偏心环空二维运动微分方程式。

牛顿流体偏心环空二维流动

牛顿流体本构方程：

$$\tau_y=\mu\frac{\partial u_z}{\partial y} \tag{3}$$

$$边界条件\begin{cases} u_z=0 & (y=\pm h)\\ u_y=V_0 & (y=h)\end{cases} \tag{4}$$

连续性方程：

$$\frac{\partial u_z}{\partial z}+\frac{\partial u_y}{\partial y}=0 \tag{5}$$

因为$\frac{\partial u_z}{\partial z}=0$，所以由(4)、(5)式有：

$$u_y=V_0=常数 \tag{6}$$

将(3)、(6)式代入(2)式有：

$$V_0\frac{\partial u_z}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{\mu}{\rho}\frac{\partial^2 u_z}{\partial y^2}-g \tag{7}$$

由于是均匀流动[3]，所以：

$$-\frac{\partial p}{\partial z}=\frac{\Delta p}{L}=常数 \tag{8}$$

在边界条件(4)式下积分(7)式有：

$$u_z=\frac{\Delta p-\rho gL}{\rho LV_0}\left[y-\frac{h\left(2e^{\frac{\rho V_0 y}{\mu}}-e^{\frac{\rho V_0 h}{\mu}}-e^{-\frac{\rho V_0 h}{\mu}}\right)}{e^{\frac{\rho V_0 h}{\mu}}-e^{-\frac{\rho V_0 h}{\mu}}}\right] \tag{9}$$

由(9)式可得流量计算公式：

$$Q=2\,\frac{\Delta p-\rho gL}{\rho LV_0}\int_0\left\{(R_p+h)\left[-\frac{2\mu h}{\rho V_0}+2h^2\frac{e^{\frac{\rho V_0 h}{\mu}}+e^{-\frac{\rho V_0 h}{\mu}}}{e^{\frac{\rho V_0 h}{\mu}}-e^{-\frac{\rho V_0 h}{\mu}}}\right]+\frac{2}{3}h^3-\frac{2\mu h}{\rho V_0}\left[h\,\frac{e^{\frac{\rho V_0 h}{\mu}}+e^{-\frac{\rho V_0 h}{\mu}}}{e^{\frac{\rho V_0 h}{\mu}}-e^{-\frac{\rho V_0 h}{\mu}}}-\frac{\mu}{\rho V_0}\right]\right\}d\varphi$$
$$+2\pi R_h LV_0 \tag{10}$$

利用(10)式也可以计算压降 Δp。由(9)式容易求出最大流速点坐标：

$$y_0=\frac{\mu}{\rho V_0}\ln\left[\frac{\mu}{2\rho V_0 h}\left(e^{\frac{\rho V_0 h}{\mu}}-e^{-\frac{\rho V_0 h}{\mu}}\right)\right] \tag{11}$$

容易证明 $y_0>0$，即最大流速点靠向井壁。

宾汉流体偏心环空二维流动

设流核中心坐标为 y_0，流核宽度为 $2b$。

对区域$-h\leqslant y\leqslant y_0-b$，有本构方程：

$$\tau_y=\tau_0+\eta\frac{\partial u_z}{\partial y} \tag{12}$$

$$\text{边界条件}\begin{cases}u_z=0 & (y=-h)\\ \dfrac{\partial u_z}{\partial y}=0 & (y=y_0-b)\end{cases} \tag{13}$$

将(12)式代入(2)式积分并整理(6)、(8)、(13)式，有：

$$u_z=\frac{\Delta p-\rho gL}{\rho LV_0}\left\{\frac{\eta}{\rho V_0}\left[e^{-\frac{\rho V_0}{\eta}(h+y_0-b)}-e^{\frac{\rho V_0}{\eta}(y-y_0+b)}\right]+y+h\right\}\quad(-h\leqslant y\leqslant y_0-b) \tag{14}$$

