Effects of spectral linewidth of ultrashort pulses on the spatiotemporal distribution of diffraction fields

XU Jingzhou, WANG Li & YANG Guozhen

Laboratory of Optical Physics, Institute of Physics, Chinese Academy of Sciences, Beijing 100080, China

Correspondence should be addressed to Wang Li (e-mail: wangli@aphy.iphy.ac.cn)

Abstract **The spatiotemporal characteristics of electromagnetic pulses with ultrabroad spectral bandwidth in the far field are analyzed by using classical scalar diffraction theory. The effects of the ratio of the frequency width to the central frequency on the diffraction spatial distribution are discussed. It is concluded that the diffraction spatial distribution of the pulsed radiation gets narrower than a monochromatic wave when the frequency width of the pulse is comparable to or larger than its central frequency.**

Keywords: pulse propagation, diffraction, ultrabroad frequency bandwidth, THz radiation.

The up-to-date laser technology can produce laser pulses with femtoseconds pulse duration and extremely high transient power density, and is applying widely to ultrafast dynamic studies in physics, chemistry and biology, as well as the property studies of materials in strong electromagnetic field. For a laser pulse with a few femtoseconds pulse duration[1], there are only a few oscillation cycles contained in the pulse. In this case, its frequency width is comparable to the central frequency. This kind of pulse can no longer be well described by a quasi-mono- chromatic wave. Christov paid attention to the propagation of ultrashort pulses in the 1980s[2]. He analyzed the temporal waveform evolution in propagation of an ultrashort pulse with Gaussian spatial and temporal profiles. Later on, Porras[3] studied the influence of the spectral bandwidth of an ultrashort pulse on its propagation in detail. Since the 1990s, the ultrashort terahertz (THz) electromagnetic radiation has been widely applied to the studies on the dynamics of transient carriers in semiconductors, time-domain spectroscopic measurements, imaging through opaque materials, etc. The unique feature of the THz radiation is that its central frequency is between the microwave and the far infrared wave, and that the central frequency is comparable to its frequency width[4]. For quite a long time, the uncertainty of the measured pulse temporal waveform puzzled researchers. Recently, it has been realized that the diffraction effects resulting from the broad frequency width of the THz pulses give rise to the variation of the pulse waveforms measured in difference experimental configurations[5—8]. Our analysis and measurements also indicated that the waveform change of a THz pulse undergoing focusing could be well understood by the classical scalar diffraction theory[9]. In despite of extensive studies made on the temporal properties so far, the spatial distribution of an ultrashort pulse with ultrabroad frequency width attracts much less attention. In this note, we discuss the far field waveforms of THz pulses emitted from the sources with different spatial distributions, in the scheme of the classical scalar diffraction theory. The single slit Fraunhofer diffraction patterns are calculated. The effects of spectral broadening on the far field spatial distribution are studied for Gaussian pulses and single slit Fraunhofer diffraction.

1 Temporal waveform of a pulse with ultrabroad frequency width in propagation

The electromagnetic radiation described by the scalar diffraction theory satisfies the following Maxwell equation:

$$\Delta U-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}U=0. \tag{1}$$

For a monochromatic wave, Kirhoff and Sommerfeld gave a solution as follows:

$$U(P_0)=\frac{1}{\mathrm{i}l}\iint_{\Sigma}U(P_1)\frac{\exp(\mathrm{i}kr_{01})}{r_{01}}\cos(\boldsymbol{n},\boldsymbol{r}_{01})\mathrm{d}s, \tag{2}$$

where P_0 is the space coordinate of the field in propagation, P_1 the space coordinate of the source, l the wavelength of the electromagnetic wave, k the wave vector, $\boldsymbol{r}_{01}$ the displacement vector from source to the field point, $\boldsymbol{n}$ the normal direction of the source surface. This equation well describes the propagation of a monochromatic electromagnetic wave except in the near field. For a time dependent electromagnetic pulse, eq. (2) can be applied to each Fourier components. After a straightforward calculation, the corresponding equation for the pulsed field is

$$u(P_0,t)=\iint_{\Sigma}\frac{\cos(\boldsymbol{n},\boldsymbol{r}_{01})}{2pcr_{01}}\frac{\partial}{\partial t}u\left(P_1,t-\frac{r_{01}}{c}\right)\mathrm{d}s. \tag{3}$$

It is indicated that the propagating field is determined by the time derivative of the field at source.

