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SU(2) symmetry in a Hubbard model with spin-orbit coupling

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We study the underlying symmetry in a spin-orbit coupled tight-binding model with Hubbard interaction. It is shown that, in the absence of the on-site interaction, the system possesses the SU(2) symmetry arising from the time reversal symmetry. The influence of the on-site interaction on the symmetry depends on the topology of the networks: The SU(2) symmetry is shown to be the spin rotation symmetry of a simply-connected lattice even in the presence of the Hubbard interaction. On the contrary, the on-site interaction breaks the SU(2) symmetry of a multi-connected lattice. This fact indicates that a discrete spin-orbit coupled system has exclusive features from its counterpart in a continuous system. The obtained rigorous result is illustrated by a simple ring system.

spin-orbit coupling, lattice fermion models, topological phases

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The spin-orbit coupling effect as an important mechanism to control spin dynamics without introducing an external magnetic field [1], has received much attention in the context of spintronics and of the attempts to build a spin-transistor since the first proposal by Datta and Das [2] in 1990. Two widely discussed spin-orbit coupling contributions are the Rashba and the Dresselhaus effects [3–5]. In the last decade, it has been shown that the spin-orbit interaction (SOI) can play an important role in generation, manipulation and detection of spin polarization and spin current (for review see ref. [6] and references therein), as well as spin transport in organic semiconductors [7]. Among many interesting questions the most important one concerns the underlying symmetry of this model, which reveals many far-reaching physical implications that are not obvious at the first glance. A paradigm example is the SU(2) symmetry discovered by Bernevig et al. [8] in a class of spin-orbit coupled models including the model with equal Rashba and Dresselhauss coupling constants and the Dresselhaus [110] model. This finding predicted that a spin precession phenomenon should be experimentally observable. Most of the previous investigations have been focused on non-interacting systems, while less attention has been paid to the existence of the electron correlations arising from the Coulomb interaction. However, the electron-electron interaction is unavoidable in practice, which influence to features of the system, such as relaxation time of the electron spins.

In this article, we study the underlying symmetry in a spin-orbit coupled tight-binding model, where only the time reversal symmetry is required. It is shown that, in the absence of the interaction between electrons, the system possesses the SU(2) symmetry arising from the time reversal symmetry. Remarkably, we find that the influence of the on-site interaction on the symmetry depends on the topology of the networks in the following way. This SU(2) symmetry is shown to be the spin rotation symmetry of a simply-connected lattice, i.e., the system contains no loops, which can be formed with a set of nodes connected by edges as shown in Figure

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1(a), so it still holds for the case of nonzero on-site interaction. On the contrary, the on-site interaction breaks the SU(2) symmetry of a multi-connected lattice. Based on the exact solution of a ring system, our result is demonstrated explicitly. This fact indicates that a discrete spin-orbit coupled system has exclusive features from its counterpart in a continuous system.

The article is organized as follows: in sect. 1, we introduce a general spin-orbit coupled Hubbard Hamiltonian with the time reversal symmetry. In sect. 2, we first construct the SU(2) operators for an on-site interaction free system by using the Kramers degeneracy. In sect. 3 we investigate the influence of the on-site correlation to the SU(2) symmetry. Sect. 4 is the conclusion and a short discussion.

1 Time reversal symmetry

The Hamiltonian H is written as follows:

$$\begin{aligned} H &= H_T + H_U, \\ H_T &= \sum_{i\neq j} c_i^\dagger T_{ij} c_j + \text{H.c.} + \sum_i \mu_i c_i^\dagger c_i, \\ H_U &= \sum_i U_i n_{i\uparrow} n_{i\downarrow}, \end{aligned} \tag{1}$$

where $c_i^\dagger$ and c_i are the creation and annihilation fermion operators at the ith site that have two components,

$$c_i^\dagger = \left(c_{i\uparrow}^\dagger, c_{i\downarrow}^\dagger\right),\ c_i = \begin{pmatrix} c_{i\uparrow} \\ c_{i\downarrow} \end{pmatrix}. \tag{2}$$

Here H_T describes the motion of free particles, while H_U represents the on-site interaction of opposite spin electrons. Unlike the simple Hubbard model, $T_{ij}/\left|T_{ij}\right|$ is no longer the unit matrix arising from the coupling between momentum (and/or position) operators and spin operators. In this work, we do not restrict the model to be in a certain explicit form, but to possess the time reversal symmetry. Therefore the conclusion of this article is available to the Rashba and Dresselhaus types of spin-orbit interactions.