对区域$y_0+b\leqslant y\leqslant h$，有本构方程：

$$\tau_y=-\tau_0+\eta\frac{\partial u_z}{\partial y} \tag{15}$$

$$\text{边界条件}\begin{cases}u_z=0 & (y=h)\\ \dfrac{\partial u_z}{\partial y}=0 & (y=y_0+b)\end{cases} \tag{16}$$

将(15)式代入(2)式积分并整理(6)、(8)、(16)式，有：

$$u_z=\frac{\Delta p-\rho gL}{\rho LV_0}\left\{\frac{\eta}{\rho V_0}\left[e^{\frac{\rho V_0}{\eta}(h-y_0-b)}-e^{\frac{\rho V_0}{\eta}(y-y_0-b)}\right]+y-h\right\}\quad(y_0+b\leqslant y\leqslant h) \tag{17}$$

流核流速为：

$$u_0=u_z\big|_{y=y_0-b}=\frac{\Delta p-\rho gL}{\rho LV_0}\left\{\frac{\eta}{\rho V_0}\left[e^{-\frac{\rho V_0}{\eta}(h+y_0-b)}-1\right]+y_0-b+h\right\}\quad(y_0-b\leqslant y\leqslant y_0+b) \tag{18}$$

y_0由下式确定：

$$y_0=-\frac{\eta}{\rho V_0}\ln\left\{\frac{2\rho V_0(h-b)}{\eta\left[e^{\frac{\rho V_0}{\eta}(h-b)}-e^{-\frac{\rho V_0}{\eta}(h-b)}\right]}\right\} \tag{19}$$

容易证明，$y_0>0$，即流核中心靠向井壁。b由下式确定：

$$b=\frac{\tau_0}{\dfrac{\Delta p}{L}-\rho g} \tag{20}$$

流量由下式确定：

$$\begin{aligned}Q=2\,&\frac{\Delta p-\rho gL}{\rho LV_0}\int_0^{\pi}\Big\{(R_p+h)\Big[\Big(\frac{\eta}{\rho V_0}e^{-\frac{\rho V_0}{\eta}(h+y_0-b)}+h\Big)(y_0-b+h)\\&-\Big(\frac{\eta}{\rho V_0}\Big)^2\big(1-e^{-\frac{\rho V_0}{\eta}(h+y_0-b)}\big)+\frac{1}{2}\left[(y_0-b)^2-h^2\right)\Big]+\Big[\frac{\eta}{\rho V_0}e^{-\frac{\rho V_0}{\eta}(h+y_0-b)}+h\Big]\frac{1}{2}\left[(y_0-b)^2-h^2\right]\\&-\Big(\frac{\eta}{\rho V_0}\Big)^2\Big[y_0-b-\frac{\eta}{\rho V_0}+\Big(h+\frac{\eta}{\rho V_0}\Big)e^{-\frac{\rho V_0}{\eta}(h+y_0-b)}\Big]+\frac{1}{3}\left[(y_0-b)^3+h^3\right]+2bu_0(R_p+h+y_0)\\&+(R_p+h)\Big[\Big(\frac{\eta}{\rho V_0}e^{\frac{\rho V_0}{\eta}(h-y_0-b)}-h\Big)(h-y_0-b)-\Big(\frac{\eta}{\rho V_0}\Big)^2\big(e^{\frac{\rho V_0}{\eta}(h-y_0-b)}-1\big)+\frac{1}{2}\big(h^2-(y_0+b)^2\big)\Big]\\&+\Big[\frac{\eta}{\rho V_0}e^{\frac{\rho V_0}{\eta}(h-y_0-b)}-h\Big]\frac{1}{2}\left[h^2-(y_0+b)^2\right]-\Big(\frac{\eta}{\rho V_0}\Big)^2\Big[\Big(h-\frac{\eta}{\rho V_0}\Big)e^{\frac{\rho V_0}{\eta}(h-y_0-b)}-y_0-b+\frac{\eta}{\rho V_0}\Big]\\&+\frac{1}{3}\left[h^3-(y_0+b)^3\right]\Big\}\mathrm{d}\varphi+2\pi R_hLV_0\end{aligned} \tag{21}$$

