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Analysis for twinning and slip in face-centered cubic crystals under axisymmetric co-deformation

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Abstract The maximum work principle of Bishop-Hill was developed to analyze the axisymmetric co-deformation in face-centered cubic crystals (f.c.c.) for twinning on $\{111\}\langle 112\rangle$ and slip on $\{111\}\langle 110\rangle$ systems. The influence of ξ, the ratio of critical resolved shear stress for twinning to slip, on the yield stress states and corresponding active slip or/and twinning systems for orientations in the standard stereographic triangle of cubic crystal was investigated systematically. The Taylor factors and the anisotropy of yield strength for three important orientations [100], [110] and [111] in orientation space were analyzed. It is found that the yield strength asymmetry for the case of axisymmetric deformation of tension and compression can be explained based on the microscopic theory of crystal plasticity. The concept of orientation factor for twinning ability was proposed and the deformation mechanism map in the orientation space was established for the case of axisymmetric deformation. The deformation texture formation and development of f.c.c. crystals with low stacking fault energy for axisymmetric tension can be explained qualitatively on the basis of analyzed results.

Keywords: twinning, slip, face-centered cubic crystals, axisymmetric co-deformation, Bishop-Hill maximum work principle, yield strength anisotropy.

In general, for plastic deformation in face-centered cubic (f.c.c.) crystals, the $\{111\}\langle 110\rangle$ slip and $\{111\}\langle 112\rangle$ twinning mechanisms are investigated independently in theory. As pointed out by Kalidindi[1], there exist very few modeling and simulation results for polycrystals that deform by slip and twinning simultaneously compared with the large number of modeling and simulation literatures for polycrystalline metals and alloys that deform by slip alone. This is attributed to the complexity associated with introduction of twinning deformation mechanism in the framework of crystal plasticity models. There are little investigations on the twinning and slip co-deformation and corresponding active twinning and slip systems for arbitrarily orientated crystals due to the complexity. However, both slip and twinning can take place in many cases for f.c.c. metals and alloys with medium and low stacking fault energy[2−8]. In fact, twinning and slip occurred simultaneously for pure silver and Co-Fe or Cu-Al alloys under conditions of axisymmetric deformation of tension[9,10]. Therefore, it is very important to study the axisymmetric co-deformation in f.c.c. crystals theoretically. Taking account of slip and twinning together, the single crystal yield surfaces (SCYS) and their characteristics of f.c.c. crystals were analyzed systematically by Chen *et al.*[11] when ξ, the ratio of critical resolved shear stress (CRSS) for $\{112\}\langle 111\rangle$ twinning to $\{110\}\langle 111\rangle$ slip, is changed.

In the present study, the maximum work principle of Bishop-Hill[12,13], which was mainly applied to slip deformation, has been developed to analyze the axisymmetric co-deformation of f.c.c. crystals for twinning and slip. The influence of the CRSS ratio ξ on the yield stress states and corresponding active slip or/and twinning systems for arbitrary orientations in the standard stereographic triangle of cubic crystal has been investigated systematically while the Taylor factors and yield strength anisotropy for some important orientations in orientation space have been analyzed, which will be very useful to the further study of the mechanism of microscopic plastic deformation for twinning and slip in f.c.c. crystals.

1 Analysis of Bishop-Hill maximum work principle

1.1 Co-yield vertices for slip and twinning in f.c.c. crystals

For plastic deformation of single crystals, it obeys the Schmid law. When a single crystal is deformed in the experiment of single axis tension, only a shear is required to fulfill this tensile deformation since the lateral dimensions can be deformed freely. In this case, the shear system with the maximum resolved shear stress operates. On the other hand, for an arbitrarily orientated crystal in polycrystalline aggregates, 5 independent shears are required to accommodate 5 independent strain components. It has been confirmed in ref. [11] that the single crystal yield surfaces are different when the CRSS ratio ξ is changed if the twinning mechanism is introduced into the plastic deformation of f.c.c. crystals. The complete yield stress states have been established for different slip or/and twinning deformation mechanism. The results show that only slip occurs when $\xi > 2/\sqrt{3}$, there are 56 yield stress states that can be subdivided into 5 basic groups according to the symmetry of crystal structure. Only twinning happens when $\xi < 1/\sqrt{3}$, there are 25 yield stress states that may be classified into 4 groups. Both slip and twinning take place together when $1/\sqrt{3} \leqslant \xi \leqslant 2/\sqrt{3}$. Furthermore, there are only two types of mixed yield surfaces. One type consists of 259 stress states when $\sqrt{3}/2 < \xi < 2/\sqrt{3}$, which can be divided into 21 groups and another 259 ones as well when $1/\sqrt{3} < \xi < \sqrt{3}/2$, which can be subdivided into 19 groups. Between the two types of yield stress states, 139 ones are common and 120 ones are different. Therefore, just the four kinds of cases need to be considered when ξ is within the different ranges.

1.2 Analysis of Bishop-Hill maximum work principle for axisymmetric co-deformation

For the case of axisymmetric deformation of tension, it can be described by an elongation $\delta\varepsilon_{11}$ in the x_1-direction and two contractions $\delta\varepsilon_{22}$ and $\delta\varepsilon_{33}$ in the x_2- and x_3-directions. The deformation tensor may be written in the form

$$\delta\varepsilon_{kl}^{s} = \begin{bmatrix} \delta\varepsilon_{11} & 0 & 0 \\ 0 & -\delta\varepsilon_{11}/2 & 0 \\ 0 & 0 & -\delta\varepsilon_{11}/2 \end{bmatrix}, \qquad (1)$$

The superscript s refers to the coordinate system defined by the axes of the strain tensors. Using the full constrained model of Taylor[14], the grains in polycrystalline aggregates experience the same strain that is also the macroscopic strain. In order to apply the maximum work principle of Bishop-Hill, the macroscopic strain tensor with respected to the specimen coordinate system must be transformed into the crystal coordinate system. According to the tensor transformed rule

$$\delta\varepsilon_{ij}^{c} = a_{ik}a_{jl}\delta\varepsilon_{kl}^{s} \qquad (i,j,k,l=1,2,3). \qquad (2)$$

For an arbitrarily orientated crystal, since all the possible yield stress states (including slip or /and twinning yield stresses) are known, the actual stress state is selected by the criterion of maximum work principle of Bishop-Hill

$$\delta w = \sigma_{ij}\delta\varepsilon_{ij}^{c} = \sigma_{k}\delta\varepsilon_{k}^{c} = \delta w_{\max}. \qquad (3)$$

Correspondingly, the active systems are decided. The a_{ik} in eq. (2) is the orientation matrix of the crystal coordinate system with regards to the specimen coordinate system while the superscript c stands for the crystal system. The Taylor factor is defined as

$$M = \delta w/\left(\delta\varepsilon_{11}^{s}\cdot\tau_{cs}\right), \qquad (4)$$

here τ_{cs} is the critical shear stress for $\{111\}\langle 110\rangle$ slip. For an arbitrarily orientated crystal, the Taylor factor is proportional to the work needed to bring about the unit deformation, hence, it is a measure of the relative yield strength as a function of crystal orientation. It means that, for different orientations with the same deformation, the greater the M value is, the more difficult the deformation will be.

