





# Stochastic modelling of soil moisture dynamics in a grassland of Qilian Mountain at point scale

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Stochastic modeling of soil moisture dynamics is crucial to the quantitative understanding of plant responses to water stresses, hydrological control of nutrient cycling processes, water competition among plants, and some other ecological dynamics, and thus has become a hotspot in ecohydrology at present. In this paper, we based on the continuously monitored data of soil moisture during 2002-2005 and daily precipitation date of 1992-2006, and tried to make a probabilistic analysis of soil moisture dynamics at point scale in a grassland of Qilian Mountain by integrating the stochastic model improved by Laio and the Monte Carlo method. The results show that the inter-annual variations for the soil moisture patterns at different depths are very significant, and that the coefficient of variance (CV) of surface soil moisture (20 cm) is almost continually kept at about 0.23 whether in the rich or poor rainy years. Interestingly, it has been found that the maximal CV of soil moisture has not always appeared at the surface layer. Comparison of the analytically derived soil moisture probability density function (PDF) with the statistical distribution of the observed soil moisture data suggests that the stochastic model can reasonably describe and predict the soil moisture dynamics of the grassland in Qilian Mountain at point scale. By extracting the statistical information of the historical precipitation data in 1994-2006, and inputting them into the stochastic model, we analytically derived the long-term soil moisture PDF without considering the inter-annual climate fluctuations, and then numerically derived the one when considering the inter-annual fluctuation effects in combination with a Monte-Carlo procedure. It was found that, though the peak position of the probability density distribution significantly moved towards drought when considering the disturbance forces, and its width was narrowed, accordingly its peak value was increased, no significant bimodality was observed in the soil moisture dynamics under the given intensity of random fluctuation disturbance.

soil moisture dynamics, stochastic modelling, Monte Carlo method, probability density function, grassland ecosystem

Soil moisture nonlinearly affects and links most of the fundamental processes acting in hydrologic cycles, such as evaporation, transpiration, deep percolation, and runoff. It synthesizes the actions of climate, soil, and vegetation on the water balance and the dynamic impacts of water balance on plants, thus is vital to the healthy running of any ecosystems and has been widely recognized as a key variable in ecohydrology<sup>[1–4]</sup>. Soil moisture dynamics directly or indirectly controls meteorological processes, vegetation dynamics, soil biogeochemistry,

groundwater dynamics, and the exchanges of nutrients and contaminants in the soil-plants-atmosphere-continuum (SPAC)<sup>[5-7]</sup>. Accordingly, multi-scale understanding of the temporal and spatial dynamics of soil moisture is crucial to the studies of several other eco-

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logical and hydrological processes. Due to the facts that no simple linearity exists between the interactions of climate, soil and vegetation type, and that there are a lot of different complex processes (from physical to biological) and a large number of uncertain variables (e.g. the high intermittence of rainfall, the atmospheric turbulence, the heterogeneous soil structure and topography, and the presence of plant roots and organic matter) involved in soil moisture dynamics, probabilistic and stochastic nature of the dynamics are unavoidable [8–10]. As a result, the idea of probability and possibility is necessary for the studies of soil moisture dynamics to explore the complex relationships between the components of SPAC and to describe the multi-scale and nonlinear features of the dynamics [111].

Stochastic modelling of soil moisture dynamics has becoming a hotspot in ecohydrology at present [2,4,6,12-28]. The notable stochastic model of soil moisture dynamics proposed by Rodriguez-Iturbe et al. [9] is one of the most typical and widely cited frameworks. It represents the input and output of soil moisture in a very physically realistic manner by fully considering the intermittent nature of precipitation and nonlinear dependent of infiltration, evapotranspiration and leakage to soil moisture statues, trough which the soil moisture probability density function (PDF) can be obtained analytically for different climate, soil conditions and vegetation types in steady-state [9]. The model and the derived PDF are very sensible to the climate characters, soil attributes, vegetation types, and morphological properties, and thus is a powerful tool for quantificationally estimating the responses of vegetation to water stresses, analyzing the hydrological control of soil nutrient recycling, and understanding the plant water competition and some other ecosystem dynamics<sup>[4]</sup>. Many studies have proved that the model is suitable in a large range of climates [3,13,29-31], and the widely application of the stochastic model, in fact, has promoted a lot of meaningful ecohydrological results, e.g. the model was used to explore the spatiotemporal relationships between the vegetation, climate and soil moisture in savannas by Rodriguez-Iturbe, and the results showed that the responses of vegetation to stochastic soil moisture availability contribute a lot to the dominance or possible coexistence of herbaceous and woody plant [32,33]; using the model, ecohydrological studies carried by Fernandez-Illescas suggested that soil texture in water controlled environments plays a major role in modulation of the impacts that interannual rainfall fluctuations have on the fitness of vegetation types, or in other words, the optimal soil texture for a given vegetation type changes with rainfall amount [30]. This framework was further improved by Laio later in the terms of representations of evapotranspiration and leakage losses toward a more realistic description of soil moisture dynamics, especially in conditions of drought [13]. Based on the improved framework, Porporato proposed a measure of vegetation water stress, namely dynamic water stress, which combines the mean intensity, duration, and frequency of the periods of soil water deficit<sup>[14]</sup>. More recently, the Monte Carlo procedure was introduced in order to numerically study the stochastic soil moisture dynamics, e.g. a Monte Carlo procedure was implemented in studying effects of random interannual fluctuations on the probability distribution of the mean value of soil moisture during a growing season, and as a result, the emergence of a bimodal behavior driven by the variability of the climatic parameters was theoretically observed a similar application of Monte Carlo procedure in Fernandez-Illescas' study confirmed that the soil texture plays a major role in the modulation of the impacts that interannual rainfall fluctuations have on the fitness and coexistence of trees and grasses [30].

