Global canonical symmetry in a quantum system*

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Abstract Based on the phase-space path integral for a system with a regular or singular Lagrangian the generalized canonical Ward identities under the global symmetry transformation in extended phase space are deduced respectively, thus the relations among Green functions can be found. The connection between canonical symmetries and conservation laws at the quantum level is established. It is pointed out that this connection in classical theories, in general, is no longer always preserved in quantum theories. The advantage of our formulation is that we do not need to carry out the integration over the canonical momenta in phase-space generating functional as usually performed. A precise discussion of quantization for a nonlinear sigma model with Hopf and Chern-Simons terms is reexamined. The property of fractional spin at quantum level has been clarified.

Keywords: path-integral quantization, constrained Hamiltonian system, Ward identity, symmetry and conservation laws.

The connection between global symmetries and conservation laws are usually referred to as Noether's first theorem, and Noether's second theorem or Noether identities refers to a local symmetry of a system in classical theory. Noether identity corresponds to the Ward (or Ward-Takahashi) identity in quantum theory. Noether theorems and Ward identities are formulated in terms of Lagrange's variables in configuration space[1]. In ref. [2] the canonical symmetry for a system with singular Lagrangian in classical theory has been established. The Ward identities play an important role in modern quantum field theories, and these identities have been generalized to the supersymmetry[3] and superstring and other problems. All these derivations for Ward identities in the functional integration method are usually discussed by using configuration space generating functional[4], which is valid for the case where the phase-space path integral can be simplified by carrying out explicit integration over canonical momenta; then the phase-space generating functional can be represented in the form of a functional integral only over the coordinates (or field variables) of the expression containing a certain Lagrangian (or effective Lagrangian) in configuration space. In the case where the "mass" depends on coordinates[5] or on coordinates and momenta[6], one obtained effective Lagrangian in configuration-space path integral which shows singularities with a δ-function. Generally, for a constrained Hamiltonian system, it is very difficult or even impossible to carry out the integration over

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canonical momenta. Phase-space path integral is much more fundamental than the configuration-space path integral[7]. Therefore, the study of canonical symmetry of the system in the phase space has more important sense. In ref. [8] the canonical Ward identities for a local symmetry of a system in phase space have been derived. In the persent paper, the global symmetry in phase space for a quantum system will be further investigated.

In this paper, the quantal global symmetry in phase space for a system with a regular and singular Lagrangian will be discussed respectively. In sec. !, the generalized canonical Ward identities for global symmetry are deduced, and the relations among Green function can be obtained immediately. In sec. 2, Noether theorem at the quantum level has been established, and for a certain case the global symmetries of the system imply corresponding conservation laws in quantum theories. In general, the connection between global canonical symmetries and conservation laws in classical theories is no longer preserved in quantum theories. In sec. 3, the nonlinear sigma model with Hopf and Chern-Simons terms is reexamined[9], and a precise treatment of the quantization for those models is given. The property of fractional spin at the quantum level has been explained.

1 Global canonical symmetries and Ward identities

Let us first consider a physical field defined by the field variable $\varphi(x)$ and the motion of the field described by a regular Lagrangian $\mathscr{L}(\varphi, \varphi_{,\mu})$. Our metric conventions are $g_{\mu\nu}=\mathrm{diag}(+,-,-,-)$. The canonical Hamiltonian $H_c=\int d^3x\,\mathscr{H}_c=\int d^3x(\pi\dot{\varphi}-\mathscr{L})$ is a functional of independent canonical variables $\varphi(x)$ and $\pi(x)$, where $\pi(x)=\partial\mathscr{L}/\partial\dot{\varphi}(x)$ is a canonical momentum conjugate to $\varphi(x)$. We adopt the path integral quantization for the system, the phase-space generating functional of Green functions in the form of a functional integral is[8]

$$Z[J, K]=\int \mathscr{D}\varphi\mathscr{D}\pi \exp\left\{i\left[I^p+\int d^4x(J\varphi+K\pi)\right]\right\}, \tag{1}$$

