ON THE EQUAL-THICKNESS VARI-CROSS-SECTION ULTRASONIC FLEXURAL VIBRATION ROD*

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I. INTRODUCTION

For designing the thick equal-cross-section ultrasonic flexural vibration rod, the Timoshenko Theory[1] works satisfactorily[2, 3]. For the vari-cross-section ultrasonic flexural vibration rod, however, there has not been an accurate and convenient design theory to date. That is, no frequency equation is available. In the present study, we extend the Timoshenko Theory to the vari-cross-section rod. For equal-thickness vari-cross-section exponent-mode rods, the frequency equation was derived. Experiments were carried out to verify the equation. The theoretical calculation shows good agreement with the experimental results.

II. BASIC RELATIONS ON EQUAL-THICKNESS EXPONENT-MODE ROD

1. *Outline of the Equal-Thickness Exponent-Mode Rod*

Shown in Fig. 1 with solid line is an exponent-mode or exponentially-tapered-cross-section rod. It has the height of h, length of L, width of $b_z = b_0 e^{-\beta z}$, and cross-section-area of $S = b_0 h e^{-\beta z}$, where β is a constant greater than zero, z the coordinate in the length direction, and the origin of the coordinate is at the thick end of the rod.

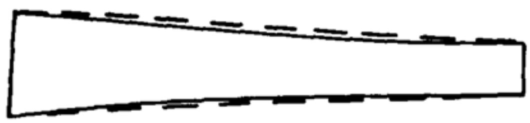

Fig. 1. Scheme of the vari-cross-section rod, in which the solid line stands for exponent-mode rod, while the dashed line for the wedge-mode rod.

The definitions of the signs used here are as follows: $K_L^2 = \omega^2 \rho / E$, $K_S^2 = \omega^2 \rho / (K' G)$, $K^2 = K_L^2 + K_S^2$, $K_h = 12\omega^2 \rho / (h^2 E)$, $\omega = 2\pi f$, where f is the resonance frequency, ρ the density of the material used, E the longitudinal elastic constant, G the shearing elastic constant, K' the shear coefficient of the cross section

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(for a rectangular cross section, $K' = 0.8333$); $S_1 = \frac{1}{2}\sqrt{2(g^2 + K^2) - \beta^2}$, $S_2 = \sqrt{2(g^2 - K^2) + \beta^2}$, $g^2 = \sqrt{(K_L^2 - K_S^2) + 4K_h^4}$; A, B, C, and D are constants related to the boundary conditions.

The material used in this study was 45# carbon steel and $E = 21.6 \times 10^{10}$ N/M², $G = 8.4 \times 10^{10}$ N/M², and $\rho = 7.8 \times 10^3$ kg/M³.

2. *The Frequency Equation*

The frequency equation is of the following form:

$$(A_i \mathrm{ch} S_2 L + B_i \mathrm{sh} S_2 L)\cos S_1 L + (C_i \mathrm{ch} S_2 L + D_i \mathrm{sh} S_2 L)\sin S_1 L - 1 = 0, \quad (1)$$

where $i = 1$ or 2, corresponding to the fixed-free and free-free boundary conditions, respectively.

Table 1

Coefficients Under the Two Boundary Conditions

i	A_i	B_i	C_i	D_i
1	$\frac{1}{2}\left(\frac{R_1}{R_2} + \frac{R_2}{R_1}\right)$	$\frac{4\beta g^4}{S_2 R_1 R_2}$	$\frac{4\beta g^4}{S_1 R_1 R_2}$	$\frac{2\beta^2 g^4}{S_1 S_2 R_1 R_2} + \frac{2K^2 - \beta^2}{4 S_1 S_2}$
2	1	0	0	$\frac{1}{4S_1 S_2}\left[(g^2 + K^2)\frac{R_2}{R_1} - (g^2 - K^2)\frac{R_1}{R_2} - \beta^2\right]$

In Table 1, $R_1 = 4K_L^2 - 2(g^2 + K^2)$, $R_2 = 4K_L^2 + 2(g^2 - K^2)$.

When $S_2 L \geqslant 6$, Eq. (1) can be modified to

$$L \approx \frac{n\pi - \delta_{i0}}{S_1}, \quad (2)$$

where n takes positive integers, $\delta_{i0} = \mathrm{tg}^{-1}\frac{A_i + B_i}{C_i + D_i}$.

3. *The Effect of β Value on the Resonance Frequency*

Table 2 shows the effect of β value on the resonance frequency. The frequencies were calculated for a both-end-free rod, 150 mm in length and 20 mm in height. From Table 2 it can be seen that the smaller the β, the less its effect on the resonance frequency. When $\beta = 0$, the rod becomes of equal cross section.

III. Experiment and Analysis

The experimental result on the free-free rods is shown in Table 3. The parameters of

Table 2

Frequencies Corresponding to Different β Values

β	Frequency (Hz)		
	$n=3$	$n=4$	$n=5$
0.0	20091	29750	39878
2.0	20093	29752	39880
10.0	20148	29800	39921

Sample 1 are : $L=0.15$ m, $h=0.02$ m, and $\beta=4.62$/m. For Sample 2, $L=0.2$ m, $h=0.015$ m, and $\beta=3.47$/m. The displacement nodal line coordinate indicates such positions at which the vibration displacement equals zero. z_1 is the measured coordinate while z_2 is calculated from the above-mentioned equations.

Table 3

The Bending Vibration Displacement-Nodal-Line Coordinate of Sample Rods

Rod No.	n	z_1				$(z_1-z_2)/z_1$			
1	3	14.2	52.7	96.5	136.6	−4.87	−1.02	−0.65	−0.92
2	3	18.9	69.3	127.3	178.8	−3.79	+0.79	+0.33	+0.89

Listed in Table 4 are the resonance frequencies of exponent-mode samples and corresponding wedge-mode samples. The so-called wedge-mode rod is shown in Fig. 1 with dashed line.

Table 4

Comparison Between Measured and Calculated Resonance Frequencies

Rod No.	n	Exponent-Mode			Wedge	Errors	
		f_1(H2)	f_2(H2)	f_3(H2)	f_4(H2)	e_1(%)	e_2(%)
1	3	20105	20103	20033	20158	+0.3	+0.6
	4	29760	29761	29660	29932	+0.6	+0.9
2	2	5302	5303	5485	5469	+3.0	−3.0
	5	21877	21877	21567	22028	+0.7	+2.1
	6	28744	28744	28600	28939	+0.7	+1.2

In Table 4, f_1 is the resonance frequency calculated from Eq.(1), f_2 from Eq.(2), while f_3 and f_4 are the measured resonance frequencies for the exponent-mode rod and wedge rod, respectively; e_1 is the error between f_4 and f_2, while e_2 between f_4 and f_3. It can be found from Table 4 that, in our case, the frequency from Eq.(2) does not show much difference from that from Eq.(1), although Eq.(2) is much simpler in calculating. It can also be seen that the measured resonance frequency of the wedge-mode rods is very close to that of the exponent-mode ones. These two types of rods are the same in thickness, length, thick-end cross-section-area, and thin-end cross-section-area, but differ in the varying mode of cross-section.

IV. Conclusion

The frequency Eq. (1) coincides with the result of the experiment fairly well. The modified frequency Eq. (2) shows very little difference but is more convenient. To a certain extent, it will make designing and processing much easier if we use the exponential mode as a designing model while actually making a wedge rod.

References

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