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An accessibility measure for the combined travel demand model

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Abstract The accessibility-oriented transport planning method is critical for many high population density cities in China. Most definitions of accessibility only consider spatial separation and ignore the influence of traveler choice on accessibility. In this paper, the combined travel demand model is employed in transport planning. The travelers' choice behavior of the model is based on the random utility theory. The model overcomes inconsistence problem of the sequential four-step model on travel behavior and congestion effects. Based on the same random utility theory, an accessibility measure is proposed, which can be integrated into the combined travel demand model. The accessibility measure can reflect the traveler's choices at different stages (travel choice, destination choice, mode choice and route choice). The properties of the accessibility measure are discussed through a numerical example.

Keywords accessibility, random utility theory, combined travel demand model, urban transport planning

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1 Introduction

Currently, the mobility-oriented planning methods hold the dominating position in urban transport planning of China. Urban Road Transport Planning and Designing Standard requires that: urban road network planning should be adapted to the expansion of land use and be conducive to the development of motorization and rapid transportation trends [1]. Normally, indices, such as average travel speed, average density of road network, etc., are used to evaluate transport planning schemes. However, with the rapid development of economics and the dramatic increase in car ownership, the mobility-oriented planning method can hardly solve the deteriorating problems of traffic congestion and environment pollution in the high population density cities of China. Thus, the accessibility-oriented transportation planning becomes a new trend of researches.

A few problems remain open in the research of the accessibility-oriented transportation planning method. First, the accessibility is not fully recognized; second, no suitable measure is available to quantify accessibility. This paper aims to put forward an accessibility measure for the urban transportation planning and provide a theoretical basis and data basis for the accessibility-oriented transportation planning. Based on the combined travel demand model, the accessibility measure in this paper can reflect the influence congestion and the behavioral factors of mode choice, destination choice and so on, which are

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not considered in previous studies on accessibility. The rest of the paper is organized as follows. In section 2, a literature review is given. Section 3 introduces the combined travel demand model used to calculate the accessibility measure. In section 4, a method for calculating the accessibility measure is proposed. Section 5 analyzes the characteristics of the proposed accessibility measure through an example. Section 6 concludes the paper and indicates the further research.

2 Literature review

Accessibility is an important performance measure of urban transportation system and land-use efficiency [2]. Research on the accessibility can be traced back to 1953. Shimble [3] proposed the following equation to calculate the accessibility:

$$A_i = \sum_{j=1, j\neq i}^{n} d_{ij},$$

where d_{ij} is the length of the shortest route between zones i and j. Wachs and Kumagai [4] and Vickerman [5] proposed to use the accumulated opportunity index to calculate accessibility:

$$A_i = \sum_j W_j a_j,$$

where a_j is the number of opportunity of zone j. If $c_{ij} \leqslant c^*_{ij}$, then $W_j = 1$, otherwise W_j=0. c_{ij} is the impedance between zones i and j, and c^*_{ij} is the impedance threshold corresponding to effective opportunity. According to the classification by Handy and Niemeier [6], these measures belong to isochronous indicators, while the measures based on gravity model and utility proposed by Hansen [7], Neuburger [8], Ben-Akiva and Lerman [9] belong to another kind. The basic form of gravity model-based measure is $A_i = \sum_j a_j f(c_{ij})$, where $f(c_{ij})$ is the impedance function between zones i and j. The destination-attraction models proposed by Stewart and Warntz [10] and Hansen [7] belong to this kind also. Based on the measure proposed by Weibull [11], Shen [12] divided the travelers into commuters and telecommuters and calculated their accessibilities respectively. The basic form of the utility-based measure is

$$A = E(\max_{k\in K} U_k), \tag{1}$$

where K is the set of choices which travelers can choose, and for each choice, $k \in K$, has its utility, defined as random variable U_k. The advantage of utility-based measure is that with different assumptions of traveler choice behavior, different forms of utility function can be employed, so that different travel choices which the traveler faces can be fully considered. Other studies on accessibility measures can be found in review articles by Jones [13] and Bhat et al. [14].

