Quantum Interference, Lasing Without Inversion and Enhancement of the Index of Refraction

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Received January 11, 1993.

Abstract Lasing without inversion can be obtained in the medium consisting of atoms of three-level cascade decay system by choosing the incoherently pumping rates if atomic coherence is produced by an external electromagnetic field at two lower levels. Discussions are made on strong external field and resonance cases, respectively. Meanwhile, based on numerical calculations, it is found that a large index of refraction with vanishing absorption and dispersion can be achieved for the probe light of a certain frequency in this medium.

Keywords: cascade decay system, lower-level atomic coherence, lasing without inversion, index of refraction.

1 Introduction

One of the central issues in modern laser techniques is the generation of a laser in X-ray domain. However, since the population inversion between two states connecting an ultrashort wavelength transition is difficult to achieve by conventional pumping schemes, an ultrashort wavelength laser, in general, is hard to realize. Moreover, even if a population inversion is reached between two such states, the strong spontaneous emission will yield a large phase noise, as the spontaneous emission rate relates to the laser wavelength in a cubic inverse ratio. Consequently, it is impossible for an output ultrashort wavelength laser to have a narrow linewidth. Recently, it is theoretically recognized that under proper conditions a buildup of coherent radiation is achievable in some multi-level systems, even if population inversion is absent. This new kind of laser mechanism, termed 'lasing without inversion'[1-4], may provide us with an important route to realize an ultrashort laser. Meanwhile, as it does not need a large population in the upper lasing level, the spontaneous emission phase noise is quite small and then the laser generated in this way will have a very narrow natural linewidth[2,5].

Atomic-coherence-induced reduction of absorption[6] has been observed in recent experiments. Another interesting phenomenon, which is closely related to it, as revealed by Scully *et al.*[7], is the enhancement of the refractive index of a light in a non-absorption medium. The usual absorption-dispersion relation tells us that when the refractive index is large, the absorption of light by the medium is large too. Harris *et al.*[8] found that with the help of atomic coherence created by an external field, the absorption of light by the medium can be made vanished under certain conditions, but at the same time the refractive index is unity and the dispersion is large. From numerical calculations, Scully *et al.* showed that by properly adjusting the external conditions (e.g. the intensity of external field, incoherently pumping rates, etc.), a high refractive index can be obtained with vanishing absorption. It can be used[7] to develop a new kind of optical material and applied to laser particle accelerator[9].

In this paper, we study a three-level cascade decay system with atomic coherence produced by an external electromagnetic field at two lower levels.

2 Atomic Coherence and Polarization

Consider a closed three-level cascade system as shown in Fig. 1, where $r_1(r_2)$ is the decay rate from level $|1\rangle$ to $|2\rangle$ (from level $|2\rangle$ to $|3\rangle$), $\lambda_1(\lambda_2)$ is the incoherently pumping rate in the inverse direction, $\omega_p(\omega_c)$ is the frequency of probe (external) field. The $|1\rangle \leftrightarrow |3\rangle$ transition is dipole-forbidden. We take g and G to represent the coupling coefficients of the atom with the probe and external fields, respectively, i.e. $g=\mu_{12} E_p$ and $G=\mu_{23}E_c$, where μ_{12} and μ_{23} are the matrix elements of atomic dipole transitions, with the subscripts p and c denoting the probe and external fields, respectively. The equations of motion for atomic density matrix elements in the rotating frame[10] can be written as

$$\dot{\rho}_{12}=-(r_{12}+i\Delta_1)\rho_{12}+ig(\rho_{22}-\rho_{11})-iG^*\rho_{13}, \tag{1a}$$

$$\dot{\rho}_{13}=-(r_{13}+i\Delta_3)\rho_{13}+ig\rho_{23}-iG\rho_{12}, \tag{1b}$$

$$\dot{\rho}_{23}=-(r_{23}+i\Delta_2)\rho_{23}+ig^*\rho_{13}+iG(\rho_{33}-\rho_{22}), \tag{1c}$$