Suppose that the pulse propagates along the z-axis. r is the distance between P_0 and P_1 in X-Y plane. When paraxial ($z^2>>r^2$) and far field ($z>>r^2/\lambda$) conditions are satisfied, eq. (3) can be approximated as

$$u(P_0,t)\approx\frac{1}{2\mathrm{i}pcz}\iint_{\Sigma}\mathrm{d}s\int\mathrm{d}w\ U(P_1,w)w$$

$$\times\exp\left[\mathrm{i}w\left(\frac{z}{c}+\frac{r^2}{2zc}-t\right)\right]. \tag{4}$$

Fig. 1 shows the calculated on-axis waveforms of THz pulses in propagation from different sources, which have the same temporal waveform but different spatial distributions that take symmetrical profile $F(r)$. It is clearly shown that, for emitting sources with different spatial distributions, the temporal waveforms of the THz pulses change their shapes in different ways in the early stage of propagation but all approach the similar waveforms in far field and become the time derivative of the waveforms at sources.

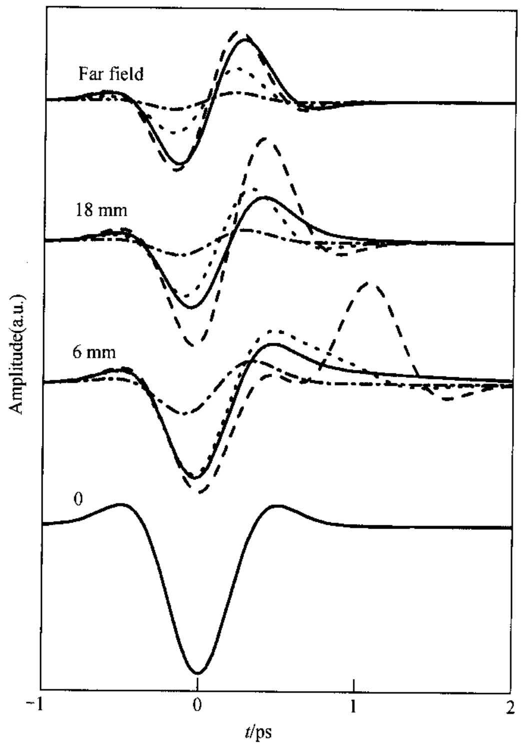

Fig. 1. Evolution of on-axis waveforms of THz pulses in propagation with different spatial distributions at source. Spatial distributions at source ($z=0$): solid line, $F(r)=\exp(-r^2/4)$; dash line, $F(r)=1$, $-2<r<2$; dot line, $F(r)=\cos(\pi r/4)$, $-2<r<2$; dash dot line, $F(r)=\exp(-2r)$. All the units are measured in centimeter.

2 Spatiotemporal distributions of Fraunhofer diffraction of electromagnetic pulses with ultrabroad frequency width

For Fraunhofer diffraction from a slit, the diffraction field of a monochromatic wave satisfies the equation

$$u(q) = C\frac{\sin\left(\frac{wa}{2c}\sin(q)\right)}{\sin(q)}\exp(ikz), \qquad (5)$$

where C is a coefficient independent of frequency and space arguments, q the diffraction angle, a the slit width, and z the distance between the planes where the object point and the slit are located. For a coherent electromagnetic pulse with ultrabroad frequency width such as THz pulse, its Fraunhofer diffraction field consists of the superposition of all the frequency components,

$$u(q,t) = C\int_{-\infty}^{+\infty} U(w)\frac{\sin\left(\frac{wa}{2c}\sin(q)\right)}{\sin(q)} \times\exp(ikz)\exp(-iwt)\mathrm{d}w. \qquad (6)$$