The existing accessibility measures view the spatial separation (or travel cost) as the main factor influencing accessibility. Some studies considered the influence of destination attraction or different traffic modes. But the influence of travelers' choice behavior and the way it can be applied into transportation planning model are rarely studied. Accessibility is an important concept in traffic analysis and transportation planning. This paper proposes an accessibility measure to reflect the traveler choice at different stages (travel choice, destination choice, mode choice and route choice) based on the combined travel demand model. With the unified traveler choice model as a basis and through explicitly considering the congestion effect, this measure can overcome the inconsistence problem of traveler choice assumption in traditional sequential four-step model.

3 The combined travel demand model

Traditional demand analysis method in transportation planning is a sequential four-step model [15]: trip generation, trip distribution, mode split and traffic assignment. Although it has been widely used in practice, the four-step model has some problems. For instances, travelers' choice behavior is inconsistent, and travel time is inconsistent with congestion effect among different steps [15]. To deal with these problems, Oppenheim [16] proposed the combined travel demand model (CTDM) based on unified traveler choice behavior. This model views travelers as consumers in trip, and travelers' travel choices are relevant to the travel utility and the traveler's budget. The travel choice in CTDM is based on the nested logit model of discrete choice theory, with which CTDM could overcome the inconsistence problems of choice behaviors in four-step model. Furthermore, CTDM models tackle the choices of all stages by a mathematical programming to keep the consistency of congestion effect at different stages. The following gives a brief introduction to CTDM.

The travel choices of travelers are divided into several stages in CTDM:

1. In zone i, given time and purpose, a potential traveler will decide to travel or not, and $P_{t/i}$ is the probability of travel.
2. Given the choice of previous stage, the probability of choosing destination j is $P_{j/i}$.
3. Given the choice of previous stage, the probability of choosing travel mode m is $P_{m/ij}$.
4. Given the choice of previous stage, the probability of choosing route r is $P_{r/ijm}$.

This structure is illustrated as Figure 1. Given N_i the number of all potential travelers in zone i, we have $T_{ijmr} = N_i \cdot P_{ijmr} = N_i \cdot P_{t/i} \cdot P_{j/i} \cdot P_{m/ij} \cdot P_{r/ijm}, \forall i, j, m, r$, where T_{ijmr} is volume on route r by travel mode m between origin-destination (O-D) ij. P_{ijmr} is probability corresponding to T_{ijmr}. The probability at each stage, i.e. $P_{t/i}, P_{j/i}, P_{m/ij}, P_{r/ijm}$, conforms to probability equation based on the following logit model:

$$\frac{T_i}{N_i} = P_{t/i} = \frac{\mathrm{e}^{\beta_t(h_i+\tilde{W}_{t/i})}}{1+\mathrm{e}^{\beta_t(h_i+\tilde{W}_{t/i})}}, \forall\ i, \tag{2}$$

$$\frac{T_{ij}}{T_i} = P_{j/i} = \frac{\mathrm{e}^{\beta_d(h_{ij}+\tilde{W}_{j|i})}}{\sum_l \mathrm{e}^{\beta_d(h_{il}+\tilde{W}_{l/i})}}, \forall\ i, j, \tag{3}$$

$$\frac{T_{ijm}}{T_{ij}} = P_{m/ij} = \frac{\mathrm{e}^{\beta_m(h_{ijm}+\tilde{W}_{m/ij})}}{\sum_n \mathrm{e}^{\beta_m(h_{ijn}+\tilde{W}_{n/ij})}}, \forall\ i, j, m, \tag{4}$$

$$\frac{T_{ijmr}}{T_{ijm}} = P_{r/ijm} = \frac{\mathrm{e}^{-\beta_r g_{ijmr}}}{\sum_k \mathrm{e}^{-\beta_r g_{ijmk}}}, \quad \forall\ i, j, m, r, \tag{5}$$

where T_i is volume of travel from zone i; T_{ij} is volume from origin i to destination j; T_{ijm} is volume by mode m from origin i to destination j; $\beta_r, \beta_m, \beta_d, \beta_t$ are parameters of different stages in logit model; g_{ijmr} is generalized route cost; h_i, h_{ij}, h_{ijm} are constants in utility functions at different stages related to social and economic indices; $\tilde{W}_{t/i}, \tilde{W}_{j/i}, \tilde{W}_{m/ij}$ are expected values of actual utilities at every stage.