$$\dot{\rho}_{11}=-r_1\rho_{11}+\lambda_1\rho_{22}+ig\rho_{21}-ig^*\rho_{12}, \tag{1d}$$

$$\dot{\rho}_{22}=-(r_2+\lambda_1)\rho_{22}+r_1\rho_{11}+\lambda_2\rho_{33}+ig^*\rho_{12}-ig\rho_{21}+iG\rho_{32}-iG^*\rho_{23}, \tag{1e}$$

$$\dot{\rho}_{33}=r_2\rho_{22}-\lambda_2\rho_{33}+iG^*\rho_{23}-iG\rho_{32}, \tag{1f}$$

where

$$\Delta_1=\omega_{12}-\omega_p,\quad \Delta_2=\omega_{23}-\omega_c,\quad \Delta_3=\Delta_1+\Delta_2=\omega_{13}-\omega_p-\omega_c,$$

$$r_{12}=\frac{r_1+r_2+\lambda_1}{2},\quad r_{13}=\frac{r_1+\lambda_2}{2},\quad r_{23}=\frac{r_2+\lambda_1+\lambda_2}{2}. \tag{1g}$$

If the energy flux of fields p and c have Lorentz line shapes

$$J_i(\omega)=\frac{J_i^0\pi/\Delta_i}{(\omega-\omega_i)^2+\Delta_i^2},\quad i=p \text{ or } c, \tag{2}$$

the effects of linewidths of fields p and c can be taken into account by replacing r_{ij} by[11]

$$r_{12}=\frac{r_1+r_2+\lambda_1}{2}+\Delta_p,$$
$$r_{13}=\frac{r_1+\lambda_2}{2}+\Delta_p+\Delta_c,$$
$$r_{23}=\frac{r_2+\lambda_1+\lambda_2}{2}+\Delta_c. \tag{3}$$

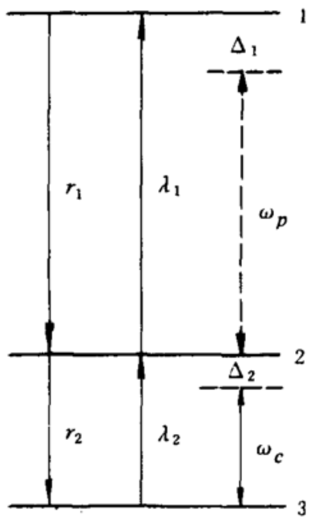

Fig. 1. The cascade system with atomic coherence in two lower levels.

We assume that the probe field is weak so that the polarization amplitude can be calculated up to only the linear term of g. Thus, the terms including g in Eqs. (1c−1f) can be omitted. Atomic polarization induced at probe field frequency is given by[10]

$$P=N\mu_{12}^{*}\,\rho_{12}, \tag{4}$$

where N is atomic density.

Solving Eqs. (1) in steady state ($\dot{\rho}=0$), we have

$$\rho_{23}=\frac{iG(\rho_{33}-\rho_{22})}{r_{23}+i\Delta_2}, \tag{5}$$

from Eq.(1c), and

$$\rho_{11}=\frac{\lambda_1(\lambda_2+A)}{r_1r_2+r_1\lambda_2+\lambda_1\lambda_2+A(\lambda_1+2r_1)}, \tag{6a}$$

$$\rho_{22}=\frac{r_1(\lambda_2+A)}{r_1r_2+r_1\lambda_2+\lambda_1\lambda_2+A(\lambda_1+2r_1)}, \tag{6b}$$

$$\rho_{33}=\frac{r_1(r_2+A)}{r_1r_2+r_1\lambda_2+\lambda_1\lambda_2+A(\lambda_1+2r_1)}, \tag{6c}$$

where

$$A=\frac{2r_{23}|G|^2}{r_{23}^2+\Delta_2^2}, \tag{6d}$$

from Eqs. (1d-1f), with the help of Eq. (5). Using Eqs. (5) and (6), we can solve Eqs. (1a) and (1b) and find

$$\rho_{12}=igBB_1[(\rho_{22}-\rho_{11})+B_2B_3|G|^2(\rho_{33}-\rho_{22})], \tag{7}$$

where

$$B_1=\frac{1}{r_{12}+i\Delta_1},\quad B_2=\frac{1}{r_{13}+i\Delta_3},\quad B_3=\frac{1}{r_{23}+i\Delta_2},\quad B=\frac{1}{1+B_1B_2|G|^2}. \tag{7a}$$

