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论 文

基于部分纠缠信道的高维多粒子态分等级可控隐形传态

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摘要　量子隐形传态是量子通信独有的量子信息处理方式. 最近, 基于纠缠信道的分等级量子可控隐形传态引起广泛关注. 本文提出了以四粒子χ态、四粒子Cluster态等不同多粒子纠缠态为信道的单量子比特分等级可控隐形传态方案. 研究部分纠缠信道下m粒子qudit态(d维量子态)分等级可控隐形传态方案. 发送方对手中2m个粒子执行广义Bell基测量. 为完成原未知量子态的重建, 高等级的接收方只需依据其他控制方之中任一控制方的测量结果, 选取执行相应局域幺正操作, 低等级接收方则需依据所有控制方的测量结果选择执行相应局域幺正操作. 接收方通过引入附加粒子和执行联合幺正演化概率重建原未知量子态. 由于采用高维部分纠缠态为纠缠信道完成分等级量子可控隐形传态, 方案具有信道容量高、可行性强的优点.

关键词　分等级量子可控隐形传态, 高维纠缠, 附加粒子, 联合幺正演化

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1　引言

以量子态为信息载体的量子通信拥有一些经典通讯中没有对应的信息处理方法, 包括量子秘钥分发[1–3]、量子安全直接通信[4–11]、量子态远程制备[12–19]、量子态远程操控[20–28]、量子态多方远程制备[29–35]以及量子隐形传态[36–38]. 基于预先共享量子纠缠信道非定域相关性, 量子隐形传态可以完成任意未知量子态隐形传送.

自从Bennett等人[36]提出第一个量子隐形传态方案以来, 量子隐形传态引起广泛关注. 一方面, 以不同纠缠态为量子纠缠信道, 针对任意未知量子态, 特别是多粒子态, 提出了量子隐形传态方案[36–44]. 另一方面, 基于纠缠离子对和纠缠光子对, 人们完成了量子隐形传态实验验证[45–47]. 为提高网络中量子隐形传态安全, 1998年, Karlsson和Bourennane[48]提出了量子可控隐形传态方案. 在量子可控隐形传态中, 接收方只有与控制方合作, 在控制方的控制下才能完成原未知量子态重

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建. 此后, 人们提出了多种不同的量子可控隐形传态协议[49−53]. 原有的量子可控隐形传态方案大都为对称的可控隐形传态协议, 即协议中所有接收方的地位是平等的, 任一接收方通过与所有发送方、控制方合作均可完成原量子态重建. 2010年, Wang等人[54]提出了分等级量子可控隐形传态协议, 在分等级量子可控隐形传态中, 高等级的接收方只需依据其他控制方之中任一控制方的测量结果即可完成原未知量子态的重建, 而低等级接收方则需要其他所有控制方合作才能完成原量子态重建. 分层级量子可控隐形传态可用于某些特殊的量子通信任务中, 如银行依据可信度对通信参与者分级, 其中高等级参与者只需与部分其他参与者合作即可获取账户机密信息, 而低等级参与者则需与所有其他参与者合作才能获取账户机密信息. 在分层级量子可控隐形传态中, 按接收方可重建原未知量子态能力分级, 高等级接收方仅需已知部分其他控制方测量结果即可完成原未知量子态重建, 而低等级接收方则需与其他所有控制方合作才能完成原未知量子态的重建. 由于在量子通讯网络中的重要应用, 学者开展了不同纠缠信道下任意未知单量子比特态分等级可控隐形传态方案的研究. 2010年, Wang等人[54,55]提出了基于4粒子χ态和6粒子Cluster态的未知单量子比特分等级可控隐形传态方案. 2011年, 他们提出了以多粒子Graph态为纠缠信道的单量子比特分等级可控隐形传态协议[56]. 2013年, Shukla和Pathak[57]提出了基于4粒子Ω态和4粒子Cluster态的分层级量子隐形传态协议. 2019年, Bich和An[58]提出了基于多粒子纠缠信道的3个未知单量子比特态分等级可控隐形传态协议. Zha和Miao[59]提出了基于8粒子纠缠态的任意未知双量子比特态分等级可控隐形传态方案.

基于高维量子系统的高维量子通信具有容量高、安全性强等优点, 引起了研究者的广泛关注. 2000年, Zeilinger课题组[60]证明了N维最大纠缠态对于局域实在性的违背大于二维最大纠缠态对于局域实在性的违背, 对于局域实在性的违背随维度N的增加而增加. 2002年, Gisin课题组[61]将量子秘钥分发协议推广到高维, 提出基于d维量子系统的秘钥分发协议. Liu等人[62]提出了基于高维量子态的量子多方超密编码协议. 2005年, Wang等人[63]提出了基于量子超密编码的量子安全直接通信方案. 2008年, Tian等人[64]提出了基于非最大纠缠Cluster态的任意高维双粒子态隐形传态方案. 2016年, Chen和Lu[65]提出了基于高维纠缠态确定性实现高维非定域量子门方案. 2017年, 基于光子自旋和轨道角动量自由度编码, Karimi课题组[66]完成了分发距离超过300 m的高维量子秘钥分发实验. 2018年, Morandotti课题组[67]基于光子时间、频率自由度, 完成了高维Cluster态制备. Guo等人[68,69]完成了基于光子极化、路径自由度高维杂化量子纠缠态的制备实验以及基于四维纠缠态的量子超密编码实验. 2019年, Fonseca[70]开展了噪声环境下的高维量子隐形传态研究. Liu等人[71]提出了基于光子-原子杂化量子系统的高维Bell分析协议. 2020年, Hu等人[72]实现了32维路径纠缠态的制备, 使用多芯光纤技术完成了分发距离超过11 km的高维量子纠缠分发[73]. Liu课题组[74]基于光子路径自由度完成了高维量子态隐形传态实验. Wang等人[75]开展了基于高维三粒子GHZ态的高维可控隐形传态方案中控制方控制能力的研究. 2021年, 叶天语和胡家莉[76]提出了基于高维量子系统的量子安全多方求和协议.