Using the maximum work principle of Bishop-Hill and all the possible yield stress states (including slip or/and twinning yield stresses), the 5 dimensional yield stress states (the definition of notations is seen in reference [15]) and corresponding active slip or/and twinning systems (the definition of notations is seen in ref. [11]) for arbitrary orientations in the standard stereographic triangle of cubic crystal orientations under axisymmetric co-deformation have been calculated for the four cases of $\xi > 2/\sqrt{3}$, $\sqrt{3}/2 < \xi < 2/\sqrt{3}$,

$1/\sqrt{3} < \xi < \sqrt{3}/2$ and $\xi < 1/\sqrt{3}$. The Taylor factors and yield strength anisotropy for some important orientations have been analyzed.

2 Results and discussion

2.1 The influence of ξ on the yield stress states and active systems

First, the axisymmetric plastic deformation of tension is considered. In the case of deformation by slip alone ($\xi > 2/\sqrt{3}$, as shown in Table 1), the results show that the standard stereographic triangle (it can represent all tension or compression axial orientations when symmetry is considered) can be divided into 5 certain regions, each of them is activated by a specific yield stress states respectively. Further study shows that they belong to the 5 basic groups of Bishop-Hill slip yield vertices[12,13]. For crystalline materials deforming by slip/twinning, only 5 independent active systems are necessary in order to accomplish an arbitrary shape change (such as FC Taylor model). The active slip systems associated with the yield vertices in Table 1 are 6 or 8 systems, and there are more than one combination of the 5 independent active systems. In this case, the operated slip systems are uncertain, i.e. the ambiguity in selection of the active slip systems arises.

Table 1 The yield stress states, the active systems and the numbers of sets of 5 independent systems in the standard stereographic triangle for tension deformation when $\xi > 2/\sqrt{3}$

5 stress states	Yield stress states (multiplied by $\sqrt{6}\tau_{cs}$)	Active systems	Sets of 5 independent systems
1	$(1/2,-1/2,0,0,0)$	$s_3\ s_6\ s_9\ s_{12}\ s_{14}\ s_{17}\ s_{20}\ s_{23}$	32
2	$(1/4,-1/4,0,1/2,1/2)$	$s_3\ s_4\ s_{12}\ s_{14}\ s_{17}\ s_{22}$	4
3	$(1/2,0,0,0,1/2)$	$s_3\ s_6\ s_9\ s_{12}\ s_{14}\ s_{17}\ s_{19}\ s_{22}$	36
4	$(0,0,1/2,1/2,1/2)$	$s_4\ s_8\ s_{12}\ s_{17}\ s_{21}\ s_{22}$	6
5	$(0,0,0,0,1)$	$s_1\ s_4\ s_8\ s_{11}\ s_{14}\ s_{17}\ s_{19}\ s_{22}$	32

When $\sqrt{3}/2 < \xi < 2/\sqrt{3}$, both slip and twining happens simultaneously. In order to calculate the yield stress states, the first value of ξ taken here is 1, which corresponds to identical CRSS's on the $\{111\}\langle 110\rangle$ slip and $\{111\}\langle 112\rangle$ twinning systems, the stress states and corresponding active slip or/and twinning systems can be obtained. Substituting ξ as a parameter into the yield condition equations associated with $\{111\}\langle 112\rangle$ twinning systems, the coordinates of the vertices can be calculated analytically as a function of ξ, as shown in Table 2. The results show that the standard stereographic triangle can be divided into 12 regions associated with different type of basic yield vertices. The region near the [100] orientation is activated by a yield vertex of Bishop-Hill slip (the stress state 1 in Table 2), which is the same as the one for pure slip (the stress state 1 in Table 1). All the corresponding active systems are slip systems and only slip occurs within the region. Both slip and twinning take places simultaneously within the other regions since the corresponding active systems associated with the other stress states include slip and twinning systems together. It is found that the stress states 1, 2, 3, 5, 7, 8 and 12 labeled by an asterisk in Table 2 fulfill the yield condition within the whole range of $1/\sqrt{3} < \xi < 2/\sqrt{3}$ while the stress states 4, 6, 9, 10 and 11 just satisfy the condition within the range of $\sqrt{3}/2 < \xi < 2/\sqrt{3}$ [11]. It also should be pointed out that there are more than one combination of the 5 independent active systems associated with the stress states 1, 2, 3, 5, 6, 10 and 12, the operated slip or/and slip systems are uncertainty and the ambiguity exists in the selection of the active systems. There are only 5 active systems associated with the yield stress states 4, 7, 8, 9 and 11, the active systems are certain and there exists no ambiguity.

Table 2 The yield stress states, the active systems and the numbers of sets of 5 independent systems in the standard stereographic triangle for tension deformation when $\sqrt{3}/2 < \xi < 2/\sqrt{3}$