Substituting Eq. (7) into Eq. (4), we obtain the slowing-varying complex polarization amplitude $P_0 = Pe^{i\omega_p t}$ as

$$\mathrm{Re}\,P_0 = N|\mu_{12}|^2 E_p^2 \left[(\rho_{22}-\rho_{11}) \frac{\Delta_1 C_1 - r_{12} C_2}{D_1 C_3} + |G|^2 (\rho_{33}-\rho_{22}) \frac{C_1 C_5 + C_2 C_4}{D_1 D_2 D_3 C_3} \right], \quad (8a)$$

$$\mathrm{Im}\,P_0 = N|\mu_{12}|^2 E_p^2 \left[(\rho_{22}-\rho_{11}) \frac{r_{12} C_1 + \Delta_1 C_2}{D_1 C_3} + |G|^2 (\rho_{33}-\rho_{22}) \frac{-C_1 C_4 + C_2 C_5}{D_1 D_2 D_3 C_3} \right], \quad (8b)$$

where

$$C_1 = 1 + |G|^2 \frac{r_{12} r_{13} - \Delta_1 \Delta_3}{D_1 D_2}, \quad C_2 = |G|^2 \frac{r_{13}\Delta_1 + r_{12}\Delta_3}{D_1 D_2}, \quad C_3 = C_1^2 + C_2^2,$$
$$C_4 = -r_{12} r_{13} r_{23} + r_{12}\Delta_2\Delta_3 + r_{13}\Delta_1\Delta_2 + r_{23}\Delta_1\Delta_3,$$
$$C_5 = r_{13} r_{23}\Delta_1 + r_{12} r_{13}\Delta_2 + r_{12} r_{23}\Delta_3 - \Delta_1\Delta_2\Delta_3,$$
$$D_1 = r_{12}^2 + \Delta_1^2, \quad D_2 = r_{13}^2 + \Delta_3^2, \quad D_3 = r_{23}^2 + \Delta_2^2. \quad (9)$$

Eqs. (8) are the complex polarization amplitude $P_0 = ReP_0 + i\,\mathrm{Im}P_0$ of interest. Since the expressions are very complicated, we shall analyze the conditions of lasing without inversion and the feasibilty of achieving a large refractive index with vanishing absorption and dispersion in some cases where it is easy to perform the experimental examination.

3 Lasing Without Inversion

The equation of motion for the electric field amplitude of a weak probe field is[10]

$$\dot{E}_p^0 = -kE_p^0 + 2\pi i \omega_p P_0, \quad (10)$$

where k is the loss rate due to the leakage of the cavity. If k is small, $\mathrm{Im}P_0 < 0$ will lead to the amplification of the probe field. Lasing without inversion requires

$$\rho_{11} < \rho_{22}, \quad (11a)$$

$$\mathrm{Im}P_0 < 0. \quad (11b)$$

From Eqs. (6), we immediately obtain the noninversion condition

$$\lambda_1 < r_1. \quad (12)$$

In the following, we study how to realize $\mathrm{Im}P_0 < 0$ when (i) the external field used to create atomic coherence is very strong ($|G|$ is large) or (ii) both the probe and external fields are on resonance with respective atomic transitions (but the large $|G|$ condition is released).