Although a fairly large number of research results have been achieved in the past decade, some aspects of the stochastic modelling research of soil moisture dynamics are still in progress. The researches combining high temporal resolution data of soil moisture and stochastic modeling procedure to deal with soil moisture dynamics are relatively few so far<sup>[34]</sup>. Most of the traditional researches are still mainly concentrated on the soil moisture dynamics at weekly, ten-daily, or monthly resolutions, but few attentions were paid on the soil moisture dynamics and their statistical characters at daily or finer time resolutions[35-37]. Stochastic modelling researches of soil moisture dynamics are much fewer in China as compared with its international counterpart [11]. In this paper, we based on the continuously monitored data of soil moisture during 2002-2005 and daily precipitation date of 1994-2006, to make a probabilistic analysis of soil moisture dynamics at point scale in a grassland of Oilian Mountain by integrating the stochastic model improved by Laio [13] and the Monte Carlo method, and expected that these efforts would be a

useful addition in promoting the development of related researches in China.

### 1 Study area and methods

### 1.1 Description of study area

The research was carried out in the Pailugou Watershed of Qilian Mountains, an egg-shaped, small but typical mountain valley in Northwest China. It has an area of 2.95 km<sup>2</sup>, a body length of 4.25 km, a longitudinal slope of 1:4.19, and an elevation range of 2600 - 3800 m. The climate in this region is temperate, with mean annual temperature of 0.5°C and relative humidity of 60%. The mean annual precipitation is 368 mm, mainly occurs from May to October. The average annual sunshine duration is 1892.6 h, and the average value of the daily radiation is 110.28 kW·m<sup>2</sup>. The soil types are dominated by montane grey drab soil, montane chestnut soil and sub-alpine shrub meadow soil, with an average depth of 0.5 m. The natural vegetation in the grassland ecosystem is mainly composed of Stipa spp., Agropyrom cristatum, Artemisia frigida, Achnatherum sibiricum and Festuca ovina Linn. The observation site, located at 100°17′E, 38°24′N, with an elevation of 2700 m above sea level, is typical of mountain grassland with moderate slope in the watershed of Qilian Mountain. The average height of grasses surrounding the observation site is about 8-15 cm, and the corresponding coverage is about 95%. Plant roots distributed mainly (more than 95%) in the top 40 cm layer; the soil is rich in humus organic matter in this layer and beneath the layer where a lot of gravel was mixed in with the soil. Generally, the observation site is typical in the ecosystem it represented considered in terms of soil texture, soil depth, grassland coverage, and vegetation types.

### 1.2 Data observation and preparation

An IMKO ENVIS environmental measurement system was installed in the observation site, to continually monitor microclimate factors, such as air temperature, precipitation, wind velocity at two planes, global radiation, reflected radiation, net radiation, relative humidity, soil temperature and moisture profile (surface, 20, 40, 60, 80, 120 and 160 cm), and the geothermal flux. Sensor types are listed as below: Incoming and reflected radiation, CM7B, KIPP & ZOEN, Holland; Net radiation, TYPE 8110, Wein GmbH & Co. KG, Austria; Air pres-

sure, PTB100, Vaisala, Finland; Geothermal flux, HFT-3 and HFP01, Campbell, Britain; Wind velocity and direction, RITA and LISA, Siggelkow Geratebau, Germany; Precipitation, RG50, SEBA Hydrometrie, Germany; Soil volume moisture, T8, IMKO, Germany. In this research, synchronous observation precipitation data (2002–2005) are measured and recorded by automatic precipitation sensor (RG50, SEBA Hydrometrie, Germany), and volumetric soil water content are measured by TDR sensors (T8, IMKO, Germany) at 20, 40, 60, 80, 120 and 160 cm in depth. All the observation data were automatically collected and saved by ENVIS system at 30 min intervals, and were daily averaged to be consistent with the time-step of soil moisture dynamics modeling in this research. The TDR-measured volumetric soil water contents were converted to relative soil moisture using (1) as below  $^{[9]}$ :

$$s = \frac{V_{\rm w}}{V_{\rm a} + V_{\rm w}} = \frac{\theta}{n},\tag{1}$$

where  $V_{\rm w}$  and  $V_{\rm a}$  are the volume of water and air, respectively;  $\theta$  is the measured volumetric water content; n is the soil porosity. The historical precipitation data (1994–2002) were obtained from a near weather station located at the mountain pass of Pailugou watershed (the horizontal and vertical distance between the weather station and the observation site are about 500 m and 100 m, respectively), and adjusted according to the empirical relationship between the precipitation and altitude [38].

