叶形四边形产热体焓耗散率最小导热构形优化

朱鸿威^{1,2,3}, 陈林根^{1,2,3*}, 冯辉君^{1,2,3}, 戈延林^{1,2,3}

1. 武汉工程大学热科学与动力工程研究所, 武汉 430205;

2. 湖北省绿色化工装备工程技术研究中心, 武汉 430205;

3. 武汉工程大学机电工程学院, 武汉 430205

* E-mail: lingenchen@hotmail.com, lgchenna@yahoo.com

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摘要 将构形理论与焓理论相结合, 对已有文献建立的基于常见叶脉形仿生四边形的叶形四边形产热体进行了研究. 导出了焓耗散率最小时叶形四边形产热体的最优构形, 研究了分支高导热材料划分的区间数 n 和分支高导热材料与中心高导热材料的夹角 θ 对叶形四边形产热体最优构形的影响, 比较了叶形四边形产热体在焓耗散率最小和最大温差最小下的构形差异. 结果表明: 叶形四边形产热体在焓耗散率最小时的最优构形下平均传热温差最小; 随着区间数($n \geq 20$)的增加, 叶形四边形产热体的最优长宽比和最小平均温差略微下降, 因此增加产热体内高导热通道的复杂程度能降低平均传热温差; 叶形四边形产热体以焓耗散率和最大温差为优化目标的最优构形有区别, 但都在分支高导热材料垂直于中心高导热材料时最佳, 且焓耗散率最小下的平均温差更小, 因此以焓耗散率最小为优化目标能更有效地降低产热体内的平均传热温差.

关键词 构形理论, 焓理论, 叶形四边形产热体, 热传导, 最优构形, 广义热力学优化

1 引言

电子元件在工作时会产生大量热量, 因此有效散热成为电子器件发展的重要问题之一. 例如, 运用传热优化理论来对电子元件产热体的散热结构进行优化, 将大大提高电子器件的热稳定性和可靠性. 构形理论^[1~14]和焓理论^[15~39]是传热优化理论中具有代表性的理论. 构形理论可以描述和解释多种自然现象, 也可以用于很多工程问题设计, 多学科、多目标、多尺度构形优化是其发展的前沿和趋势. 具体进展详见综述文

献[2~7,11,15]. Bejan^[1]首次基于最大温差最小给出了构形优化的解析方法. 为了弥补传热过程缺少势的概念, 过增元院士等人^[16]定义了焓, 提出并发展了焓理论. 焓理论是完全不同于构形理论的新的传热优化理论, 已经在导热、对流、辐射等传热问题和各种热力循环的性能优化中得到了广泛应用, 具体进展详见综述文献[20~23,39]. 魏曙环等人^[40]首次将二者结合, 研究结果表明以焓耗散率最小为优化目标得到的最优构形更能有效地降低产热体内的平均传热温差.

众多学者基于最大温差最小研究了各种产热体的

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Zhu H W, Chen L G, Feng H J, et al. Constructal optimization of heat conduction with minimum entransy dissipation rate for leaf-shaped quadrilateral heat-generating body (in Chinese). Sci Sin Tech, 2022, 52: 1111–1120, doi: [10.1360/SST-2021-0238](https://doi.org/10.1360/SST-2021-0238)

图 2 (网络版彩图)基于任意分支连接件 $M_i D_i$ 的单元体
Figure 2 (Color online) Element unit based on any branch connector $M_i D_i$ [46].

可知该单元体是梯形, 高度为 δ_i ($\delta_i = (L_1 \sin \theta) / n$), 当 n 足够大($n \gg 1$)时, $\delta_i \ll w_i$, 因此在单元体上产生的热流垂直于分支连接件 $M_i D_i$, 整个单元体内产生的热量通过 D_0 通道沿 x 轴流向点 M_i . 考虑到 $\delta_i \ll w_i$, 梯形单元体“abcd”可近似替换为矩形区域“1234”.

