基于晶格缺陷的谷极化三维弹性声子晶体的拓扑界面传播特性

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摘要 晶格缺陷(包括旋错和位错等)广泛存在于各种材料,并呈现出优异的物理和力学性能.在经典波动体系,晶格缺陷态首先应用于二维光学系统,实现了晶格缺陷激发的谷极化界面态和束缚态.本文设计了一种三维弹性声子晶体,其单胞在第一布里渊区的 $K-H$ 方向线性简并.打破单胞的镜像对称性,该三维弹性声子晶体沿第一布里渊区 $K-H$ 方向的简并线打开而形成完全带隙,激发出谷极化量子霍尔效应.将晶格缺陷态引入具有谷极化量子霍尔效应的三维弹性声子晶体,晶格畸变导致单胞谷极化拓扑相反转而形成界面,实现了弹性波在三维弹性声子晶体的稳健界面传播.基于晶格缺陷的谷极化三维弹性声子晶体拓扑界面态的实现,突破了传统经典波动系统拓扑波导设计的局限性,为三维复杂拓扑波导器件设计提供了良好的技术支撑.

关键词 三维弹性声子晶体, 谷极化量子霍尔效应, 晶格缺陷, 拓扑界面态

近年来,声学超材料因其奇特的物理性质如负折射、亚波长成像、强反射等受到了广泛的关注^[1~5].随后,源于凝聚态物理的拓扑态成为声子晶体领域新的研究热点^[6~13].在二维经典波动体系,研究人员首先通过环形流场^[14,15]、旋转陀螺仪^[16~18]和时空调制^[19]等方法打破声子晶体的时间反演对称性,激发类量子霍尔效应,实现弹性波和声波的非互易单向传播.时间反演对称性破坏机制需要同步协调所有单元的周期性运动,其控制难度较大,并且单元运动的能量损耗和噪声使其应用场景有限.近期,研究人员通过打破波动系统的空间反演对称性来构造拓扑波导.一类以量子自旋霍尔效应为代表^[20~27],通过声子晶体的晶格变形,将布里渊区中心偶然的双狄拉克点打开,产生具有赝自旋特性的无带隙拓扑界面态.另一类以谷霍尔效应^[28~34]为代表,通过破坏声子晶体的镜像对称性,将布里渊区

角点的单狄拉克点打开,产生基于谷极化效应的无带隙拓扑界面态.上述拓扑声子晶体的拓扑相由拓扑不变量和体能带的贝利相位所决定,遵循体边对应原则.

广泛存在于各种材料的晶格缺陷使材料呈现出意想不到的物理和力学性能.近期,研究人员将晶格缺陷引入二维经典波动系统,研究晶格缺陷激发的拓扑界面态和束缚态.Wang等人^[35]将晶格旋错和位错引入谷极化光子晶体,晶格畸变诱发的谷极化拓扑相演变产生了类似于畴壁的拓扑界面,并实验观察到了光子拓扑界面态.Liu等人^[36]和Peterson等人^[37]在高阶拓扑非平庸蜂窝晶格中增加或者减少 60° 扇形结构,使其局部呈现五边形或者七边形对称性,并在结构的中心观察到受拓扑保护的束缚态.然而,截至目前,晶格缺陷所激发的拓扑态研究主要集中于二维经典波动系统,在三维波动系统还需进一步探索.本文设计了一种三维

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弹性声子晶体,通过打破单胞的镜像对称性,实现了谷极化量子霍尔效应.接着,将旋错和位错等晶体缺陷引入谷极化三维弹性声子晶体,通过单胞谷极化拓扑相的演变构造了开环波导,实现了弹性波的稳健界面传播.不同于传统拓扑波导,晶格缺陷激发的拓扑波导不需要闭环或贯穿整个域以完全界定两类具有不同拓扑相的结构,拓展了拓扑波导的设计思路,可为新型三维拓扑调控器件设计提供技术支撑.