虽然已有一些量子分等级可控隐形传态方案, 但是大多数方案都是基于多粒子最大纠缠态的任意单量子比特分等级可控隐形传态方案[57,58]. 本文提出基于高维部分纠缠态的任意m粒子qudit态(d维量子态)分等级可控隐形传态方案. 发送方对手中$2m$个粒子执行广义Bell基测量. 高等级的接收方只需依据其他控制方之中任一控制方的测量结果, 选取执行相应局域幺正操作, 低等级接收方则需依据所有控制方的测量结果选择执行相应局域幺正操作. 接收方引入附加粒子并对手中粒子执行联合幺正演化即可概率重建原未知量子态. 由于使用高维部分纠缠态为量子纠缠信道实现任意高维多粒子态分等级可控隐形传态, 方案具有容量高、可行性强的优点.

2 基于部分纠缠态的任意单粒子qudit态分等级可控隐形传态

广义Bell基可表示为[74]

$$|\varphi_{rs}\rangle = \frac{1}{\sqrt{d}}\sum_{j=0}^{d-1} \mathrm{e}^{\frac{2\pi \mathrm{i}}{d}js}|j,j\oplus r\rangle, \tag{1}$$

其中, $r,s=0,\cdots,d-1$表示d^2个广义Bell态$|\varphi_{rs}\rangle$, $j\oplus r$表示$j+r$模d. 广义X基可表示为[74]

$$|l_X\rangle = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} e^{\frac{2\pi i}{d} lj} |j\rangle, \tag{2}$$

其中, $l = 0, \cdots, d-1$表示d个相互正交量子态$|l_x\rangle$.

任意单粒子qudit态可表示为[73]

$$|\chi\rangle_{\chi_1} = \alpha_0|0\rangle + \alpha_1|1\rangle + \cdots + \alpha_{d-1}|d-1\rangle, \tag{3}$$

其中, $|\alpha_0|^2 + |\alpha_1|^2 + \cdots + |\alpha_{d-1}|^2 = 1$满足归一化条件. 为完成任意单粒子qudit态$|\chi\rangle_{\chi_1}$可控隐形传态, 发送方Alice和Bob, Charlie, David共享四粒子纠缠态. 假设通讯方共享的四粒子纠缠态为部分纠缠态:

$$|\psi\rangle = \sum_{j_1,j_2=0}^{d-1} \beta_{j_1} |j_1, j_1 \oplus j_2, j_2, j_2\rangle_{\text{ABCD}}, \tag{4}$$

其中, $|\beta_0|^2 + |\beta_1|^2 + \cdots + |\beta_{d-1}|^2 = \frac{1}{d}$. 粒子A属于发送方Alice, 粒子B, C, D分别属于Bob, Charlie, David.

建立量子纠缠信道后, 发送方对手中粒子执行广义Bell基测量. 若选择高等级接收方Bob重建原未知量子态, 则由控制方Charlie和David执行广义Z基测量, Bob依据发送方以及两个控制方Charlie和David之中任一控制方的测量结果选取执行相应的局域幺正操作, 即可完成原量子态重建; 若选取低等级接收方Charlie (David)重建原未知量子态, 则控制方Bob执行广义Z基测量, David (Charlie)执行广义X基测量, Charlie (David)依据发送方以及所有控制方的测量结果, 才能选取执行相应的局域幺正操作, 完成原量子态重建.

任意单粒子分等级可控隐形传态原理图如图1所示, 粒子χ_1, A, B, C, D分别属于Alice, Bob, Charlie和David. 双实线表示发送方、控制方与接收方之间的经典通信. 粒子χ_1, A, B, C, D组成的复合系统状态可表示为

$$\begin{aligned}|\Psi\rangle &= |\chi\rangle_{\chi_1} \otimes |\psi\rangle_{\text{ABCD}} \\ &= \left(\sum_{j=0}^{d-1} \alpha_j |j\rangle\right)_{\chi_1} \otimes \left(\sum_{j_1,j_2=0}^{d-1} \beta_{j_1} |j_1, j_1 \oplus j_2, j_2, j_2\rangle\right)_{\text{ABCD}} \\ &= \sum_{j,j_1,j_2=0}^{d-1} \alpha_j \beta_{j_1} |j, j_1, j_1 \oplus j_2, j_2, j_2\rangle_{\chi_1\text{ABCD}}.\end{aligned} \tag{5}$$

为完成任意单粒子qudit态$|\chi\rangle_{\chi_1}$可控隐形传态, 发送方Alice对手中粒子χ_1和A执行广义Bell基测量. 粒子χ_1, A, B, C, D组成的复合系统状态可改写为

$$\begin{aligned}|\Psi\rangle = &\sum_{r,s,j,j_2=0}^{d-1} e^{-\frac{2\pi i}{d} js} \alpha_j \beta_{j\oplus r} |\varphi_{rs}\rangle_{\chi_1\text{A}} \\ &\times |j \oplus r \oplus j_2, j_2, j_2\rangle_{\text{BCD}}.\end{aligned} \tag{6}$$

若Alice广义Bell基测量结果为$|\varphi_{rs}\rangle_{\chi_1\text{A}}$($r, s$=0, ⋯, d–1), 剩余粒子B, C, D状态演化为

$$|\Phi\rangle = \sum_{j,j_2=0}^{d-1} e^{-\frac{2\pi i}{d} js} \alpha_j \beta_{j\oplus r} |j \oplus r \oplus j_2, j_2, j_2\rangle_{\text{BCD}}. \tag{7}$$