12 stress states	Yield stress states (multiplied by $\sqrt{6}\tau_{cs}$)	Active systems	Sets of 5 independent systems
1*	$(1/2, -1/2, 0, 0, 0)$	$s_3\ s_6\ s_9\ s_{12}\ s_{14}\ s_{17}\ s_{20}\ s_{23}$	32
2*	$(3/4-\sqrt{3}\xi/4, \sqrt{3}\xi/4-3/4, 0, \sqrt{3}\xi/2-1/2, \sqrt{3}\xi/2-1/2)$	$s_3\ s_{12}\ s_{14}\ s_{17}\ t_6\ t_{11}$	4
3*	$(\sqrt{3}\xi/4-1/4, 1/4-\sqrt{3}\xi/4, \sqrt{3}\xi/2-1, 1/2, 1/2)$	$s_3\ s_4\ s_{14}\ s_{22}\ t_6\ t_{11}$	6
4	$(1/4, 3/4-\sqrt{3}\xi/2, 0, \sqrt{3}\xi-3/2, 3/2-\sqrt{3}\xi/2)$	$s_{14}\ s_{17}\ s_{22}\ t_6\ t_{11}$	1
5*	$(1/2, \sqrt{3}\xi/2-1, 0, 0, \sqrt{3}\xi/2-1/2)$	$s_3\ s_6\ s_9\ s_{12}\ s_{14}\ s_{17}\ t_8\ t_{11}$	40
6	$(\sqrt{3}\xi/2-1/2, 0, 0, 0, 3/2-\sqrt{3}\xi/2)$	$s_{14}\ s_{17}\ s_{19}\ s_{22}\ t_8\ t_{11}$	6
7*	$(\sqrt{3}\xi/8-1/4, 1/4-\sqrt{3}\xi/8, 3\sqrt{3}\xi/4-1, 1/2, 1/2)$	$s_4\ s_{22}\ t_6\ t_7\ t_{11}$	1
8*	$(0, -\sqrt{3}\xi/4+1/2, \sqrt{3}\xi/2-1/2, \sqrt{3}\xi/2-1/2, -\sqrt{3}\xi/4+1)$	$s_8\ s_{22}\ t_6\ t_7\ t_{11}$	1
9	$(1/2-\sqrt{3}\xi/4, 1/2-\sqrt{3}\xi/4, \sqrt{3}\xi/2-1/2, \sqrt{3}\xi-3/2, 3/2-\sqrt{3}\xi/2)$	$s_8\ s_{17}\ s_{22}\ t_6\ t_{11}$	1
10	$(1-\sqrt{3}\xi/2, 0, 0, 0, \sqrt{3}\xi/2)$	$s_{14}\ s_{17}\ s_{19}\ s_{22}\ t_3\ t_6$	6
11	$(1/2-\sqrt{3}\xi/4, 1/2-\sqrt{3}\xi/4, 1-\sqrt{3}\xi/2, 0, \sqrt{3}\xi/2)$	$s_8\ s_{17}\ s_{19}\ s_{22}\ t_6$	1
12*	$(0, 1/2-\sqrt{3}\xi/4, 0, 0, 1/2+\sqrt{3}\xi/4)$	$s_8\ s_{11}\ s_{19}\ s_{22}\ t_3\ t_6$	6

When $1/\sqrt{3} < \xi < \sqrt{3}/2$, slip and twining take place together (as shown in Table 3). The results show that the standard triangle can be divided into 11 regions associated with different types of basic yield vertices. Similar to $\sqrt{3}/2 < \xi < 2/\sqrt{3}$, the yield stress state within the region near the [100] orientation is still the particular case of Bishop-Hill slip yield vertices (the stress state 1 in Table 3), which is the same as the one in Tables 1 and 2. Therefore, the crystal within the region near [100] orientation is deformed by slip while the crystal within the others regions by both slip and twinning. The stress states 1, 2, 3, 7, 8, 9 and 11 fulfill the yield condition within the whole range of $1/\sqrt{3} < \xi < 2/\sqrt{3}$ while the other stresses only satisfy the condition within the range of $1/\sqrt{3} < \xi < \sqrt{3}/2$ [11]. For stress states 1, 2, 3, 4, 6, 7 and 11, the ambiguity in selecting active systems exists while the selection is certain for the other stress states.

Compared Table 2 with Table 3, it is found that the stress states 1, 2, 3, 5, 7, 8, 12 in

Table 2 and the stress states 1, 2, 3, 7, 8, 9, 11 in Table 3 is common, respectively. It can be concluded that they belong to the 139 common yield stress states, which satisfy the yield condition for $1/\sqrt{3} < \xi < 2/\sqrt{3}$. The other stress states not labeled by asterisk belong to the 120 different ones, which satisfy the yield condition within the different ranges of ξ [11].

In the case of twinning alone for $\xi < 1/\sqrt{3}$, as shown in Table 4, the standard stereographic triangle can be divided into 4 regions that correspond to different types of the basic yield stress states. There does not exist ambiguity in the selection of active systems associated with the yield stress state 3.

Table 3 The yield stress states, the active systems and the numbers of sets of 5 independent systems in the standard stereographic triangle for tension deformation when $1/\sqrt{3} < \xi < \sqrt{3}/2$

11 stress states	Yield stress states (multiplied by $\sqrt{6}\tau_{cs}$)	Active systems	Sets of 5 independent systems
1*	$(1/2,-1/2,0,0,0)$	$s_3\ s_6\ s_9\ s_{12}\ s_{14}\ s_{17}\ s_{20}\ s_{23}$	32
2*	$(3/4-\sqrt{3}\xi/4,\sqrt{3}\xi/4-3/4,0,\sqrt{3}\xi/2-1/2,\sqrt{3}\xi/2-1/2)$	$s_3\ s_{12}\ s_{14}\ s_{17}\ t_6\ t_{11}$	4
3*	$(\sqrt{3}\xi/4-1/4,1/4-\sqrt{3}\xi/4,\sqrt{3}\xi/2-1,1/2,1/2)$	$s_3\ s_4\ s_{14}\ s_{22}\ t_6\ t_{11}$	6
4	$(1-\sqrt{3}\xi/2,\sqrt{3}\xi-3/2,0,0,3\sqrt{3}\xi/2-3/2)$	$s_{14}\ s_{17}\ t_3\ t_6\ t_8\ t_{11}$	6
5	$(\sqrt{3}\xi/2-1/2,0,\sqrt{3}\xi-3/2,3/2-\sqrt{3}\xi,\sqrt{3}/2\xi)$	$s_{14}\ s_{22}\ t_3\ t_6\ t_{11}$	1
6	$(\sqrt{3}\xi/2-1/2,3/4-\sqrt{3}\xi/2,0,0,3/4)$	$s_{19}\ s_{22}\ t_3\ t_6\ t_8\ t_{11}$	6
7*	$(1/2,\sqrt{3}\xi/2-1,0,0,\sqrt{3}\xi/2-1/2)$	$s_3\ s_6\ s_9\ s_{12}\ s_{14}\ s_{17}\ t_8\ t_{11}$	40
8*	$(\sqrt{3}\xi/8-1/4,1/4-\sqrt{3}\xi/8,3\sqrt{3}\xi/4-1,1/2,1/2)$	$s_4\ s_{22}\ t_6\ t_7\ t_{11}$	1
9*	$(0,1/2-\sqrt{3}\xi/4,\sqrt{3}\xi/2-1/2,\sqrt{3}\xi/2-1/2,1-\sqrt{3}\xi/4)$	$s_8\ s_{22}\ t_6\ t_7\ t_{11}$	1
10	$(\sqrt{3}\xi/4-1/4,1/2-\sqrt{3}\xi/4,\sqrt{3}\xi/2-1/2,0,3/4)$	$S_8\ s_{19}\ s_{22}\ t_6\ t_{11}$	1
11*	$(0,1/2-\sqrt{3}\xi/4,0,0,\sqrt{3}\xi/4+1/2)$	$s_8\ s_{11}\ s_{19}\ s_{22}\ t_3\ t_6$	6