1 三维弹性声子晶体的几何模型和单胞能带

1.1 三维弹性声子晶体的几何结构

图1(a)为三维弹性声子晶体的整体结构,其中单胞(最小的周期性结构单元)的晶格常数 $a=34\text{ mm}$,用菱形虚线框标记.单胞以六角晶格形式在 xy 平面周期性排列,并沿 z 轴叠加.图1(b)为三维弹性声子晶体单胞的放大图,单胞的高度参数 $H_0=6.3\text{ mm}$, $H_1=15.3\text{ mm}$.图1(c)为单胞俯视图,其中 z 方向的连接杆半径 $r=3.5\text{ mm}$,A和B圆柱的半径分别标记为 R_a 和 R_b ,A和B圆柱之间连接杆的宽度 $W=2.8\text{ mm}$.该三维弹性声子晶体的材质为铝,杨氏模量 $E=68.9\text{ GPa}$,密度 $\rho=2700\text{ kg/m}^3$,泊松比 $\nu=0.33$.

1.2 单胞的能带结构

如果A和B圆柱的半径相等,即 $R_a=R_b=8.45\text{ mm}$ 时,三维弹性声子晶体单胞的能带结构如图2(a)所示.由于单胞在 xy 平面具有 C_{6v} 对称性,因此单胞的能带在第一布里渊区沿K-H方向线性简并,如图2(a)中菱形虚线框所示.改变A和B圆柱的半径,使得 $R_a=7.67\text{ mm}$ 和 $R_b=9.23\text{ mm}$ (记为I型单胞),此时,由于单胞在 xy 平面的镜像对称性被打破,因此K-H方向的简并线将被打开,形成频率范围为98.2~110.02 kHz的完全带隙,如图2(b)所示.带隙能带在布里渊区的K点分别用 p 和 q 标记,其数值分别为94.058和110.42 kHz.当 $R_a=9.23\text{ mm}$ 和 $R_b=7.67\text{ mm}$ (记为II型单胞)时,三维弹性声子晶体的单胞具有完全相同的带隙.用“+”和“-”表示谷涡旋的逆时针和顺时针旋转方向.图2(c)左侧表示I型单胞($R_a=7.67\text{ mm}$ 和 $R_b=9.23\text{ mm}$)布里渊区K点对应的谷涡旋模态,上能带谷态为 q^+ ,下能带谷态为 p^- .图2(c)右侧表示II型单胞($R_a=9.23\text{ mm}$ 和 $R_b=7.67\text{ mm}$)布里渊区K点对应的谷涡旋模态,上能带谷态为 p^- ,下能带谷态为 q^+ .从图2(c)可知,当 R_a 和 R_b 发生交换时,位于左侧的上下谷涡

旋模态反转为右侧下上谷涡旋模态,这意味着谷霍尔相变.事实上,沿K-H方向上的所有点都有谷涡旋模态反转的现象.图2(d)表示单胞的第一布里渊区,其中三棱柱表示三维弹性声子晶体的最小不可约布里渊区.

为了量化描述布里渊区K点的谷霍尔相变过程,引入非零谷陈数.由 $k\cdot p$ 微扰理论^[38]推导可得,当 $k_z=0$ 时,有效哈密顿量^[39]可以表示为

$$\delta H_m = v_D \delta k_x \delta_x + v_D \delta k_y \delta_y + \beta \delta_R^2 \delta_z, \quad (1)$$