由以上分析, 可得单元体中的导热微分方程为

$$\frac{\partial^2 T}{\partial y^2} + \frac{q'''}{k_0} = 0. \quad (1)$$

$$T(x, y) - T_{M_i} = \frac{q'''}{2k_0} \left(\frac{L_1 \sin \theta}{n} y - y^2 \right) + \frac{q''' L_1 \sin \theta}{2n k_c D_0} \left(\frac{(2i-1)H_1}{n \sin \theta} x - x^2 \right). \quad (8)$$

由于温度关于 x 轴对称分布, 当 $y < 0$ 时, 式(8)中的 H_1 由 $-H_1$ 代替, 单元体的焐耗散率为

$$\begin{aligned} \dot{E}_{h\phi, A_i} &= 2 \int_0^{\frac{L_1 \sin \theta}{2n}} \int_0^{\frac{(i-0.5)H_1}{n \sin \theta}} q''' \left[\frac{q'''}{2k_0} \left(\frac{L_1 \sin \theta}{n} y - y^2 \right) + \frac{q''' L_1 \sin \theta}{2n k_c D_0} \left(\frac{(2i-1)H_1}{n \sin \theta} x - x^2 \right) \right] dx dy \\ &= 2 \left[\frac{q'''^2 L_1^3 \sin^2 \theta (i-0.5)H_1}{24k_0} + \frac{q'''^2 L_1^2 (i-0.5)^3 H_1^3}{6n^5 k_c D_0 \sin \theta} \right]. \end{aligned} \quad (9)$$

图3中为中心高导热材料[46], 热量从单元体的 M_i 点输入通道 D_1 并沿 x 轴向A点. 两个邻接点 M_i 和 M_{i+1} 之间的温差分布由下式决定:

$$\frac{\partial^2 T}{\partial x^2} = 0. \quad (10)$$

图 3 (网络版彩图)中心高导热材料
Figure 3 (Color online) Center high thermal conductivity material [46].

相应的边界条件为

$$\frac{\partial T}{\partial y} = 0, \quad y = \delta_i / 2 = L_1 \sin \theta / 2n, \quad (2)$$

$$T = T_i(x), \quad y = 0, \quad (3)$$

其中, $T_i(x)$ 是 $M_i D_i$ 条状路径中心的温度, 当 $y > 0$ 时求解式(1)得

$$T(x, y) = \frac{q'''}{2k_0} \left(\frac{L_1 \sin \theta}{n} y - y^2 \right) + T_i(x). \quad (4)$$

$M_i D_i$ 通道中的导热微分方程为

$$k_c D_0 \frac{d^2 T_i}{dx^2} + q''' \frac{L_1 \sin \theta}{n} = 0. \quad (5)$$

相应的边界条件为

$$\frac{dT_i}{dx} = 0, \quad x = w_i = (i-0.5)H_1 / n \sin \theta, \quad (6)$$

$$T_i = T(0, 0) = T_{M_i}, \quad x = 0. \quad (7)$$

由式(5)~(7)解得 $T_i(x)$ 代入式(4), 可得

边界条件为

$$-k_c D_1 \frac{dT}{dx} = q'' \left(\frac{i}{n} \right)^2 L_1 H_1, \quad x = (i-0.5)L_1 / n(M_i), \quad (11)$$

$$T = T_{M_{i+1}}, \quad x = (i+0.5)L_1 / n(M_{i+1}). \quad (12)$$

由式(10)~(12), 可得

$$T - T_{M_{i+1}} = -\frac{q''^2 L_1^2 H_1}{D_1 n^2 k_c} (x - (i+0.5)L_1 / n). \quad (13)$$

用 $(i-0.5)L_1 / n$ 代替方程式(13)中的 x , 可得 M_i 和 M_{i+1} 点之间的温差为

$$T_{M_i} - T_{M_{i+1}} = \frac{q''^2 L_1^2 H_1}{D_1 n^2 k_c}. \quad (14)$$

点 M_n 和A之间的温差分布由下式决定:

$$\frac{\partial^2 T}{\partial x^2} = 0. \quad (15)$$

边界条件为

$$-k_c D_1 \frac{dT}{dx} = q'' \left(\frac{i}{n} \right)^2 L_1 H_1, \quad x = (i-0.5)L_1 / n(M_i), \quad (16)$$

$$T = T_0, \quad x = L_1(A). \quad (17)$$

由式(15)~(17), 可得

$$T - T_0 = -\frac{q'' L_1 H_1}{D_1 k_c} (x - L_1). \quad (18)$$

用 $(n-0.5)L_1/n$ 代替方程式(18)中的 x , 可得 M_n 和 A 之间的温差为

$$T_{M_n} - T_0 = \frac{q'' L_1^2 H_1}{2n D_1 k_c}. \quad (19)$$

由 D_1 通道上相邻两点间隔的能耗散等于热流与温差之积, 可得

$$\dot{E}_{h\phi, M_n, A} = \dot{q}_{M_n, A} (T_{M_n} - T_0) = \frac{q''^2 L_1^3 H_1^2}{2n D_1 k_c}, \quad (20)$$

$$\dot{E}_{h\phi, M_i, M_{i+1}} = \dot{q}_{M_i, M_{i+1}} (T_{M_{i+1}} - T_{M_i}) = \frac{q''^2 i^4 L_1^3 H_1^2}{D_1 n^5 k_c}. \quad (21)$$