Table 4 The yield stress states, the active systems and the numbers of sets of 5 independent systems in the standard stereographic triangle for tension deformation when $\xi < 1/\sqrt{3}$

4 stress states	Yield stress states (multiplied by $\sqrt{6}\tau_{ct}$)	Active systems	Sets of 5 independent systems
1	$(1/2,-1/2,0,0,0)$	$t_2\ t_3\ t_5\ t_6\ t_8\ t_9\ t_{11}\ t_{12}$	56
2	$(0,0,-1/2,1/2,1/2)$	$t_2\ t_3\ t_4\ t_6\ t_{10}\ t_{11}$	6
3	$(-1/8,1/8,-1/4,1/2,1/2)$	$t_4\ t_6\ t_7\ t_{10}\ t_{11}$	1
4	$(0,1/4,0,0,3/4)$	$t_3\ t_6\ t_7\ t_8\ t_{10}\ t_{11}$	6

Changing the macroscopic strain tensor components to the opposite sign, the axisymmetric deformation of compression can be analyzed by the same method. The results are summarized for $\xi > 2/\sqrt{3}$, $\sqrt{3}/2 < \xi < 2/\sqrt{3}$, $1/\sqrt{3} < \xi < \sqrt{3}/2$, $\xi < 1/\sqrt{3}$ in Tables 5—8. Compared with Tables 1—4, it is found that in the case of slip alone for $\xi > 2/\sqrt{3}$, the required yield stress states for compression are just negatives of those for tension while it is not the case for $\xi < 2/\sqrt{3}$. It can be explained by the symmetry of slip and twinning yield surfaces in stress space. The slip yield surfaces is symmetric about the origin since the slip direction can be positive and negative while the twinning yield surfaces is asymmetric about the origin since the twinning direction is unidirectional. Due to the opposite strain tensor components between tension and compression, the yield stress states for tension are just negatives of those for compression if only slip occurs. As far as slip is concerned, there exist the opposite stress states. Naturally, the corresponding active slip systems have the opposite slip direction. However, in the case that both slip and twinning take place or only twinning happens for $\xi < 2/\sqrt{3}$, this relationship dose not exist since the opposite stresses violate the yield condition.

When $\sqrt{3}/2 < \xi < 2/\sqrt{3}$, as shown in Table 6, both slip and twining take places simultaneously. The results show that the standard triangle can be divided into 11 regions associated with different types of basic yield vertices. The yield stress state within the region near the [111] orientation is the particular case of Bishop-Hill slip (the stress state 7 in Table 6), which is the same as the one for pure slip (the stress state 4 in Table 5). Only slip occurs within the region since the corresponding active systems are slip systems while both slip and twinning take places within the other regions since the corresponding active systems associated with the other yield stress states include slip and twinning systems

Table 5 The yield stress states, the active systems and the numbers of sets of 5 independent systems in the standard stereographic triangle for compression deformation when $\xi > 2/\sqrt{3}$

5 stress states	Yield stress states (multiplied by $\sqrt{6}\tau_{cs}$)	Active systems	Sets of 5 independent systems
1	$(-1/2, 1/2, 0, 0, 0)$	$s_2\ s_5\ s_8\ s_{11}\ s_{15}\ s_{18}\ s_{21}\ s_{24}$	32
2	$(-1/4, 1/4, 0, -1/2, -1/2)$	$s_2\ s_5\ s_{10}\ s_{15}\ s_{16}\ s_{24}$	4
3	$(-1/2, 0, 0, 0, -1/2)$	$s_2\ s_5\ s_7\ s_{10}\ s_{15}\ s_{18}\ s_{21}\ s_{24}$	36
4	$(0, 0, -1/2, -1/2, -1/2)$	$s_5\ s_9\ s_{10}\ s_{16}\ s_{20}\ s_{24}$	6
5	$(0, 0, 0, 0, -1)$	$s_2\ s_5\ s_7\ s_{10}\ s_{13}\ s_{16}\ s_{20}\ s_{23}$	32

In the case of mixed slip and twining for $1/\sqrt{3} < \xi < \sqrt{3}/2$ (as shown in Table 7), the results show that the standard triangle can be divided into 11 regions. Similar to $\sqrt{3}/2 < \xi < 2/\sqrt{3}$, the stress state within the region near the [111] orientation is still the particular case of Bishop-Hill slip (the state 9 in Table 7), which is the same as the one in Tables 5 and 6. Therefore, the crystal within the region is deformed by slip while the crystal within the others regions by mixed slip and twinning.

Compared Table 6 with Table 7, it is found that stress states 1, 2, 3, 5, 7, 10, 11 in Table 6 and the stress states 1, 2, 3, 6, 9, 10, 11 in Table 7 labeled by asterisk is common, respectively. It can be concluded that they belong to the 139 common yield stress states, which satisfy the yield condition within the whole range of $1/\sqrt{3} \leqslant \xi \leqslant 2/\sqrt{3}$. The other stress states not labeled by asterisk belong to the 120 different ones, which satisfy the yield condition within the different range of ξ.

In the case that only twinning occurs ($\xi < 1/\sqrt{3}$, as shown in Table 8), the standard stereographic triangle can be divided into 4 regions that correspond to different basic groups of yield stress states. There is no ambiguity in the selection of active systems associated with the yield stress state 2.

Table 6 The yield stress states, the active systems and the numbers of sets of 5 independent systems in the standard stereographic triangle for compression deformation when $\sqrt{3}/2 < \xi < 2/\sqrt{3}$