利用(21)式也可以计算压降 Δp。

幂律流体偏心环空二维流动

设最大流速点坐标为 y_0,对区域 $-h\leqslant y\leqslant y_0$,本构方程为:

$$\tau_y=k\left(\frac{\partial u_z}{\partial y}\right)^n \tag{22}$$

将(22)式代入(2)式整理后有:

$$V_0\frac{\partial u_z}{\partial y}=\frac{\Delta p}{\rho L}+\frac{1}{\rho}kn\left(\frac{\partial u_z}{\partial y}\right)^{n-1}\frac{\partial^2 u_z}{\partial y^2}-g \tag{23}$$

$$\text{边界条件}\begin{cases}u_z=0 & (y=-h)\\ \dfrac{\partial u_z}{\partial y}=0 & (y=y_0)\end{cases} \tag{24}$$

同理,对区域 $y_0\leqslant y\leqslant h$,本构方程为:

$$\tau_y=-k\left(-\frac{\partial u_z}{\partial y}\right)^n \tag{25}$$

将(25)式代入(2)式整理后有:

$$V_0\frac{\partial u_z}{\partial y}=\frac{\Delta p}{\rho L}+\frac{1}{\rho}kn\left(-\frac{\partial u_z}{\partial y}\right)^{n-1}\frac{\partial^2 u_z}{\partial y^2}-g \tag{26}$$

$$\text{边界条件}\begin{cases}u_z=0 & (y=h)\\ \dfrac{\partial u_z}{\partial y}=0 & (y=y_0)\end{cases} \tag{27}$$

y_0 可由最大流速点处的连续性条件求出。偏微分方程(23)、(26)的求解需用数值方法。下面利用差分法求解上述方程。

算例:已知 $R_p=0.025\text{m}$,$R_h=0.0175\text{m}$,$L=15\text{m}$,$e=0.0131\text{m}$,$\varphi=0°$,$\Delta p=0.22\text{MPa}$,$n=0.5$,$k=1.5\text{Pa}\cdot\text{s}^n$,$\rho=1.5\text{g/cm}^3$,$V_0=0.0421\text{m/s}$

计算结果见图 2 中曲线 2。曲线 1 是相同条件下的非渗透性偏心环空轴向流速分布。由图 2 可以看出,由于井壁渗透,环空最大流速点移向井壁一侧,即 $y_0>0$ 恒成立。这在上述两节中已由解析法加以证明。由于井壁渗透,使得偏心环空中靠井壁一侧的流速剖面比较丰满。这在注水泥作业中,对提高靠井壁一侧的注水泥顶替效率是有利的。