3.1 *Strong External Field Case*

This case corresponds to recent experiments in examining atomic-coherence-induced reduction of absorption[6]. Under the approximation $|G| \gg r_1, r_2, \lambda_1, \lambda_2, \Delta_1, \Delta_2, \Delta_p, \Delta_c$, Eqs. (8) are simplified into

$$\mathrm{Re}P_0 = N|\mu_{12}|^2 E_p^0 \frac{|G|^2}{(\lambda_1+2r_1)D_1D_2C_3}\left[(\lambda_1-r_1)(\Delta_1+\Delta_2)-\frac{r_1}{2r_{23}}\Delta_2(\lambda_2-r_2)\right], \tag{13a}$$

$$\mathrm{Im}P_0 = N|\mu_{12}|^2 E_p^0 \frac{-r_{13}|G|^2}{(\lambda_1+2r_1)D_1D_2C_3}\left[(\lambda_1-r_1)+\frac{r_1}{2r_{23}}(\lambda_2-r_2)\right], \tag{13b}$$

where $\mathrm{Im}P_0$ depends on the detunings Δ_1 and Δ_2 through quantities D_1, D_2 and C_3.

Using Eq. (13b), $\mathrm{Im}P_0<0$ yields

$$(\lambda_1-r_1)+\frac{r_1}{2r_{13}}(\lambda_2-r_2)>0. \tag{14}$$

Therefore, lasing without inversion will be realized when λ_1 and λ_2 satisfy

$$\begin{cases} \left(1+\dfrac{r_2-\lambda_2}{2r_{13}}\right)r_1<\lambda_1<r_1, & (15a)\\ \lambda_2>r_2. & (15b)\end{cases}$$

The left- and right-hand sides of Eq. (15a) are the gain and noninversion conditions, respectively, while (15b) is the compatibility condition of Eq. (15a), i.e. it is necessary for the weak probe field to have a gain in the absence of population inversion.

3.2 *Resonance Case*

For strong external field, $\mathrm{Im}P_0 \propto |G|^{-2}$, and then the achievable gain coefficient of the weak probe field is small. In this subsection we consider the resonant situation ($\Delta_1=\Delta_2=0$, without the large-$|G|$ approximation). Prasad *et al.*[10] recently analyzed the cascade system with atomic coherence in two upper levels and found that a weak probe field can be amplified if its frequency is properly detuned to atomic transition. From the discussions given below, we find that lasing without inversion will occur when both fields are tuned on resonance, if atomic coherence exists in two lower levels.

For $\Delta_1=\Delta_2=0$, the imaginary part of polarization is simplified into (the real part is zero)

$$\mathrm{Im}P_0 = N|\mu_{12}|^2 E_p^0 \frac{-1}{r_{12}r_{13}r_{23}[r_1r_2+r_1\lambda_2+\lambda_1\lambda_2+(\lambda_1+2r_1)A]C_1}\times$$

$$\left[\left(\lambda_2+\frac{2|G|^2}{r_{23}}\right)(\lambda_1-r_1)r_{13}r_{23}+|G|^2r_1(\lambda_2-r_2)\right], \tag{16}$$

where $A=2|G|^2/r_{23}$ and $C_1=1+|G|^2/r_{12}r_{13}$. From Eqs. (11b), (12) and (16), we obtain the conditions of lasing without inversion

$$\begin{cases} \left[1+\frac{|G|^2(r_2-\lambda_2)}{(2|G|^2+r_{23}\lambda_2)r_{13}}\right]r_1<\lambda_1<r_1, & (17a) \\ \lambda_2>r_2\,. & (17b) \end{cases}$$

It is obvious that Eqs.(17) recover Eqs.(15) when $|G|$ is large. Eqs.(17) tell us that lasing without inversion may take place in the resonance case where atomic coherence is established in two lower levels in a cascade system.

Basing on the results discussed above, we find that the cascade system is different from the Δ system proposed by Imamoglu *et al.*[3] in realizing lasing without inversion. In the latter system, one requires conditions not only for the incoherently pumping rates, but also for atomic decay rates, while in the cascade system with lower-level coherence, no requirement is placed on atomic decay rates.

4 Enhancement of Refractive Index With Vanishing Absorption and Dispersion

The index of refraction of a light in a medium relates to atomic polarization by

$$n^2-1=\frac{1}{\varepsilon_0 E_p^0}\,\mathrm{Re}P_0, \tag{18}$$

which may be approximated as

$$n-1\approx\frac{1}{2\varepsilon_0 E_p^0}\,\mathrm{Re}P_0. \tag{19}$$

The dispersion is determined by $dn/d\omega_p$.