According to the discrete choice model, the expected values of actual utilities at every stage can be calculated as follows:

$$\tilde{W}_{t/i} = \frac{1}{\beta_d} \ln \sum_j \mathrm{e}^{\beta_d(h_{ij}+\tilde{W}_{j/i})}, \forall\ i, \tag{6}$$

$$\tilde{W}_{j/i} = \frac{1}{\beta_m} \ln \sum_m \mathrm{e}^{\beta_m(h_{ijm}+\tilde{W}_{m/ij})}, \forall\ i, j, \tag{7}$$

$$\tilde{W}_{m/ij} = \frac{1}{\beta_r} \ln \sum_r \mathrm{e}^{-\beta_r g_{ijmr}}, \forall\ i, j, m. \tag{8}$$

For the choice of no travel, the utility is set at zero, so the expected value of actual utility corresponding to the choice of travel is

$$\tilde{W}_i = \frac{1}{\beta_t} \ln(1+\mathrm{e}^{\beta_t(h_i+\tilde{W}_{t/i})}), \forall\ i. \tag{9}$$

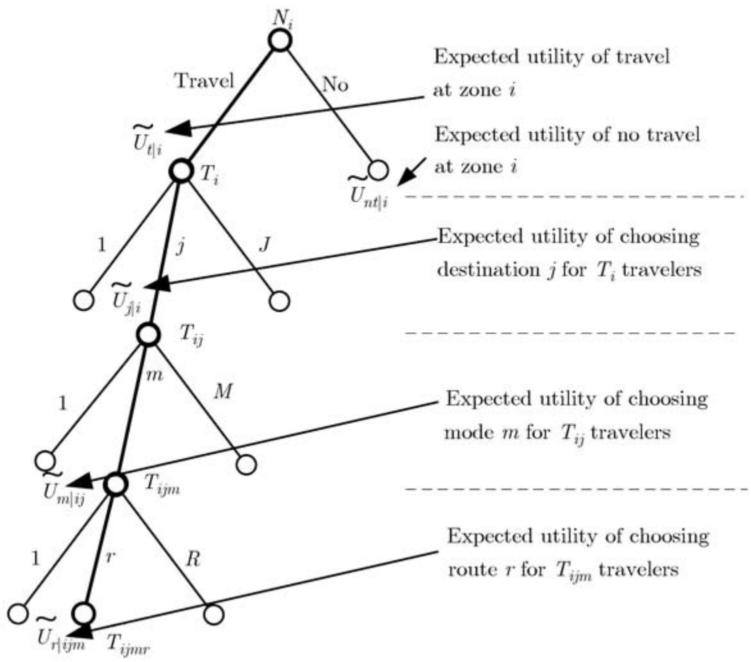

Figure 1 The structure of stratified choice in the combined travel demand model.

The nonlinear programming model of CTDM [16] is

$$\begin{aligned}\mathrm{Min}U_{\mathrm{TDMR}}(T_i,T_{i0},T_{ij},T_{ijm},T_{ijmr}) &= \sum_m\sum_{a_m}\int_0^{\sum_{ijr}T_{ijmr}\delta_{ijr}^{a_m}} g_{a_m}(\omega)d\omega - \sum_{ijm}h_{ijm}T_{ijm}\\ &-\sum_{ij}h_{ij}T_{ij}-\sum_i h_iT_i+\frac{1}{\beta_r}\sum_{ijmr}T_{ijmr}\ln T_{ijmr}+\frac{1}{\beta'_m}\sum_{ijm}T_{ijm}\ln T_{ijm}+\frac{1}{\beta'_d}\sum_{ij}T_{ij}\ln T_{ij}\\ &+\frac{1}{\beta'_t}\sum_i T_i\ln T_i+\frac{1}{\beta_t}\sum_i T_{i0}\ln T_{i0},\end{aligned} \tag{10a}$$