11 stress states	Yield stress states (multiplied by $\sqrt{6}\tau_{cs}$)	Active systems	Sets of 5 independent systems
1*	$(1/2-\sqrt{3}\xi/2, 3/2-\sqrt{3}\xi/2, 0,0,0)$	$s_2\ s_5\ s_8\ s_{11}\ t_1\ t_4\ t_7\ t_{10}$	44
2*	$(1/4-3\sqrt{3}\xi/8, 3\sqrt{3}\xi/8-1/4, \sqrt{3}\xi/4-1/2, \sqrt{3}\xi/2-1, \sqrt{3}\xi/2-1)$	$S_5\ s_{24}\ t_1\ t_4\ t_{10}$	1
3*	$(-\sqrt{3}\xi/8, \sqrt{3}\xi/8, \sqrt{3}\xi/4-1/2, -1/2, -1/2)$	$s_5\ s_{10}\ s_{16}\ s_{24}\ t_1$	1
4	$(1/4-\sqrt{3}\xi/4, \sqrt{3}\xi/4-1/4, 0, 1/2-\sqrt{3}\xi/2, \sqrt{3}\xi/2-3/2)$	$s_2\ s_5\ s_{10}\ s_{16}\ t_1$	1
5*	$(1/4-3\sqrt{3}\xi/8, 1/4-\sqrt{3}\xi/8, \sqrt{3}\xi/4-1/2, \sqrt{3}/2\xi-1, -1/2)$	$s_5\ s_{10}\ s_{24}\ t_1\ t_4$	1
6	$(1/2-\sqrt{3}\xi/2, 0,0,0, \sqrt{3}\xi/2-3/2)$	$s_2\ s_5\ s_7\ s_{10}\ t_1\ t_4$	6
7*	$(0,0,-1/2,-1/2,-1/2)$	$s_5\ s_9\ s_{10}\ s_{16}\ s_{20}\ s_{24}$	6
8	$(\sqrt{3}\xi/2-1, 0,0,0, -\sqrt{3}\xi/2)$	$s_2\ s_5\ s_7\ s_{10}\ t_9\ t_{12}$	6
9	$(\sqrt{3}\xi/4-1/2, 1/2-\sqrt{3}\xi/4, 0, \sqrt{3}\xi/2-1, -\sqrt{3}\xi/2)$	$s_2\ s_5\ s_{10}\ s_{16}\ t_{12}$	1
10*	$(0,0,\sqrt{3}\xi/2-1, \sqrt{3}\xi/2-1, -\sqrt{3}\xi/2)$	$s_5\ s_{10}\ s_{16}\ s_{20}\ t_9\ t_{12}$	6
11*	$(0, 1/2-\sqrt{3}\xi/4, 0,0, -1/2-\sqrt{3}\xi/4)$	$s_2\ s_5\ s_{13}\ s_{16}\ t_9\ t_{12}$	6

Table 7 The yield stress states, the active systems and the numbers of sets of 5 independent systems in the standard stereographic triangle for tension deformation when $1/\sqrt{3} < \xi < \sqrt{3}/2$

11 stress states	Yield stress states (multiplied by $\sqrt{6}\tau_{cs}$)	Active systems	Sets of 5 independent systems
1*	$(1/2-\sqrt{3}\xi/2, 3/2-\sqrt{3}\xi/2, 0,0,0)$	$s_2\ s_5\ s_8\ s_{11}\ t_1\ t_4\ t_7\ t_{10}$	44
2*	$(1/4-3\sqrt{3}\xi/8, 3\sqrt{3}\xi/8-1/4, \sqrt{3}\xi/4-1/2, \sqrt{3}\xi/2-1, \sqrt{3}\xi/2-1)$	$s_5\ s_{24}\ t_1\ t_4\ t_{10}$	1
3*	$(-\sqrt{3}\xi/8, \sqrt{3}\xi/8, \sqrt{3}\xi/4-1/2, -1/2, -1/2)$	$s_5\ s_{10}\ s_{16}\ s_{24}\ t_1$	1
4	$(-1/8, 1/8, \sqrt{3}\xi/2-3/4, \sqrt{3}\xi/2-1, -\sqrt{3}\xi/2)$	$s_5\ s_{10}\ s_{16}\ t_1\ t_{12}$	1
5	$(1/4-\sqrt{3}\xi/4, 1/2-\sqrt{3}\xi/4, 0, 1/2-\sqrt{3}\xi/2, -3/4)$	$s_2\ s_5\ s_{16}\ t_1\ t_{12}$	1
6*	$(1/4-3\sqrt{3}\xi/8, 1/4-\sqrt{3}\xi/8, \sqrt{3}\xi/4-1/2, \sqrt{3}\xi/2-1, -1/2)$	$s_5\ s_{10}\ s_{24}\ t_1\ t_4$	1
7	$(1/8-\sqrt{3}\xi/4, 3/8-\sqrt{3}\xi/4, \sqrt{3}\xi/2-3/4, \sqrt{3}\xi-3/2, -\sqrt{3}\xi/2)$	$s_5\ s_{10}\ t_1\ t_4\ t_{12}$	1
8	$(1/2-\sqrt{3}\xi/2, 3/4-\sqrt{3}\xi/2, 0,0,-3/4)$	$s_2\ s_5\ t_1\ t_4\ t_9\ t_{12}$	6
9*	$(0,0,-1/2,-1/2,-1/2)$	$s_5\ s_9\ s_{10}\ s_{16}\ s_{20}\ s_{24}$	6
10*	$(0,0,\sqrt{3}\xi/2-1, \sqrt{3}\xi/2-1, -\sqrt{3}\xi/2)$	$s_5\ s_{10}\ s_{16}\ s_{20}\ t_9\ t_{12}$	6
11*	$(0, 1/2-\sqrt{3}\xi/4, 0,0, -1/2-\sqrt{3}\xi/4)$	$s_2\ s_5\ s_{13}\ s_{16}\ t_9\ t_{12}$	6

Table 8 The yield stress states, the active systems and the numbers of sets of 5 independent systems in the standard stereographic triangle for compression deformation when $\xi < 1/\sqrt{3}$

4 stress states	Yield stress states (multiplied by $\sqrt{6}\tau_{ct}$)	Active systems	Sets of 5 independent systems
1	$(0,1,0,0,0)$	$t_1\ t_2\ t_4\ t_5\ t_7\ t_8\ t_{10}\ t_{11}$	56
2	$(-1/8,1/8,-1/4,-1/2,-1/2)$	$t_1\ t_4\ t_5\ t_{10}\ t_{12}$	1
3	$(0,1/4,0,0,-3/4)$	$t_1\ t_2\ t_4\ t_5\ t_9\ t_{12}$	6
4	$(0,0,-1/2,-1/2,-1/2)$	$t_4\ t_5\ t_8\ t_9\ t_{10}\ t_{12}$	6

It may be concluded that regardless of axisymmetric tension and compression, the standard stereographic triangle of cubic crystal for the four cases can be divided into some certain regions associated with specific yield stress states. Some of these regions contain just 5 active systems, and some contain 6 or 8 systems. Since rotation of the crystals during straining depends on the activity of a particular combination of systems, for a given crystal orientation in which there are only five active systems, there is no ambiguity in selecting active systems. Correspondingly, the development of deformation texture is certain. For crystal orientations in which there are 6 or 8 systems, the uncertainty in selecting active systems leads to an uncertainty of texture development. A certain rule should be followed in the selection of the active systems during simulation of deformation texture[16,17].