The time reversal operator for a spin–1/2 particle takes the form

$$\mathcal{T} = -i\sigma^y K, \tag{3}$$

where K denotes the complex conjugation operator satisfying

$$K\,(\text{c-number}) = (\text{c-number})^* K, \tag{4}$$

and $\sigma^\alpha (\alpha = x, y, z)$ are the Pauli matrices. The Hubbard Hamiltonian H possesses the time reversal symmetry if $\mathcal{T}$ commutes with all the matrices T_{ij}, i.e., $\left[\mathcal{T}, T_{ij}\right] = 0$ for arbitrary $\{i, j\}$. In the Appendix, we will show that the T_{ij} should have the form

$$T_{ij} = t_{ij} \exp\left(\mathrm{i}\frac{\theta_{ij}\widehat{n}_{ij}}{2} \cdot \overrightarrow{\sigma}\right), \tag{5}$$

which can be determined by the specific model. Where t_{ij} is a real number, θ_{ij} is an arbitrary angle and $\widehat{n}_{ij}$ is a unit vector. Obviously, T_{ij}/t_{ij} is a unitary matrix and represents the spin rotation operation.

As is well known, the spin operators of the whole system are defined as:

$$s^\alpha = \sum_{i=1}^N s_i^\alpha,\ s_i^\alpha = \frac{1}{2} c_i^\dagger \sigma_\alpha c_i, \tag{6}$$

which obey the following commutation relations

$$\left[s^\alpha, s^\beta\right] = \mathrm{i}\varepsilon_{\alpha\beta\gamma} s^\gamma, \tag{7}$$

where $\varepsilon_{\alpha\beta\gamma}$ is the Levi-Civita symbol.

At first, we concentrate on the case of the Hamiltonian of eq. (1) without the spin-orbit interaction. In the absence of the spin-orbit interaction, $\theta_{ij} = 0$, we have

$$\left[\overrightarrow{s}, H_{\theta_{ij}=0}\right] = 0, \tag{8}$$

i.e., the system $H_{\theta_{ij}=0}$ possesses the SU(2) spin rotation symmetry. The conservation of $\overrightarrow{s}$ leads to the spin inversion symmetry along an arbitrary direction, which is in accordance with the result of the time reversal symmetry. According to the general analysis, the spin inversion symmetry is broken when the spin-orbit interaction is switched on. However, the time reversal symmetry still holds for nonzero θ_{ij}. The objective of this article aims at the inverse problem, namely: Can the present system acquire a SU(2) symmetry from the time reversal symmetry? We will find it possible under certain conditions.

2 On-site interaction free case

We start with the case of zero U, but nonzero θ_{ij}. Due to the time reversal symmetry of the present system, the Hamiltonian can always be diagonalized in the form

$$H_T = \sum_{k,\lambda=1,2} \epsilon_k f_{k\lambda}^\dagger f_{k\lambda} = \sum_k \epsilon_k f_k^\dagger f_k, \tag{9}$$

where $\lambda = 1, 2$ labels the two fold degeneracy, k labels the energy levels and the corresponding two component fermion operators

$$f_k^\dagger = \left(f_{k1}^\dagger, f_{k2}^\dagger\right),\ f_k = \begin{pmatrix} f_{k1} \\ f_{k2} \end{pmatrix}. \tag{10}$$

The Kramers degeneracy allows us to construct the operators

$$F^\alpha = \sum_{k=1}^N F_k^\alpha,\ F_k^\alpha = \frac{1}{2} f_k^\dagger \sigma_\alpha f_k, \tag{11}$$

which obey the SU(2) commutation relations

$$\left[F^\alpha, F^\beta\right] = \mathrm{i}\varepsilon_{\alpha\beta\gamma} F^\gamma. \tag{12}$$

In this sense, eq. (9) describes a non-interacting Fermi gas of spin–1/2 particles. The two Kramers degenerate states can service as the two components of a pseudo spin. Obviously, we have

$$\left[\overrightarrow{F}, H_T\right] = 0, \tag{13}$$

which means that the system H_T possesses a new type of SU(2) symmetry. Note that the construction of $\{F^\alpha\}$ is not unique, since any linear transformation of $\left(f_{k1}^\dagger, f_{k2}^\dagger\right)$ cannot change the facts of eqs. (12) and (13). In other words, H_T is invariant under the local rotation of the pseudo spins $\left\{F_k^\alpha\right\}$.