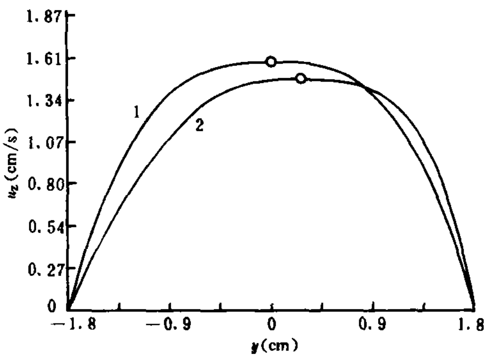

图 2　环空流场分布

结　　论

(1)本文导出了非牛顿流体在偏心环空中二维流动的运动微分方程式。

(2)提出了牛顿流体、宾汉流体在渗透性偏心环空或井眼内二维流动的速度场分布公式及其流量、压降计算公式。

(3)用数值方法求解了幂律流体在渗透性

偏心环空或井眼内二维流动的速度场分布。

(4)井壁渗透使环空中靠井壁一侧的流速剖面变得丰满，因而有利于提高该侧的注水泥顶替效率。

符号说明

b——流核宽度的一半，m；

e——套管偏心距，m；

g——重力加速度，9.8m/s^2；

h——环空间隙宽度的一半，$h=0.5(e\cos\varphi+\sqrt{R_h^2-e^2\sin^2\varphi}-R_p)$，m；

k——幂律流体稠度系数，Pa·s^n；

L——井深，m；

n——幂律流体流性指数，无量纲；

Δp——偏心环空两端压降，Pa；

Q——流量，m^3/s；

R_h——井眼半径，m；

R_p——套管外半径，m；

V_0——渗透速度，m/s；

μ——牛顿流体动力粘滞系数，Pa·s；

τ_0——宾汉流体屈服值，Pa；

η——宾汉流体塑性粘度，Pa·s；

ρ——流体密度，kg/m^3。

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首台240道数字地震仪在川投入生产

从80年代中期起，地震勘探野外资料采集全部实现数字化的四川气区近日又添新装备。从美国引进的MDX-18X型240道数字地震仪今年6月1日正式投入三维地震资料采集，采集资料经初评合格率达100%，一级品率为71%。

屈永志

Zheng Yonggang: **Two Dimensional Flowing of Non-Newtonian Fluid in Eccentric Annulus With Permeable Well Wall,NGI** 13(5), 1993: 42～46

The two-dimentional motion differential which fits for non-Newtonian fluid in eccentric annulus with permeable well wall is studied in this paper. The analytic solution for flowing field and the numerical solution for power law flow field of Newtonian and Bingham fluids flowing in eccentric annulus are proposed. The formulas used to calculate the pressure fall and flow rate of Newtonian and Bingham fuids flowing in actual well hole are also derived.

Subject Headings: non-Newtonian fluid, permeable well wall, eccentric annulus, two-dimentional flowing.

DRILLING/PRODUCTION TECHNOLOGY AND EQUIPMENT

Zhu Qingchen, Zheng Yong and Zhou Xa: **The Technique and Method to Determine the Safety of Working Derrick,NGI** 13(5), 1993: 47～49

Through testing field derrick, setting up data base and determination model and analysing the tested data with computer, the technique concepts of initial bend stress, data normal distribution examination(by x^2), establishing regression equation and determining present bearing capacity at the given confidence level are proposed out to slove the safety evaluation problem with working derrick A, which is reasonable and reliable determination technique.

Subject Headings: drilling, working derrick, bearing capacity, safety determination, analytical method.

Li Qian and Chen Zhongshi: **Frictional Resistance Computation of Setting Casing in Highly—Deviated Well,NGI** 13(5), 1993: 50～54

In the light of the equilibrium differencial equation of elastic beam in bearied force and deformation condition, based on deriving frictional resistance computation model of casing unit in two-dimention bend section, the one in three-dimention bend section is derived out by approximately analysing. The calculated hook load is idendical with that of actually testing is verified by examining the setting casing data in highly-deviated well of Sichuan Long 40—1.

Subject Headings: highly-deviated well, setting casing frictional resistance, friction resistance, axial force, calculation model.

Zhou Houan: **Discussion on Drilling Waste Water Disposal in Sichuan Petroleum Administration, NGI** 13(5), 1993: 54～58

The treatment present situation of drilling waste water in some gas fields of Sichuan Petroleum Administration is described, and the source, composition and characteristics of drilling waste water are analysed in this paper. Why the standardized rates of chemical oxygen demand (COD) and chromaticity are both lower is discussed, and the countermeasures of prevention and cure are posed out combining the existing problems in this paper.

Subject Headings: oil and gas fields in Sichuan, drilling waste water, chemical oxygen demand, waste water disposal, countermeasures of prevention and cure.

Wang Yuwen, Zhang Lanyou and Sun Jiazheng: **Study on the Methods for Defermining Gas Well Reasonable Output and Froecasting New Well Productivity,NGI** 13(5), 1993: 59～62

Starting from the actual need of conceptual design of Carboniferous gas development in Datianchi structure belt in East Sichuan, this paper analyses and studies the gas well reasonable output and the