Taking $\Delta_2=0$, we immediately find from Eq. (8a) that $\mathrm{Re}P_0$ is an odd function of probe detuning Δ_1, and then when $\mathrm{Re}P_0\neq0$ at a certain detuning Δ_1^0, a positive value of $\mathrm{Re}P_0$ can be obtained at one of the two points $\pm\Delta_1^0$. Thus, the conditions for achieving an enhancive refractive index with vanishing absorption and dispersion can be written in the form

$$\mathrm{Im}P_0=0, \tag{20a}$$

$$\frac{d\mathrm{Re}P_0}{d\Delta_1}=0, \tag{20b}$$

$$\mathrm{Re}P_0\neq0. \tag{20c}$$

For strong external field, $\mathrm{Re}P_0\propto|G|^{-2}$, and the refractive index cannot have a nontrivial enhancement. It is also clear that the $\Delta_1=\Delta_2=0$ case cannot lead to a large n (since $\mathrm{Re}P_0=0$). Under such a situation, we perform numerical calculations: Fig. 2 shows the absorption and dispersion without atomic coherence, while Figs. 3 and 4 show them with atomic coherence included. The three sets of physical parameters used in Figs. 2—4 are

$$r_2 \equiv 2r, \quad r_1 = \lambda_2 = \lambda_1 = 0.2r, \ G = 0, \tag{21a}$$

$$r_2 \equiv 2r, \ r_1 = \lambda_2 = \lambda_1 = 0.2r, \ G = 0.5r, \tag{21b}$$

$$r_2 \equiv 2r, \ r_1 = \lambda_2 = 0.2r, \ \lambda_1 = 0.39r, \ G = 0.5r. \tag{21c}$$

Other parameters are set at zero in all the three cases.

Figure 2 indicates that the light absorption by the medium cannot be made vanished at any light frequency if atomic coherence is absent. Fig. 3 shows that a large refractive index is obtainable at zero absorption, but the dispersion is still large. In Fig. 4, we have a large refractive index with vanishing absorption and dispersion. We note that it makes more sense from the practical point of view to have a refractive index large with vanishing absorption and at the same time with the dispersion as small as possible, since a nonzero linewidth always accompanies the probe light, though it is usually small for a laser (in the calculations we have taken it to be zero). Moreover, it is worth pointing out that the above figures have not only demonstrated the feasibility of achieving a large refractive index at zero absorption and dispersion, but also the feasibility of suppressing the absorption by the establishment of atomic coherence (in a proper frequency domain).

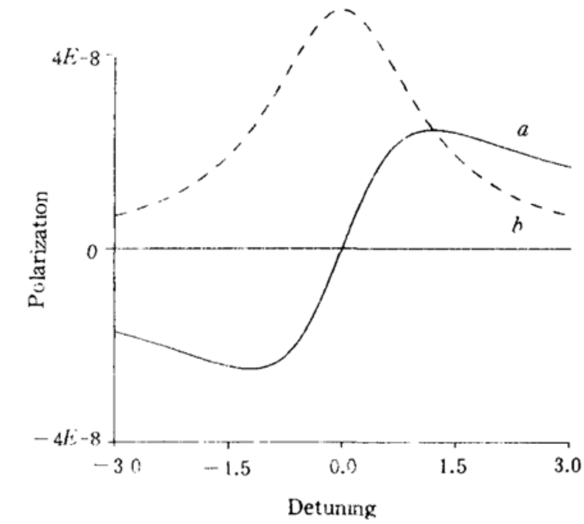

Fig. 2. The curves for absorption and dispersion in no-coherence case. In Figs. 2 — 4, the polarization is in unit of $N|\mu_{12}|^2 E_p^0/2r$ and the detuning Δ_1 in unit of r, the curves labelled by a and b represent ReP_0 and ImP_0, respectively. The parameters used in this figure are given in Eq. (21a).