An interesting question is: Among all the sets of the conserved quantities $\{F^\alpha\}$, which one is close to the familiar physical quantity and is feasible to measure in experiment. It will be shown that the geometry and the spin-dependent interaction play an important role in this issue. It is also the main goal of the present article. To this end, we firstly investigate the Hamiltonian H_T on a simply-connected network. It can be observed that, by taking an arbitrary node as a starting point there exists a unique path to another. A schematic illustration of the simply-connected network is presented in Figure 1(a). This characteristic feature of such networks allows H_T to be rewritten as:

$$\mathcal{H}_T = \sum_{i\neq j} t_{ij} d_i^\dagger d_j + \text{H.c.} + \sum_i \mu_i d_i^\dagger d_i, \tag{14}$$

by absorbing the unitary matrices in T_{ij} into the fermion operators $d_i^\dagger$ and d_j. For instance, one can take the transformation

$$c_i^\dagger T_{ij} c_j = t_{ij} c_i^\dagger \mathrm{e}^{\mathrm{i}\theta_{ij}\widehat{n}_{ij}\cdot\overrightarrow{\sigma}} c_j = t_{ij} d_i^\dagger d_j, \tag{15}$$

by the definition

$$d_i^\dagger = c_i^\dagger, d_j = \mathrm{e}^{\mathrm{i}\theta_{ij}\widehat{n}_{ij}\cdot\overrightarrow{\sigma}} c_j. \tag{16}$$

The equivalent Hamiltonian (14) represents a non-spin-orbit interaction system. Accordingly, one can construct the corresponding SU(2) operators

$$\mathcal{S}^\alpha = \sum_{i=1}^N \mathcal{S}_i^\alpha, \mathcal{S}_i^\alpha = \frac{1}{2} d_i^\dagger \sigma_\alpha d_i, \tag{17}$$

satisfying the following commutation:

$$\left[\mathcal{S}^\alpha, \mathcal{S}^\beta\right] = i\varepsilon_{\alpha\beta\gamma}\mathcal{S}^\gamma. \tag{18}$$

Similar to eq. (8), we have

$$\left[\overrightarrow{\mathcal{S}}, H_T\right] = 0. \tag{19}$$

The physics of $\overrightarrow{\mathcal{S}}$ can be understood by the following relationship between operators $\mathcal{S}_i^\alpha$ and s_i^α

$$\mathcal{S}_i^\alpha = u_i s_i^\alpha u_i^\dagger, \tag{20}$$

where u_i is a unitary matrix of the form $\mathrm{e}^{\mathrm{i}\overrightarrow{\gamma}_i\cdot\overrightarrow{\sigma}}$ appearing in the transformation

$$d_i = u_i c_i. \tag{21}$$

This indicates that the operators $\mathcal{S}^\alpha$ act as the real spin operators under the local transformation $\{u_i\}$. In this sense, one can apply the theory of itinerant electron magnetism on the spin-orbit coupling system on a simply-connected lattice. The ferromagnetic state (all the spins being aligned parallel) with respect to the operators $\mathcal{S}^\alpha$ in a system with nonzero θ_{ij} is equivalent to the spin helix state with respect to the operators s^α [8]. We will give an extensive discussion about this issue after taking the on-site interaction into account in the next section.

On the other hand, the equivalent Hamiltonian (14) can be diagonalized in the form

$$H_T = \sum_{k,\sigma} \epsilon_k d_{k\sigma}^\dagger d_{k\sigma}, \tag{22}$$

where

$$\begin{aligned} d_{k\sigma} &= \sum_j \mathcal{D}_j^k d_{j\sigma}, \\ \sum_j \left(\mathcal{D}_j^k\right)^* \mathcal{D}_j^{k'} &= \delta_{kk'}, \\ \sum_k \left(\mathcal{D}_j^k\right)^* \mathcal{D}_{j'}^k &= \delta_{jj'}. \end{aligned} \tag{23}$$

Comparing eqs. (9) and (22), we find that one can have

$$d_k = f_k. \tag{24}$$

From eq. (23), we obtain the identity

$$\sum_k d_{k\sigma}^\dagger d_{k\sigma'} = \sum_j d_{j\sigma}^\dagger d_{j\sigma'}, \tag{25}$$

which leads to

$$\overrightarrow{\mathcal{S}} = \overrightarrow{F}. \tag{26}$$

Then we can conclude that the SU(2) symmetry obtained from the time reversal symmetry of a simply-connected system is essentially spin rotational symmetry. The conservative quantity $\overrightarrow{F}$ is connected to an experimental observable $\overrightarrow{\mathcal{S}}$, which means a persistent spin helix. It is hardly observed in practice since a natural material with simply-connected geometry is rare. However, artificial lattices, such as arrays of quantum dots in semiconductor heterostructures [9,10] confining the conduction electrons, or optical lattices—stable periodic arrays of potentials created by the standing waves of laser light [11], can implement this task.