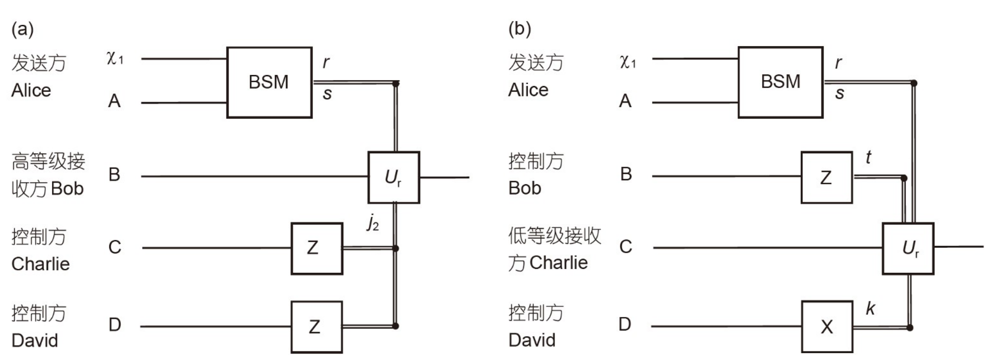

图 1 任意单粒子qudit态分等级量子可控隐形传态示意图. 其中, 粒子χ_1和A属于Alice, 粒子B属于Bob, 粒子C属于Charlie, 粒子D属于David. 双实线表示发送方、控制方与接收方之间的经典通信. (a) Bob为接收方时分等级量子可控隐形传态示意图. U_r表示与发送方以及控制方测量结果相应的局域幺正操作. (b) Charlie为接收方时分等级量子可控隐形传态示意图

Figure 1 Quantum circuit for hierarchically controlled teleportation of an arbitrary single-qudit state. Particles χ_1 and A belong to Alice, particle B belongs to Bob, particle C belongs to Charlie and particle D belongs to David. Classical communication between the sender, controller and the receiver is represented by double lines. (a) Quantum circuit for the situation that the partners agree to let Bob reconstruct the original state. U_r denotes the unitary operation according to the sender and the controller's measurement result. (b) Quantum circuit for the situation that the partners agree to let Charlie reconstruct the original state.

与单量子比特分等级可控隐形传态类似[54], 若通讯方同意选取Bob作为接收方, 控制方Charlie和David对手中粒子C和D执行Z基测量. 若Charlie (David)测量结果为$|j_2\rangle_C$($|j_2\rangle_D$), 则粒子B状态转化为(未归一化)

$$|\phi\rangle = \sum_{j=0}^{d-1} e^{-\frac{2\pi i}{d}js}\alpha_j\beta_{j\oplus r}\left|j\oplus r\oplus j_2\right\rangle_B. \quad (8)$$

高等级接收方Bob只需依据两个控制方Charlie和David其中任一控制方的测量结果$|j_2\rangle$以及发送方Alice的测量结果$|\varphi_{rs}\rangle$, 对手中粒子B选择执行相应的局域幺正操作:

$$U_r = \sum_{m=0}^{d-1} e^{\frac{2\pi i}{d}ms}|m\rangle\langle m\oplus r\oplus j_2|, \quad (9)$$

将粒子B状态转化为对应量子态(未归一化):

$$|\phi_1\rangle = U_r|\phi\rangle = \sum_{j=0}^{d-1}\alpha_j\beta_{j\oplus r}|j\rangle_B. \quad (10)$$

与部分纠缠信道下的量子可控隐形传态类似, 为重建原未知量子态Bob引入初态为$|0\rangle_a$的附加量子比特a, 对粒子B, a执行联合幺正演化U_{max}, 并对附加粒子a执行Z基测量即可概率重建原未知量子态. 若$|\beta_n|^2 = \min\left\{|\beta_0|^2, |\beta_1|^2, \cdots, |\beta_{d-1}|^2\right\}$, 则在基$\left\{|0\rangle_B|0\rangle_a, \cdots, |d-1\rangle_B|0\rangle_a, |0\rangle_B|1\rangle_a, \cdots, |d-1\rangle_B|1\rangle_a\right\}$下, 联合幺正演化为[26]

$$U_{max} = \begin{pmatrix}
\frac{\beta_n}{\beta_r} & 0 & \cdots & 0 & -\sqrt{1-\left(\frac{\beta_n}{\beta_r}\right)^2} & 0 & \cdots & 0 \\
0 & \frac{\beta_n}{\beta_{1\oplus r}} & \cdots & 0 & 0 & -\sqrt{1-\left(\frac{\beta_n}{\beta_{1\oplus r}}\right)^2} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \frac{\beta_n}{\beta_{(d-1)\oplus r}} & 0 & 0 & \cdots & -\sqrt{1-\left(\frac{\beta_n}{\beta_{(d-1)\oplus r}}\right)^2} \\
\sqrt{1-\left(\frac{\beta_n}{\beta_r}\right)^2} & 0 & \cdots & 0 & \frac{\beta_n}{\beta_r} & 0 & \cdots & 0 \\
0 & \sqrt{1-\left(\frac{\beta_n}{\beta_{1\oplus r}}\right)^2} & \cdots & 0 & 0 & \frac{\beta_n}{\beta_{1\oplus r}} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \sqrt{1-\left(\frac{\beta_n}{\beta_{(d-1)\oplus r}}\right)^2} & 0 & 0 & \cdots & \frac{\beta_n}{\beta_{(d-1)\oplus r}}
\end{pmatrix}. \quad (11)$$

可将量子态$|\phi_1\rangle_B|0\rangle_a$转化为

$$U_{max}|\phi_1\rangle_B|0\rangle_a = \sum_{j=0}^{d-1}\alpha_j\beta_{j\oplus r}|j\rangle_B\left(\frac{\beta_n}{\beta_{j\oplus r}}|0\rangle + \sqrt{1-\left(\frac{\beta_n}{\beta_{j\oplus r}}\right)^2}|1\rangle\right)_a. \quad (12)$$

对粒子B, a执行联合幺正演化后, Bob对附加粒子a执行Z基测量. 若测量结果为$|0\rangle_a$, 则粒子B状态为$|\chi\rangle_B$, 量子态分等级可控隐形传态成功; 否则量子态分等级隐形传态失败.