subject to:

$$\sum_r T_{ijmr}=T_{ijm},\quad \forall\ i,j,m, \tag{10b}$$

$$\sum_m T_{ijm}=T_{ij},\quad \forall\ i,j, \tag{10c}$$

$$\sum_j T_{ij}=T_i,\quad \forall\ i, \tag{10d}$$

$$T_i+T_{i0}=N_i,\quad \forall\ i, \tag{10e}$$

$$T_{i0}>0,\ T_i>0,\ T_{ij}>0,\ T_{ijm}>0,\ T_{ijmr}>0,\quad \forall\ i,j,m,r, \tag{10f}$$

where $\delta_{ijr}^{a_m}$ is link-route indicator matrix; g_{a_m} is generalized link cost function, including time and fare. $\frac{1}{\beta'_m}=\frac{1}{\beta_m}-\frac{1}{\beta_r}$; $\frac{1}{\beta'_d}=\frac{1}{\beta_d}-\frac{1}{\beta_m}$; $\frac{1}{\beta'_t}=\frac{1}{\beta_t}-\frac{1}{\beta_d}$. The first four terms of the object function are direct utilities related to route, mode, destination and travel choices, while the last five terms are related to probabilities of choices at every stage. Constraints (10a)–(10e) implicate flow conservation, and constraint (10f) ensures the solution (volume) to be positive.

It is easy to prove that the Karush-Kuhn-Tucher (KKT) condition of the optimal solution in the model satisfies the condition of the probabilities of travel choices at all stages (eqs. (2)–(5)). The existence and uniqueness of the solution have been proven [16]. This problem can be solved using the method of partial linearization [17]. It is worth noting that an important assumption of the above model is that the Jacobian matrix of link cost function is symmetric. The asymmetric condition can be formulated as a problem of variational inequalities, and the studies on related model and algorithm are going on.

4 Accessibility measure

The proposed accessibility measure according to the utility theory is based on CTDM, so that it can reflect travelers' behaviors at different stages (travel choice, destination choice, model choice, route choice). Meanwhile, travelers' choice of CTDM is based on discrete choice behavior theory, so it is also in favor of adapting the utility-based accessibility measure. According to random utility theory, Ben-Akiva and Lerman [9] proposed an accessibility measure as eq. (1). The accessibility measure in the equation has the following properties:

1. The accessibility measure is monotonic with regard to the size of choice set. In other words, if the optional travel choices in the set increase in number, the accessibility will not decrease.

2. The accessibility measure is monotonic with regard to the average utility of set, that is, if the utility of a choice in the set increases, the accessibility will not decrease.

For the first property, if the number of routes to a destination increases, its accessibility should be improved. The increasing choices can be the increase of destinations, travel modes or potential routes. For the second property, if a certain travel mode (for example bus) is improved (travel time decreases), then the network accessibility will be improved. Here, the utility of choices should be in inverse proportion to travel time, that is to say, if travel time decreases, the utility will increase.

Assuming that random variable U_k conforms to Gumbel distribution. Then the explicit expression of the accessibility measure is available. Let $U_k = V_k + \varepsilon_k$, where ε_k is a random variable of Gambel distribution. Then we have $E(U_k) = V_k$. The accessibility measure is

$$A = E\left(\underset{k\in K}{\mathrm{Max}}\, U_k\right) = \frac{1}{\mu}\ln\sum_{k\in K}\mathrm{e}^{\mu V_k}, \tag{11}$$

where μ is Gambel distribution parameter. The corresponding choice model is called Multinomial Logit model. According to eqs. (6)–(9), the expected utilities at each stage are

