OPERATORS IN Ⓗ ARE *M*-HYPONORMAL

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Suppose that T is an operator in Ⓗ, then T can determine a normal operator

$$C = 1/2[T + T^* + i\sqrt{4T^*T-(T+T^*)^2}]. \quad (1)$$

Lemma. *Let T be an operator in Ⓗ on a Hilbert space $\mathscr{H}$. Suppose that C is defined as in Eq. (1) and λ is a complex scalar. Then in case* $\mathrm{Im}\lambda>0$, *for all $x\in\mathscr{H}$, we have*

$$\|(T-\lambda)\ (C^*-\lambda)^{-1}x\|\leqslant\|x\|.$$

In case $\mathrm{Im}\lambda<0$, *for all $x\in\mathscr{H}$, we have*

$$\|(T-\lambda)(C-\lambda)^{-1}x\|\leqslant\|x\|.$$

Theorem. *Let T be an operator in Ⓗ on $\mathscr{H}$, then for all $\lambda\in\mathbf{C}$ and all $x\in\mathscr{H}$, we have*

$$\|(T-\lambda)^*x\|\leqslant 3\|(T-\lambda)x\|, \quad (2)$$

that is, T is M-hyponormal.

Corollary 1 (Putnam-Fuglede Type Theorem). *Suppose that T and S^* are operators in Ⓗ. If there is an operator W such that $TWS=W$, then $\mathscr{N}(W)^{\perp}$ and $Cl\mathscr{R}(W)$ reduce S and T respectively, and $S|\mathscr{N}(W)^{\perp}$, $T|\mathscr{CLR}(W)$ are normal. Consequently, $T^*WS^*=W$.*

Corollary 2. *If T is an operator in Ⓗ, and T^* is a dominant operator, then T must be normal.*

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SOME QUESTIONS ON INCREMENTS OF A WIENER PROCESS

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Hanson and Russo considered some forms of increments of a Wiener process (*Ann. Probability*, **11** (1983), 609—623). And they proposed five questions. Soon they answered the first question by themselves. We try to answer the rest. Let $\{W(t),t\geqslant 0\}$ be a standard Wiener process and

$$d(T,t)=\{2t[\log(T/t)+\log\log t]\}^{1/2}.$$

The following theorem answers Questions 2—4.

Theorem 1.

$$\limsup_{T\to\infty}\ \sup_{0\leqslant t\leqslant T}\ \sup_{0\leqslant s\leqslant t}\ |W(T)-W(T-s)|/d(T,t)=1 \qquad a.s.$$

Combining this theorem with theorem 2.2 (*Ann. Probability*, **11** (1983), 1009—1015), we can sharpen the latter.

Theorem 2. *Suppose that a_T is measurable and that $0<a_T\leqslant T$ for all $T>0$. Then the set of limit points (as $T\to\infty$) of*

$$\frac{W(T)-W(T-a_T)}{\{2a_T[\log(T/a_T)+\log\log T]\}^{1/2}}$$

is $[-1,1]$ with probability one.

Theorem 3 partially answers Question 5.

Theorem 3. *Under Conditions*

(i) *$a_T\to\infty$ continuously as $T\to\infty$,*

(ii) $\lim \dfrac{\log T/a_T}{\log\log a_T}=\infty$,

we have

$$\lim_{T\to\infty}\ \sup_{0\leqslant t\leqslant T-a_T}\ |W(t+a_T)-W(t)|/d(t+a_T,a_T)=1 \qquad a.s.$$

$$\lim_{T\to\infty}\ \sup_{0\leqslant t\leqslant T-a_T}\ \sup_{0\leqslant s\leqslant a_T}\ |W(t+s)-W(t)|/d(t+a_T,a_T)=1 \qquad a.s.$$

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