On the contrary, for a multi-connected system illustrated in Figure 1(b), the above analysis is invalid since one cannot find a set of unitary matrices $\{u_i\}$ to implement the transformation of eq. (21). Then for an on-site free system with the time reversal symmetry, it always possesses a SU(2) symmetry, but the physics of the symmetry depends on the underlying topology of the network.

3 On-site interaction effect on the symmetry

In the previous part, we have found that a SU(2) symmetry in a spin-orbit coupling system obeys the time reversal symmetry when the electron-electron interaction is absent. This indicates the macroscopic emergence of certain physical features (as ferromagnet, antiferromagnet, spin helix, etc.) in a long time scale, especially in a multi-particle system. Nevertheless, the spin-dependent interaction between particles may break the symmetry. Now we turn to investigate the influence of the on-site interaction H_U on the SU(2) symmetry. Applying the transformation $d_i = u_i c_i$ to H_U on a simply-connected system, we have

$$H_U = \sum_i U_i c_{i\uparrow}^\dagger c_{i\uparrow} c_{i\downarrow}^\dagger c_{i\downarrow} = \sum_i U_i d_{i\uparrow}^\dagger d_{i\uparrow} d_{i\downarrow}^\dagger d_{i\downarrow}, \quad (27)$$

which means it is invariant under the transformation. Consequently for a simply-connected system, we still have

$$\left[\overrightarrow{\mathcal{S}}, H_U\right] = \left[\overrightarrow{\mathcal{S}}, H\right] = 0. \quad (28)$$

This has many implications on a spin-orbit coupling system. Mathematically, it can be treated as a normal Hubbard model. Then all the conclusions for the Hubbard model on a simply-connected lattice are completely available for the present system. Here we only give a subtle conclusion from the Lieb theorem [12]. In the following, we present a statement for a Hubbard model with the spin-orbit interaction on a simply-connected network by simply modifying the abstract in ref. [12]. In the attractive Hubbard Model (and some extended versions of it), the ground state is proved to have spin angular momentum $\mathcal{S} = 0$ for every (even) electron filling. In the repulsive case, with a bipartite lattice and a half-filled band, the ground state has $\mathcal{S} = 1/2\,(|B| - |A|)$, where $|B|$ ($|A|$) is the number of sites in the B (A) sublattice. In both cases the ground state is unique. We believe that such kind of rigorous result obtained from the simple Hubbard model is useful for understanding the feature of the present model, thereby providing a general guiding principle for spintronics.

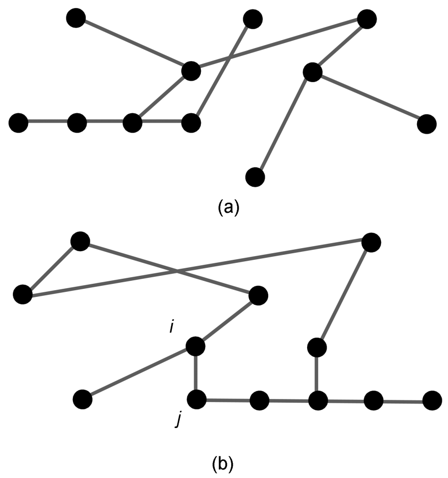

Figure 1 Schematic illustration of (a) the simply-connected network and (b) the multi-connected network. If the connection between i and j is broken, the topology is changed from multi- to simply-connected one, which will affect the SU(2) symmetry of the system, especially in the presence of the on-site correlation.

Now we consider the case of a multi-connected system. Since the transformation $d_i = u_i c_i$ cannot eliminate the nonzero θ_{ij} term completely, we cannot judge the commutation relation $\left[\overrightarrow{F}, H\right] = \left[\overrightarrow{F}, H_U\right]$ in a general manner. However, a single example can provide the conclusion that the on-site interaction breaks the SU(2) symmetry generated by the operators $\{F^\alpha\}$, although the time reversal symmetry still holds.