## 2 Stochastic model of soil moisture dynamics

#### 2.1 Stochastic model

The Stochastic model of soil moisture dynamics used in this research is the one that has been further improved and modified by Laio et al. [13], which included both the probabilistic representation of the rainfall input and the state-dependent losses for evapotranspiration and leakage. Soil water balance at a point is expressed by the stochastic differential equation:

$$nZ_r \frac{ds}{dt} = I[s(t), t] - E[s(t), t] - L[s(t), t],$$
 (2)

where n is the soil porosity (dimensionless); s is the relative soil moisture content (dimensionless);  $Z_r$  is the depth of active soil or root depth (L); I[s(t),t] is the infiltration from rainfall (L/T), equal to the amount of the rainfall rate subtracts the interception by canopy cover;

E[s(t),t] and L[s(t),t] are the rates of evapotranspiration and leakage (L/T), respectively.

In this simple representation, the occurrence of rainfall was idealized as a series of point events in continuous time, arising according to a Poisson process of rate  $\lambda$ , each carrying a random amount of rainfall h, extracted from an exponential distribution with mean  $\alpha$ . Soil is treated as a storage medium with porosity n and depth  $Z_r$ : rainfall results in an infiltration depth into the soil, I(s,t), which is taken to be the minimum of h and  $nZ_r(1-s)$  to reflect the fact that only a fraction of h can infiltrate when the rainfall amount exceeds the storage capacity of the soil column; excessive rainfall produces runoff according to the mechanism of saturation from below. Canopy interception is included in the model by assuming a threshold of rainfall depth,  $\Delta$ , below which no water effectively penetrates the canopy. All model results are interpreted at the daily timescale  $\frac{[2,3]}{}$ .

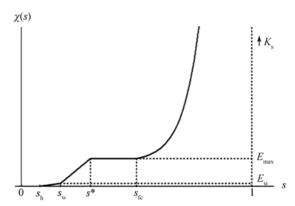
The term E[s(t),t] incorporates losses due to evaporation from the soil and transpiration from the plant. At the daily timescale it may span three regimes. Regime 1, or soil evaporation regime, defines E[s(t),t] as a slow linear rise from 0 at the hygroscopic point  $s_h$  to  $E_w$  at the wilting point  $s_w$ . Regime 2, or stressed evapotranspiration regime, has a relatively moderate, linear rise in E[s(t),t]from  $E_{\rm w}$  at  $s_{\rm w}$  to  $E_{\rm max}$  at  $s^*$ , where  $s^*$  is the soil moisture level at which the plant begins to close stomata in response to water stress. Regime 3 is the unstressed evapotranspiration regime, where evapotranspiration remains constant at  $E_{\text{max}}$  when soil moisture are between  $s^*$  and soil field capacity  $s_{fc}$ , and plants are in good water conditions. When the soil moisture higher than soil field capacity  $s_{\rm fc}$ , the leakage, L[s(t),t], is dominant [2,3] (Figure 1).

Assuming no interaction with the underlying soil layers and water table, L[s(t),t] represents vertical percolation with unit gradient

$$L[s(t),t)] = \frac{K_s}{e^{\beta(1-s_{fc})} - 1} [e^{\beta(s-s_{fc})} - 1],$$
 (3)

where  $K_s$  is the saturated hydraulic conductivity;  $\beta = 2b + 4$ , b is the pore size distribution index, both of them are empirical fitting parameters and related to the shape of the water retention curve, and are usually directly measured by Guelph-permeameter or indirectly estimated from water-retention curves. All the values of  $s_h$ ,  $s_{fc}$ ,  $s_w$  and  $s^*$  are related to the corresponding soil matric potentials,  $s_w$  and  $s^*$  are also related to the vegetation

types. The analytical expressions for the steady state PDF of soil moisture as well as the expressions for the water balance components during the growing season are given under the assumption of statistically homogeneity in growing season climate<sup>[13]</sup>.



**Figure 1** Behavior of soil water losses (evapotranspiration and leakage), x(s) as function of relative soil moisture for typical climate, soil and vegetation characteristics in semiarid ecosystems, and after ref. [16].

#### 2.2 Parameter estimation

Many of related parameters in the model were gathered from in situ measured data, and others of them were determined by indirect estimation methods. The cutting ring method was used to determine soil porosity n and field capacity  $s_{\rm fc}$  of root zone (3 repeats at the soil surface, 20 cm and 40 cm depths, respectively, and take the average). Hygroscopic point  $s_h$  was determined by the water vapor balance adsorption method (saturated K<sub>2</sub>SO<sub>4</sub> solution, 3 repeats and take the average). The wilting point  $s_{\rm w}$  was indirectly estimated by the experiential expressions:  $s_w = s_h \times 1.5$ ; the saturated soil hydraulic conductivity  $K_s$  was measure by using of the tension infiltrometer (Holland, Eijkelkamp) in undisturbed field conditions;  $s^*$  and another soil parameters  $\beta$  were determined according to related reference<sup>[13]</sup>. Synchronous precipitation information was extracted from the 2002 -2005 precipitation databank; the historical precipitation information were extracted from the receive data at a near weather station, and minor calibration was done by referring to the experiential relationship between precipitation and elevation. The depth of active soil or root zone depth, defined as the soil depth range in which 95% below-ground biomass were distributed, was determined by filed investigation. The canopy interception  $\Delta$  was determined by the water injection method. The  $E_{\rm w}$ at  $s_{\rm w}$  and  $E_{\rm max}$  at  $s_{\rm fc}$  were determined through statistical analysis of the lysimeter observations during the 20022005 growing season.