沿 D_1 通道上的能耗散率为通道上所有间隔间能耗散率之和:

$$\dot{E}_{h\phi, M_1, A} = \dot{E}_{h\phi, M_n, A} + \sum_{i=1}^n \dot{E}_{h\phi, M_i, M_{i+1}} = \frac{(n+1)(2n+1)(3n^2+3n-1)q''^2 L_1^3 H_1^2}{30 D_1 n^4 k_c} + \frac{q''^2 L_1^3 H_1^2}{2n D_1 k_c}. \quad (22)$$

叶形四边形产热体总能耗散率为所有单元体内能耗散率与沿 D_1 通道上的能耗散率之和:

$$\begin{aligned} \dot{E}_{h\phi, A_1} &= \dot{E}_{h\phi, M_1, A} + 2 \sum_{i=1}^n \dot{E}_{h\phi, A_i} = \frac{(n+1)(2n+1)(3n^2+3n-1)q''^2 L_1^3 H_1^2}{30 D_1 n^4 k_c} + \frac{q''^2 L_1^3 H_1^2}{2n D_1 k_c} \\ &\quad + \frac{(2n^2-1)q''^2 L_1^2 H_1^3}{24 n^3 k_c D_0 \sin \theta} + \frac{q''^2 L_1^3 \sin^2 \theta H_1}{24 n^2 k_0}. \end{aligned} \quad (23)$$

单元体高导热材料占比为

$$\alpha_0 = \frac{A_{c0}}{A_{i0}} = \frac{D_0 w_i}{\delta_i w_i} = \frac{n D_0}{L_1 \sin \theta}. \quad (24)$$

叶形四边形产热体的高导材料占比为

$$\alpha_1 = A_{c1} / A_1, \quad (25)$$

$$A_{c1} = 2 \sum_{i=1}^n \frac{(i-0.5)D_0 H_1}{n \sin \theta} + D_1 L_1, \quad (26)$$

其中, A_1 是叶形四边形产热体的总面积, A_{c1} 是所有单元体内高导热材料占的总面积与中心高导热材料所占面积之和.

由式(24)~(26), 可得

$$\alpha_1 = \alpha_0 + \frac{D_1}{H_1}. \quad (27)$$

将 $A_1 = H_1 \times L_1$ 代入式(23), 化简可得

$$\dot{E}_{h\phi, A_1} = \frac{(n+1)(2n+1)(3n^2+3n-1)q''^2 A_1^2}{30 n^4 k_c (\alpha_1 - \alpha_0)} \frac{L_1}{H_1} + \frac{q''^2 A_1^2}{2n k_c (\alpha_1 - \alpha_0)} \frac{L_1}{H_1} + \frac{q''^2 A_1^2 \sin^2 \theta}{24 n^2 k_0} \frac{L_1}{H_1} + \frac{(2n^2-1)q''^2 A_1^2}{24 n^2 k_c \alpha_0 \sin^2 \theta} \frac{H_1}{L_1}. \quad (28)$$

叶形四边形产热体的总能耗散率可以对长宽比 H_1/L_1 进行优化. 令 $\hat{k} = k_c / k_0$, 可得最佳长宽比和相应的最小总能耗散率分别为

$$\left(\frac{H_1}{L_1} \right)_{\text{opt}} = \sqrt{\frac{\hat{k} \alpha_0 \sin^4 \theta}{(2n^2-1)} + \frac{4(n+1)(2n+1)(3n^2+3n-1)\alpha_0 \sin^2 \theta}{5n^2(2n^2-1)(\alpha_1 - \alpha_0)} + \frac{12n\alpha_0 \sin^2 \theta}{(2n^2-1)(\alpha_1 - \alpha_0)}}, \quad (29)$$

$$\dot{E}_{h\phi, A_1, \min} = \frac{(2n^2-1)q''^2 A_1^2}{12n^2 k_c \sin \theta} \sqrt{\frac{\hat{k} \sin^2 \theta}{(2n^2-1)\alpha_0} + \frac{4(n+1)(2n+1)(3n^2+3n-1)}{5n^2(2n^2-1)(\alpha_1 - \alpha_0)\alpha_0} + \frac{12n}{(2n^2-1)(\alpha_1 - \alpha_0)\alpha_0}}. \quad (30)$$

