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News & Views

Excitation of the isoscalar giant monopole resonance and incompressibility of nuclear matter: resolution of a long-standing puzzle

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Ever since the discovery of the isoscalar giant monopole resonance (ISGMR) [1] and isoscalar giant dipole resonance (ISGDR) [2,3] compression modes, there was much interest in determining the incompressibility and symmetry energy of nuclear matter, which define to a large extent its equation of state (EOS).

The incompressibility of nuclear matter, $K_{\rm nm}$, is of fundamental importance because of its implication in various phenomena ranging from collective excitations of nuclei to collapse and supernova explosions of giant stars. It also governs formation of neutron stars (mass, radius, crust) and is an important ingredient in the study of nuclear properties. It is defined as follows [4]:

$$K_{\rm nm} = \left[9\rho^2(\mathrm{d}^2(E/A)/\mathrm{d}\rho^2)\right]_{\rho=\rho_0}, \quad (1)$$

where E/A is the binding energy per nucleon, ρ is the nuclear density and ρ_0 is the nuclear density at saturation. This relation does not lead to a direct measurement of $K_{\rm nm}$. Experimentally, one measures the excitation energies of the ISGMR and ISGDR. These can be related to K_A, the compression modulus of the nucleus, which is defined as:

$$K_A = \left[r^2\left(\mathrm{d}^2(E/A)/\mathrm{d}r^2\right)\right]_{r=R_0}, \quad (2)$$

which scales with the nuclear radius. In a hydrodynamic model, the ISGMR and ISGDR compression modes can be understood as hydrodynamic density oscillations. The density oscillation for the ISGMR is isotropic and is best described as a “breathing mode”. In the case of the ISGDR, the compression waves traverse the nucleus back and forth in a “squeezing mode” pattern. Stringari [5] considered two different models for the description of the nuclear motion: the hydrodynamical model and the generalised scaling (fluid-dynamical) model. He derived the excitation energies of the ISGMR and ISGDR in the generalised scaling approach to be:

$$E_{\rm ISGMR} = \hbar\left[K_A/(m < r^2 >)\right]^{1/2}, \quad (3)$$

$$E_{\rm ISGDR} = \hbar\left[7/3(K_A + \ (27/25)\varepsilon_{\rm F})/(m < r^2 >)\right]^{1/2}, \quad (4)$$

where m is the nucleon mass, $\langle r^2 \rangle$ is the mean square radius of the nucleus, $\varepsilon_{\rm F}$ is the Fermi energy and K_A the nucleus incompressibility.

In a macroscopic approach [6], using the semi-empirical mass formula, the second derivative of E/A as in Eq. (2) yields the following expression for K_A:

$$K_A = K_{\rm nm} + K_{\rm surf}A^{-1/3} + K_{\rm sym}((N-Z)/A)^2 + K_{\rm Coul}Z^2A^{-4/3}, \quad (5)$$

where $K_{\rm nm}$, $K_{\rm surf}$, $K_{\rm sym}$ and $K_{\rm Coul}$ are the second derivatives of coefficients of volume, surface, neutron-excess, and Coulomb terms, respectively, in the mass formula with respect to the radial coordinate of the nucleus as in Eq. (2). The Coulomb term, $K_{\rm Coul}$ can be obtained analytically assuming a uniform density for the nucleus within a sphere of radius $R_{\rm C}$. If one takes the equilibrium condition of the nuclear ground state into account, one obtains a similar relation but with coefficients $K_{\rm s}$, K_δ and $K_{\rm C}$, which are different from the coefficients $K_{\rm surf}$, $K_{\rm sym}$ and $K_{\rm Coul}$, and do not have the simple interpretation of being the second derivatives of coefficients of surface, neutron-excess, and Coulomb terms in the mass formula, respectively; see Eqs. (6.3) and (6.5) of Ref. [6].

During the late seventies and in the eighties, the ISGMR was studied at several laboratories and mainly with inelastic proton and alpha scattering. The amount of data available allowed the extraction of the ISGMR excitation energies from which the nuclear compression modulus could be determined for each of the nuclei studied. The nuclear matter incompressibility, $K_{\rm nm}$, and the coefficients $K_{\rm s}$ and K_δ could be obtained by a three-parameter fit of the experimentally observed systematics of the ISGMR [7]. Including different sets of Sn, spherical Sm nuclei, ^{208}Pb and ^{24}Mg, values for $K_{\rm nm}$ ranging between 270 and 300 MeV were obtained. These values agreed within the errors with the results obtained by the Texas A&M group using a different set of inelastic alpha scattering data and the results obtained by the Grenoble group using inelastic ^{3}He scattering data. The analysis of both of these experiments yielded a value of $K_{\rm nm}$ of around 270 MeV. It should be pointed out, however, that the three-parameter fit based on Eq. (5) has some ambiguities as has been discussed in the literature, and theoretical efforts have been made to link ISGMR energies and $K_{\rm nm}$ [6]. The value of the nuclear-matter incompressibility obtained from the three-parameter fit using Eq. (5) for the nucleus compression modulus, which in turn is determined from the ISGMR excitation energies according to Eq. (3), is indeed higher than the “commonly accepted” value (210 ± 30) MeV obtained by Blaizot [6] through

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reproducing the ISGMR excitation energies in ^{208}Pb and ^{90}Zr in random-phase approximation (RPA) calculations.