It is indicated from eq. (6) that the integration over w affects not only the temporal profile of the electromagnetic wave but also its spatial distribution. When $U(w)$ is no longer a usual d-like function in the case of monochromatic or quasi monochromatic wave, and the spectral width is comparable to the central frequency, the spatial distribution of $u(q, t)$ will deviate from the typical Fraunhofer patterns. Fig. 2 shows the calculated results that describe the temporal and spatial distributions in Fraunhofer diffraction region of a THz pulse (shown in the insert) passing a slit with 1 cm width. The on-axis temporal profile becomes the time derivative of waveform at source. The waveforms in off-axis area keep the similar shape as that at source but the pulse duration is broadened. The positive and negative peaks move back- and forwards, respectively, in the reference frame moving along the pulse. These features indicate the strong diffraction effects of the low frequency components. For THz pulse discussed in fig. 2, the temporal waveforms at both on-axis and off-axis points are time symmetric. Taking the symmetric point as the time zero in the reference frame moving along with the pulse, it is noticed in fig. 2 that the far field spatial distribution of the pulsed diffraction field varies at different parts of the pulse and gets narrower as approaching the time zero. When the time interval from the time zero is less than 1 ps, the spatial distribution of the electrical amplitude takes on a single peak. As the time interval goes beyond 1 ps, the spatial distribution develops into a two-peak structure, and the two peaks move apart from each other. Furthermore, the time integration of the pulse energy takes a single peak in space, rather than the square of a *Sinc* function in the case of a monochromatic wave. After a systemic study on the propagating pulses with different spectral widths, it is found that the spatial energy distribution of the electromagnetic pulses takes a multi-peak structure in general, but the side peaks gradually disappear as the frequency width increases. At last, only the central peak remains when the frequency width becomes comparable to the central frequency. This phenomenon is just the counterpart in space domain of what happens in mode-locking laser. The increased frequency width results in convergence of pulse energy towards the center of the diffraction pattern by coherent superposition of the individual frequency components.

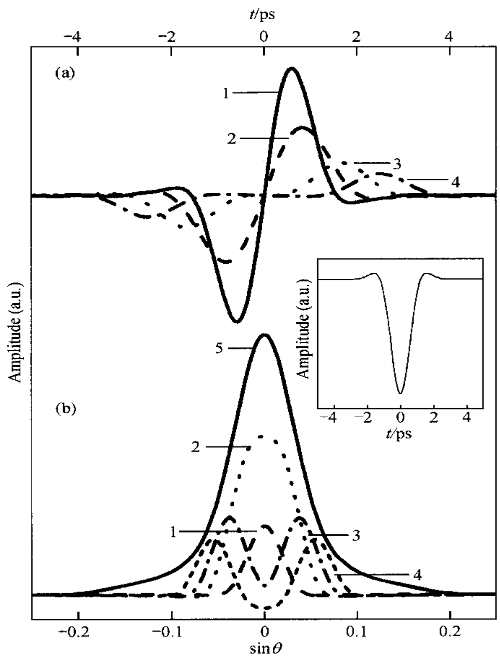

Fig. 2. The temporal and spatial distributions of Fraunhofer diffraction from a slit for a typical THz pulse. (a) The temporal profiles of radiation field at different off-axis conditions. 1, sin (q) = 0; 2, sin(q) =0.025; 3, sin(q) =0.05; 4, sin(q) = 0.075. (b) 1—4, Spatial distributions of radiation field with time interval of t = 0.1, 1, 1.5 and 2 ps from the time zero, respectively; 5, energy angular distribution, i.e. the diffraction patterns in the far field. The insert is the temporal waveform of the THz pulse at source.

