CLUSTER ANALYSIS OF THE SPECIES OF *Ustilago* IN CHINA

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The smut fungi are destructive parasites of plants, especially of some important cereal crops. Mycologists have no consensus about the basic criteria for the classification of smut fungi. The genus *Ustilago* (Pers.) Roussel (1806) is a large genus of around 300 species, which are rather cosmopolitan. However, mycologists have different points of view about the classification of the species under this genus.

A hierarchical clustering analysis was tried with 51 species-specimen under the genus *Ustilago* hitherto collected and identified from China by many authors. The result is based on 33 morphological and biological coding characters as well as with average method of agglomerative strategy. Some controvertible questions have arisen during the analysis. From the result of cluster analysis, the 51 species-specimens under *Ustilago* are classified into 3 phenons. *Ustilago esculenta* P. Henn is more rational to be kept under this genus, instead of a new one *Yenia*. *Ustilago ocrearum* is likely to be a variant of *Ustilago koenigiae* while *Ustilago kusanoi* a variant of *Ustilago minima*.

A trial on the use of the computerized analysis reveals the validity of numerical taxonony of fungi. However, further development in this line will attract attention of taxonomical mycologists and facilitate their laborious work.

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FINITE ELEMENT METHODS FOR THE NUCLEAR WASTE-DISPOSAL CONTAMINATION IN POROUS MEDIA

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This letter describes a 2-dimensional finite element model used to simulate flow and transport processes in geologic media. The model was developed by the Nuclear Regulatory Commission, for the use in the analysis of deep geologic nuclear waste-disposal facilities. The mathematical model is a fully transient, compressible and 2-dimensional model which solves the nonlinear parabolic system. The considered processes are:

The fluid flow

$$-\nabla \cdot (\rho \underline{u}) - \rho q + \rho R_s' = \frac{\partial}{\partial t} (\Phi \rho);$$

the heat transport

$$-\nabla \cdot (\rho H \underline{u}) + \nabla \cdot (E_H \nabla T) - \rho q_i - \rho q H - \rho q_H = \frac{\partial}{\partial t} [\Phi \rho U + (1-\Phi) \rho_R C_{pr} T];$$

the dominant-species miscible displacement (brine)

$$-\nabla \cdot (\rho \hat{C} \underline{u}) + \nabla \cdot (\rho E_c \nabla \hat{C}) - \rho q \hat{C} - \rho q_c + \rho R_s = \frac{\partial}{\partial t} (\Phi \rho \hat{C});$$

the trace-species miscible displacement (radionuclide)

$$-\nabla \cdot (\rho C_i \underline{u}) + \nabla \cdot (\rho E_c \nabla \hat{C}_i) - \rho q C_i - \rho q_{C_i} + \rho q_{0i} + \sum_{j=1}^{N} k_{ij} \lambda_j K_j \Phi \rho C_j - \lambda_i K_i \Phi \rho C_i$$

$$= \frac{\partial}{\partial t} (\Phi \rho K_i C_i), \quad i = 1, 2, \cdots, N,$$

where $\underline{u} = -(k(x)/u(\hat{C}))\nabla p$ is Darcy flux, $\rho = \rho_c e^{C_w(p-p_0)}$ is the fluid density, p the pressure, $\hat{C}$ the brine concentration, C_i the radionuclide concentration of the component i, and T the temperature. We assume that no fluid flow occurs across the boundary and Dirichlet boundary condition for T is zero.

Let h_p, h_c and h_T be positive. Let $N_h = N_{hp} \subset W^{1,\infty}(\Omega)$ denote a standard finite element space of degree at least k, $M_h = M_{hc} \subset W^{1,\infty}(\Omega)$ at least l and $R_h = R_{hr} \subset W^{1,\infty}(\Omega) \cap H_0^1(\Omega)$ at least r.

At first, we determine the approximations for the pressure, Darcy velocity and the brine concentration by P_h, $\underline{U}_h$ and $\hat{C}_h$ respectively. P_h: $J \to N_h$ is considered as the solution of

$$(\Phi_1 \frac{\partial P_h}{\partial t}, v) + (a(\hat{C}_h)\nabla P_h, \nabla v) - (C_w \nabla P_h \cdot (a(\hat{C}_h)\nabla P_h), v)$$

$$= (-q + R_s'(\hat{C}_h), v), \forall v \in N_h, t \in J. \quad (1)$$

$\underline{U}_h = -a(\hat{C}_h)\nabla P_h$. $\hat{C}_h$: $J \to M_h$ is regarded as the solution of