发送方执行广义Bell基测量后, 剩余粒子B, C, D状态演化为与广义Bell基测量结果$|\varphi_{rs}\rangle_{\chi_1 A}$相应量子态$|\Phi\rangle = \sum_{j,j_2=0}^{d-1} e^{-\frac{2\pi i}{d}js}\alpha_j\beta_{j\oplus r}|j\oplus r\oplus j_2, j_2, j_2\rangle_{BCD}$. 若通讯方选取Charlie (David)作为接收方, Bob和David (Charlie)作为控制方. 不失一般性, 我们考虑Charlie作为接

收方的情况. 为完成量子态可控隐形传态, 控制方Bob对手中粒子B执行Z基测量.

若Bob的Z基测量结果为$|t\rangle_{\mathrm{B}}(t=0,1,\cdots,d-1)$, 则粒子C和D状态演化为

$$|\xi\rangle=\sum_{j=0}^{d-1}\mathrm{e}^{-\frac{2\pi\mathrm{i}}{d}js}\alpha_j\beta_{j\oplus r}\times|t\oplus(d-(j\oplus r)),t\oplus(d-(j\oplus r))\rangle_{\mathrm{CD}}. \tag{13}$$

控制方David对手中纠缠粒子D执行广义X基测量. 粒子C和D状态可改写为

$$|\xi\rangle=\sum_{j=0}^{d-1}\mathrm{e}^{-\frac{2\pi\mathrm{i}}{d}js}\mathrm{e}^{-\frac{2\pi\mathrm{i}}{d}k(t-j-r)}\alpha_j\beta_{j\oplus r}\times|t\oplus(d-(j\oplus r))\rangle_{\mathrm{C}}|k_x\rangle_{\mathrm{D}}. \tag{14}$$

若David广义X基测量结果为$|k_x\rangle_{\mathrm{D}}$ $(k=0,1,\cdots,d-1)$, 则接收方Charlie手中粒子C状态演化为

$$|\phi'\rangle=\sum_{j=0}^{d-1}\mathrm{e}^{-\frac{2\pi\mathrm{i}}{d}j(s-k)}\alpha_j\beta_{j\oplus r}|t\oplus(d-(j\oplus r))\rangle_{\mathrm{C}}. \tag{15}$$

为完成原量子态的重建, 低等级接收方Charlie需要所有控制方Bob和David的测量结果$|t\rangle_{\mathrm{B}}$, $|k_x\rangle_{\mathrm{D}}$以及发送方Alice的广义Bell基的测量结果$|\varphi_{rs}\rangle_{\chi_1\mathrm{A}}$, 才能对手中粒子C选取执行相应的局域幺正操作:

$$U_{\mathrm{r}}'=\sum_{m=0}^{d-1}\mathrm{e}^{\frac{2\pi\mathrm{i}}{d}m(s-k)}|m\rangle\langle t\oplus(d-(m\oplus r))|. \tag{16}$$

将粒子C状态转化为对应量子态(未归一化):

$$|\phi_1\rangle=U_{\mathrm{r}}'|\phi'\rangle=\sum_{j=0}^{d-1}\alpha_j\beta_{j\oplus r}|j\rangle_{\mathrm{C}}. \tag{17}$$

与Bob作为接收方重建原未知量子态类似, Charlie通过引入初态为$|0\rangle_{\mathrm{a}}$的附加粒子a, 对粒子B, a执行联合幺正演化$U_{\max}$, 并对附加粒子a执行Z基测量来重建原未知量子态. 对附加粒子a的测量结果为$|0\rangle_{\mathrm{a}}$, 粒子C状态演化为原未知量子态$|\chi\rangle_{\mathrm{C}}$, 分等级量子可控隐形传态成功; 否则量子可控隐形传态失败. 与文献[26]类似, 部分纠缠信道下任意单粒子qudit态分等级量子可控隐形传态成功率为$d|\beta_n|^2$, 其中

$$|\beta_n|^2=\min\left\{|\beta_0|^2,|\beta_1|^2,\cdots,|\beta_{d-1}|^2\right\}.$$

3 部分纠缠信道下的任意*m*粒子qudit态分等级可控隐形传态

任意m粒子qudit态可表示为[74]

$$|\chi\rangle_{\chi_1\cdots\chi_m}=\sum_{j_1,\cdots,j_m=0}^{d-1}\alpha_{j_1\cdots j_m}|j_1\cdots j_m\rangle, \tag{18}$$

其中,

$$\sum_{j_1,\cdots,j_m=0}^{d-1}\left|\alpha_{j_1\cdots j_m}\right|^2=1 \tag{19}$$

满足归一化条件. 为完成任意m粒子qudit态$|\chi\rangle_{\chi_1\cdots\chi_{m-1}}$分等级可控隐形传态, 通讯方共享$m$个多粒子纠缠态. 假设Alice, Bob, Charlie, David所共享多粒子纠缠信道m个部分纠缠态:

$$\begin{aligned}|\psi'\rangle&=|\psi\rangle^{\otimes m}\\&=\sum_{\substack{j_{11},j_{12},\cdots,\\j_{m1},j_{m2}=0}}^{d-1}\beta_{j_{11}}|j_{11},j_{11}\oplus j_{12},j_{12},j_{12}\rangle_{\mathrm{A_1B_1C_1D_1}}\cdots\\&\quad\beta_{j_{m1}}|j_{m1},j_{m1}\oplus j_{m2},j_{m2},j_{m2}\rangle_{\mathrm{A_mB_mC_mD_m}},\end{aligned} \tag{20}$$

其中, $\sum_{j_{11}=0}^{d-1}\left|\beta_{j_{11}}\right|^2=\sum_{j_{21}=0}^{d-1}\left|\beta_{j_{21}}\right|^2=\cdots=\sum_{j_{m1}=0}^{d-1}\left|\beta_{j_{m1}}\right|^2=\frac{1}{d}$满足归一化条件.