where

$$I^p=\int d^4x\,\mathscr{L}^p=\int d^4x(\pi\dot{\varphi}-\mathscr{H}_c) \tag{2}$$

is a canonical action of the system. Here we have also introduced the exterior sources K with respect to the field canonical momenta π which does not alter the calculation of Green functions. Path integrals in phase space are more fundamental than configuration-space path integral; the latter is fit for a Hamiltonian quadratic in the canonical momenta, whereas the former is fit for arbitrary Hamiltonian density.

Let $F(\varphi, \pi)$ be a functional of canonical variables $\varphi(x)$ and $\pi(x)$. One can define the following functional integral:

$$Z_F[J, K]=\int \mathscr{D}\varphi\mathscr{D}\pi F(\varphi, \pi) \exp\left\{i\left[I^p+\int d^4x(J\varphi+K\pi)\right]\right\}, \tag{3}$$

for the case $J=K=0$, the expection value of the operator $\hat{F}(\hat{\varphi}, \hat{\pi})$ on the ground state of the field is just given by equation (3).

Consider an infinitesimal global transformation in extended phase space:

$$\begin{cases} x'^{\mu}=x^{\mu}+\Delta x^{\mu}=x^{\mu}+\varepsilon_{\sigma}\tau^{\mu\sigma}(x, \varphi, \pi), \\ \varphi'(x')=\varphi(x)+\Delta\varphi(x)=\varphi(x)+\varepsilon_{\sigma}\xi^{\sigma}(x, \varphi, \pi), \\ \pi'(x')=\pi(x)+\Delta\pi(x)=\pi(x)+\varepsilon_{\sigma}\eta^{\sigma}(x, \varphi, \pi), \end{cases} \tag{4}$$

where ε_{σ} ($\sigma=1, 2, \cdots, r$) are infinitesimal arbitrary parameters, $\tau^{\mu\sigma}$, ξ^{σ} and η^{σ} are some functions of x, $\varphi(x)$ and $\pi(x)$. For example, the conformal and internal transformation of the fields are a special case of transformation (4). Under transformation (4), the variation of canonical action (2) is given by[2]

$$\delta I^{p}=\int \mathrm{d}^{4}x\varepsilon_{\sigma}\left\{\frac{\delta I^{p}}{\delta\varphi}(\xi^{\sigma}-\varphi_{,\mu}\tau^{\mu\sigma})+\frac{\delta I^{p}}{\delta\pi}(\eta^{\sigma}-\pi_{,\mu}\tau^{\mu\sigma}) + \partial_{\mu}[(\pi\dot{\varphi}-\mathscr{H}_{c})\tau^{\mu\sigma}]+D[\pi(\xi^{\sigma}-\varphi_{,\mu}\tau^{\mu\sigma})]\right\}, \tag{5}$$

where $D=\mathrm{d}/\mathrm{d}t$ and

$$\frac{\delta I^{p}}{\delta\varphi}=-\dot{\pi}-\frac{\delta H_{c}}{\delta\varphi}, \qquad \frac{\delta I^{p}}{\delta\pi}=\dot{\varphi}-\frac{\delta H_{c}}{\delta\pi}. \tag{6}$$

It is supposed that the Jacobian of transformation (4) is equal to unity. If the functional F and canonical action are invariant under transformation (4), because the functional integral (3) is invariant under transformation (4), we have