$$\tilde{U}_{t/i} = h_i + \tilde{W}_{t/i}, \forall\, i, \tag{12a}$$

$$\tilde{U}_{nt/i} = 0, \forall\, i, \tag{12b}$$

$$\tilde{U}_{j/i} = h_{ij} + \tilde{W}_{j/i}, \forall\, i, j, \tag{12c}$$

$$\tilde{U}_{m/ij} = h_{ijm} + \tilde{W}_{m/ij}, \forall\, i, j, m, \tag{12d}$$

$$\tilde{U}_{r/ijm} = -g_{ijmr}, \forall\, i, j, m, r. \tag{12e}$$

According to the definition of accessibility in eq. (11), the accessibility measures at different levels are

the measure at network level: $$A = \sum_i N_i\tilde{W}_i \Big/ \sum_i N_i, \tag{13a}$$

the measure at traffic zone level: $$A_i = \tilde{W}_{t/i}, \forall\, i, \tag{13b}$$

the measure at O-D level: $$A_{ij} = \tilde{W}_{j/i}, \forall\, i, j, \tag{13c}$$

the measure at travel mode level between O-D: $$A_{ijm} = \tilde{W}_{m/ij}, \forall\, i, j, m. \tag{13d}$$

By above equations, the accessibility of different levels (entire network, traffic zone, O-D, travel mode among O-D) could be calculated, which makes it possible to analyze accessibility at different levels. In next section, the properties of proposed accessibility measure will be further discussed using a numerical example.

5 Numerical examples

A trial network used in the example is illustrated in Figure 2, which is composed of five nodes, seven links and two O-D pairs (i.e. 1–4 and 1–5).

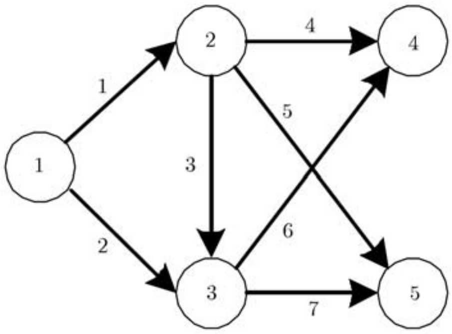

Figure 2 Trial network.

Link parameters, including free flow travel time and capacity, are listed in Table 1, where subscript 'c' means car and subscript 't' means bus. It is assumed that bus network and car network are the same without interaction. The population of zone 1 is 200 (N_1), and other parameters are defined as follows: the attraction parameter of zone 1 is 5.0 (h_1); the attraction parameters of O-D pairs 1–4 and 1–5 are 3.5 (h_{14}) and 3.8 (h_{15}) respectively, h_{14c}, h_{14t}, h_{15c}, h_{15t} are equal to 3.5, 3.6, 3.8, 3.4, and β_r, β_m, β_d, β_t are equal to 2.0, 1.0, 0.5, 0.2. The link cost functions of bus and car are

$$t_{a_c}(v_{a_c}) = t^0_{a_c}\left[1 + 0.15\left(\frac{v_{a_c}}{C_{a_c}}\right)^{4.0}\right], \tag{14}$$

$$t_{a_t}(v_{a_t}) = t^0_{a_t} + 0.06\left(\frac{v_{a_t}}{C_{a_t}}\right)^{2.0}, \tag{15}$$

where v_{a_c} and v_{a_t} are link volumes of car and bus; $t^0_{a_c}$, $t^0_{a_t}$, C_{a_c}, C_{a_t} are listed in Table 1 (fare is not considered here, so the link cost equals generalized link cost).

Consider two scenarios: normal and abnormal. In abnormal scenario, capacity of link 1 of car network and bus network declines to 1.0 ($C_{1_c} = 1.0$; $C_{1_t} = 1.0$). The equilibrium solutions, which can be calculated using partial linearization method [17], are listed in Tables 2 and 3. Comparing to normal scenario, the trip production in zone 1 decreases in abnormal situation; demands of O-D pairs 1–4 and 1–5 decline; meanwhile, the usage of bus increases. The results indicate the capacity of CTDM to analyze the travel choice, destination choice and mode split.