所有发送方共享多粒子纠缠信道, Alice拥有纠缠粒子A_1, $\cdots$, A_m, Bob拥有B_1, $\cdots$, B_m, Charlie拥有粒子C_1, $\cdots$, C_m, David拥有粒子D_1, $\cdots$, D_m. 建立纠缠信道后, 发送方Alice对手中粒子χ_j, A_j $(j=1,\cdots,m)$执行广义Bell基测量. 如果选择高等级接收方Bob重建原量子态, 则Bob只需依据Charlie, David两控制方中任一控制方测量结果以及Alice广义Bell基的测量结果即可选取相应的局域幺正操作, 重建原量子态. 如果通讯方决定由低等级接收方Charlie (David)重建原量子态, 则Charlie (David)需依据所有控制方Bob和David (Charlie)的测量结果以及Alice的测量结果才能完成原量子态重建.

任意多粒子分等级可控隐形传态原理图如图2所示, 双实线表示发送方、控制方与接收方之间的经典通信. U_{r}表示接收方依据发送方以及控制方的测量结果所选择的局域幺正操作. 由粒子χ_1, $\cdots$, χ_m, A_1, $\cdots$, D_1, $\cdots$, A_m, $\cdots$, D_m所组成复合系统量子态可表示为

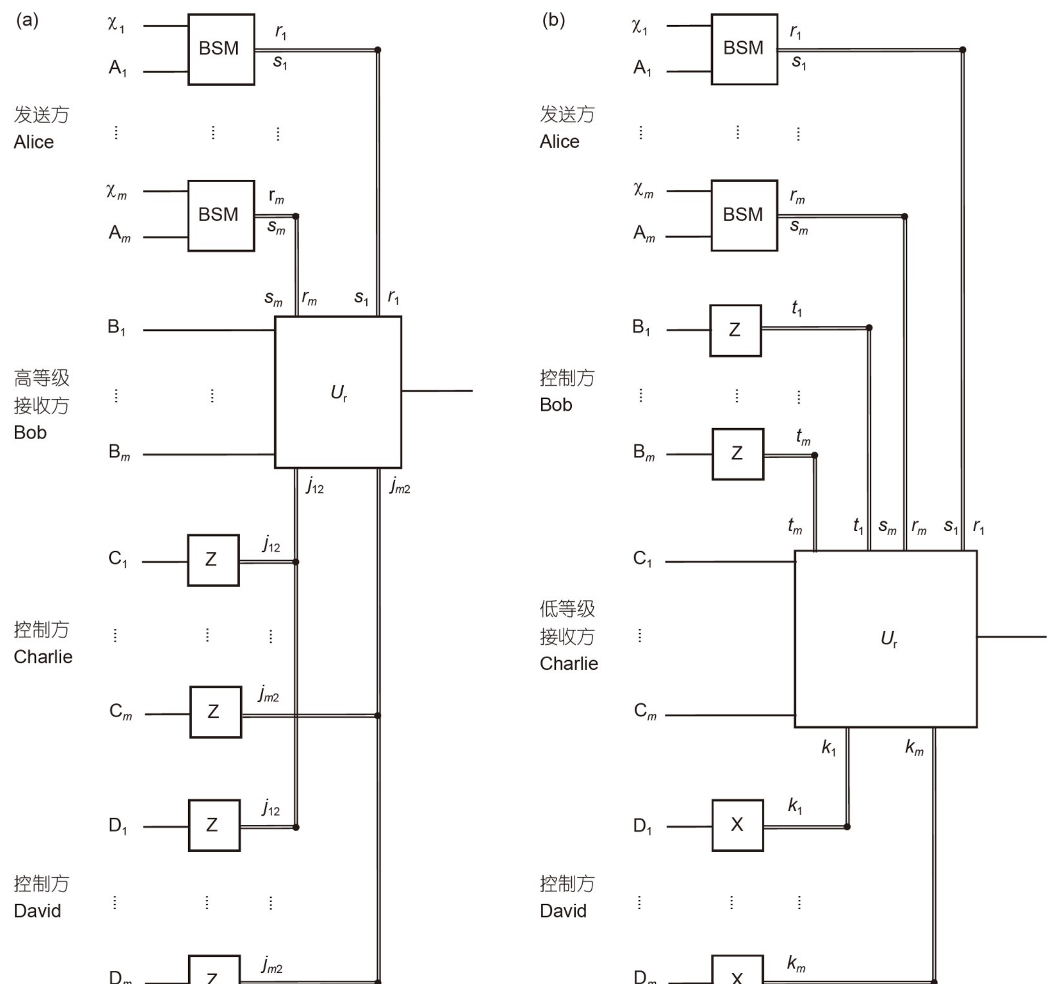

图 2 任意多粒子qudit态分等级量子可控隐形传态示意图. 粒子χ_l, A_l, B_l, C_l, D_l (l=1, ···, m)分别属于Alice, Bob, Charlie和David. 双实线表示发送方、控制方与接收方之间的经典通信. (a) Bob为接收方时分等级量子可控隐形传态示意图. (b) Charlie为接收方时分等级量子可控隐形传态示意图

Figure 2 Quantum circuit for hierarchically controlled teleportation of an arbitrary multi-qudit state. Particles χ_l, A_l, B_l, C_l, D_l (l=1, ···, m) belong to Alice, Bob, Charlie and David. Classical communication between the sender, controller and the receiver is represented by double lines. (a) Quantum circuit for the situation that the partners agree to let Bob reconstruct the original state. (b) Quantum circuit for the situation that the partners agree to let Charlie reconstruct the original state.