$$\begin{aligned} Z_{F}[J, K]&=\int \mathscr{D}\varphi\mathscr{D}\pi F(\varphi, \pi)\Big(1+\mathrm{i}\varepsilon_{\sigma}\int \mathrm{d}^{4}x\{J(\xi^{\sigma}-\varphi_{,\mu}\tau^{\mu\sigma})+K(\eta^{\sigma}-\pi_{,\mu}\tau^{\mu\sigma}) \\ &\quad +\partial_{\mu}[(J\varphi+K\pi)\tau^{\mu\sigma}]\}\Big)\exp\left\{\mathrm{i}\left[I^{p}+\int \mathrm{d}^{4}x(J\varphi+K\pi)\right]\right\} \\ &=\left(1+\mathrm{i}\varepsilon_{\sigma}\int \mathrm{d}^{4}x\left\{J\left(\xi^{\sigma}-\tau^{\mu\sigma}\partial_{\mu}\frac{\delta}{\delta J}\right)+K\left(\eta^{\sigma}-\tau^{\mu\sigma}\partial_{\mu}\frac{\delta}{\delta K}\right)\right.\right. \\ &\quad \left.\left.+\partial_{\mu}\left[\tau^{\mu\sigma}\left(J\frac{\delta}{\delta J}+K\frac{\delta}{\delta K}\right)\right]\right\}\right)_{\substack{\varphi\to-\mathrm{i}\delta/\delta J \\ \pi\to-\mathrm{i}\delta/\delta K}} Z_{F}[J, K]. \end{aligned} \tag{7}$$

Consequently, we obtain the following results: if $F(\varphi, \pi)$ and I^{p} are invariant under transformation (4), then the functional integral (3) satisfies

$$\int \mathrm{d}^{4}x\left\{J\left(\xi^{\sigma}-\tau^{\mu\sigma}\partial_{\mu}\frac{\delta}{\delta J}\right)+K\left(\eta^{\sigma}-\tau^{\mu\sigma}\partial_{\mu}\frac{\delta}{\delta K}\right) +\partial_{\mu}\left[\tau^{\mu\sigma}\left(J\frac{\delta}{\delta J}+K\frac{\delta}{\delta K}\right)\right]\right\}_{\substack{\varphi\to-\mathrm{i}\delta/\delta J \\ \pi\to-\mathrm{i}\delta/\delta K}} Z_{F}[J, K]=0. \tag{8}$$

So if I^p is invariant under transformation (4), but F is a variance one, we can proceed the same way to obtain

$$\int d^4x\left\{\frac{1}{i}\left[\frac{\delta F}{\delta\varphi}\left(\xi^\sigma-\tau^{\mu\sigma}\partial_\mu\frac{\delta}{\delta J}\right)+\frac{\delta F}{\delta\pi}\left(\eta^\sigma-\tau^{\mu\sigma}\partial_\mu\frac{\delta}{\delta K}\right)\right]+J\left(\xi^\sigma-\tau^{\mu\sigma}\partial_\mu\frac{\delta}{\delta J}\right)\right.$$
$$\left.+K\left(\eta^\sigma-\tau^{\mu\sigma}\partial_\mu\frac{\delta}{\delta K}\right)+\partial_\mu\left[\tau^{\mu\sigma}\left(J\frac{\delta}{\delta J}+K\frac{\delta}{\delta K}\right)\right]\right\}_{\substack{\varphi\to -i\delta/\delta J\\ \pi\to -i\delta/\delta K}} Z_F[J, K]=0. \tag{9}$$

We take $F=1$ in expression (9), from (1), (3) and (9), we obtain a result that the generating functional $Z[J, K]$ of Green function should satisfy the following identities:

$$\int d^4x\left\{J\left(\xi^\sigma-\tau^{\mu\sigma}\partial_\mu\frac{\delta}{\delta J}\right)+K\left(\eta^\sigma-\tau^{\mu\sigma}\partial_\mu\frac{\delta}{\delta K}\right)+\partial_\mu\left[\tau^{\mu\sigma}\left(J\frac{\delta}{\delta J}+K\frac{\delta}{\delta K}\right)\right]\right\}_{\substack{\varphi\to -i\delta/\delta J\\ \pi\to -i\delta/\delta K}} Z[J,K]=0. \tag{10}$$

Expressions (8), (9) and (10) are called canonical Ward identities for global symmetry transformation in phase space. Functionally differentiating (10) with respect to $J(x)$ many times and setting exterior sources equal to $J=K=0$, we can obtain some relationships among the Green functions in which one does not need to carry out the integration over the canonical momenta in generating functional (1).