We exemplify the above analysis by taking a simple multi-connected network, a ring system as an example. This may shed light on the role of the topology of the network. Recently, the semiconductor quantum rings received a lot of theoretical attention due to spin-related transport phenomena [13–16]. The Hamiltonian of the ring reads

$$\begin{aligned} H^{\text{ring}} &= H_T^{\text{ring}} + H_U^{\text{ring}}, \\ H_T^{\text{ring}} &= -J\sum_{j=1}^{N-1} c_j^\dagger c_{j+1} - J c_1^\dagger \mathrm{e}^{-\mathrm{i}\frac{\pi\sigma_y}{2}} c_N + \text{H.c.}, \\ H_U^{\text{ring}} &= U\sum_{j=1}^{N} c_{j\uparrow}^\dagger c_{j\uparrow} c_{j\downarrow}^\dagger c_{j\downarrow}. \end{aligned} \quad (29)$$

Here the spin-orbit interaction only takes place on the tunneling between sites 1 and N. Despite its simplicity, it reveals the common properties of the underlying symmetry for more complex systems. It acts as a Möbius system [17]. For the solution, by taking the transformation

$$\mathrm{e}^{-\mathrm{i}\frac{(j-1)\pi}{2N}} c_{j\uparrow} = a_j, \quad (30)$$

$$\mathrm{e}^{-\mathrm{i}\frac{(N+j-1)\pi}{2N}} c_{j\downarrow} = a_{N+j},\ j \in [1, N], \quad (31)$$

the original Hamiltonian is mapped into

$$\mathcal{H}_T^{\text{ring}} = -J\sum_{j=1}^{2N}\left(\mathrm{e}^{\mathrm{i}\frac{\pi}{2N}} a_j^\dagger a_{j+1} + \text{H.c.}\right), \quad (32)$$

where we take the boundary condition $a_{2N+1}^\dagger = a_1^\dagger$. It represents a $2N$-site ring penetrated by a half magnetic flux quanta, which is schematically illustrated in Figure 2. Such a Hamiltonian can be diagonalized in the form

$$\mathcal{H}_T^{\text{ring}} = -2J\sum_k \cos\left(k + \frac{\pi}{2N}\right) a_k^\dagger a_k, \quad (33)$$

by using the following Fourier transformation

$$a_j = \frac{1}{\sqrt{2N}}\sum_k \mathrm{e}^{\mathrm{i}kj} a_k, \quad (34)$$

where $k = \pi n/N$, $n \in [1, 2N]$ denotes the momentum. Note that the momentum shift by $\pi/(2N)$ in the dispersion relation ensures the Kramers degeneracy. Then the corresponding SU(2) operators (11) and the on-site interaction H_U^{ring} can

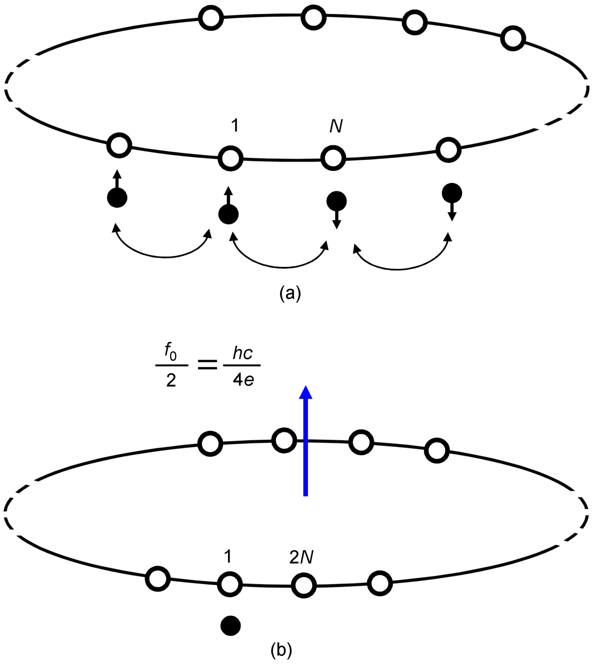

Figure 2 (Color online) Schematic illustration of (a) spin$-\frac{1}{2}$ fermionic system of an N-site ring with spin-orbit interaction on the bond between 1 and N, and (b) its equivalent system, which is a spinless fermionic system of a $2N$-site ring penetrated by a half magnetic flux quanta.