### 2.3 Monte Carlo simulation

The statistical information of the mean value of soil moisture <s> during a growing season can be used to characterize the impact of climate on water balance and vegetation in arid and semiarid regions. Several research cases have identified that both the autocorrelations and the cross correlations are very weak for the average storm depth  $\alpha$  and frequency of storm arrivals  $\gamma$ , and that the histograms of frequency distribution of the parameters are always unimodal and the coefficients of variation of them are similar in value. Two parameter gamma distributions provided a good fit to the histograms [3], and thus, these parameters was modeled as independent gamma-distributed random variables in the paper. A Monte Carlo procedure was implemented to numerically estimate the PDF of <s> resulting from the random interannual fluctuations of  $\alpha$  and  $\gamma$ , and then compared it with that derived under the condition of not considering the interannual climate fluctuations. The multi-year precipitation information, including the average storm depth  $\alpha$ , frequency of storm arrivals  $\gamma$ , and the corresponding coefficient of variance were extracted from the precipitation records during 1994-2006 growing seasons.

### 3 Results

# 3.1 Precipitation pattern and its statistical characteristics

The seasonal distribution of precipitation in the research region is not even. Precipitation is highly concentrated in May through October, during which the precipitation occupies about 89.1% and 70% of the annual precipitation in terms of precipitation amount and occurrence frequency, respectively (Figure 2). About 78.8% of all precipitation events are 0-5 mm and only 11.2% are >5 mm (Figure 3(a)). Events of 0-5 mm, 5-10 mm, 10-15 mm, 15-20 mm, 20-25 mm and >25 mm account for about 30%, 27.6%, 17.4%, 11.7%, 8% and 6.8% of the annual precipitation amounts, respectively. Obviously, the account percentage for precipitation is decreasing with the increase of event size class (Figure 3(b)). Dry days and periods (intervals between precipitation events) are dominated by those with the shortest duration. On average, 55.5% of the dry days are those among periods of 0-10 d, only 26%, 9.4%, 7.6% and

1.5% are 11-20 d, 21-30 d, 31-40 d and >40 d, respectively (Figure 4(a)); 86.1% of the dry periods are 0-10 d, and only 13.9% are >10d (Figure 4(b)). It is clear that both the percent of dry days and the frequency of dry periods decrease with the increasing dry day classes.

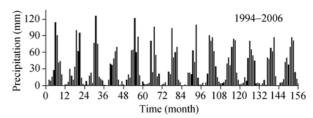
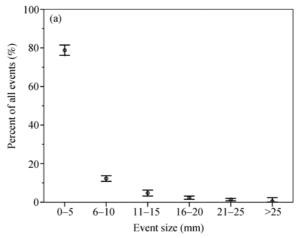
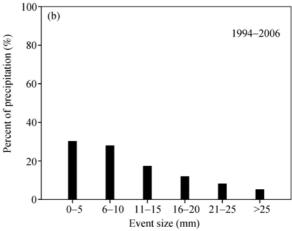


Figure 2 Monthly precipitation amounts distribution in the studied region from 1994 to 2006.

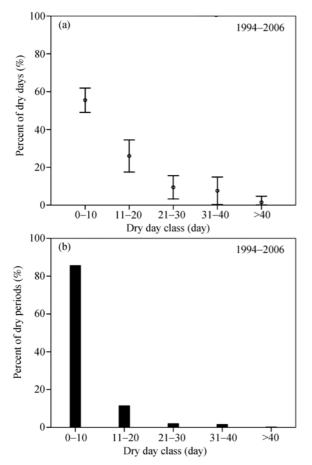




**Figure 3** Statistical distribution of the precipitation events of the years 1994–2006. (a) Frequency of daily precipitation events in six size classes; (b) frequency of daily precipitation amounts in six size classes.

# 3.2 Soil moisture dynamics and probability distribution characteristics

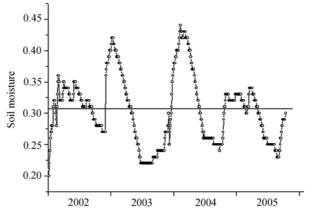
(i) Soil moisture dynamics. Inner-annual variations. The relative soil moisture in the root zone (0-40 cm) is



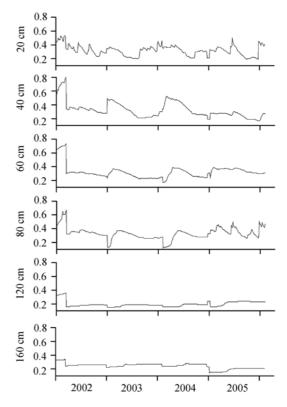
**Figure 4** Statistical distributions of dry days and dry periods in years 1994—2006. (a) Frequency of dry days between daily precipitation events in five size classes; (b) frequency of dry periods in five size classes.

comparatively high (almost 40% or more) at the very beginning stage of growing season, that decrease during the next months with the increasing soil evaporation and vegetation transpiration (due to the warming soil temperature and faster growing of vegetation), and then increase at the end stage of growing season for the contrary reasons (Figure 5). Because of the coincidence of the growing and rainy season, the soil water content at the observation point appeared to be very sensitive to precipitation events and evapotranspiration losses. Soil moisture in the surface soil layer (0-20 cm) appeared to be more sensitive to precipitation events compared with that in the lower layers, and correspondently, its fluctuation is much more significant than that in the lower layers. The variance pattern of soil moisture in the depth of 20-80 cm are similar to the average trends of the soil moisture profile, but the fluctuation intensity is obviously weakening with soil depth. Inner-annual variation of soil moisture beneath 80 cm depth is very

slight due to almost no effects of precipitation disturbance on it (Figure 6). 4 typical months, namely August 2002, May 2003, July 2004 and September 2005, were selected randomly among the months of 2002—2005. Further analysis of the soil moisture dynamics in the typical months shows that the trends of soil-moisture change were not all the same at depths of 0—40 cm, 60 cm and 80 cm, e.g. in September 2005, the soil moisture in the root zone (0—40 cm) was decreasing due to the