Furthermore, Blaizot et al. [4] determined the value of the compression modulus K_A of ^{208}Pb, obtained from a Hartree-Fock calculation with a constraint on the r.m.s. radius, as a function of K_{nm}. Results obtained with a variety of Skyrme interactions were also presented. It turns out that all these results are consistent, and a linear behaviour of K_A as a function of K_{nm} is obtained:

$$K_A = 0.64K_{nm} - 3.5. \quad (6)$$

Uchida et al. [8] studied the ISGDR in ^{208}Pb via inelastic α scattering at 400 MeV. Using the excitation energy of the high-energy ISGDR peak (E_x = (23.0 ± 0.3) MeV), a K_A = (130 ± 5) MeV was obtained using Eq. (4). If the ISGMR excitation energy (E_x = (13.5 ± 0.2) MeV) is used instead in Eq. (3), the K_A value of (134 ± 4) MeV is obtained. These values are very close, which indicates a consistency between the two compressional modes. Using the empirical relationship between K_A and K_{nm} for ^{208}Pb, a K_A value of ~130 MeV obtained for the ISGDR results implies a K_{nm} ~210 MeV. This is in agreement with the earlier result by Blaizot [6].

One of the intriguing puzzles in the last decade was the discrepancy found between the nuclear incompressibility determined from the closed-shell nuclei ^{208}Pb and ^{90}Zr and that determined from nuclei far from closed shells such as the Cd and Sn nuclei. Systematics of the moment ratios m_1/m_0 for the ISGMR strength distributions in the Sn isotopes were determined [9] from inelastic alpha scattering at 400 MeV. The experimental results were compared with results from calculations with K_{nm} that could fairly reproduce the ISGMR excitation energies in ^{208}Pb and ^{90}Zr. These included nonrelativistic RPA calculations (without pairing) by Colò et al. [10] and relativistic RPA calculations by Piekarewicz [11]. The calculations overestimate the experimental ISGMR energies in the Sn isotopes significantly.

A similar situation arose when trying to understand the ISGMR systematics in the Cd isotopes. Systematics of the moment ratios m_1/m_0 for the ISGMR strength distributions in the Cd isotopes were determined [12] from inelastic alpha scattering at 400 MeV. The experimental results were compared with relativistic RPA calculations performed using the FSUGold (K_{nm} = 230 MeV) [13] and NL3 (K_{nm} = 271 MeV) [14] effective interactions. They were also compared with results from non-relativistic calculations performed using the SLy5 parameter set in the Hartree-Fock (HF)-BCS + Quasiparticle RPA (QRPA) formalism with and without the mixed pairing interaction [15]. As in the Sn case, the calculations overestimate the experimental ISGMR energies in the Cd isotopes significantly. These results are in contrast with similar calculations performed with the same effective interactions, which reproduce the ISGMR excitation energies in ^{90}Zr and ^{208}Pb. At the time, the disagreement left us with an open puzzling question concerning the softness of the Sn and Cd isotopes; see e.g., Ref. [11].

The resolution of this puzzle came only very recently by Li et al. [16]. This followed the development of fully self-consistent QRPA plus Quasiparticle-Vibration Coupling model (QPVC), based on the Skyrme-Hartree-Fock-Bogoliubov (SHFB) framework. In this model, both QPVC effects and pairing effects are considered self-consistent. In Fig. 1, the experimental ISGMR strength distributions are compared with QRPA and QRPA + QPVC calculations using the SV-K226 Skyrme force. Clearly, including QPVC improves the agreement with the data suggesting that QPVC effects are crucial for reaching a unified description of the ISGMR in Ca, Sn, and Pb isotopes simultaneously.

In Table 1, taken from Ref. [16], the deviations of theoretical ISGMR energies, calculated in QRPA and QPVC using the Skyrme parameter sets SkP, SkM*, SV-K226, KDE0, SV-bas, SV-K241, and SAMi, from experimental data are listed. Including QPVC effects, good agreement between theoretical calculations and experimental results is obtained with SkM* and SV-K226 for ^{48}Ca, with

