SCIENCE CHINA
Information Sciences

• LETTER •

February 2022, Vol. 65 129202:1–129202:3
https://doi.org/10.1007/s11432-019-2788-5

Social conformity creates consensus and strong diversity of Hegselmann-Krause opinion dynamics

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Received 5 September 2019/Revised 21 November 2019/Accepted 26 December 2019/Published online 1 February 2021

Citation Cheng C, Yu C B. Social conformity creates consensus and strong diversity of Hegselmann-Krause opinion dynamics. Sci China Inf Sci, 2022, 65(2): 129202, https://doi.org/10.1007/s11432-019-2788-5

Dear editor,

Over the past decade, "opinion dynamics" of social networks have obtained considerable attention from control theory, sociology, and physics [1–3]. One of the most commonly studied models is the Hegselmann-Krause (H-K) model [4], which is boundedly confident, so that each agent only updates his/her opinion by averaging all the opinions of neighbors. However, the model has been criticized for a considerable time because limiting behaviour such as weak diversity has been pointed out in [5]. Weak diversity refers to the strong convergence of the sub-clusters owing to the bounded confidence of the model when the group opinion evolves into multiple clusters, so that individuals in the sub-cluster hold only one opinion. On the contrary, strong diversity is defined as a continuous and stable distribution within the opinion space that disappears in a fully connected network and cannot be obtained in the bounded confidence model. However, strong diversity is more effective in explaining small and tiny differences of opinions among individuals in groups or subgroups in real life.

The classical H-K model cannot generate strong diversity because each agent of the H-K model has a single opinion on a given issue. However, in many cases, for example, when a candidate tries to gain recognition from voters, the agent's expressed opinion may differ from his/her inner belief [6, 7]. In the field of psychology, the fact that a discrepancy exists between the expressed and private opinion of an agent is well acknowledged. Various factors such as political correctness and conformity pressure contribute to this phenomenon. Researchers have proposed many social experiments to examine the effect of conformity, for example Asch's famous conformity experiment [8]. According to David Myers [9], two types of conformity exist: compliance and acceptance. (i) Sometimes we conform to an expectation or a request without really believing in what we are doing. This insincere, outward conformity is compliance. (ii) Sometimes we genuinely believe in what the group has persuaded us to do. This sincere, inward conformity is called acceptance.

In this study, we attempt to introduce social conformity behavior to enrich the process of an individual's opinion formation and expression. Additionally, we introduce it as an effective patch to address the limitations of the H-K model to reveal the mechanism for consensus and strong diversity of opinion evolution in social networks. Therefore, we propose a novel multi-agent model based on bounded confidence and conformity theory to describe both types of conformity behaviors, namely compliance and acceptance. We theoretically prove the ideal condition for consensus and capture the phenomenon of strong diversity in our new model.

Model definition. For a population of n agents, by defining $V = \{1, 2, \ldots, n\}$, let $y_i(t)$ and $\hat{y}_i(t)$, $i \in V$, represent an individual i's private and expressed opinions at time $t = 0, 1, \ldots, \infty$. The opinions are scaled to be $y_i(t), \hat{y}_i(t) \in [0, 1]$. Because of the information asymmetry, for any individual $i \in V$, only the expressed opinions of other individuals $\hat{y}_j(t)$, $j \in V$, $j \neq i$ can be observed.

Model I: acceptance.

• At each step, individual i first expresses an opinion $\hat{y}_i(t)$, and then observes others' expressed opinions, $\hat{y}_j(t)$, $j \neq i$.

• Individual i updates both his/her private opinion $y_i(t+1)$ and expressed opinion $\hat{y}_i(t+1)$ based on the bounded confidence and conformity pressure.