其中, v_D 是 $R_a=R_b=8.45\text{ mm}$ 时形成简并狄拉克点的群速度, $\delta_{i=x,y,z}$ 表示不同赝自旋的泡利矩阵, $\delta k = (\delta k_x, \delta k_y)$ 是动量空间中偏离K点的动量.打破晶格镜像对称性,形成了带宽为 $|2\beta \delta_R^2|$ 的带隙,其中 $\delta_R^2 = R_a^2 - R_b^2$, β 表征带隙打开系数.因此,当 $k_z=0$ 时,通过哈密顿量 δH 可得到非平庸的贝利曲线^[40]:

$$\Omega(\delta k) = \frac{1}{2} \beta \delta_R^2 \left[\delta_k^2 + (\beta \delta_R^2 / v_D)^2 \right]^{-3/2} \frac{1}{v_D}. \quad (2)$$

在布里渊区内对其积分可以得到非零拓扑电荷:

$$C_K = \frac{1}{2\pi} \int \Omega_K(\delta k) d^2 k = \text{sgn}(2\beta \delta_R^2) / 2. \quad (3)$$

对于K'谷,由时间反演对称性可得 $C_{K'} = -C_K$,因此量子化的谷陈数 $|C_V| = |C_K - C_{K'}| = 1$.对于一个不同谷霍尔绝缘体组成的界面,两侧的拓扑荷的变化为 $|\Delta C_V| = 1$,也就意味着会产生手性的界面传输态.

2 基于晶格旋错的谷极化三维弹性声子晶体的界面态

2.1 基于晶格旋错的谷极化三维弹性声子晶体的五棱柱模型

在图3(a)所示谷极化三维弹性声子晶体正六棱柱模型的基础上,删除一个 $2\pi/6$ 扇区(用阴影区域表示),并将扇区两侧的点阵重新连接,可得到如图3(b)所示三维弹性声子晶体的五棱柱模型.注意,在五棱柱模型和正六棱柱模型中, xy 平面内连接梁的长度相等.从图3(b)可以看出,三维弹性声子晶体的五棱柱模型具有一个局部界面,如虚线所示.界面上部单胞的圆柱的几何参数为 $R_a=7.67\text{ mm}$ 和 $R_b=9.23\text{ mm}$,界面下部单胞的圆柱的几何参数为 $R_a=9.23\text{ mm}$ 和 $R_b=7.67\text{ mm}$.缺陷促使单胞由原来的I型($R_a=7.67\text{ mm}$ 和 $R_b=9.23\text{ mm}$)逐渐演变

图 1 三维弹性声子晶体结构示意图。(a) 整体结构；(b) 三维弹性声子晶体单胞放大图；(c) 三维弹性声子晶体单胞俯视图
Figure 1 Schematic diagram of the three-dimensional elastic phononic crystal. (a) The overall structure; (b) enlarged view of the unit cell of the three-dimensional elastic phononic crystal; (c) vertical view of the unit cell of the three-dimensional elastic phononic crystal

图 2 三维弹性声子晶体原胞能带结构图。(a) 单胞半径 $R_a=R_b=8.45$ mm, 能带沿K-H方向形成两重简并线；(b) 单胞半径 $R_a=7.67$ mm和 $R_b=9.23$ mm, K-H方向简并线打开形成完全带隙；(c) 单胞半径 $R_a=7.67$ mm、 $R_b=9.23$ mm和单胞半径 $R_a=9.23$ mm、 $R_b=7.67$ mm两种情况下, 布里渊区K点对应的谷涡旋模式；(d) 第一布里渊区示意图

Figure 2 Band structure of the unit cell of the three-dimensional elastic phononic crystal. (a) Band structure of the unit cell with $R_a=R_b=8.45$ mm. A two-fold nodal line along the K-H direction is formed. (b) Band structure of the unit cell with $R_a=7.67$ mm and $R_b=9.23$ mm. The two-fold nodal line along the K-H direction is lifted, yielding a complete bandgap. (c) The valley vortex features of these two cases with $R_a=7.67$ mm, $R_b=9.23$ mm and $R_a=9.23$ mm, $R_b=7.67$ mm at the K point of the first Brillouin zone. (d) Schematic of the first Brillouin zone