The system described by the singular Lagrangian $\mathscr{L}(\varphi, \partial_\mu\varphi)$ is subject to some inherent phase space constraint and is called a constrained Hamiltonian system. Let Λ_k $(k=1, 2, \cdots, K)$ be first-class constraints, and θ_i $(i=1, 2, \cdots, I)$ be second-class constraints. The gauge conditions connecting the first-class constraints are $\Omega_k(k=1, 2, \cdots, K)$. The phase-space generating functional of a system with a singular Lagrangian is[8]

$$Z[J, K]=\int\mathscr{D}\varphi\mathscr{D}\pi\mathscr{D}\lambda_m\mathscr{D}C_l\mathscr{D}\overline{C}_k\exp\left\{i\left[I_{\rm eff}^p+\int d^4x(J\varphi+K\pi)\right]\right\}, \tag{11}$$

where

$$I_{\rm eff}^p=\int d^4x\,\mathscr{L}_{\rm eff}^p=\int d^4x(\mathscr{L}^p+\mathscr{L}_m+\mathscr{L}_{gh}), \tag{12}$$

$$\mathscr{L}_m=\lambda_k\Lambda_k+\lambda_l\Omega_l+\lambda_i\theta_i, \tag{13}$$

$$\mathscr{L}_{gh}=\int d^4y\left[\overline{C}_k(x)\{\Lambda_k(x), \Omega_l(y)\}C_l(y)+\frac{1}{2}\overline{C}_i(x)\{\theta_i(x), \theta_j(y)\}C_j(y)\right] \tag{14}$$

and $\lambda_m=(\lambda_k, \lambda_l, \lambda_i)$, λ_m can be determined by using the consistency requirement of constraint and gauge conditions. Let us denote $\varphi=(\varphi, \lambda_m, C_l, \overline{C}_k)$. Thus, expression (11) can be simplified as

$$Z[J, K]=\int\mathscr{D}\varphi\mathscr{D}\pi\exp\left\{i\left[I_{\rm eff}^p+\int d^4x(J\varphi+K\pi)\right]\right\}. \tag{11a}$$

For a system with a singular Lagrangian, one can still proceed in the same way to obtain canonical Ward identities under the global symmetry transformation in phase space for such a system, but in this case one must use $I^p_{\rm eff}$ instead of I^p in expressions (1), (5) and (7).

2 Global canonical symmetries and quantal conservation laws

The global canonical symmetries in connection with the conservation laws in classical theories have been discussed in a previous work[2]. Here this problem at the quantum level will be further studied. Let us first consider a system with regular Lagrangian. It is supposed that the canonical action (2) is invariant under the global transformation (4). Now we localize transformation (4) and consider the following local transforamation connected with transformation (4):

$$\begin{cases} x^{\mu'}=x^\mu+\Delta x^\mu=x^\mu+\varepsilon_\sigma(x)\,\tau^{\mu\sigma}(x,\varphi,\pi), \\ \varphi'(x')=\varphi(x)+\Delta\varphi(x)=\varphi(x)+\varepsilon_\sigma(x)\,\xi^\sigma(x,\varphi,\pi), \\ \pi'(x')=\pi(x)+\Delta\pi(x)=\pi(x)+\varepsilon_\sigma(x)\,\eta^\sigma(x,\varphi,\pi), \end{cases} \tag{15}$$

where $\varepsilon_\sigma(x)$ $(\sigma=1,2,\cdots,r)$ are infinitesimal arbitrary functions and their values and derivatives will vanish on the boundary of time-space domain. Under transformation (15) the variation of canonical action (2) is given by

