Brownian ratchets driven by flashing noise strength

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Abstract The stochastic transport driven by flashing multi-noise sources in a ratchet is studied. The stationary current versus the noise strength and the colored noise correlated time τ is obtained. At a fixed flip rate and strength of the colored noises, the novel phenomenon of the current reversal occurs as τ reaches a certain value. More than one reversal point exists when a flashing colored noise source and a flashing white noise source are simultaneously taken into account.

Keywords: flashing force, ratchet, current reversal, Langevin equation.

THE energy consumed in biological activity comes from the hydrolysis of adenosine triphosphate (ATP). With the new progress in experimental research of protein motors[1], various mechanisms have been proposed and discussed. The noise-induced transport mechanisms have attracted great attention[2—6]. Within them, the flashing ratchet mechanism[7,8], in which the overdamped Brownian particles move in a flashing periodic potential with white noise source, could be applied to describing the directed motion even though the current reversal did not occur. On the other hand, the flashing white noise with fixed periodic potential can also induce directed motion, but cannot lead to current reversal[9].

In this note, we shall extend our study to a general situation, in which the overdamped Brownian particles move in a spatially periodic potential $U(x)$ with flashing noise sources including the colored noise. We have found that with the increase of the colored noise correlation time, the current changes its sign from negative to positive, which is quite different from flashing white noise case considered previously[9].

Let us consider the overdamped motion of a Brownian particle in a one-dimensional periodic potential $U(x)$ with period $L = 1$. Its dynamical behavior is determined by the Langevin equation

$$\dot{x}(t) = f(x) + z(t)\varepsilon(t) + g(z(t))\xi_0(t), \tag{1}$$

where the periodic force $f(x) = -\frac{\mathrm{d}U(x)}{\mathrm{d}x}$, ξ_0 is zero-mean flashing Gaussian white noise with the correlation function

$$\langle \xi_0(t)\xi_0(t')\rangle = 2D\delta(t-t').$$

The non-equilibrium fluctuations are modeled by zero-mean exponential correlated colored noise, namely,

$$\langle \varepsilon(t)\rangle = 0, \quad \langle \varepsilon(t)\varepsilon(t')\rangle = \frac{Q}{\tau}\exp\left(-\frac{|t-t'|}{\tau}\right), \tag{2}$$

where Q is a measure of the strength of colored noise and τ is the correlation time. The colored noise $\varepsilon(t)$ satisfies the following Langevin equation

$$\dot{\varepsilon}(t) = -\frac{1}{\tau}\varepsilon(t) + \frac{\sqrt{2Q}}{\tau}\xi(t), \tag{3}$$

where $\xi(t)$ is again a Gaussian white noise source with

$$\langle \xi(t)\rangle = 0, \langle \xi(t)\xi(t')\rangle = \delta(t-t').$$

From eq. (1) we have

$$x(t+h) = x(t) + \int_t^{t+h} f(x(s))\mathrm{d}s + \int_t^{t+h} z(t)\varepsilon(s)\mathrm{d}s + \int_t^{t+h} g(z(t))\xi_0(s)\mathrm{d}s. \tag{4}$$

To expand $f(x(t))$ to the first order, we obtain

$$x(t+h) = x(t) + f(x(t))h + \tau\left(1-\exp\left(-\frac{h}{\tau}\right)\right)z(t)\varepsilon(t) + \frac{\sqrt{2Q}}{\tau}z(t)\omega_1 + \sqrt{2Dh}g(z(t))R. \tag{5}$$

Here, ω_1, R are zero-mean Gaussian random variables,

$$\omega_1 = \int_t^{t+h}\mathrm{d}t'\int_t^{t'}\exp\left(-\frac{s-t'}{\tau}\right)\xi(s)\mathrm{d}s. \tag{6}$$

Solving eq. (3) we get

$$\varepsilon(t+h) = -\exp\left(-\frac{h}{\tau}\right)\varepsilon(t) + \frac{\sqrt{2Q}}{\tau}\omega_0, \tag{7}$$

where ω_0 is defined as

$$\omega_0 = \int_t^{t+h}\exp\left(\frac{s-t-h}{\tau}\right)\xi(s)\mathrm{d}s. \tag{8}$$

Carry out the following transformations

$$\omega_0 = \langle \omega_0^2\rangle^{\frac{1}{2}}R_1, \tag{9}$$

and

$$\omega_1 = \frac{\langle \omega_0\omega_1\rangle}{\langle \omega_0^2\rangle^{\frac{1}{2}}}R_1 + \left[\langle \omega_1^2\rangle - \frac{\langle \omega_0\omega_1\rangle^2}{\langle \omega_0^2\rangle}\right]^{\frac{1}{2}}R_2. \tag{10}$$