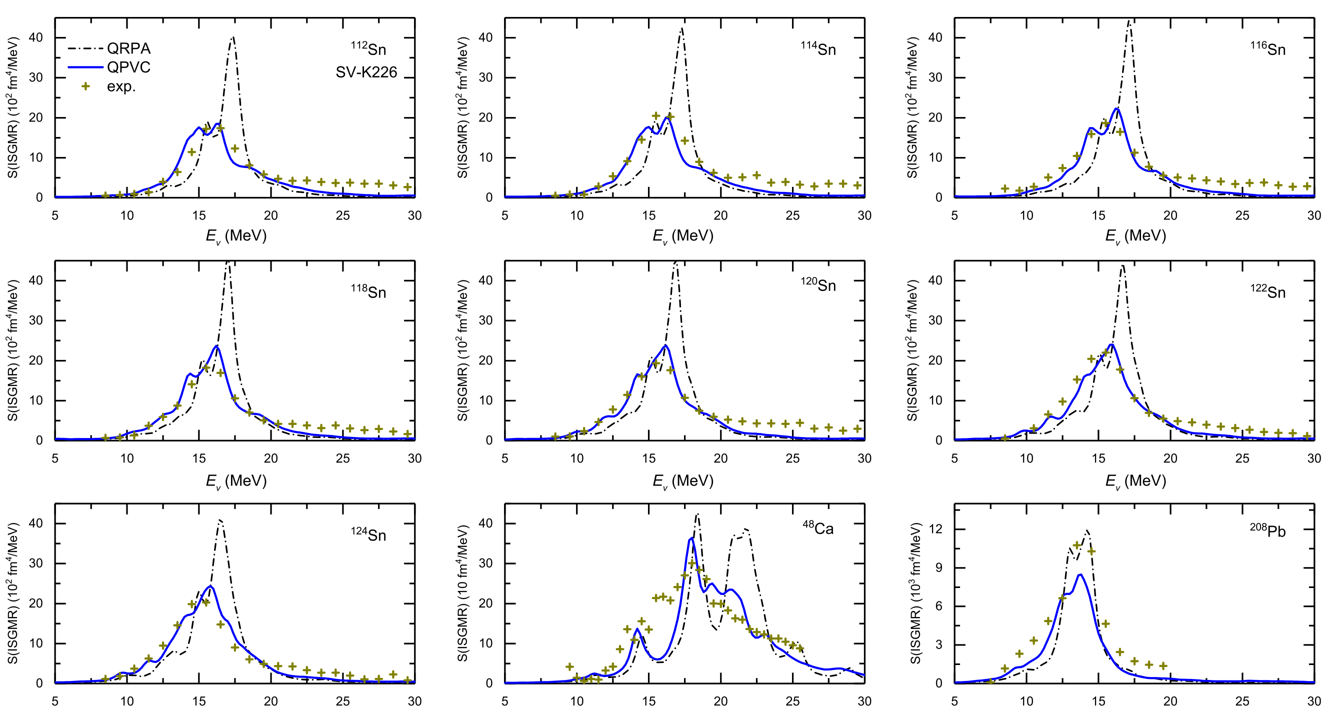

Fig. 1. (Color online) ISGMR strength functions in even-even $^{112-124}$Sn, ^{48}Ca, and ^{208}Pb isotopes, calculated either by QRPA using a smoothing function with a Lorentzian having a width of 1 MeV (dash-dotted (black) line), or QRPA + QPVC (solid (blue) line). The SV-K226 Skyrme force is used. The experimental data are given by green crosses and obtained from Refs. [8,9,17] for ^{A}Sn, ^{48}Ca, and ^{208}Pb, respectively. Figure is taken from Ref. [16], where additional details about the measurements and calculations can be found.

Table 1
The deviation of ISGMR energies from experimental data ($|E_{\text{theo}} - E_{\text{exp}}|$ (MeV)) in ^{48}Ca, ^{120}Sn, and ^{208}Pb, calculated by QRPA + QPVC using the Skyrme parameter sets SkP, SkM*, SV-K226, KDE0, SV-bas, SV-K241, and SAMi[a].

	SkP	SkM*	SV-K226	KDE0	SV-bas	SV-K241	SAMi
K_∞	201	217	226	229	233	241	245
QRPA + QPVC							
^{48}Ca	0.70	0.25	0.36	0.51	0.67	0.90	1.07
^{120}Sn	0.67	0.14	0.02	0.18	0.36	0.68	0.82
^{208}Pb	0.94	0.37	0.25	0.06	0.08	0.31	0.48

[a]The experimental data for ^{120}Sn, ^{48}Ca, and ^{208}Pb are taken from Refs. [8,9,17], respectively. Table is taken from Ref. [16].

SkM*, SV-K226, and KDE0 for ^{120}Sn, and with SVK226, KDE0, and SV-bas for ^{208}Pb. It can be concluded that calculations with SV-K226 and KDE0 describe all three nuclei very well at the same time, with K_∞ = 226 and 229 MeV, respectively. This removes the puzzle regarding the softness of Cd and Sn nuclei relative to ^{208}Pb and is consistent with the constraint (240 ± 20) MeV, obtained previously from the ISGMR of ^{90}Zr and ^{208}Pb in RPA calculations with Skyrme forces with different density dependence and calculations with relativistic functionals [18]. Recently, this conclusion has been corroborated by Litvinova [19] in which the inclusion of beyond-mean-field correlations of QPVC type allowed for a simultaneous description of the ISGMR in nuclei of Pb, Sn, Zr, and Ni mass regions.

Conflict of interest

The author declares that he has no conflict of interest.

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