当 $n \gg 1$ 时, 式(30)中根号内的第一项“ $\frac{\hat{k} \sin^2 \theta}{(2n^2-1)\alpha_0}$ ”,

相比其他项很小, 可以忽略不计. 因此, 式(30)存在能耗

散率最小的最佳 α_0 , 可得

$$(\alpha_0)_{\text{opt}} = \frac{\alpha_1}{2}, \quad (31)$$

$$E_{h\phi, A_1, \min, \min} = \frac{(2n^2-1)q''A_1^2}{12n^2k_c \sin\theta} \sqrt{\frac{2\hat{k}\sin^2\theta}{(2n^2-1)\alpha_1} + \frac{16(n+1)(2n+1)(3n^2+3n-1)}{5n^2(2n^2-1)\alpha_1^2} + \frac{48n}{(2n^2-1)\alpha_1^2}}. \quad (32)$$

由式(32)可得叶形四边形产热体的最小当量热阻和最小平均温差分别为

$$R_{h,m} = \frac{(2n^2-1)}{12n^2k_c \sin\theta} \sqrt{\frac{2\hat{k}\sin^2\theta}{(2n^2-1)\alpha_1} + \frac{16(n+1)(2n+1)(3n^2+3n-1)}{5n^2(2n^2-1)\alpha_1^2} + \frac{48n}{(2n^2-1)\alpha_1^2}}, \quad (33)$$

$$(\Delta T)_{\min} = \frac{(2n^2-1)q''A_1}{12n^2k_c \sin\theta} \sqrt{\frac{2\hat{k}\sin^2\theta}{(2n^2-1)\alpha_1} + \frac{16(n+1)(2n+1)(3n^2+3n-1)}{5n^2(2n^2-1)\alpha_1^2} + \frac{48n}{(2n^2-1)\alpha_1^2}}. \quad (34)$$

由式(24)~(27), (29)和(31), 可得叶形四边形产热体的最优构形为

$$(H_1)_{\text{opt}} = A_1^{0.5} \left(\frac{\hat{k}\alpha_1 \sin^4\theta}{4n^2-2} + \frac{4(n+1)(2n+1)(3n^2+3n-1)\sin^2\theta}{5n^2(2n^2-1)} + \frac{12n\sin^2\theta}{2n^2-1} \right)^{0.25}, \quad (35)$$

$$(L_1)_{\text{opt}} = A_1^{0.5} \left(\frac{\hat{k}\alpha_1 \sin^4\theta}{4n^2-2} + \frac{4(n+1)(2n+1)(3n^2+3n-1)\sin^2\theta}{5n^2(2n^2-1)} + \frac{12n\sin^2\theta}{2n^2-1} \right)^{0.25}, \quad (36)$$

$$\left(\frac{D_1}{D_0} \right)_{\text{opt}} = \frac{n}{\sin\theta} \sqrt{\frac{\hat{k}\alpha_1 \sin^4\theta}{4n^2-2} + \frac{4(n+1)(2n+1)(3n^2+3n-1)\sin^2\theta}{5n^2(2n^2-1)} + \frac{12n\sin^2\theta}{2n^2-1}}. \quad (37)$$

3 数值计算与分析

根据文献[46]取 $A_1=5 \times 10^3 \text{ mm}^2$, $k_c/k_0=470$, $k_0=0.8 \text{ W/mK}$, $\alpha_1=0.15$ 和 $q''=2 \times 10^4 \text{ W/m}^2$. 图4(a)给出了不同角度下最优长宽比和区间数的关系. 图4(b)给出了不同角度下最小平均温差和区间数的关系. 图5(a)给出了 $n=30$ 时最优长宽比和角度的关系. 图5(b)给出了 $n=30$ 时最小平均温差和角度的关系.

由图4(a)可知区间数大于20时, 最优长宽比随区间数增加略微下降, 但不同角度下的最优长宽比差别较大. 由图4(b)可知区间数大于20时, 最小平均温差随

区间数增加略微下降, 因此增加产热体内高导热通道的复杂程度能降低平均传热温差. 由图5可知平均传热温差最小的最佳角度为 $\pi/2$, 且角度为最佳值时最优长宽比最大. 表1为叶形四边形产热体以焓耗散率最小为优化目标的最优构形. 表2为叶形四边形产热体以最大温差最小为优化目标的最优构形[46].