图3 基于晶格旋错的谷极化三维弹性声子晶体。(a) 三维弹性声子晶体的六棱柱模型, 阴影区域为 $2\pi/6$ 扇区; (b) 三维弹性声子晶体五棱柱模型
Figure 3 Valley-polarized three-dimensional elastic phononic crystals induced by disclinations. (a) The hexagonal prism model of the three-dimensional elastic phononic crystal, and the shaded area indicates a $2\pi/6$ sector; (b) pentaprism model of the three-dimensional elastic phononic crystal

为II型($R_a=9.23$ mm和 $R_b=7.67$ mm), 即局部界面两侧的单元具有不同的谷极化, 如图2(c)。因此, 从I型单元到II型单元的转变中, 能带经历了打开-闭合-打开的过程, 虽然局部界面上部I型单元和下部II型单元具有相似的能带结构, 但其能带所对应的模式完全相反, 发生能带反转现象, 意味着谷霍尔相变。这表明, 基于晶格旋错的局部界面等价于由两种具有相反相位解耦的晶格组成的传统谷极化界面。不同于一般的体边对应原则, 晶格畸变诱发的谷极化拓扑相演变产生了类似于畴壁的拓扑界面, 在该界面可实现受拓扑保护的谷极化界面态^[41~48]。

2.2 谷极化三维弹性声子晶体五棱柱模型的本征态

为研究谷极化三维弹性声子晶体五棱柱模型界面的本征态, 对其本征方程进行全波数值模拟。图4(a)为带隙附近的本征频率。在带隙范围内, 谷极化三维弹性声子晶体五棱柱模型具有表面模式(虚线框所示)和谷极化界面模式(实线框所示)。图4(c)~(e)分别为谷极化三维弹性声子晶体五棱柱模型的界面模式、表面模式和体模式的本征位移场。图4(c)显示, 在界面模式中, 弹性波动能量主要集中在基于晶格旋错的谷极化界面, 而界面以外的区域, 弹性波动能量几乎为0。在带隙外, 弹性波能量分布于整个模型, 以体态的形式存在, 如图4(e)。作为对比, 我们还分析了三维弹性声子晶体六棱柱模型的特征频率。如图4(b)所示, 三维弹性声子晶体六棱柱模型在其带隙内只有表面模式(虚线框所示), 而

没有界面模式。这是因为谷极化三维弹性声子晶体六棱柱模型全部由I型单元组成, 并没有形成拓扑界面; 而谷极化三维弹性声子晶体五棱柱模型的晶格畸变导致I型单元逐渐演化为II型单元, 并最终形成了拓扑界面。因此, 不同于传统谷极化界面的闭环或者贯穿整个域以完成界定两类具有不同拓扑相的单元, 本文所提的基于旋错缺陷的拓扑界面高度可设计, 可在结构的任意位置终止, 不需要形成闭环。

具有K-H方向线性简并的三维弹性声子晶体的五棱柱模型如图5(a)所示。该模型中, 单元内A和B圆柱的半径相等, $R_a=R_b=8.45$ mm, 具有镜像对称性。该模型的本征频率如图5(b)所示。由于该模型单元的本征频率沿K-H方向线性简并, 未形成完全带隙, 因此该模型的本征模式在简并频率附近具有圆点所示的体模式, 而没有谷极化三维弹性声子晶体五棱柱模型的带隙, 以及带隙内的表面模式和界面模式。如图5(c)所示, 在体模式下, 该模型的弹性波动能量散布于整个声子晶体, 未形成能量集中的波导。

2.3 谷极化三维弹性声子晶体五棱柱模型的拓扑界面传输特性

为了验证谷极化三维弹性声子晶体五棱柱模型的拓扑界面传输特性, 本节在模型的中心施加频率为109.6 kHz的点激励, 如图6(a)所示。图6(c)显示, 弹性波主要沿着拓扑界面稳健地传播, 在偏离拓扑界面的方向呈指数衰减。为进一步研究弹性波沿拓扑界面传播