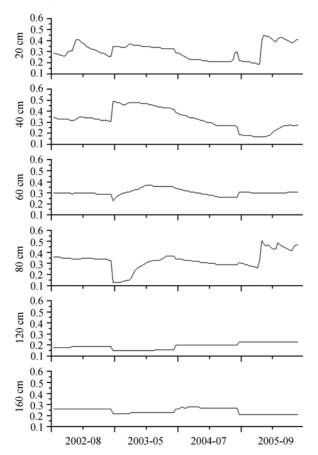


**Figure 5** Evolution of daily-averaged soil moisture in root zone during the 2002 to 2005 growing seasons.



**Figure 6** Evolutions of daily-averaged soil moisture at different depths of the soil profile from 2002 to 2005.

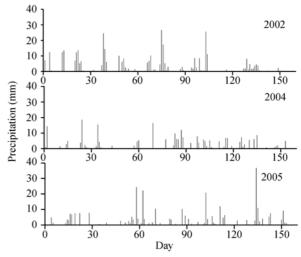
relatively infrequent precipitation and continually rising temperature, while beneath 40 cm that was increasing because of the thawing of seasonal frozen soil. In mid-September 2005, the soil moisture in the root zone (0-40 cm) was significantly changed by a few relatively large precipitation events, however, that at the depth of 60 cm almost had not been affected by those events (Figure 7).



**Figure 7** Detailed evolutions of the daily-averaged soil moisture at different depths of the soil profile during the random selected months in 2002–2005.

Inter annual variations. The average of the observed soil moisture at the depths of 20, 40, 60, 80, 120 and 160 cm are 0.32, 0.33, 0.32, 0.33, 0.20 and 0.24, respectively. Variance analysis shows that significant inter-annual variation exits for the soil moisture patterns at different depths (20 cm,  $F_{3,572}$ =13.86, p<0.01; 60 cm,  $F_{3,572}$ =60.30, p<0.01; 80 cm,  $F_{3,572}$ =76.28, p<0.01; 120 cm,  $F_{3,572}$ =37.32, p<0.01 and 160 cm,  $F_{3,572}$ =366.01, p<0.01). Based on the precipitation anomaly percentage for the growing seasons of 1994—2006 [(P = growing season precipitation)/(long-term average growing season precipitation)/(long-term average growing season precipitation)/(long-term average growing season precipita-

tion)], 2002, 2004, 2005 years were selected to be the representative years of rich raining year (P>15%), poor raining year (P < -15%), and normal rainy year(-5% < P≤ 5%), respectively. Descriptive statistical analysis shows that the maximum CV (coefficients of variances, 0.42) in 2002 (rich raining year) happened at the depth of 40 cm, that in 2004 (0.28, poor raining year) happened at the depth of 40 cm, and in 2005 (0.25, normal raining year) happened at the depth of 20 cm. Further analysis shows that the maximum CV for 2002-2005 happened at the depth of 20-40 cm. The CV of soil moisture in root zone for the rich raining year (0.42) was significantly greater than that for poor (0.21) or normal raining year (0.19). Significant differences ( $F_{2,15}$ =4.064, p<0.05) exist among the different hydrological years in the CVs of growing season soil moisture at the in situ soil moisture profile. The coefficient for 2002-2005 is 0.27, close to the average variance level (Table 1). Analysis of the daily precipitation time series for 2002, 2004 and 2005 shows that: there are 40, 36 and 45 small rainfall events (≤5 mm) and 12, 6 and 8 large events (≥10 mm) occurred in the 3 years, with the corresponding precipitation of 62.4, 72.1, and 85.9 mm and 193.1, 76.6, and 147 mm, respectively. Obviously, the differences in the happening frequency of large precipitation events among the different hydrological years should be. at least partly, responsible for the significant differences in soil moisture patterns of the years (Figure 8).

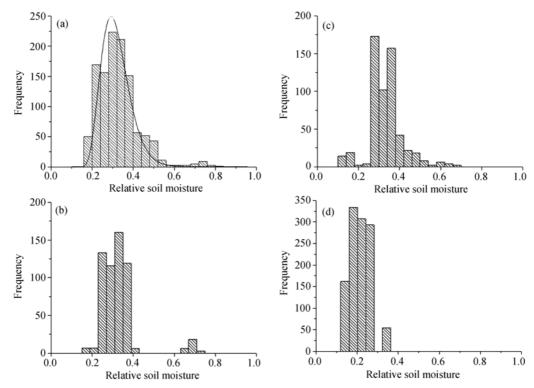


**Figure 8** Daily precipitation during the growing seasons from 2002 to 2004.

(ii) Probabilistic distribution characteristics. The probabilistic distribution of the root-zone (0-40 cm)