R_1, R_2 are all Guassian random numbers with zero-mean value and standard deviation value. The second-order moments of ω_0, ω_1 can be written as

$$\langle \omega_0^2\rangle = \frac{\tau}{2}\left[1-\exp\left(-\frac{2h}{\tau}\right)\right], \tag{11}$$

$$\langle \omega_1^2\rangle = \tau^2\left[h - \frac{3}{2}\tau + 2\tau\exp\left(-\frac{h}{\tau}\right) - \frac{\tau}{2}\exp\left(-\frac{2h}{\tau}\right)\right], \tag{12}$$

and,

$$\langle \omega_0 \omega_1 \rangle = \frac{\tau^2}{2}\left[1 - \exp\left(-\frac{h}{\tau}\right)\right]^2. \tag{13}$$

In order to describe the flashing behavior of the fluctuations, a dichotomous process termed as the function $z = z(t)$ is introduced. For this process, the function $z(t)$ takes only the two values z_1 and z_2 (in this note $z_1 = 0, z_2 = 1$), and λ is the transition rate between values of z_1 and z_2. For each time interval between t_i and $t_i + \lambda^{-1}$, we give a homogeneously distributed random number r in the range of $(0,1)$; the function $z(t)$ then can be produced as

$$z(t) = z_1, \text{ for } 0 < r \leqslant 0.5,$$

and otherwise

$$z(t) = z_2.$$

The average relaxation time $\tau_0 = 2\lambda^{-1}$.

We make the transition time between the two states random, so the steady current of a particle can be described as

$$J = \lim \frac{\langle \dot{x}(t) \rangle}{L} = \lim \left| -\frac{\partial U(x)}{\partial x} \right|. \tag{14}$$

In our model we assume an asymmetric periodic potential with the following form (fig. 1):

$$U(x) = -\frac{1}{2\pi}(\sin(2\pi x) + 0.25\sin(4\pi x)). \tag{15}$$

We have arrived at a general framework for flashing noise sources model. In the following, we shall consider several examples.

It is known that the colored noise becomes a Gaussian white noise when its correlation time turns to zero. And if we set $g(z) = 1 - z$, the Langevin equation (1) becomes

$$\dot{x}(t) = f(x) + z(t)\xi_1(t) + (1 - z(t))\xi_2(t), \tag{16}$$

with

$$\langle \xi_i(t) \rangle = 0, \quad \langle \xi_i(t)\xi_j(t') \rangle = 2D_i\delta_{ij}\ \delta(t - t'),\ i,j = 1,2. \tag{17}$$

In this case it is just corresponding to the two-state diffusion noise model proposed in ref. [9], in which the system oscillates between two temperatures stochastically. Brownian Particles jump between two heat sources, absorb heat from the high temperature heat bath and lose heat to the low temperature heat bath, and in the meanwhile some energy is transduced into useful work which leads to the directed movement.

In fig. 1, we plot the curves of the current as a function of D_1 for different D_2. When the difference between D_1 and D_2 decreases, the current decreases too. And the current approaches zero when D_1 is equal to D_2. Our results fit very well with those of reference [9].

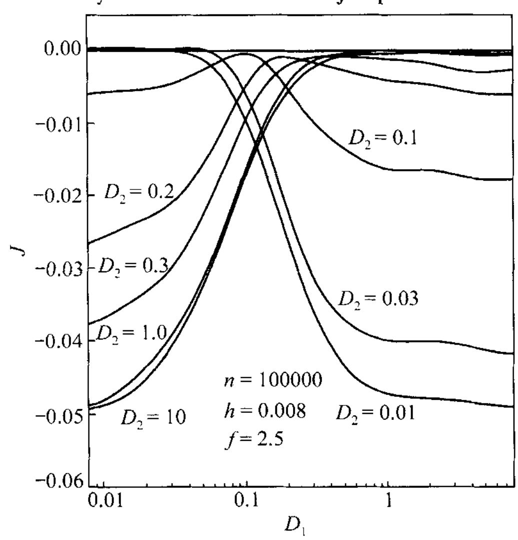

Fig. 1. The probability current J vs. flashing white noise intensity D_1 for fixed n = 100 000, h = 0.008, flipping rate γ = 2.5, step number m = 1 000 and selected values of flashing thermal noise strength D_2 = 0.01, 0.03, 0.1, 0.2, 0.3, 1.0 and 10.

In eq. (1), if $g(z) = 0$, the two-state model can be extended from white noise source to colored noise source, and the Langevin equation then takes the form

$$\dot{x}(t) = f(x) + z(t)\varepsilon(t), \tag{18}$$

where $z(t)\varepsilon(t)$ is zero-mean exponential correlated Ornstein-Uhlenbeck flashing colored noise.