2.2 The influence of ξ on the Taylor factor and anisotropy of yield strength

Using the obtained slip or/and twinning yield stress states and Bishop-Hill maximum work principle[12,13], the analytical expressions of Taylor factor can be calculated for different cases. For simplicity, the results are not listed in this paper. In order to understand comprehensively the influence of ξ on the Taylor factor and yield strength anisotropy in f.c.c. crystals under axisymmetric plastic deformation, the Taylor factor *M* for three important orientations [111], [110] and [100] in the standard stereographic triangle have been calculated for the four cases of $\xi > 2/\sqrt{3}$, $\sqrt{3}/2 < \xi < 2/\sqrt{3}$, $1/\sqrt{3} < \xi < \sqrt{3}/2$ and $\xi < 1/\sqrt{3}$, as shown in Table 9.

Table 9 The Taylor factor *M* for three important orientations in the standard triangle for axisymmetric deformation

Deformation mode	Orientations	$\xi > 2/\sqrt{3}$	$\sqrt{3}/2 < \xi < 2/\sqrt{3}$	$1/\sqrt{3} < \xi < \sqrt{3}/2$	$\xi < 1/\sqrt{3}$
Axisymmetric tension	[111]	$3\sqrt{6}/2$	$3\sqrt{3}\xi/4\times\sqrt{6}$	$3\sqrt{3}\xi/4\times\sqrt{6}$	$3\sqrt{3}\xi/4\times\sqrt{6}$
	[110]	$3\sqrt{6}/2$	$(1/2+\sqrt{3}\xi/2)\times\sqrt{6}$	$(1/2+\sqrt{3}\xi/2)\times\sqrt{6}$	$\sqrt{3}\xi\times\sqrt{6}$
	[100]	$\sqrt{6}$	$\sqrt{6}$	$\sqrt{6}$	$\sqrt{3}\xi\times\sqrt{6}$
Axisymmetric compression	[111]	$3\sqrt{6}/2$	$3\sqrt{6}/2$	$3\sqrt{6}/2$	$3\sqrt{3}\xi/2\times\sqrt{6}$
	[110]	$3\sqrt{6}/2$	$(1+\sqrt{3}\xi/4)\times\sqrt{6}$	$(1+\sqrt{3}\xi/4)\times\sqrt{6}$	$5\sqrt{3}\xi/4\times\sqrt{6}$
	[100]	$\sqrt{6}$	$\sqrt{3}\xi/2\times\sqrt{6}$	$\sqrt{3}\xi/2\times\sqrt{6}$	$\sqrt{3}\xi/2\times\sqrt{6}$

It can be known through calculation that in the case of slip alone for $\xi > 2/\sqrt{3}$, the [111] and [110] orientations with $M = 3\sqrt{6}/2$ is 1.5 times of the [100] orientation with $M = \sqrt{6}$. When $1/\sqrt{3} < \xi < 2/\sqrt{3}$, $M_{[111]}$ and $M_{[110]}$ decreases as the value of ξ decreases and $M_{[111]}$ decreases faster than $M_{[110]}$ while $M_{[100]}$ remains unchanged, which can be known from the analytical expressions of Taylor factor in Table 9. When $\xi < 1/\sqrt{3}$, twinning is the only deformation mechanism for all orientations in the standard stereographic triangle. The [111] orientation with $M_{[111]} = 1.84\xi\sqrt{3}$ is 25% lower than that of $M = 2.45\xi\sqrt{3}$ for [100] and [110] orientations. Further study reveals that the analytical expression of $M_{[111]}$ remains unchanged for $\sqrt{3}/2 < \xi < 2/\sqrt{3}$, $1/\sqrt{3} < \xi < \sqrt{3}/2$ and $\xi < 1/\sqrt{3}$, $M_{[111]}$ decreases continuously with decreasing ξ while the decreasing rate remains unchanged. Though the analytical expression of $M_{[110]}$ for $1/\sqrt{3} < \xi < 2/\sqrt{3}$ differ from that for $\xi < 1/\sqrt{3}$, it can be found that the value of $M_{[110]}$ at the boundary varies continuously and no abrupt change occurs while the decreasing rate of $M_{[110]}$ for $\xi < 1/\sqrt{3}$ is two times as great as that for $1/\sqrt{3} < \xi < 2/\sqrt{3}$ with decreasing ξ. The value of $M_{[100]}$ for $\xi > 1/\sqrt{3}$ remains unchanged while $M_{[100]}$ for $\xi < 1/\sqrt{3}$ decreases as fast as $M_{[110]}$. In fact, for all orientations in the standard stereographic triangle of cubic crystal, it can be known through calculation that the change of Taylor factor is continuous with decreasing ξ, no abrupt change happens even if ξ is located within the different range. Certainly, the change rate of Taylor factor may be different for different orientations.

The maximum difference of yield strength among the three orientations is 0.5 times if only slip happens while the minimum difference is 0.25 times only twining happens. The results show that the introduction of twinning has influences on the yield strength anisotropy compared with pure slip. The anisotropy is decreased for the case of tension. In fact, the same conclusion can be obtained by calculating the Taylor factor *M* for arbitrary orientations in the standard stereographic triangle of cubic crystal for the case that ξ is located at four different ranges.

The Taylor factor for axisymmetric deformation of compression can be analyzed by the same method. For $\xi > 2/\sqrt{3}$, slip is the only deformation mechanism, the Taylor factors for compression are identical for tension, which is consistent with the analysis result of yield stress. For $1/\sqrt{3} < \xi < 2/\sqrt{3}$, $M_{[111]}$ decreases rapidly and $M_{[100]}$ remains unchanged for tension while $M_{[100]}$ decreases rapidly and $M_{[111]}$ remains changed for compression. For $\xi < 1/\sqrt{3}$, twinning is the only mechanism,

$M_{[111]}=3.674\sqrt{3}\xi$ is three times of $M_{[100]}=1.225\sqrt{3}\xi$. The difference of yield strength among the three orientations is the minimum if only slip occurs while the difference is the maximum if only twinning happens. The results show that the anisotropy is enhanced by the introduction of twinning for the case of compression.