 Table 1
 Descriptive statistic results on the relative soil moisture during the different hydrological years

Year	No.	Depth (cm)	Sample size –	Relative soil moisture					CV
				Maximum	Minimum	Average	Median	SD	
2002 (Rich rainy year)	1	20	139	0.53	0.23	0.35	0.33	0.08	0.23
	2	40	139	0.80	0.28	0.40	0.34	0.15	0.38
	3	60	139	0.73	0.26	0.37	0.31	0.16	0.42
	4	80	139	0.67	0.30	0.38	0.35	0.09	0.25
	5	120	139	0.36	0.16	0.21	0.19	0.07	0.31
	6	160	139	0.34	0.24	0.27	0.26	0.03	0.11
	7	20	130	0.40	0.21	0.31	0.31	0.06	0.19
	8	40	130	0.52	0.19	0.36	0.35	0.10	0.28
2004	9	60	130	0.39	0.17	0.29	0.29	0.06	0.20
(Poor rainy year)	10	80	130	0.37	0.12	0.28	0.29	0.07	0.19
	11	120	130	0.24	0.16	0.18	0.19	0.02	0.11
	12	160	130	0.28	0.23	0.26	0.25	0.02	0.08
	13	20	153	0.50	0.19	0.31	0.32	0.08	0.25
	14	40	153	0.30	0.17	0.25	0.27	0.04	0.16
2005 (Normal rainy year)	15	60	153	0.39	0.23	0.34	0.36	0.03	0.09
	16	80	153	0.51	0.26	0.37	0.37	0.06	0.17
	17	120	153	0.24	0.16	0.21	0.23	0.03	0.13
	18	160	153	0.21	0.14	0.19	0.21	0.03	0.15
	19	20	575	0.53	0.19	0.32	0.32	0.07	0.23
	20	40	575	0.80	0.17	0.33	0.29	0.12	0.36
2002—2005 Average	21	60	575	0.73	0.17	0.32	0.31	0.10	0.30
	22	80	575	0.67	0.12	0.33	0.33	0.08	0.26
	23	120	575	0.36	0.15	0.20	0.19	0.04	0.21
	24	160	575	0.34	0.14	0.24	0.25	0.04	0.16



**Figure 9** Probability distributions of relative soil moisture in different soil layers. (a) The observed and derived probability distributions in the root zone layer (0-40 cm); (b) observed probability distributions in the layer of 40-60 cm; (c) observed probability distributions in the layer of 80-120 cm.

soil moisture for the growing seasons of 2002-2006 appeared to have a single peak, which emerged at s=0.28 and the range is relatively wide (s=0.2-0.6) (Figure 9(a)). The probabilistic distributions of the subsurface soil layer (40-60 cm) are largely concentrated on the range of 0.18-0.42 (the peak appeared at s=0.32, and there also have very little probability distributed at the range of 0.62-0.74) (Figure 9(b)), and that of the lower layer (in the depth of 60-80 cm) largely concentrated on the range of 0.1-0.7 (the peak appeared at s=0.35) (Figure 9(c)). The probabilistic distribution range of the soil moisture at the depth of  $80\sim160$ cm are relatively narrow (0.12-0.28) and the peak happened at s=0.18 (Figure 9(d)).

### 3.3 Stochastic modelling of soil moisture dynamics

(i) Parameterization schemes. Analysis of soil mechanical component shows that clay (<0.002 mm), fine silt (0.002—0.05 mm) and the sand grains (>0.05 mm) constitute about 0.9%, 80.3%, and 18.8% of the material at the observation point, respectively. The pore size distribution index is 5.39, the bulk density of soil is 0.74 g/cm³ and the total porosity is about 64.1%. The capillary water-holding capacity is 0.59 g/cm³ and the saturated hydraulic conductivity is 1188 cm/d. The hydraulic point, the wilting point, the water stress happened point and the field capacity are 0.09, 0.15, 0.47 and 0.95, respectively. More detailed information of the parameterization is listed in Table 2.

(ii) Stochastic modeling. The statistical information of precipitation in 2002-2005 was input into the stochastic model of soil moisture dynamics improved by Laio, and then the PDF of soil moisture is derived to evaluate the effect of rainfall forcing (Figure 9(a)). Comparison of analytically derived soil moisture PDF with the histogram distribution of the observed soil moisture data suggests that the stochastic model can reasonably describe and predict the soil moisture dynamics of the grassland in Qilian Mountain at point scale (both the peak position and width of the root zone soil moisture PDF in the growing seasons, are similar between the modeling and observation results). Based on this recognition, we analytically derived the long-term soil moisture PDF without considering the inter-annual climate fluctuations, by extracting the statistical information of the historical precipitation data in 1994-2006, and inputting those information in the stochastic model (Figure

10(a)). It is shown that the peak position and width of the soil moisture PDF appeared to be  $\langle s \rangle = 0.28$ , and 0.16-0.55. To consider the effects of inter-annual fluctuation on the soil moisture dynamics, a Monte-Carol procedure was used to numerically derive the PDF of the long-term soil moisture (Figure 10(b)). A very interesting result was that no significant changes happened to the shape of the long-term average soil moisture PDF during the growing season after adding the disturbances of inter-annual fluctuations of climate. However, the peak position was significantly moved toward drought (from s=0.28 to s=0.18) and its value increased from 8.0 to 9.5, correspondingly, the peak width was narrowed from 0.17-0.51 to 0.13-0.29.