be expressed explicitly as:

$$F^{\alpha} = \frac{1}{2}\sum_{k}\left(a_k^{\dagger}, a_{2\pi-k-\pi/N}^{\dagger}\right)\sigma_{\alpha}\begin{pmatrix} a_k \\ a_{2\pi-k-\pi/N}\end{pmatrix},$$
$$k = n\pi/N,\ n \in [0, N-1], \tag{35}$$

and

$$\mathcal{H}_U^{\text{ring}} = U\sum_{j=1}^{N} a_j^{\dagger}a_j a_{j+N}^{\dagger}a_{j+N}, \tag{36}$$

respectively. After a lengthy but straightforward algebra, we have

$$\left[F^{\alpha}, H^{\text{ring}}\right] \neq 0. \tag{37}$$

It is clear that the time reversal symmetry is not the sufficient condition for a SU(2) symmetry, since the validity of the symmetry depends on the geometry of the system as well as the on-site correlation. Only the coexistence of the closed loop in the network and the on-site interaction between particles can break the SU(2) symmetry.

4 Conclusion

In conclusion, we studied the underlying symmetry for a spin-orbit coupled tight-binding model with the time reversal symmetry. We found that the characteristics of the symmetry strongly depend on the topology of the network and the on-site interaction. It follows that a discrete spin-orbit coupled system has exclusive features from its counterpart in a continuous system. It is shown that, in the case of zero on-site interaction, the system possesses the SU(2) symmetry arising from the Kramers degeneracy. The influence of the on-site interaction on the symmetry depends on the geometry of the networks: The SU(2) symmetry is shown to be the spin rotation symmetry of a simply-connected lattice, so it still holds for the case of nonzero U. We also investigate the multi-connected system based on the exact solution of a simple ring. Our result showed that the on-site interaction can break the SU(2) symmetry of a multi-connected lattice.

Regarding the reason why the topology and on-site correlation affects the symmetry, it may due to its gauge characteristic. It has been pointed out that one can regard the Rashba and the Dresselhaus spin-orbit interaction in two-dimensional semiconductor heterojunctions as a non-Abelian gauge field, or the Yang-Mills field [18]. The Yang-Mills field generates a physical field due to which the wave function acquires a spin-dependent phase factor. Therefore it is not surprising that the topology of the system affects the symmetry.

Appendix

In this Appendix, we will prove that the form of T_{ij} in eq. (5) cover all the possible Hubbard Hamiltonian (1) possessing the time reversal symmetry. Without loss of generality, we define the 2×2 matrices T_{ij} as:

$$T_{ij} = \begin{pmatrix} A_{ij} & B_{ij} \\ C_{ij} & D_{ij}\end{pmatrix}, \tag{a1}$$

where A_{ij}, B_{ij}, C_{ij}, and D_{ij} are arbitary complex numbers. It has been shown that if the time reversal operator $\mathcal{T}$ commutes with all the matrices T_{ij}, i.e., $\left[\mathcal{T}, T_{ij}\right] = 0$ for arbitrary $\{i, j\}$, then the Hubbard Hamiltonian H is $\mathcal{T}$ symmetric. Therefore, we have

$$A_{ij} = D_{ij}^{*},\ B_{ij} = -C_{ij}^{*}, \tag{a2}$$

which leads to the eq. (5)

$$T_{ij} = t_{ij}\exp\left(\mathrm{i}\frac{\theta_{ij}\widehat{n}_{ij}}{2}\cdot\vec{\sigma}\right), \tag{a3}$$

where

$$t_{ij} = \sqrt{\left|A_{ij}\right|^2 + \left|B_{ij}\right|^2}, \tag{a4}$$

$$\theta_{ij} = 2\arccos\left[\frac{\mathrm{Re}\left(A_{ij}\right)}{\sqrt{\left|A_{ij}\right|^2 + \left|B_{ij}\right|^2}}\right], \tag{a5}$$

$$\widehat{n}_{ij} = \left(\sin\varphi_{ij}\cos\phi_{ij}, \sin\varphi_{ij}\sin\phi_{ij}, \cos\varphi_{ij}\right), \tag{a6}$$

and

$$\phi_{ij} = \arctan\left[\frac{\mathrm{Re}\left(B_{ij}\right)}{\mathrm{Im}\left(B_{ij}\right)}\right], \varphi_{ij} = \arctan\left[\frac{\mathrm{Im}\left(A_{ij}\right)}{\left|B_{ij}\right|}\right]. \tag{a7}$$

Consequently, the expression of T_{ij} in eq. (5) are general form, which cover all the possible Hamiltonian H possessing the time reversal symmetry.

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