For non-flashing colored noise[10], the current is positive for fixed Q and a certain potential $U(x)$, and the current $J(\tau)$ starts out from zero at $\tau = 0$, reaches a maximum, and approaches zero again as $\tau \to \infty$. The increase and decrease occur monotonically. In this case an effective potential leads to a bias force even though the potential is flat on a larger scale. And the effective average force is determined by the symmetry of the potential. The directed motion is always in the positive direction. The mechanism of the two-state diffusion model is different from that mentioned above. And the direction of the flashing-induced motion is also opposite to that of the colored noise source.

For flashing colored noise, it is necessary to synthesize the two kinds of mechanisms for us to understand the transport behavior of the particles. Some new features have been found as compared to the flashing white noise (see fig. 2) and the non-flashing colored noise[10]. First, it is noticed that with fixed τ, the current $J(Q)$ exhibits a bell-shaped maximum as Q varies from 0 to ∞. Moreover, the maximum moves toward larger Q values as the correlation time increases. Second, with forward ratchet (expressed in eq. (17)), the current may be positive or negative, depending on the correlation time τ. It is easy to see that with the increase of τ the current sign changes from negative to positive at almost the same value of τ for different Q, while the current increases with Q. There is a reversal point τ_c; if $\tau > \tau_c$, the current is negative, and if $\tau < \tau_c$, the current is positive. Third, the current vanishes both as $Q \to 0$ and $Q \to \infty$, but for $\tau \to 0$ the current does not vanish when $Q \to \infty$; it becomes a constant which corresponds to the case of white noise (cf. figure 1(a)).

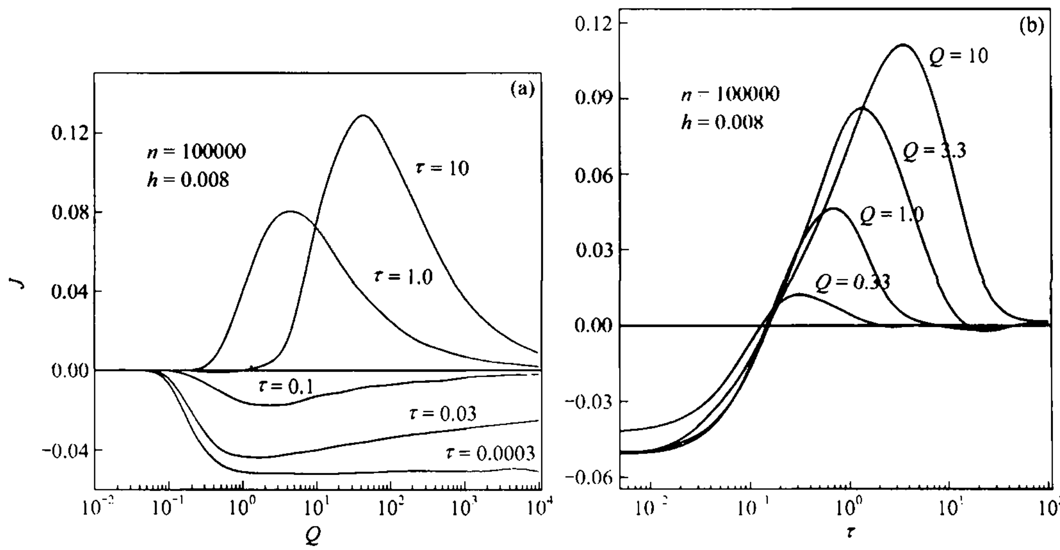

Fig. 2. (a) Plot of the current J as a function of flashing colored noise strength Q for fixed realization number n = 100 000 with time step $h = 0.008$, step number m = 1 000, flipping rate $\gamma = 2.5$ and selected value of colored noise correlation time $\tau = 0.0003$, $\tau = 0.03, 0.1, 1.0$, and 10. (b) The current J as a function of the correlation time τ of the flashing colored noise and selected values of the intensity Q of the noise $Q = 0.33, 1.0, 3.3$ and 10 with the fixed value of n = 100 000, $\gamma = 2.5$, $h = 0.008$, m = 1 000.

If we choose $g(z) = 1 - z$, the Langevin equation (1) has the form

$$\dot{x}(t) = f(x) + z(t)\varepsilon(t) + (1 - z(t))\xi(t). \tag{19}$$

So colored noise and white noise sources act alternatively and are both flashing.