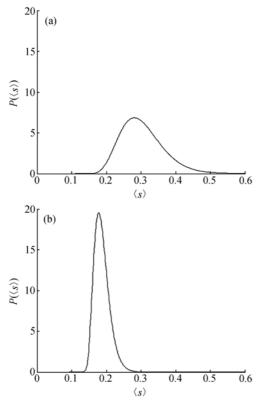
Table 2 Values of the stochastic model parameters for the grassland ecosystem in Qilian Mountain

Parameters	Symbols(units)	Values
Empirical fitting parameters of soil properties	β	14.8
Soil porosity	n	0.64
Hygroscopic point	$S_{\rm h}$	0.09
Permanent wilting point	$S_{ m W}$	0.15
Soil moisture level at which plants begin closing stomata	s*	0.47
Soil field capacity	$s_{ m fc}$	0.95
Threshold of canopy-intercepted precipitation	$\Delta(\text{mm})$	0.10
Depth of active soil or root depths	$Z_r(cm)$	40.0
Arrival rate of rainfall events during 2002 – 2005	$\gamma(d^{-1})$	0.42
Mean rainfall depth during 2002-2005	$\alpha$ (mm)	5.36
Arrival rate of rainfall events during 1994—2006	$\gamma(d^{-1})$	0.38
Mean rainfall depth during 1994-2006	$\alpha$ (mm)	5.68
Interannual CV of mean rainfall depths during 1994–2006	Cv-α	0.12
Interannual CV of the rainfall arrival rates during 1994—2006	Cv-γ	0.15
Average daily evapotranspiration rate at $s = s_{fc}$	$E_{\rm max}$ (cm/d)	0.40
Average daily evapotranspiration rate at $s = s_w$	$E_w$ (cm/d)	0.02
Saturated hydraulic conductivity	$K_s$ (cm/d)	1188

### 4 Discussion

### 4.1 Vertical heterogeneity of soil moisture

Soil evaporation and vegetation transpiration are mainly and directly liable to the soil moisture status. Due to the differences in the magnitude and frequency of incoming precipitation pulses, the soil textures, and the vegetation types, soil moisture dynamics frequently display a significant vertical heterogeneity, and this kind of hetero-



**Figure 10** PDFs of the average soil moisture during the growing season. (a) Without considering random fluctuation effects; (b) when considering random fluctuation effects.

geneity again affect and determine the hydraulic redistribution, such as the precipitation infiltration, holding, and runoff. On a longer time scale, vertical heterogeneity with depth varies between wet versus dry years. For example, for the Central Plains Experimental Range in Colorado, predicted water availability for plants in dry years is limited to only the top 40 cm, whereas in wet years, water availability extends to below 1 m<sup>[39]</sup>. Climate of the grassland ecosystem in Qilian Mountain is chill, semiarid and semihumid, and belongs to the temperate mountain forest-to-steppe climate. Precipitation is almost the only source of soil water supply in this ecosystem except the melting snow and thawing frozen earth which occasionally happened at the very beginnings of some growing seasons. The precipitation events in this region are dominated by those of 0-5 mm, and the amount and frequency of which have only very small interannual variability. Accordingly, the CVs of surface soil moisture (20 cm) are almost continually kept at about 0.23 no matter in the rich or poor rainy years. It is notable that the maximal CV of soil moisture has not always appeared at the surface layer. Corresponding depths of maximum occurred are very different in different hydrological years, e.g. that in rich rainy year (0.42 occurred at the depth of 60 cm) are significantly higher than that in poor rainy year (0.28, 40 cm) or that in normal rainy year (0.25, 20 cm). This interesting phenomenon may be contributed by the relatively large differences in the happening frequencies of large precipitation events, e.g. more large rainfall events happened in rainy year lead to larger range of the infiltration depths. Because surface soil layer is always first wetted by rainfall when it arrived at the ground, and soil evaporation and grassland transpiration also mainly occur in this layer (due to most of the grassland vegetation roots distributed in the top 0-20 cm soil layer), it was traditionally regarded that the soil moisture in this layer is always most sensitive to the precipitation events, and thus the maximal CV of soil moisture should, of cause, occur in the layer  $\frac{[40-43]}{}$ . The result get in this study seems a little different to the traditional ones. This differentiation may be a result of scaling uncertainty or the high differences in terms of observation time-resolution, e.g. most of the traditional observations of soil moisture are at a weekly, ten-daily, or monthly time-resolution, the soil moisture data used in this study, however, were taken at 30 min intervals. Similar reports or papers on this subject are still limited, thus whether the result is universal remains open.

### 4.2 Soil moisture PDF at point scale

Climate factors determines the seasonal precipitation patterns and the main vegetation types of a region, largely affects the soil processes, and thus effectively controls the shape of soil moisture PDF, especially the peak position of density, e.g. for a typical tropical climate with very frequent rainfall of moderate intensity, quite deep soil, and a high value of maximum evapotranspiration, most of the probability density is concentrated around values of soil moisture just below field capacity; for a typical hot arid region with rare precipitation, shallow sandy soil and a mixture of trees, shrubs and grasses, the probability density is largely concentrated at very low values of soil moisture and the vegetation is very frequently udders water stress; and for a forest in a temperate region, the shape of the density recalls that of the tropical forest, but with a less pronounced peak at high soil moisture values [9]. The soil moisture density of the grassland ecosystem in Qilian Mountain is more open (0.16-0.55) with middle-low average values ( $\langle s \rangle = 0.28$ ), and is fairly similar with that of a typical cold region with an arid climate. The special probabilistic distribution characteristics of soil moisture in this region should be largely contributed to the frequent but moderate rainfall and the low values of maximum evapotranspiration (due to the relatively low temperature and high air humidity).