$$\delta I^p=\int {\rm d}^4x\varepsilon_\sigma(x)\left\{\frac{\delta I^p}{\delta\varphi}(\xi^\sigma-\varphi_{,\mu}\tau^{\mu\sigma})+\frac{\delta I^p}{\delta\pi}(\eta^\sigma-\pi_{,\mu}\tau^{\mu\sigma})+\partial_\mu[(\pi\dot{\varphi}-\mathscr{H}_{\rm c})\tau^{\mu\sigma}]\right.$$
$$\left.+D[\pi(\xi^\sigma-\varphi_{,\mu}\tau^{\mu\sigma})]\right\}+\int{\rm d}^4x\left\{[(\pi\varphi-\mathscr{H}_{\rm c})\tau^{\mu\sigma}]\partial_\mu\varepsilon_\sigma(x)+\pi(\xi^\sigma-\varphi_{,\mu}\tau^{\mu\sigma})D\varepsilon_\sigma(x)\right\}. \tag{16}$$

Because the canonical action I^p is invariant under the global transformation (4), the first integral in expression (16) is equal to zero.

Suppose that the Jacobian of transformation (15) is equal to unity. The generating functional (1) is invariant under transformation (15). We perform the integration by parts of the right-hand side of the remaining part in (16) and use the boundary condition of $\varepsilon_\sigma(x)$, after which we substitute the result into (1) and functionally differentiate it with respect to $\varepsilon_\sigma(x)$. We obtain

$$\int\mathscr{D}\varphi\mathscr{D}\pi\{\partial_\mu[(\pi\dot{\varphi}-\mathscr{H}_{\rm c})\tau^{\mu\sigma}]+D[\pi(\xi^\sigma-\varphi_{,\mu}\tau^{\mu\sigma})]-M^\sigma\}\exp\left\{{\rm i}\left[I^p+\int{\rm d}^4x(J\varphi+K\pi)\right]\right\}=0, \tag{17}$$

where

$$M^\sigma=J(\xi^\sigma-\varphi_{,\mu}\tau^{\mu\sigma})+K(\eta^\sigma-\pi_{,\mu}\tau^{\mu\sigma}) \tag{18}$$

Functionally differentiating (17) with respect to $J(x)$ n times, one obtains

$$\int\mathscr{D}\varphi\mathscr{D}\pi(\{\partial_\mu[(\pi\dot{\varphi}-\mathscr{H}_{\rm c})\tau^{\mu\sigma}]+D[\pi(\xi^\sigma-\varphi_{,\mu}\tau^{\mu\sigma})]+M^\sigma\}\varphi(x_1)\varphi(x_2)\cdots\varphi(x_n)$$
$$-{\rm i}\sum_j\varphi(x_1)\cdots\varphi(x_{j-1})\varphi(x_{j+1})\cdots\varphi(x_n)N^\sigma\delta(x-x_j))\exp\left\{{\rm i}\left[I^p+\int{\rm d}^4x(J\varphi+K\pi)\right]\right\}=0, \tag{19}$$

where

$$N^{\sigma}=\xi^{\sigma}-\varphi_{,\mu}\tau^{\mu\sigma}. \tag{20}$$

Let $J=K=0$ in eq. (19). One gets[4]

$$\begin{aligned}&\langle 0|T^{*}\{\partial_{\mu}[(\pi\dot{\varphi}-\mathscr{H}_{c})\tau^{\mu\sigma}]+D[\pi(\xi^{\sigma}-\varphi_{,\mu}\tau^{\mu\sigma})]\}\varphi(x_1)\cdots\varphi(x_n)|0\rangle\\&=i\sum_{j}\langle 0|T^{*}[\varphi(x_1)\cdots\varphi(x_{j-1})\varphi(x_{j+1})\cdots\varphi(x_n)]N^{\sigma}|0\rangle\delta(x-x_j).\end{aligned} \tag{21}$$