$$
\begin{aligned}
|\Psi\rangle &= |\chi\rangle_{\chi_1\cdots\chi_m} \otimes |\psi'\rangle_{A_1B_1C_1D_1\cdots A_mB_mC_mD_m} \\
&= \left(\sum_{j_1,\cdots,j_m=0}^{d-1} \alpha_{j_1\cdots j_m} |j_1\cdots j_m\rangle\right)_{\chi_1} \otimes \left(\sum_{\substack{j_{11},j_{12},\cdots,\\ j_{m1},j_{m2}=0}}^{d-1} \beta_{j_{11}} |j_{11}, j_{11}\oplus j_{12}, j_{12}, j_{12}\rangle \cdots \beta_{j_{m1}} |j_{m1}, j_{m1}\oplus j_{m2}, j_{m2}, j_{m2}\rangle\right)_{A_1,\cdots,D_m} \\
&= \sum_{\substack{j_1,\cdots,j_m,\\ j_{11},\cdots,j_{m2}=0}}^{d-1} \alpha_{j_1\cdots j_m}\beta_{j_{11}}\cdots\beta_{j_{m1}} |j_1\cdots j_m\rangle_{\chi_1\cdots\chi_m} |j_{11}, j_{11}\oplus j_{12}, j_{12}, j_{12}\rangle_{A_1B_1C_1D_1} \cdots |j_{m1}, j_{m1}\oplus j_{m2}, j_{m2}, j_{m2}\rangle_{A_mB_mC_mD_m}.
\end{aligned} \tag{21}
$$

为完成m粒子qudit分等级可控隐形传态，发送方Alice对手中粒子$\chi_j, A_j\ (j=1,\cdots,m)$执行$m$个广义Bell基测量. 粒子$\chi_1,\cdots,\chi_m$, $A_1,\cdots,D_1,\cdots$, $A_m,\cdots,D_m$所组成复合系统量子态可改写为

$$\begin{aligned}|\Psi\rangle = &\sum_{\substack{r_1,s_1,\cdots,r_m,s_m\\ j_1,j_{12},\cdots,j_m,j_{m2}=0}}^{d-1} e^{-\frac{2\pi i}{d}j_1s_1}\cdots e^{-\frac{2\pi i}{d}j_ms_m}\alpha_{j_1\cdots j_m}\\ &\times\beta_{j_1\oplus r_1}\cdots\beta_{j_m\oplus r_m}|\varphi_{r_1s_1}\rangle_{\chi_1A_1}\cdots\\ &\times|\varphi_{r_ms_m}\rangle_{\chi_mA_m}|j_1\oplus r_1\oplus j_{12},j_{12},j_{12}\rangle_{B_1C_1D_1}\cdots\\ &\times|j_m\oplus r_m\oplus j_{m2},j_{m2},j_{m2}\rangle_{B_mC_mD_m}.\end{aligned}\tag{22}$$

若Alice对粒子$\chi_j, A_j\ (j=1,\cdots,m)$的广义Bell基测量结果为$|\varphi_{r_ms_m}\rangle_{\chi_mA_m}\ (r_m,s_m=0,\cdots,d-1)$, 则剩余粒子$B_1, C_1, D_1,\cdots,B_m, C_m, D_m$处于状态(未归一化):

$$\begin{aligned}|\Phi\rangle = &\sum_{j_1,j_{12},\cdots,j_m,j_{m2}=0}^{d-1} e^{-\frac{2\pi i}{d}j_1s_1}\cdots e^{-\frac{2\pi i}{d}j_ms_m}\\ &\times\alpha_{j_1\cdots j_m}\beta_{j_1\oplus r_1}\cdots\beta_{j_m\oplus r_m}\\ &\times|j_1\oplus r_1\oplus j_{12},j_{12},j_{12}\rangle_{B_1C_1D_1}\cdots\\ &\times|j_m\oplus r_m\oplus j_{m2},j_{m2},j_{m2}\rangle_{B_mC_mD_m}.\end{aligned}\tag{23}$$

与任意单粒子qudit态分等级可控隐形传态类似, 高等级接收方Bob只需依据控制方Charlie和David其中任一控制方的测量结果以及发送方Alice的测量结果即可选取执行相应的局域幺正操作. 若Charlie和David手中纠缠粒子$C_l, D_l\ (l=1,\cdots,m)$的Z基测量结果为$|j_{l2}\rangle_C, |j_{l2}\rangle_D\ (l=1,2,\cdots,m)$, 则剩余粒子$B_1,\cdots,B_m$的状态为(未归一化)

$$\begin{aligned}|\phi\rangle = &\sum_{j_1,\cdots,j_m=0}^{d-1} e^{-\frac{2\pi i}{d}j_1s_1}\cdots e^{-\frac{2\pi i}{d}j_ms_m}\alpha_{j_1\cdots j_m}\beta_{j_1\oplus r_1}\cdots\beta_{j_m\oplus r_m}\\ &\times|j_1\oplus r_1\oplus j_{12}\rangle_{B_1}\cdots|j_m\oplus r_m\oplus j_{m2}\rangle_{B_m}.\end{aligned}\tag{24}$$

Bob只需依据控制方Charlie和David其中任一控制方的测量结果$|j_{l2}\rangle\ (l=1,2,\cdots,m)$以及发送方Alice的测量结果$|\varphi_{r_ms_m}\rangle_{\chi_mA_m}\ (r_m,s_m=0,\cdots,d-1)$即可选取执行相应的局域幺正操作:

$$\begin{aligned}U_r = &\sum_{l_1,\cdots,l_m=0}^{d-1} e^{\frac{2\pi i}{d}l_1s_1}\cdots e^{\frac{2\pi i}{d}l_ms_m}|l_1,\cdots,l_m\rangle\\ &\times\langle l_1\oplus r_1\oplus j_{12},\cdots,l_m\oplus r_m\oplus j_{m2}|.\end{aligned}\tag{25}$$

将粒子$B_1,\cdots,B_m$状态转化为对应量子态(未归一化):

$$\begin{aligned}|\phi_1\rangle &= U_r|\phi\rangle\\ &= \sum_{j_1,\cdots,j_m=0}^{d-1}\alpha_{j_1\cdots j_m}\beta_{j_1\oplus r_1}\cdots\beta_{j_m\oplus r_m}|j_1\cdots j_m\rangle_{B_1\cdots Bm}.\end{aligned}\tag{26}$$