3 Narrowing of diffraction distribution due to increased frequency width

(i) Fraunhofer diffraction from a slit. As an example, consider a THz pulse with the following Gaussian spectral distribution:

$$U(w) = \exp\left[-\frac{(w-w_0)^2}{{w_h}^2}\right]. \qquad (7)$$

For THz pulses with different ratio of w_h / w_0, the dimension of Fraunhofer diffraction pattern from a slit of these THz pulses is compared with that of the zero order Fraunhofer diffraction pattern of a monochromatic wave with frequency w_0. The calculated ratio of full width at half maximum (FWHM) for the two cases is shown in fig. 3, where the quantities measured in experiments are used, i.e. the time integrated energy and the power are taken for THz pulse and monochromatic wave, respectively. It is clearly seen in fig. 3 that there is little difference in the zero order Fraunhofer distributions for pulsed and monochromatic waves when w_h/w_0 is small, but when $w_h/w_0>0.5$, the spatial distribution of the Fraunhofer diffraction pattern of THz pulse becomes much narrower than that of monochromatic wave.

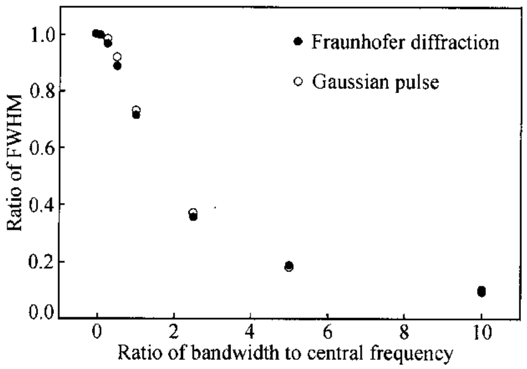

Fig. 3. The relation of the energy spatial distribution and the pulse frequency width for electromagnetic pulses with ultrabroad linewidth (●, Fraunhofer diffraction; ○, Gaussian pulse). The ordinate gives the ratio of beam sizes for the ultrabroad band pulse and the monochromatic wave, the abscissa the ratio of pulse's bandwidth to the central frequency.

(ii) Diffraction of Gaussian beam. For an electromagnetic pulse with a Gaussian intensity distribution and ultrabroad frequency width, the spatial distribution of electrical field can be approximated as

$$E(z,r,t)=\int \mathrm{d}w\,\frac{U(w)wa_0}{2\mathrm{i}cz}\exp\left(-\frac{w^2a^2r^2}{4c^2z^2}\right)\times\exp\left(\mathrm{i}kz+\frac{\mathrm{i}kr^2}{2z}\right)\exp(-\mathrm{i}\,w\,t), \tag{8}$$

where a_0 is the waist radius, r the distance from the object point to the propagation axis. It can be found after comparing eq. (8) to eq. (4) that the two expressions are almost the same except a spatial decay factor. Fig. 3 also shows ratios of waist sizes of the THz pulse, which has frequency components with Gaussian spatial distribution and the same spectral distribution as eq. (8), and that of a monochromatic wave. Just as expected, the variation of the ratio follows the same way as in the case of the Fraunhofer diffraction from a slit when frequency width of the THz pulse changes.

In conclusion, the spatiotemporal properties of an electromagnetic pulse with ultrabroad frequency width are considerably different from a quasi-monochromatic wave. For broadband pulse, the Fourier components coherently superpose in time and space, which results in not only the change of the temporal waveform but also the narrower spatial distribution in the far field or undergoing focusing. Therefore, in applications such as information transfer through long distance and distance ranging, using the pulse with ultrabroad frequency width can effectively extend the operation distance or obtain better signal-to-noise ratio under the same condition. Also, in the signal pro- cessing and microelectronics industry where the higher spatial resolution is always in high demand, the pulse with ultrabroad frequency width could be used to image the image pixel into smaller size.

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