Fixing t and letting

$$t_1, t_2, \cdots, t_m\to\infty,\quad t_{m+1}, t_{m+2}, \cdots, t_n\to-\infty$$

and using the reduction formula[4], we find that expression (21) can be reduced to

$$\langle \text{out}, m|\{\partial_{\mu}[(\pi\dot{\varphi}-\mathscr{H}_{c})\tau^{\mu\sigma}]+D[\pi(\xi^{\sigma}-\varphi_{,\mu}\tau^{\mu\sigma})]\}|n-m, \text{in}\rangle=0. \tag{22}$$

Since m and n are arbitrary,

$$\partial_{\mu}[(\pi\dot{\varphi}-\mathscr{H}_{c})\tau^{\mu\sigma}]+D[\pi(\xi^{\sigma}-\varphi_{,\mu}\tau^{\mu\sigma})]=0. \tag{23}$$

We now take a cylinder in 4-dimensional space, of which the axis is directed along t axis and the upper and lower bottoms V_1 and V_2 are two space-like hypersurfaces $t=t_1$ and $t=t_2$ respectively. If we assume that the fields have a configuration which vanishes rapidly at spatial infinity, using Gauss theorem we obtain quantal conserved quantities:

$$Q_R^{\sigma}=\int_V \mathrm{d}^3x[\pi(\xi^{\sigma}-\varphi_{,k}\tau^{k\delta})-\mathscr{H}_{c}\tau^{o\sigma}]\ (\sigma=1, 2, \cdots, r). \tag{24}$$

Consequently, we obtain: for a system with a regular Lagrangian, if the canonical action of this system is invariant under the global transformation in an extended phase space and the Jacobian of the corresponding transformation is equal to unity, then there are some conserved quantities (24) for such a system. These results can be considered a realization of the classical canonical Noether theorem at quantum level. This result holds true for the anomalies-free theories.

For a system with a singular Lagrangian, the phase-space generating function of Green function can be written as expression (11).

If the effective canonical action I^{p}_{eff} is invariant under the global transformation (4) and the Jacobian of the corresponding transformation (15) is equal to unity then one can still proceed in the same way to obtain a result that there are also some quantal conserved quantities for a system with a singular Lagrangian:

$$Q_s^{\sigma}=\int_V \mathrm{d}^3x[\pi(\xi^{\sigma}-\varphi_{,k}\tau^{k\sigma}-\mathscr{H}_{\text{eff}})]\ (\sigma=1, 2, \cdots, r), \tag{25}$$

where $\mathscr{H}_{\text{eff}}$ is an effective Hamiltonian density connecting with the effective Lagrangian density $\mathscr{L}^{p}_{\text{eff}}$. This result corresponds to canonical Noether theorem in classical theory[2]. In general, because $\mathscr{H}_{\text{eff}}$ is different from the canonical Hamiltonian $\mathscr{H}_{c}$, the conserved quantities (25) are different from (24). For a system with a singular Lagrangian, the existence of conserved quantities (25) at quantum level one needs not only to require that the canonical action be invariant under the global transformation in extended phase space, but also to require that the constrained conditions be also invariant under corresponding transformation; thus, one can be sure that the effective canonical action is invariant under such transformation. From (25) one can easily see that the connection between the symmetries and conservation laws in classical theories in general no longer preserve in quantum theories. In the classsical theories of constrained Hamiltonian systems Dirac conjectured that all the first-class constraints (primary and secondary) are generators of gauge transformations. If this conjecture holds true, the classical canonical equations must be derived from the extended Hamiltonian H_E, and the conserved quantities are also determined by H_E but not determined by the total Hamiltonian H_T. From time to time there have been objections to Dirac's conjecture[2, 11]. In the quantum theories of constrained Hamiltonian system, we have shown that the quantal canonical equations of motion are derived from the effective Hamiltonian $\mathscr{H}_{\text{eff}}$[8]. Consequently, the quantal conserved quantities are determined by $\mathscr{H}_{\text{eff}}$ whether Dirac's conjecture holds true or not. Thus, the conserved quantities (25) at the quantum leval differ from the classical ones.