图 4 三维弹性声子晶体的特征频率与位移场分布图. (a) 谷极化三维弹性声子晶体五棱柱模型的本征频率. 实线框内的圆点表示界面模态, 虚线框内的圆点表示表面模态, 其余的点表示体模态. (b) 谷极化三维弹性声子晶体六棱柱模型的本征频率. 虚线框内的圆点表示表面模态, 其余的点表示体模态. 谷极化三维弹性声子晶体五棱柱模型的界面模态(c)、表面模态(d)和体模态(e)的本征模拟位移剖面图

Figure 4 Eigenfrequencies and the simulated displacement field profiles of the three-dimensional elastic phononic crystal. (a) Eigenfrequency spectra of the valley-polarized pentagonal elastic phononic crystal. Dots in the solid line box denote interface eigenfrequencies, dots in the dotted line box denote surface eigenfrequencies, and the rest dots denote bulk eigenfrequencies. (b) Eigenfrequency spectra of the valley-polarized hexagonal elastic phononic crystal. Dots in the dotted line box denote surface eigenfrequencies, and the rest dots denote bulk eigenfrequencies. The simulated displacement field profiles of the interface (c), surface (d) and bulk (e) states of the valley-polarized pentagonal three-dimensional elastic phononic crystal

图 5 镜像对称三维弹性声子晶体特征频率和位移场分布. (a) 镜像对称三维弹性声子晶体五棱柱模型. (b) 镜像对称三维弹性声子晶体五棱柱模型的本征频率. 圆点表示模型的体模态. (c) 镜像对称三维弹性声子晶体五棱柱模型沿K-H方向线性简并能带所对应的体模态位移场分布

Figure 5 Eigenfrequencies and the simulated displacement field profiles of the three-dimensional elastic phononic crystal with reflection-symmetry. (a) The pentagonal three-dimensional elastic phononic crystal with reflection-symmetry. (b) Eigenfrequencies of the pentagonal three-dimensional elastic phononic crystal with reflection-symmetry. Dots denote bulk eigenfrequencies. (c) The simulated displacement field profiles of the pentagonal three-dimensional elastic phononic crystal with reflection-symmetry for the bulk eigenfrequency near the degenerated line along the K-H direction

的稳健性, 在图6(b)中设计了带有拐角的传播路径, 并在模型中心处施加频率为109.6 kHz的激励. 如图6(d)所

示, 弹性波绕过拐角, 沿着拓扑界面高效地传播, 犹如直线界面. 这表明, 基于晶格旋错的谷极化三维弹性声

图 6 基于晶格旋错的谷极化三维弹性声子晶体拓扑界面传输特性. 基于晶格旋错的谷极化三维弹性声子晶体的直线界面波导(a)和带有拐角的界面波导(b)示意图; 直线界面波导(c)和带有拐角的界面波导(d)在109.6 kHz激励下弹性波传播的位移场图

Figure 6 Topological interface propagation characteristics of valley-polarized three-dimensional elastic phononic crystals induced by lattice disclinations. Schematic diagrams of the linear interface (a) and the curve interface (b) of valley-polarized three-dimensional elastic phononic crystals induced by lattice disclinations. Displacement field distributions of elastic waves along the linear interface (c) and the curve interface (d) excited by the point source with a frequency of 109.6 kHz

子晶体的拓扑界面传播特性受拓扑保护, 对拐角等引起的弹性波散射和反射等具有良好的免疫力。

3 基于晶格位错缺陷的谷极化三维弹性声子晶体的拓扑界面传输

本节基于谷极化三维弹性声子晶体设计了一个由晶格位错缺陷诱导的拓扑界面。如图7(a)所示, 谷极化三维弹性声子晶体左侧的一个六边形单胞的一个“原子”嵌入右侧的一个六边形单胞, 其中左侧丢失一个“原子”的六边形单胞转换为五边形单胞, 而右侧新嵌入一个“原子”的六边形单胞转换为七边形单胞。在五边形单胞和七边形单胞之间有了一个界面。由于晶格