对比表1和2可知, 以焓耗散率最小为优化目标的最优长宽比和以最大温差最小为优化目标的最优长宽比有明显的差异, 但最佳角度都为 $\pi/2$; 当区间数增加到20以上时, 最大温差最小下的长宽比基本不变, 而焓耗散率最小下的长宽比略微下降, 且最大温差最小下

图4 (网络版彩图) (a) 不同角度下最优长宽比和区间数的关系; (b) 不同角度下最小平均温差和区间数的关系

Figure 4 (Color online) (a) The relationship between aspect ratio and the number of intervals under different angles; (b) the relationship between the minimum mean temperature difference and the number of intervals under different angles.

图 5 (a) $n=30$ 时最优长宽比和角度的关系; (b) $n=30$ 时最小平均温差和角度的关系
Figure 5 (a) The relationship between the optimal aspect ratio and the angle with $n=30$; (b) the relationship between the minimum mean temperature difference and the angle with $n=30$.

表 1 焓耗散率最小的最优构形(本文结果)
Table 1 Optimal constructs with minimum entransy dissipation rate

n	$\alpha_1=0.15$					$\alpha_1=0.10$					$\alpha_1=0.05$				
	$(\theta)_{\text{opt}}$	$\left(\frac{D_1}{D_0}\right)_{\text{opt}}$	$\left(\frac{H_1}{L_1}\right)_{\text{opt}}$	ΔT_{max} (K) ^a	$\Delta \overline{T}$ (K)	$(\theta)_{\text{opt}}$	$\left(\frac{D_1}{D_0}\right)_{\text{opt}}$	$\left(\frac{H_1}{L_1}\right)_{\text{opt}}$	ΔT_{max} (K) ^a	$\Delta \overline{T}$ (K)	$(\theta)_{\text{opt}}$	$\left(\frac{D_1}{D_0}\right)_{\text{opt}}$	$\left(\frac{H_1}{L_1}\right)_{\text{opt}}$	ΔT_{max} (K) ^a	$\Delta \overline{T}$ (K)
20	$\pi/2$	35.0	1.75	3.02	1.03	$\pi/2$	34.8	1.74	4.50	1.54	$\pi/2$	34.8	1.74	8.97	3.08
30	$\pi/2$	50.4	1.68	2.93	0.99	$\pi/2$	50.4	1.68	4.38	1.49	$\pi/2$	50.4	1.68	8.76	2.97
40	$\pi/2$	66.0	1.65	2.89	0.97	$\pi/2$	66.0	1.65	4.33	1.46	$\pi/2$	66.0	1.65	8.65	2.97
50	$\pi/2$	81.5	1.63	2.86	0.96	$\pi/2$	81.5	1.63	4.29	1.44	$\pi/2$	81.5	1.63	8.57	2.88
60	$\pi/2$	96.6	1.61	2.83	0.95	$\pi/2$	96.6	1.61	4.24	1.43	$\pi/2$	96.6	1.61	8.48	2.86

a) 本文作者由文献[46]结果导出, ΔT_{max} 为最大温差.

表 2 最大温差最小的最优构形(文献[46]结果)
Table 2 Optimal constructs with the minimum maximum temperature difference

n	$\alpha_1=0.15$					$\alpha_1=0.10$					$\alpha_1=0.05$				
	$(\theta)_{\text{opt}}$	$\left(\frac{D_1}{D_0}\right)_{\text{opt}}$	$\left(\frac{H_1}{L_1}\right)_{\text{opt}}$	ΔT_{max} (K)	$\Delta \overline{T}$ (K) ^{a)}	$(\theta)_{\text{opt}}$	$\left(\frac{D_1}{D_0}\right)_{\text{opt}}$	$\left(\frac{H_1}{L_1}\right)_{\text{opt}}$	ΔT_{max} (K)	$\Delta \overline{T}$ (K) ^{a)}	$(\theta)_{\text{opt}}$	$\left(\frac{D_1}{D_0}\right)_{\text{opt}}$	$\left(\frac{H_1}{L_1}\right)_{\text{opt}}$	ΔT_{max} (K)	$\Delta \overline{T}$ (K) ^{a)}
20	$\pi/2$	16.6	0.83	1.65	1.33	$\pi/2$	16.4	0.82	2.48	2.00	$\pi/2$	16.0	0.80	4.96	4.06
30	$\pi/2$	25.2	0.84	1.65	1.24	$\pi/2$	24.9	0.83	2.48	1.87	$\pi/2$	24.6	0.82	4.95	3.77
40	$\pi/2$	34.0	0.85	1.65	1.19	$\pi/2$	34.0	0.85	2.47	1.79	$\pi/2$	33.6	0.84	4.95	3.60
50	$\pi/2$	43.0	0.86	1.65	1.16	$\pi/2$	42.5	0.85	2.47	1.76	$\pi/2$	42.5	0.85	4.94	3.51
60	$\pi/2$	51.6	0.86	1.65	1.14	$\pi/2$	51.6	0.86	2.47	1.72	$\pi/2$	51.6	0.86	4.94	3.45