Acceptance means that both private and expressed opinions are affected by conformity pressure. For an individual i in a state of acceptance, his/her inner mind truly agrees with his/her opinion expressed towards the group. At this time, his/her private and expressed opinions are consistent. Therefore, we have $y_i(t) \equiv \hat{y}_i(t)$ for $t = 0, 1, \ldots, \infty$. Thus we can describe the acceptance model as

$$\begin{aligned}\hat{y}_i(t+1) &\equiv y_i(t+1)\\ &= \frac{1-p_i}{1+|N_i(t)|}\left[y_i(t) + \sum_{j\in N_i(t)} \hat{y}_j(t)\right] + p_i \hat{y}_{\text{avg}}(t),\end{aligned} \tag{1}$$

where $N_i(t) = \{1 \leqslant j \leqslant n, j \neq i \,|\, |y_i(t) - \hat{y}_j(t)| \leqslant \varepsilon_i\}$ represents the set of i's communicating neighbors at time t,

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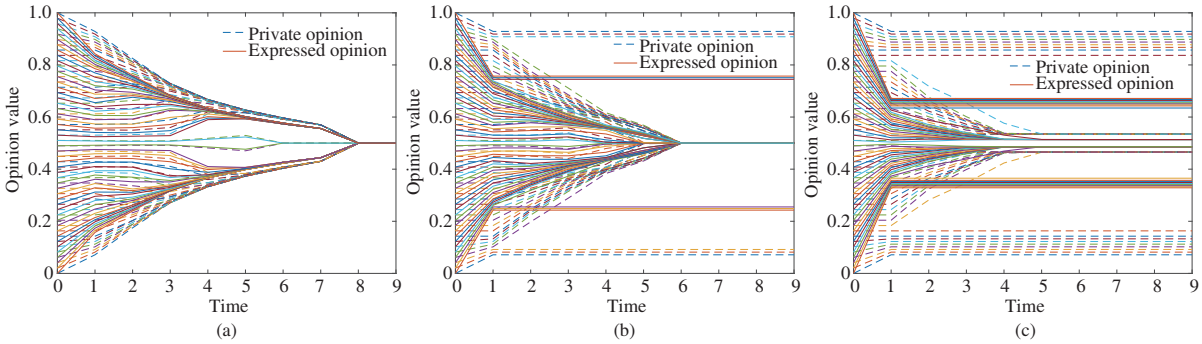

Figure 1 (Color online) Opinion evolution of $n = 50$ agents with $\varepsilon = 0.15$ and different group pressure levels. (a) $p = 0.2$; (b) $p = 0.4$; (c) $p = 0.6$.

the constant $\varepsilon_i \in (0, 1]$ is the confidence interval of agent i, and $|N_i(t)|$ is the cardinality counting of $N_i(t)$. The value $\hat{y}_{\text{avg}}(t) = \frac{1}{n}\sum_{j=1}^{n} \hat{y}_j(t)$ represents the group public opinion. Here, constant $p_i \in (0, 1]$ describes the level of pressure to conform to the public opinion. Notably, if $p_i = 0$, the model will directly revert to the classic H-K model.

Model II: compliance.

• At each step, individual i first expresses an opinion $\hat{y}_i(t)$, and then observes others' expressed opinions, $\hat{y}_j(t)$, $j \neq i$.

• Individual i updates his/her private opinion $y_i(t + 1)$ based on the bounded confidence.

• Individual i updates his/her expressed opinion $\hat{y}_i(t+1)$ under conformity pressure.

In the case of compliance, the individual i does not have conformity in his/her heart; however, he/she still expresses an opinion that is close to the public opinion because of the pressure of conformity. Individual i updates his/her private opinion as

$$y_i(t+1) = \frac{1}{1+|N_i(t)|}\left[y_i(t) + \sum_{j\in N_i(t)} \hat{y}_j(t)\right] \tag{2}$$

and determines his/her expressed opinion according to

$$\hat{y}_i(t+1) = (1-p_i)y_i(t+1) + p_i\hat{y}_{\text{avg}}(t), \tag{3}$$

where $N_i(t) = \{1 \leqslant j \leqslant n, j \neq i | \, |y_i(t) - \hat{y}_j(t)| \leqslant \varepsilon_i\}$ represents the set of i's communicating neighbors at time t.

Opinion consensus in Model I. We make the following assumption and then introduce our main result in the acceptance model.

Assumption 1. Without losing generality, we set the same confidence interval ε and pressure leve p for each agent in the group, such as $\varepsilon_i = \varepsilon \in (0, 1]$, $p_i = p \in (0, 1]$, $\forall i \in V$.