The results show that the yield strength anisotropy based on slip for axisymmetric plastic deformation is changed by the introduction of twinning. The anisotropy is decreased for the case of tension and increased for compression. Axisymmetric plastic flow can be approximated by drawing cylindrical single crystals and polycrystals through conical dies. The deformation resistance is related to the drawing stress. Using this method, Hosford[18] measured the deformation resistance for aluminum polycrystals with high stacking fault energy and a series of aluminum single crystals of different orientations while Mayer[19] for iron, copper, and Cu-7 pct Al polycrystals and single crystals with different stacking fault energy. For simplicity, only the ratio of the normalized drawing stresses near the three important orientations [100], [110] and [111] is given, 1.57∶2.20∶2.27 for aluminum and 5.18∶7.75∶7∶32 for iron, which is very close to the ratio of Taylor factors with $\sqrt{6} : 3\sqrt{6}/2 : 3\sqrt{6}/2$ (1∶1.5∶1.5) in Table 9 for the case that slip is the only mechanism when $\xi > 2/\sqrt{3}$. It can be found that the data for aluminum by Hosford[18] and iron by Mayer[19] obey the slip criterion (Though iron is body-centered cubic crystal, by duality, the results for $\{111\}\langle 110\rangle$ slip in f.c.c. crystals is identical to that for $\{110\}\langle 111\rangle$ slip in b.c.c. crystals). This is attributed that twinning is difficult for aluminum and iron metals with high stacking fault energy as a result of the high ratios of CRSS's for twinning to slip. In fact, the measured results for the other orientations also showed that deformation resistance is related to the Taylor factors. However, the results are unsatisfied for copper and Cu-7 pct Al by the treatment if only slip is considered. Assuming that both slip and twinning happen, agreement with theory can be obtained for Cu-7 pct Al. The difference between the maximum drawing stress and the minimum's for all the orientations of Cu-7 pct Al single crystals is not more than 30% while the difference exceeds 50% for iron and aluminum deforming by slip alone. The yield strength anisotropy is decreased by the introduction of twinning indeed. The experimental stress-strain curve agrees with the theory for copper by assuming equal ease of twinning and slip, i.e. $\xi = 1$. The critical resolved shear stress for twinning can be ascertained by this method while it is difficult to decide the value in experiment. However, since twinning is hard to be observed in deformation of copper at room temperature and low strain rate, further study is necessary.

The yield strength anisotropy has been analyzed for the case of axisymmetric tension or compression, in the following, the yield strength of the same orientation for tension and compression will be discussed. According to Table 9, the ratios of Taylor factor *M* of [111], [110] and [100] orientations for tension to compression have been calculated for $\xi > 2/\sqrt{3}$, $\sqrt{3}/2 < \xi < 2/\sqrt{3}$, $1/\sqrt{3} < \xi < \sqrt{3}/2$ and $\xi < 1/\sqrt{3}$, as shown in Table 10. In the case that only slip occurs for $\xi > 2/\sqrt{3}$, all the ratios for the three orientations are

1, i.e. the yield strength for tension equals that for compression, there exists no asymmetry. In fact, all the other orientations possess the same characteristic, which is consistent with the analyzed results of stress states. In the case that slip and twinning take place together for $\sqrt{3}/2 < \xi < 2/\sqrt{3}$ or $1/\sqrt{3} < \xi < \sqrt{3}/2$, the yield strength ratio of [111] orientation for tension to compression decreases with decreasing ξ while the [100] orientation has the opposite trend. In fact, they are reciprocal. This means the degree of asymmetry for the two orientations is identical, that is, the asymmetry of [111] orientation for tension to compression is identical to that of [100] orientation for compression to tension. In the case that only twinning happens for $\xi < 1/\sqrt{3}$, the ratio of [110] orientation is 4/5, which is the closest to the ratio 1 with symmetry, i.e. the asymmetry degree of [110] orientation is lower than that of [111] and [100] orientations. Furthermore, the ratios of the three orientations have maximum difference with 1, the asymmetry degree for tension to compression is the most obvious in this case. Further study shows that for all the orientations in the standard stereographic triangle of cubic crystals, the yield strength for tension is generally asymmetric to that for compression in the case that both slip and twinning take place. Therefore, the yield strength of wire texture with preferred orientation for tension should be asymmetric to that for compression.

Table 10 The yield strength asymmetry of three important orientations in the standard triangle for axisymmetric tension to compression

Orientations	The ratios of yield strength for tension to compression			
	$\xi > 2/\sqrt{3}$	$\sqrt{3}/2 < \xi < 2/\sqrt{3}$	$1/\sqrt{3} < \xi < \sqrt{3}/2$	$\xi < 1/\sqrt{3}$
[111]	1	$\sqrt{3}\xi/2$	$\sqrt{3}\xi/2$	1/2
[110]	1	$(2+2\sqrt{3}\xi)/(4+\sqrt{3}\xi)$	$(2+2\sqrt{3}\xi)/(4+\sqrt{3}\xi)$	4/5
[100]	1	$2/(\sqrt{3}\xi)$	$2/(\sqrt{3}\xi)$	2

Since twinning is easy for metals at high strain rate deformation, according to adiabatic shear band expanded energy[20], the yield strength is the most important factor on adiabatic shear sensitivity, the orientation distribution—crystallographic texture, which affects the anisotropy of crystal materials strongly, may have important influence on the adiabatic shear sensitivity. However, there is little investigation on the anisotropy of adiabatic shear sensitivity until now. The present result is useful for the study on the anisotropy of adiabatic shear sensitivity.

2.3 The orientation factor for twinning ability and its influence on tensile texture with low stacking fault energy crystals

The above results show that for some orientations in the standard stereographic triangle of cubic crystals, twinning is preferred while it is difficult for the other orientations. In order to describe the twinning ability for crystal orientations, the concept of orientation factor for twinning ability is introduced:

$$\mu = \frac{M_s - M_t}{M_s} = 1 - \frac{M_t}{M_s}. \tag{5}$$

The M_s and M_t are the Taylor factors when ξ is equal to $2/\sqrt{3}$ and $1/\sqrt{3}$, respectively. Obviously, the M_s value is greater than the M_t. For $\xi > 1/\sqrt{3}$, twinning cannot occur alone, if $M_s > M_t$, i.e. $\mu > 0$, for an arbitrary crystal orientation, twinning becomes easier as ξ decreases due to decreasing critical resolved shear stress for twinning. The lower the M_t, i.e. the larger the μ, the easier the twinning for the orientation; the larger the M_t, i.e. the lower the μ, the more difficult the twinning; if $M_s = M_t$, i.e. $\mu = 0$, only slip occurs when $\xi > 1/\sqrt{3}$ and twinning cannot happen for the orientation. It is obvious that for $\xi > 1/\sqrt{3}$, all the active systems are slip systems for $\mu = 0$ while the active systems include both slip and twinning systems for $\mu > 0$. Certainly, twinning is the only mechanism for $\xi < 1/\sqrt{3}$. The values of orientation factor μ for twinning ability in the standard stereographic triangle of cubic crystals have been calculated, as shown in Fig. 1. The slip and twinning can occur for the orientations outside the shadow regions, the twinning is difficult for the orientation inside the shadow regions. For $1/\sqrt{3} \leqslant \xi \leqslant 2/\sqrt{3}$, slip is the only deformation mechanism within the region near [100] orientation while both slip and twinning can happen together within the rest regions for axisymmetric tension; slip is the only deformation mechanism within the region near [111] orientation while both slip and twinning can occur together within the rest regions for axisymmetric compression.