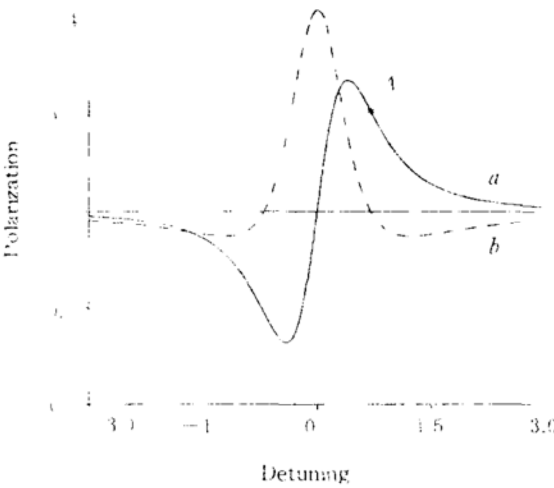

Fig. 3. The curves for absorption and dispersion with atomic coherence included. The parameters used are given in Eq. (21b). At point 1, Re$P_0 \approx 0.2$, Im$P_0 = 0$.

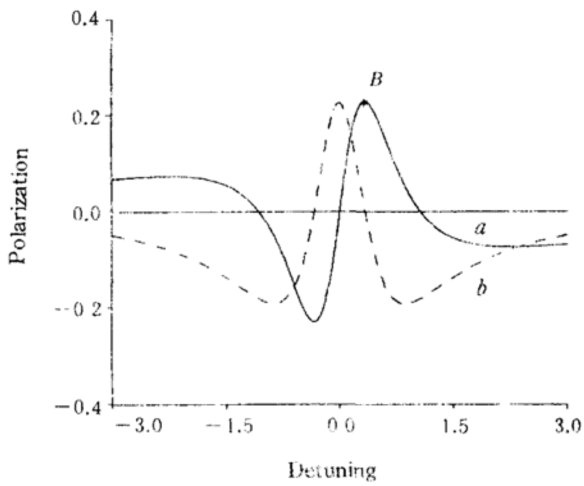

Fig. 4. The curves for absorption and dispersion with atomic coherence included. The parameters used are given in Eq. (21c). At point B, Re$P_0 \approx 0.2$, dRe$P_0/d\Delta_1 = 0$, Im$P_0 = 0$.

If atomic coherence is absent, $n-1(\propto \mathrm{Re}P_0)<<1$. A comparison between Fig. 2 and Fig. 4 reveals that atomic-coherence-induced enhancement of $\mathrm{Re}P_0$ is seven orders of magnitude. Taking r_1 to be the radiative decay rate $r_1=|\mu_{12}|^2\omega_p^3/6\pi\varepsilon_0 c^3$[7] and when $N=10^{15}\ \mathrm{cm}^{-3}$, from Fig. 4 we obtain

$$n-1 \approx \frac{1}{2\varepsilon_0 E_p^0} \cdot \frac{N|\mu_{12}|^2 E_p^0}{2r} \cdot 0.2 \approx 0.75\times 10^{12}\,\mathrm{cm}^{-3}\cdot \lambda_p^3. \tag{22}$$

For the probe light with a wavelength of $\lambda_p=0.6\,\mu\mathrm{m}$, the refractive index n will be 1.2, which is obviously larger than unity.

5 Conclusions

The three-level cascade decay system with atomic coherence produced by an external field in two lower levels is investigated. By choosing the incoherently pumping rates λ_1 and λ_2, a weak probe field in either resonance case (without the strong external field requirement) or the non-resonance case (the external field is assumed to be strong) may have a gain in the absence of population inversion.

Under proper external conditions (λ_1, λ_2, G), the refractive index of a weak light of a certain frequency can be made large in the medium with vanishing absorption. For a light with a nonzero linewidth, it is important to reduce the dispersion. We find that by properly adjusting external parameters (e.g. λ_1), a large refractive index can be obtained with vanishing absorption and dispersion.

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