Theorem 1. In Model I, if Assumption 1 holds, opinion consensus can be reached in finite time, and the upper bound of convergence time is $T^* = [\frac{n(1-\varepsilon)}{2p\varepsilon}] + 1$.

Opinion consensus in Model II. We provide an ideal condition for opinion consensus in the compliance model.

Theorem 2. In Model II, if Assumption 1 holds, considering the condition that

$$N_i(t) \neq \emptyset, \quad \forall i \in V, \ t \in \mathbb{N}, \tag{4}$$

then the consensus of private opinion $y(t)$ and expressed opinion $\hat{y}(t)$ can be reached in finite time T_1^*, and we have $y(t) = \hat{y}(t)$, for $t > T_1^*$.

Condition (4) indicates that each agent i always maintains communication with at least one of the other agents in the group. Final group consensus is guaranteed in this constant communication situation.

Fragmentation phenomenon in Model II. Note that in the classic H-K model, the distance of opinions between two agents i and j at the final stable state will be $\delta_{ij}^t = |y_i(t) - y_j(t)| > \varepsilon$, or $\delta_{ij}^t = 0$. Therefore, only weak diversity can be observed. Now, we define the distance of expressed opinions between two agents i and j as $\Delta_{ij}^t = |\hat{y}_i(t) - \hat{y}_j(t)|$, and the distance of private opinions as $\delta_{ij}^t = |y_i(t) - y_j(t)|$. Subsequently, the model presents the following properties of strong diversity.

Proposition 1. Suppose T^* is the time when Model II converges to the stable state. If $\delta_{ij}^t \to 0$, $i, j \in V$, for $t > T^*$, then we have $\Delta_{ij}^t \to 0$ and thus strong diversity of expressed opinions exists between agents i and j.

Proposition 2. In Model II, suppose that Assumption 1 holds. Then, for all $t \in \mathbb{N}$, we have

$$|\hat{y}_{\max}(t) - \hat{y}_{\min}(t)| \leqslant |y_{\max}(t) - y_{\min}(t)| \, .$$

Therefore, we find that the gap of expressed opinions is less than or equal to the gap of private opinions in the group, implying that in real life, the actual divergence of group opinions can possibly be considerably larger than the apparent divergence.

Example of Model II. Consider a simple network of n=50 agents whose initial private and expressed opinions are uniformly distributed in the space $y(0) = \hat{y}(0) \in [0, 1]$. Without loss of generality, we set $\varepsilon_i = \varepsilon$ and $p_i = p$ for all $i \in V$. The time evolution of the group with different pressure levels p is shown in Figure 1 in which $n = 50$ and $\varepsilon = 0.15$ have been set. As the pressure level p increases, the final opinions step from consensus (conformity) to fragmentation (plurality). As observed in Figure 1(a), an appropriate pressure level can guarantee the consensus of the group. However, when the pressure level gradually grows larger, the marginal agents will split from the group, and therefore, the number of marginal agents increases with the pressure level, as shown in Figures 1(b) and (c). Interestingly, the polarization of the two central clusters can be observed in Figure 1(c) with $p = 0.6$.

Conclusion. In this study, we relax the bounded confidence assumption that generates opinion clustering only with weak diversity in the H-K model by applying social conformity theory. We establish a novel model for two different types of conformity behaviors: acceptance and compliance. In the case of acceptance, we proved that the model could

converge to consensus in finite time, and in the case of compliance, an ideal condition for consensus was theoretically proposed. An important finding is that strong diversity can be captured in the case of compliance. We further investigated the reasons for strong diversity in response to the limitation of the H-K model. In the future, additional methods reflecting complex human behavior can be proposed based on this model. We may assume that the opinion updates are executed asynchronously, and the dissonance between expressed and private opinions will be further investigated. This hypothesis is similar to the "self-persuasion" theory: the intermediate process in which an individual's external behavior causes an internal state change that often occurs when people reflect on a topic and change their attitudes.

Acknowledgements This work was supported by National Natural Science Foundation of China (Grant No. 61761136005) and Australian Research Council (Grant Nos. DP-160104500, DP-190100887).

Supporting information Appendix A. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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