Table 1 Link parameters

Link No.	$t^0_{a_c}$	C_{a_c}	$t^0_{a_t}$	C_{a_t}
1	4.0	25.0	4.0	25.0
2	5.2	25.0	5.2	25.0
3	1.0	15.0	1.0	15.0
4	5.0	15.0	5.0	15.0
5	5.0	15.0	5.0	15.0
6	4.0	15.0	4.0	15.0
7	4.0	15.0	4.0	15.0

Table 2 Equilibrium solutions of link volume in normal and abnormal scenarios

Link No.	Normal		Abnormal	
	car	bus	car	bus
1	28.90	61.56	1.64	6.92
2	21.43	33.94	31.23	92.08
3	8.46	23.00	0.00	0.00
4	8.46	19.15	0.48	3.35
5	11.98	19.41	1.16	3.56
6	13.90	28.32	14.55	45.62
7	15.99	28.62	16.68	46.47

Table 3 Travel volume at each level in normal and abnormal scenarios

Volume	Normal	Abnormal
T_1	145.84	131.87
T_{10}	54.16	68.13
T_{14}	69.83	64.00
T_{15}	76.01	67.87
T_{14c}	22.36	15.03
T_{15c}	27.97	17.84
T_{14t}	47.47	48.97
T_{15t}	48.03	50.03

Table 4 The differences of accessibility measure between two scenarios

Accessibility measure	R	MR	DMR	TDMR
A	–	–	–	−1.16
A_1	–	–	–	−1.67
A_{14}	–	–	−2.05	−1.64
A_{15}	–	–	−2.11	−1.70
A_{14c}	−7.45	−2.39	−2.41	−1.95
A_{15c}	−8.10	−2.54	−2.51	−2.03
A_{14t}	–	−1.89	−1.91	−1.53
A_{15t}	–	−1.95	−1.93	−1.54

According to equilibrium solutions, the accessibility measure in two scenarios can be calculated. In Table 4, the differences in accessibility measures between two scenarios at each level are listed (the differences are equal to the value in normal scenario minus that of abnormal scenario). Meanwhile, different choice dimensions are considered. R means route choice only (for cars); MR means only choice of route and mode; DMR means the choice of route, mode and destination; TDMR means the choice of route, mode, destination and travel.

First, considering full choice dimension, i.e. TDMR, the accessibilities at all levels are lower than that of normal scenario, which is consistent with our expectation. The accessibility at upper level is not the simple sum of the ones at lower level, but is the effect of random utility function in calculation. For O-D pair 1–4, the change of the accessibility of bus is smaller than that of car. This is due to the assumption that the external congestion effect of car is higher than that of bus. Second, comparing TDMR to other choice dimensions, the differences of other choice dimensions are larger than that of TDMR due to the limitation of certain choices. If there is only route choice available, the accessibility is the worst in abnormal scenario. This result demonstrates that more travel choices will make the road network more reliable.

6 Conclusions

This paper proposes an accessibility measure based on utility theory and adapts it to CTDM. The proposed accessibility measure could reflect the choices of traveler at different stages (travel choice, destination choice, mode choice, and route choice) rather than only consider the influence of travel choice result (travel time or fare). This measure could provide a basis of quantitative analysis for accessibility-oriented transportation planning.

The main directions for future research are as follows:

1. How to obtain the accessibility measure if the Jacobian matrix of link cost function is asymmetric. In this paper, a simple condition is considered, so that CTDM could be formulated as a nonlinear programming model. If the Jacobian matrix is asymmetric, there is no corresponding nonlinear programming model. The problem will be formulated as variational inequality or fixed-point problem.

2. How to establish accessibility-oriented transportation planning methods based on the proposed accessibility measure. The resolving of the above problems requires a different view of the object of trans-

portation planning, and will change the mobility-oriented transportation planning into the accessibility-oriented transportation planning.

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