Bob引入初态为$|0\rangle_a$附加粒子a, 在对手中粒子$B_1,\cdots,B_m$, a执行联合幺正演化后, 对附加粒子执行Z基测量即可概率重建原未知量子态. 在基$\{|j_1,\cdots,j_m\rangle_{B_1,\cdots,B_m}|0\rangle_a;\ |j_1,\cdots,j_m\rangle_{B_1,\cdots,B_m}|1\rangle_a\}\ (j_1,\cdots,j_m=0,\cdots,d-1)$下, 联合幺正演化$U_{\max}$可表示为[26]

$$U_{\max}=\begin{pmatrix}\Gamma_{0\cdots0} & 0 & \cdots & 0 & -\sqrt{1-\Gamma_{0\cdots0}^2} & 0 & \cdots & 0\\ 0 & \Gamma_{0\cdots1} & \cdots & 0 & 0 & -\sqrt{1-\Gamma_{0\cdots1}^2} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \Gamma_{d-1,\cdots,d-1} & 0 & 0 & \cdots & -\sqrt{1-\Gamma_{d-1,\cdots d-1}^2}\\ \sqrt{1-\Gamma_{0\cdots0}^2} & 0 & \cdots & 0 & \Gamma_{0\cdots0} & 0 & \cdots & 0\\ 0 & \sqrt{1-\Gamma_{0\cdots1}^2} & \cdots & 0 & 0 & \Gamma_{0\cdots1} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \sqrt{1-\Gamma_{d-1,\cdots d-1}^2} & 0 & 0 & \cdots & \Gamma_{d-1,\cdots,d-1}\end{pmatrix},\tag{27}$$

其中, $\Gamma_{j_1\cdots j_m}=\dfrac{\beta_n^m}{\beta_{j_1\oplus r_1}\cdots\beta_{j_m\oplus r_m}},(j_1,\cdots,j_m=0,\cdots,d-1)$.

联合幺正演化可将粒子$B_1,\cdots,B_m$, a所组成复合系统状态转化为相应量子态:

$$\begin{aligned}&U_{\max}|\phi_1\rangle_{B_1\cdots B_m}|0\rangle_a\\&=\sum_{j_1,\cdots,j_m=0}^{d-1}\alpha_{j_1\cdots j_m}\beta_{j_1\oplus r_1}\cdots\beta_{j_m\oplus r_m}|j_1\cdots j_m\rangle_{B_1\cdots B_m}\\&\times\left(\frac{\beta_n^m}{\beta_{j_1\oplus r_1}\cdots\beta_{j_m\oplus r_m}}|0\rangle+\sqrt{1-\left(\frac{\beta_n^m}{\beta_{j_1\oplus r_1}\cdots\beta_{j_m\oplus r_m}}\right)^2}|1\rangle\right)_a.\end{aligned}\tag{28}$$

联合幺正演化后, Bob对附加粒子执行Z基测量, 若测量结果为$|0\rangle_a$, 则粒子$B_1,\cdots,B_m$状态转化为原未知量子态$|\chi\rangle_{B_1\cdots B_m}$, 分等级量子可控隐形传态成功; 否则隐形传态失败.

若通讯方选择由低等级接收方Charlie (David)重建原量子态, 则Charlie (David)需已知所有控制方Bob和David (Charlie)的测量结果以及发送方的测量结果, 才能选取对应的局域幺正操作, 完成原未知量子态的重建. 不失一般性, 我们假设通讯方选取Charlie为接收方完成原量子态重建. 与任意单粒子qudit态分等级量子可控隐形传态类似, 发送方Alice执行广义Bell基测量后, 控制方Bob对手中粒子执行Z基测量, David对手中纠缠粒子执行广义X基测量, Charlie依据所有控制方、发送方的测量结果选择执行相应的局域幺正操作, 通过引入附加粒子和执行联合幺正演化, 概率地重建原未知量子态.

发送方执行广义Bell基测量后, 测量结果为$|\varphi_{r_m s_m}\rangle_{\chi_m A_m}$ $(r_m,s_m=0,\cdots,d-1)$, 剩余粒子B_1, C_1, D_1, ⋯, B_m,C_m,D_m处于状态$|\Phi\rangle$. 控制方Bob对手中粒子$B_1,\cdots,B_m$执行Z基测量. 若Z基测量结果为$|t_1\rangle_{B_1},\cdots,|t_m\rangle_{B_m}$ $(t_1,\cdots,t_m=0,\cdots,d-1)$, 粒子$C_1$, D_1, ⋯, C_m, D_m处于状态:

$$\begin{aligned}|\xi\rangle=&\sum_{j_1,\cdots,j_m=0}^{d-1}e^{-\frac{2\pi i}{d}j_1s_1}\cdots e^{-\frac{2\pi i}{d}j_ms_m}\alpha_{j_1\cdots j_m}\beta_{j_1\oplus r_1}\cdots\beta_{j_m\oplus r_m}\\&\times|t_1\oplus(d-(j_1\oplus r_1)),t_1\oplus(d-(j_1\oplus r_1))\rangle_{C_1D_1}\cdots\\&|t_m\oplus(d-(j_m\oplus r_m)),t_m\oplus(d-(j_m\oplus r_m))\rangle_{C_mD_m}.\end{aligned}\tag{29}$$