The advantage of our derivation for the conserved quantities at the quantum level is that one does not need to carry out explicit integration over the momenta in phase-space generating functional. In general cases to carry out those integrations is very difficult or even impossible.

3 Nonlinear sigma model with a Hopf and Chern-Simons terms

Numerous recent investigations of (2+1)-dimensional gauge theories with Chern-Simons terms in the Lagrangian have revealed the occurrence of fractional spin and statistics[9, 12, 13]. It has important sense to explain the quantum Hall effect and high-T_c superconductivity. The (2+1)-dimensional nonlinear sigma model with Hopf and Chern-Simons terms was discussed in ref. [9]. Perhaps it is not precise in several points. It is worthwhile to study further. First, the canonical angular-momentum is calculated by using classical Noether theorem whether the result is valid at the quantum level. Secondly, the radiation gauge condition is inconsistent with the equation of motion of the system; thirdly, the Faddeev-Popov determinant in an appropriate gauge needs further study. Now we shall adopt the path-integral quantization for this model to explain the existence of fractional spin rigorously at the quantum level.

The Lagrangian of the nonlinear σ-model with Hopf and Chern-Simons terms is given by

$$\mathscr{L}=\frac{1}{2f}(\partial_\mu n^a)^2-\frac{\theta}{4\pi}A_\mu\varepsilon^{\mu\nu\lambda}\varepsilon^{abc}n^a\partial_\nu n^b\partial_\lambda n^c+\theta\varepsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda, \tag{26}$$

where $(n^a)^2=1$ $(a=1, 2, 3)$. The Lagrangian equation for vector field is

$$\varepsilon^{\mu\nu\lambda}\partial_\nu A_\lambda=J^\mu, \tag{27}$$

where

$$J^\mu=\frac{1}{8\pi}\varepsilon^{\mu\nu\lambda}\varepsilon^{abc}n^a\partial_\nu n^b\partial_\lambda n^c. \tag{28}$$

The canonical moment conjugating to fields n^a and A^μ are denoted by π_a and π_μ respectively. The constraints in phase space for this model are[10]

$$\Lambda_1=\pi_0\approx 0, \tag{29}$$

$$\Lambda_2=2\theta(\varepsilon^{ij}\partial_i A_j-J_0)\approx 0, \tag{30}$$

$$\theta_1=\pi_1-\theta A_2\approx 0, \tag{31}$$

$$\theta_2=\pi_2+\theta A_1\approx 0, \tag{32}$$

$$\theta_3=(n^a)^2-1\approx 0, \tag{33}$$

$$\theta_4=n^a\pi_a\approx 0. \tag{34}$$

The constraints Λ_1 and Λ_2 are first class, and the constraints θ_1, θ_2, θ_3 and θ_4 are second class. According to the theory of canonical quantization of constrained Hamiltonian system, for each first-class constraint a corresponding gauge condition should be chosen. In ref. [9] the radiation gauge condition had been taken as $\partial_i A_i\approx 0$, $A_0\approx 0$. However, this gauge condition cannot hold true simultaneously for this model, because the gauge condition must not only fix the gauge, and be preserved by the dynamical evolution of the system, but also be consistent with the equation of the motion. From the equation of motion (27) one finds

$$A_\lambda=\frac{1}{2\Box}\varepsilon_{\mu\nu\lambda}\partial^\nu J^\mu \tag{35}$$

or

$$A_\lambda(x)=\frac{1}{2}\varepsilon_{\mu\nu\lambda}\int d^2y\, G(x, y)\partial^\nu J^\mu(y), \tag{36}$$

where

$$\Box G(x, y)=\delta^{(2)}(x-y). \tag{37}$$

Thus, we see that $A_0\approx 0$ cannot be consistent with eq. (36). Owing to the consistent requirement of Coulomb gauge $\Omega_1=\partial_i A_i\approx 0$, $\partial_i A_i\approx 0$, implying another gauge constraint:

$$\Omega_2=\nabla^2 A_0-\varepsilon_{ij}\partial^i J^j\approx 0. \tag{38}$$

Thus, one must use $\Omega_2\approx 0$ instead of $A_0\approx 0$ in the radiation gauge.