For the potential $U(x)$, the behavior of the current J is depicted in fig. 3 as a function of the noise parameters D, Q and τ. In fig. 3(a), we plot a set of curves for the currents vs. Q with different values of D. The correlation time of the colored noises is fixed with $\tau = 1.0$. We noticed that with the growth of white noise strength, the curve becomes lower, i.e. for specific value of Q, the positive current decreases monotonically or even turns to negative. This phenomenon may be understood in view of competition between the flashing effect and the correlation effect of the colored noise. From fig. 3(b), we see that the behavior of the current on the white noise strength D is different for different value of τ. For small τ, the negative value of the current decreases with the increase of D, and vice versa for larger τ. In the medium range of τ, the situation is a little complex. The current may change from positive to negative monotonically or increase from negative to positive and then decrease to negative again. So there may be two reversal points when τ varies in the range of $(0, \infty)$ or when D varies in the range of $(0, \infty)$ while the rest parameters remain constant.

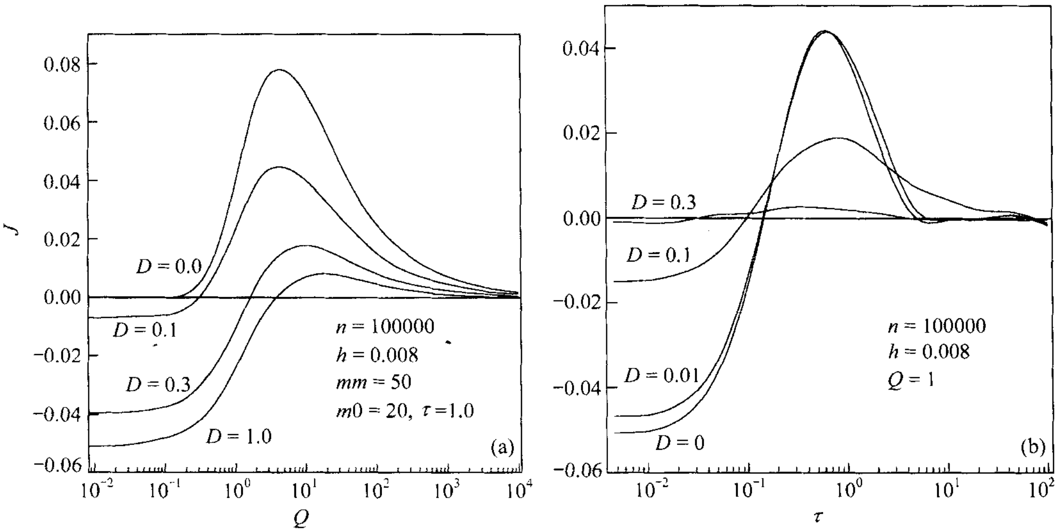

Fig. 3. (a) The probability current J is depicted vs. flashing colored noise intensity Q for fixed correlation $\tau = 0.5$, $n = 100\,000$, $\gamma = 2.5$, $h = 0.008$, $m = 1\,000$ and for selected values of white noise intensity $D = 0.0, 0.03, 0.1$ and 0.3. (b) The probability current is also shown as a function of flashing colored noise correlation time τ for fixed $Q = 1$, $n = 100\,000$, $\gamma = 2.5$, $h = 0.008$, $m = 1\,000$ and for four values of white noise intensity $D = 0.0, 0.01, 0.1$ and 0.3.

We also have studied the system with flashing colored noise and non-flashing white noise under the condition of $g(z) = 1$. In this case the white noise plays a role in making the current appear at smaller strength of the colored noise compared with the case without the white noise.

It is generally accepted that the usable work cannot be extracted if only equilibrium fluctuations exist, in accordance with the second law of thermodynamics, so the essential condition for the directed motion of the Brownian particles is due to the nonequilibrium fluctuations. Although the macroscopic forces are zero-mean, their effective average force must be biased. In these processes the flashing mechanism plays a main role in energy transduction. The two-state diffusion model is in a sense mimic to a kind of thermal engine like that of Carnot engine which is working between two heat sources at different temperatures. By exchanging energy with two heat sources, the Brownian motors can do useful work. It is, however, different from the Carnot engine, because the flashing colored noise induced transport is another kind of energy converting engine which can be thought of as an engine working in nonequilibrium noise sources. For different characteristic correlation time τ, the current may be positive or negative. This nonthermal engine might be important for the explanation of the phenomena in biological process.

We have shown that the flashing noise can induce directed transport in spatial asymmetric structures. Several examples of flashing noises have been discussed. For flashing white noises, the drift is in the opposite direction of the forward potential; namely, the current value is negative when we choose the form of the potential like that of (17). But for flashing colored noise there is a current reversal while the correlation time of colored noise changes. This result is different from that of non-flashing colored noise. We also notice that the influence of non-flashing white noises and that of flashing noises on flashing colored noises induced current are very different. Non-flashing white noise can only make the current appear at smaller strength, but flashing white noise can lead to a current reversal.

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