#### 4.3 Soil moisture PDF under random fluctuations

Hydrological and climatic time series are often characterized by small fluctuations around persistent anomalies. Rodriguez-Iturbe had reported that preferential states or bimodality may arise in the soil moisture dynamics at the continental scale as a consequence of fluctuations due to the coupling between soil surface and atmosphere (Stochastic fluctuations may give rise to separate and distinct statistical modes of soil moisture PDF, and the relative weights position of the modes are increasing with the fluctuation intensity) [8]. The possible occurrence of bimodality in the probability distribution of soil moisture induced by interannual climate fluctuation has also been reported by D'Odorico through another stochastic model of soil moisture dynamics in a completely different frameworks (in which the processes is physically interpreted at the daily timescale), e.g. for high values of the CVs of one or both of the rainfall frequency and average depth, one observes the emergence of a bimodal behavior driven by the variability of the climatic parameters, which disappear when the fluctuation become weaker<sup>[12]</sup>. This behavior suggests that the system under random fluctuation disturbance tends to switch between two preferential states, one characterized by high average soil moisture and the other characterized by low average soil moisture conditions, in other words long-term soil moisture dynamics does not necessarily tend to an equilibrium average state. The existence of bimodal behavior has also been illustrated in a field experiment by Salvucci<sup>[29]</sup>. In this research, no significant bimodality was observed in soil moisture dynamics under the given intensity of random fluctuation disturbance. By comparing the PDF derived in this case with the one derived without considering the disturbance forces, however, it was found that the peak position of the probability density distribution significantly moved towards drought when considering the disturbance forces. The results get in this research proved that the long-term soil moisture dynamics of grassland ecosystem in the Qilian Mountain also does not tend to be an equilibrium state of average (s=0.28), but to be a slightly

more drought (s=0.18).

# 4.4 Application of the stochastic model of soil moisture dynamics

The stochastic description of soil moisture dynamics may be thought of as a minimalistic description of the soil water balance, through which the probabilistic characteristics of soil moisture dynamics can be derived analytically by inputting the statistical information of precipitation. Generally, these kinds of models were developed not for prediction purposes, but rather as an efficient tool for analysis of the ecohydrological implications from the statistical data<sup>[4]</sup>. Although the model used in this study is largely based on a simple assumption of statistical-average growing seasons, and the effects of initial states on the soil moisture dynamics are also ignored  $\frac{[13]}{}$ , which are not totally consistent with the facts, the good match between the soil moisture PDFs derived from the results of the model improved by Laio and the field observations, proved that the model can be perfectly applied to this region. Accumulations of the basic and background data in arid inland river basins are usually very poor, especially in the mountainous regions of the upper reaches because of the adverse natural condition. Soil moisture data not only in spatial but also in temporal scale are very lack, and thus it is a meaningful work using the relatively rich precipitation data to derive the statistical information of soil moisture.

### 5 Conclusions

The relative soil moisture at the observation point of the grassland in the Qilian Mountain is comparatively high at the very beginning stage of growing season, that decrease during the next months and then increase again at the end stage of growing season. Significant fluctuations exist in the surface soil moisture dynamics (0-20 cm), and the variance pattern of soil moisture in the depth of 20-80 cm are similar to the average trends of the soil moisture profile, but the fluctuation intensity is obviously weakening with soil depth. The soil moisture beneath the depth of 80 cm are almost not affected by the precipitation events, consequently, its inner-annual variances are very slight. Inter-annual variations for the soil moisture patterns at different depths are also very significant, but the CV of surface soil moisture (20 cm) is almost continually kept at about 0.23 no matter in the rich or poor rainy years, and the maximal CV of soil

moisture has not always appeared at the surface layer. The intensity distribution of the root-zone (0-40 cm) soil moisture for the growing seasons of 2002 – 2005 are largely concentrated on the mid or lower levels, and appeared to be single-mode, with a peak emerging at s=0.28 and a range between s=0.2-0.6. The distributions of the subsurface soil moisture (40-80 cm) are similar to that of the root-zone soil moisture, but the range of that (0.12-0.28) is significantly narrowed for the lower layer (80— 120 cm), and the corresponding peak happens at s=0.18. Comparison of the analytically derived soil moisture PDF with the histogram distribution of the observed soil moisture data suggests that the stochastic model can reasonably describe and predict the soil moisture dynamics of grassland in the Qilian Mountain at point scale. By extracting the statistical information of the

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historical precipitation data in 1994—2006, and inputting those information in the stochastic model, we analytically derived the long-term soil moisture PDF without considering the inter-annual climate fluctuations, and then numerically derived that under the consideration of inter-annual fluctuation effects in combination with a Monte-Carlo procedure. It was found that no significant bimodality appeared in the soil moisture dynamics under the given intensity of random fluctuation disturbance, but the peak position of the probability density distribution significantly moved towards drought when considering the disturbance forces, its width were narrowed, and accordingly the peak value was increased.

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