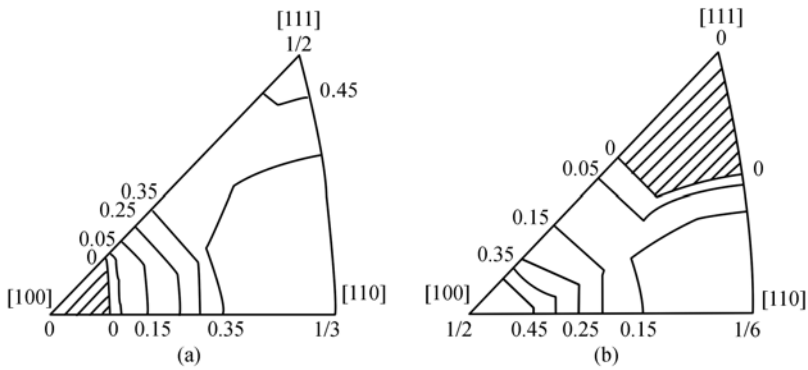

Fig. 1. The distribution of orientation factor of twinning ability for axisymmetric flow (deformation mechanism map). (a) Tension; (b) compression.

In addition to the influence on the yield strength anisotropy, the introduction of twinning has obvious influence on the development of deformation textures. It can be seen from Fig. 1 that for axisymmetric tension, twinning is favorable in the [111] orientation, less favorable in the [110] orientation and unfavorable in the [100] orientation. According to the relationship between the twinned part and matrix,

$$V_t = \Omega V, \tag{6}$$

here V is the axial orientation matrix of the non-twinned part, V_t describes the axial orientation of the twinned part, and Ω is a 3×3 matrix, the elements of which are

$$\theta_{pq} = 2v_p v_q - \delta_{pq}, \qquad \delta_{pq} = \begin{cases} 1, & p = q, \\ 0, & p \neq q, \end{cases} \tag{7}$$

here v_1, v_2 and v_3 are the direction cosines of the normal on the twinning plane.

The twinning part of $\langle 111\rangle$ orientation on the four twinning planes (111), $(\bar{1}11)$, $\langle 1\bar{1}1\rangle$ and $\langle \bar{1}\bar{1}1\rangle$ is $\langle 111\rangle$, $\langle \bar{5}\bar{1}\bar{1}\rangle$, $\langle \bar{1}\bar{5}\bar{1}\rangle$ and $\langle \bar{1}\bar{1}\bar{5}\rangle$, respectively. For $\sqrt{3}/2 < \xi < 2/\sqrt{3}$, the $\langle 111\rangle$ orientation is activated by stress state 7 or 8 listed in Table 2; for $1/\sqrt{3} < \xi < \sqrt{3}/2$, it is activated by stress state 8 or 9 listed in Table 3. The active twinning systems associated with these stress states include t_6, t_7 and t_{11}, the corresponding twinning planes are $(\bar{1}\bar{1}1)$, $(\bar{1}11)$ and $(1\bar{1}1)$, the twinning part of $\langle 111\rangle$ orientation is $\langle \bar{1}\bar{1}\bar{5}\rangle$, $\langle \bar{5}\bar{1}\bar{1}\rangle$ and $\langle \bar{1}\bar{5}\bar{1}\rangle$. According to the structure symmetry of cubic crystal (432 point groups), these orientations can be equivalently converted into $\langle 511\rangle$ orientation, which is only 16 deg from $\langle 100\rangle$.

The twinning part of $\langle 110\rangle$ orientation on the four twinning planes (111), $(\bar{1}11)$, $\langle 1\bar{1}1\rangle$ and $\langle \bar{1}\bar{1}1\rangle$ is $\langle 114\rangle$, $\langle \bar{1}\bar{1}0\rangle$, $\langle \bar{1}\bar{1}0\rangle$ and $\langle 11\bar{4}\rangle$, respectively. For $\sqrt{3}/2 < \xi < 2/\sqrt{3}$, the $\langle 110\rangle$ orientation is activated by stress state 12 listed in Table 2; for $1/\sqrt{3} < \xi < \sqrt{3}/2$, it is activated by stress state 11 listed in Table 3. The active twinning systems associated with these stress states include t_3 an t_6, the corresponding twinning plane is (111) and $(\bar{1}\bar{1}1)$, respectively, the twinning part of $\langle 110\rangle$ orientation is $\langle 114\rangle$ and $\langle 11\bar{4}\rangle$. According to the structure symmetry of cubic crystal, these orientations can be equivalently transformed into $\langle 411\rangle$ orientation, which is about 19 deg from $\langle 100\rangle$.

According to the analyzed results, for the case of the axisymmetric tension of high pure silver with low stacking fault energy, the $\langle 111\rangle$ and $\langle 100\rangle$ wire textures come into being at the beginning. The $\langle 111\rangle$ wire texture decreases and $\langle 100\rangle$ increases with increasing deformation degree, which is consistent with the experimental results on wire drawing of pure silver[9]. However, Co-8%Fe or Cu-8%Al alloys which twins more easily than silver possess a strong $\langle 111\rangle$ wire texture, which is not consistent with the experimental result for silver. Further analysis reveals that twinning occurs before the wire axis has rotated to [111] by slip. Hence the $\langle 111\rangle$ texture may not be appreciably affected in these easily twinned alloys. Though the above analysis are made for f.c.c. crystals for slip on $\{111\}\langle 110\rangle$ and twinning in $\{111\}\langle 112\rangle$ systems, by duality, the results may be applied to b.c.c. crystals for slip on $\{110\}\langle 111\rangle$ and twinning on $\{112\}\langle 111\rangle$ systems. The results of f.c.c. crystals for the case of axisymetric tension and compression are identical to those of b.c.c. crystals for the case of axisymetric compression and tension.

3 Conclusions

The influence of ξ, the ratio of critical resolved shear stress for twinning to slip on the yield stress states and the corresponding active slip or/and twinning systems for orientations in the standard stereographic triangle of cubic crystals in f.c.c. crystals was investigated systematically. The Taylor factors and yield strength anisotropy in orientation space were analyzed. The concept of orientation factor for twinning ability was proposed. The results show that the standard stereographic triangle can be subdivided into some individual regions, which are activated by some specific yield stress states. For $1/\sqrt{3} \leqslant \xi \leqslant 2/\sqrt{3}$, the yield strength anisotropy is decreased by the introduction of twinning for the case of axisymmetric tension, slip is favorable near the [100] orientation while both slip and twinning can happen together within the rest regions; the anisotropy is enhanced for the case of axisymmetric compression, slip is favorable near the [111] orientation while both slip and twinning can occur together within the rest regions. The formation and development of wire deformation textures of f.c.c. crystals with low stacking fault energy could be explained qualitatively on the proposed co-deformation model with slip and twinning.

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