$$(\Phi \frac{\partial \hat{C}_h}{\partial t}, z) + ((I + C_w a(\hat{C}h)^{-1} E_c) U_h \cdot \nabla \hat{C}_h, z) + (E_c \nabla C_{ih}, \nabla z)$$

$$= (g(\hat{C}_h), z), \quad \forall z \in M_h, t \in J. \quad (2)$$

Then we determine the approximations for the radionuclide concentration and the temperature by $C_{ih} (i = 1, 2, \cdots, N)$ and T_h respectively. C_{ih}: $J \to M_h$ is taken as the solution of

$$(\Phi K_i \frac{\partial C_{ih}}{\partial t}, z) + ((I + C_w a(\hat{C}_h)^{-1} E_c) U_h \cdot \nabla C_{ih}, z) + (E_c \nabla C_{ih}, \nabla z) + (d_{1i}(C_{ih}) \frac{\partial P_h}{\partial t}, z)$$

$$= (Q_{1i}(\hat{C}_h; C_{1h}, C_{2h}, \cdots, C_{Nh}), z), \forall z \in M_h, t \in J, i = 1, 2, \cdots, N, \quad (3)$$

T_h: $J \to R_h$ satisfies

$$(d_2(P_h) \frac{\partial T_h}{\partial t}, z) + ((C_p I + C_w a(\hat{C}_h)^{-1} \widetilde{E}_H \underline{U}_h \cdot \nabla T_h, z) + (\widetilde{E}_H \nabla T_h, \nabla z) +$$

$$+\left(d_3(P_h)\frac{\partial P_h}{\partial t}, z\right)=(Q_2(\underline{U}_h, P_h, \hat{C}_h, T_h), z), \quad z\in R_h, \ t\in J. \tag{4}$$

We suppose that the coefficients and data of Eqs. (1)–(4) are locally bounded and locally Lipschitz continuous.

Theorem. *Suppose the solution of the nuclear waste-disposal contamination is smooth, and adopt the restriction between the three spaces that*

$$h_p^{-1} h_c^{l+1} \to 0, \ h_p^h (\log h_c^{-1})^{1/2} \to 0, \ (\log h_T^{-1})^{1/2}(h_p^k + h_c^{l+1}) \to 0. \tag{5}$$

We can obtain the following error estimates

$$\|\hat{C}-\hat{C}_h\|_{L^\infty(J;L^2(\Omega))}+h_c\|\hat{C}-\hat{C}_h\|_{L^\infty(J;H^1(\Omega))}+\left\|\frac{\partial \hat{C}}{\partial t}-\frac{\partial \hat{C}_h}{\partial t}\right\|_{L^2(J;L^2(\Omega))}$$

$$+\|p-P_h\|_{L^\infty(J;H^1(\Omega))}+\left\|\frac{\partial p}{\partial t}-\frac{\partial P_h}{\partial t}\right\|_{L^2(J;L^2(\Omega))} \leqslant M\{h_p^k+h_c^{l+1}\}, \tag{6}$$

$$\sum_{i=1}^N\|C_i-C_{ih}\|_{L^\infty(J;L^2(\Omega))}+h_c\sum_{i=1}^N\|C_i-C_{ih}\|_{L^\infty(J;H^1(\Omega))}+\sum_{i=1}^N\left\|\frac{\partial C_i}{\partial t}-\frac{\partial C_{ih}}{\partial t}\right\|_{L^2(J;L^2(\Omega))}$$

$$\leqslant M\{h_p^k+h_c^{l+1}\}, \tag{7}$$

$$\|T-T_h\|_{L^\infty(J;L^2(\Omega))}+h_T\|T-T_h\|_{L^\infty(J;H^1(\Omega))}+\left\|\frac{\partial T}{\partial t}-\frac{\partial T_h}{\partial t}\right\|_{L^2(J;L^2(\Omega))}$$

$$\leqslant M\{h_p^k+h_c^{l+1}+h_T^{r+1}\}.$$

Optimal convergence rates in H^1 are demonstrated.

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THE FINITE THERMODYNAMIC CRITERIA FOR SELECTING PARAMETERS OF REFRIGERATION AND PUMPING HEAT CYCLES BETWEEN HEAT RESERVOIRS

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According to classical thermodynamics, the maximum performance coefficient of a reverse Carnot cycle between the heat source and the heat sink is

$$\varepsilon_r = T_2/(T_1-T_2), \tag{1}$$

or