It is easy to see that det $(\{\Lambda_k, \Omega_l\})$ is independent of field variables. Thus, we can omit this factor from the generating functional. Through the calculation of det $(\{\theta_i, \theta_j\})$, we find out

$$\mathscr{L}_{gh}=8\theta\overline{C}(x)(n^a(x))^2C(x). \tag{39}$$

Thus, the phase-space generating functional of Green function for this model is

$$Z[J_a, J_\mu, K^a, K^\mu]=\int \mathscr{D}n^a\mathscr{D}\pi_a\mathscr{D}A^\mu\mathscr{D}\pi_\mu \exp\left\{\mathrm{i}\int \mathrm{d}^4x(\mathscr{L}_{\mathrm{eff}}^p+J_a n^a+J_\mu A^\mu+K^a\pi_a+K^\mu\pi_\mu)\right\}, \tag{40}$$

where

$$\mathscr{L}_{\mathrm{eff}}^p=\mathscr{L}^p+\mathscr{L}_m+\mathscr{L}_{gh}, \tag{41}$$

$$\mathscr{L}_m=\lambda_k\Lambda_k+\lambda_l\Omega_l+\lambda_i\theta_i, \tag{42}$$

$$\mathscr{L}^p=\pi_a\dot{n}^a+\pi_\mu\dot{A}^\mu-\mathscr{H}_c, \tag{43}$$

$$\begin{aligned}\mathscr{H}_c=&\frac{1}{2}(\pi_a)^2-\frac{\theta^2}{8\pi^2}(\varepsilon^{ij}\varepsilon^{abc}A_i n^b\partial_j n^c)^2-\frac{1}{2}(\partial_i n^a)^2-\theta\varepsilon^{ij}(A_0\partial_i A_j+A_i\partial_j A_0)\\&+\frac{\theta}{4\pi}\varepsilon^{ij}\varepsilon^{abc}[A_0 n^a\partial_i n^b\partial_j n^c+2A_i n^a\partial_j n^b(\pi^c+\frac{\theta}{2\pi}\varepsilon^{km}\varepsilon^{cde}A_k n^d\partial_m n^e)].\end{aligned} \tag{44}$$

The effective canonical action is invariant under the spatial rotation transformation, and the Jacobian of the transformation of vactor field $A^\mu(x)$ and scalar field $n^a(x)$ under the space rotation is equal to unity, and $\tau^{o\sigma}=0$ in the space rotation, and the term $\mathscr{L}_{gh}$ does not involve the time derivative of field variables. There is no contribution to cononical momenta.

From expression (25) we obtain quantal conserved quantities under the rotation in (x_i, x_j) plan[14]:

$$M^{ij}=\int \mathrm{d}^2x\,\pi\left(x^i\frac{\partial}{\partial x_j}-x^j\frac{\partial}{\partial x_i}+D^{ij}\right)\varphi, \quad \varphi=(A^\mu,\ n^a), \tag{45}$$

where D^{ij} depending on the field belongs to the category of the representation of Lorentz group (vector representation, or spinor representation, etc.). From expression (45) we obtain a result that the conserved angular-momentum at the quantum level is identical to the canonical one J^c [9] which contains the orbital angular-momentum and spin angular-momentum of vector field. In the presence of a vortex it has been pointed out that the boundary term gives rise to the fractional spin term[9]. Here we make a precise investigation for the fractional spin. A similar problem about the angular momentum deriving from classical Noether theorem also appears in refs. [12] and [13]. One can still proceed in the same way to study those problems at the quantum level.

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