a) 本文作者由文献[46]结果导出.

的最大温差基本不变, 而焓耗散率最小下的平均传热温差略微下降; 高导热材料占比的改变不影响焓耗散率最小下和最大温差最小下的最优长宽比, 但对平均传热温差和最大传热温差有影响; 焓耗散最小下的平

均温差小于最大温差最小的平均温差.

4 结论

本文在文献[46]建立的叶形四边形产热体模型的基础上, 进一步以焓耗散率最小为优化目标, 研究了其最优构形, 分析了区间数和角度对叶形四边形产热体最优构形的影响, 比较了焓耗散率最小和最大温差最小下的构形差异, 可以得出以下结论.

(1) 叶形四边形产热体在焓耗散率最小时的最优构形下平均传热温差最小.

(2) 随着区间数($n \geq 20$)的增加, 叶形四边形产热体

的最优长宽比和最小平均温差略微下降, 因此产热体内高导热通道的复杂程度能降低平均传热温差.

(3) 叶形四边形产热体以焓耗散率和最大温差为优化目标的最优构形有区别, 但都在分支高导热材料垂直于中心高导热材料时最佳.

(4) 叶形四边形产热体以焓耗散率为优化目标下的平均温差比以最大温差为优化目标下的平均温差更小, 因此以焓耗散率最小为优化目标更能有效地降低叶形四边形产热体内的平均传热温差.

本文考虑的高导热材料基本为均质导热材料. 在今后的工作中, 将进一步考虑高导热材料的系数非均质或变导热系数对最优构形的影响.

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Constructal optimization of heat conduction with minimum entransy dissipation rate for leaf-shaped quadrilateral heat-generating body

ZHU HongWei^{1,2,3}, CHEN LinGen^{1,2,3}, FENG HuiJun^{1,2,3} & GE YanLin^{1,2,3}

¹ Institute of Thermal Science and Power Engineering, Wuhan Institute of Technology, Wuhan 430205, China;

² Hubei Provincial Engineering Technology Research Center of Green Chemical Equipment, Wuhan 430205, China;

³ School of Mechanical & Electrical Engineering, Wuhan Institute of Technology, Wuhan 430205, China

In this work, a constructal study of a leaf-shaped quadrilateral heat-generating body based on the bionic quadrilateral of the common leaf vein shape was performed by combining the constructal and entransy theories, thus allowing the optimal construct of the leaf quadrilateral heat-generating body to be derived. The effect of the number of intervals, i.e., the number of branches containing high thermal conductivity materials, n , and angle θ between the high thermal conductivity material in the branch and the center, on the optimal construction of the heat-generating body was also studied. The constructal differences of the heat-generating body were compared under the minimum entransy dissipation rate and minimum temperature difference. The results show that the mean heat transfer temperature difference of the heat-generating body is the smallest in the optimal constructal when the entransy dissipation rate is the lowest. With the increase in the number of intervals ($n \geq 20$), the optimal aspect ratio and minimum mean temperature difference of the heat-generating body decreased slightly. Therefore, increasing the complexity of the high heat conduction passage in the heat-generating body can decrease the mean heat transfer temperature difference. Different optimal constructs of the heat-generating body were obtained using the entransy dissipation rate and maximum temperature difference as the optimization objective; both provide more optimal solutions when the high thermal conductivity material of the branch and center are perpendicular. Furthermore, minimizing the entransy dissipation rate allowed for a smaller mean temperature difference to be obtained. Therefore, using the minimum entransy dissipation rate as the optimization objective can reduce the mean heat transfer temperature difference more effectively in a leaf-shaped quadrilateral heat-generating body.

constructal theory, entransy theory, leaf-shaped quadrilateral heat-generating body, heat conduction, optimal construct, generalized thermodynamic optimization

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