控制方David对手中纠缠粒子$D_1,\cdots,D_m$执行广义X基测量. 若David广义X基测量结果为$k_1,\cdots,k_m$ $(k_1,\cdots,k_m=0,\cdots,d-1)$, 则剩余粒子$C_1,\cdots,C_m$状态为

$$\begin{aligned}|\phi'\rangle=&\sum_{j_1,\cdots,j_m=0}^{d-1}e^{-\frac{2\pi i}{d}j_1(s_1-k_1)}\cdots\\&\times e^{-\frac{2\pi i}{d}j_m(s_m-k_m)}\alpha_{j_1\cdots j_m}\beta_{j_1\oplus r_1}\cdots\beta_{j_m\oplus r_m}\\&\times|t_1\oplus(d-(j_1\oplus r_1))\rangle_{C_1}\cdots|t_m\oplus(d-(j_m\oplus r_m))\rangle_{C_m}.\end{aligned}\tag{30}$$

接收方Charlie需依据所有控制方的测量结果$|t_j\rangle_{B_j},|k_j\rangle_{D_j}$ $(j=1,\cdots,m)$以及发送方的测量结果$|\varphi_{r_js_j}\rangle_{\chi_jA_j}$选取相应的局域幺正操作:

$$\begin{aligned}U_r'=&\sum_{l_1,\cdots,l_m=0}^{d-1}e^{\frac{2\pi i}{d}(s_1-k_1)}\cdots e^{\frac{2\pi i}{d}(s_m-k_m)}|l_1,\cdots,l_m\rangle\\&\times\langle t_1\oplus(d-(l_1\oplus r_1)),\cdots,t_m\oplus(d-(l_m\oplus r_m))|.\end{aligned}\tag{31}$$

将粒子$C_1,\cdots,C_m$状态转化为量子态:

$$\begin{aligned}|\phi_1\rangle&=U_r'|\phi'\rangle\\&=\sum_{j_1,\cdots,j_m=0}^{d-1}\alpha_{j_1\cdots j_m}\beta_{j_1\oplus r_1}\cdots\beta_{j_m\oplus r_m}|j_1\cdots j_m\rangle_{C_1,\cdots,C_m}.\end{aligned}\tag{32}$$

通过引入初态为$|0\rangle_{aux}$的附加量子比特a, 对粒子$C_1,\cdots,C_m$, a执行联合幺正演化$U_{\max}$, 并对附加粒子执行Z基测量, 接收方Charlie可概率完成原量子态重建. 与概率隐形传态类似, 量子分等级可控隐形传态成功率为$d^m|\beta_n|^{2m}$, 其中

$$|\beta_n|^2=\min\left\{|\beta_0|^2,|\beta_1|^2,\cdots,|\beta_{d-1}|^2\right\}.$$

4 结论

量子通信实际应用中, 最大纠缠信道为理想状况. 由于受噪声环境影响, 最大纠缠信道往往演化为部分纠缠信道. 基于部分纠缠信道的分等级量子可控隐形传态考虑了部分纠缠信道对可控隐形传态隐形, 通过附加量子比特和联合幺正演化消除部分纠缠信道对分

等级量子可控隐形传态的影响，与最大纠缠信道下的分等级量子可控隐形传态相比，具有可行性强的优点. 此外，已有的分等级量子可控隐形传态大都是基于二维量子系统的任意单量子比特态的可控隐形传态，没有考虑基于高维纠缠系统的高维多粒子态的分等级可控隐形传态. 在远程量子通信中, 高维量子系统可携带更多信息. 与基于二维量子系统的量子态分等级可控隐形传态相比，基于高维量子系统的量子态分等级可控隐形传态可携带更多信息, 具有信道容量高的优点.

当系数$\beta_0=\cdots=\beta_{d-1}=\frac{1}{\sqrt{d}}$，量子纠缠信道由多粒子最大纠缠态组成. 与文献[54,55]类似, 所有通信方合作可实现确定的分等级任意高维多粒子态可控隐形传态. 演化矩阵$U_{\max}$为单位矩阵$I_{2d^m\times 2d^m}$，即接收方不需要对手中纠缠粒子和附加粒子a执行联合幺正演化, 接收方依据控制方与发送方的测量结果选择对手中粒子执行相应的单粒子操作即可完成原未知量子态重建.

本文提出了基于部分纠缠信道的任意m粒子qudit态分等级可控隐形传态方案. 发送方对手中粒子执行m个广义Bell基测量, 通过与发送方Alice合作, 高等级接收方(Bob)只需已知其他两个控制方Charlie和David中任一控制方的测量结果即可完成原未知量子态重建; 低等级接收方Charlie (David)则需已知所有控制方Bob和David (Charlie)的测量结果才能选择执行相应的局域幺正演化, 完成原未知量子态重建. 接收方通过引入附加量子比特，执行联合幺正演化以及对附加量子比特执行Z基测量，完成基于部分纠缠信道下的任意高维多粒子态概率分等级可控隐形传态. 由于采用部分纠缠信道完成高维多粒子态概率分等级可控隐形传态, 方案具有信道容量高、可行性强的优点.

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Probabilistic hierarchically controlled teleportation of an arbitrary *m*-qudit state with a pure entangled quantum channel

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Quantum teleportation is a unique quantum information processing method in quantum communication. Researchers have shown considerable interest in hierarchically controlled teleportation by using quantum entangled states as the quantum channel. Protocols for the hierarchically controlled teleportation of an arbitrary single-qubit via a four-qubit χ state and a four-qubit cluster state have been proposed. We present a protocol for the hierarchically controlled teleportation of an arbitrary *m*-qudit (*d*-dimensional quantum system) state by using a partially entangled quantum state as the quantum channel. The sender performs *m* generalized Bell-state measurements on her 2*m* particles. To reconstruct the original state, the upper-grade receiver only needs to apply a unitary operation in accordance with one of the receivers' measurement results, and the lower-grade receivers need to perform a unitary operation according to all the other receivers' measurement results. The receivers can reconstruct the original state probabilistically by introducing an auxiliary qubit and performing a collective unitary transformation. The protocol has the advantage of having high channel capacity by using a high-dimensional entangled state as the quantum channel.

hierarchically controlled teleportation, high-dimensional quantum entanglement, auxiliary qubit, collective unitary transformation

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