畸变, 位于界面上部的I型单胞逐渐演变为界面下部的II型单胞。由于I型单胞和II型单胞具有相反的谷极化拓扑相, 因此该拓扑界面是受拓扑保护的。在拓扑界面左侧的五边形单胞施加频率为109.6 kHz的激励, 弹性波沿拓扑界面从五边形单胞高效地传向七边形单胞; 而偏离拓扑界面的方向, 弹性波动能量呈指数衰减, 如图7(c)所示。为了进行比较, 在没有拓扑缺陷的弹性声子板上设计了传统的(非拓扑)波导, 方法是沿着期望路径置换质量, 使得平庸界面两侧也是蓝色的“声原子”。图7(b)为具有平庸界面的三维弹性声子晶体结构几何示意图。在图7(b)中五角星的部分施加频率为109.6 kHz的激励, 发现在平凡的三维弹性声子晶体中, 弹性波能量

图 7 三维弹性声子晶体拓扑与平凡界面传输特性. (a) 基于晶格位错的谷极化三维弹性声子晶体的拓扑界面示意图; (b) 具有平凡波导的三维弹性声子晶体几何结构示意图; (c) 拓扑界面波导在109.6 kHz激励下弹性波传播的位移场; (d) 平凡界面波导在109.6 kHz激励下弹性波传播的位移场

Figure 7 Topological and trivial interface propagation characteristics of the three-dimensional elastic phononic crystals. (a) Schematic diagram of the valley-polarized three-dimensional elastic phononic crystal with topological interfaces induced by lattice dislocations; (b) schematic diagram of the three-dimensional elastic phononic crystal with a trivial waveguide; (c) displacement field of elastic waves along the topological interface excited by the point source with a frequency of 109.6 kHz; (d) the displacement field of elastic waves along the trivial interface excited by the point source with a frequency of 109.6 kHz

只集中在激励处,不能沿着缺陷界面连续传播,弹性波的传输被很好地阻隔,如图7(d)所示.这表明,基于晶格位错的谷极化三维弹性声子晶体的拓扑界面具有良好的传输特性.由于晶格位错可发生于声子晶体的任意位置,因此晶格位错缺陷极大地拓展了三维弹性声子晶体拓扑波导的设计思路.

4 结论

本文将晶格缺陷引入谷极化三维弹性声子晶体,晶格畸变引起的谷极化单胞演化最终形成了受拓扑保护的界面态.沿拓扑界面,弹性波能高效、稳健地传播,对拐角等引起的弹性波散射和反射具有很强的免疫力.传统拓扑界面由具有不同拓扑相的两类单胞构造而成,界面需贯穿整个域或形成闭环.然而,基于晶格缺陷的界面由晶格畸变引起的单胞拓扑相演变而形成,不需

要贯穿整个域或形成闭环,具有较大的设计自由度.

我们的工作从二维开始,将基于晶格缺陷的拓扑界面态范围扩展到三维.与最近实现的二维拓扑缺陷态相比,这种由板状单元堆叠的三维弹性拓扑声子晶体沿着垂直方向上是周期性的,允许在多个工作平面上工作和在复杂的工作条件下运行.因此,开发一种基于晶格缺陷的高维度弹性拓扑声子晶体是非常理想的,三维拓扑缺陷态的实现突破了二维系统的局限,丰富了高维度拓扑态的内容.

虽然本文的研究工作聚焦于谷极化三维弹性声子晶体,但是晶格缺陷激发的拓扑界面态对进一步研究三维非堆叠结构的拓扑缺陷态有重要意义,可拓展到其他波动系统,如三维声学系统和三维光学系统等,以指导声波和光波波导设计,为发展下一代三维经典波动调控器件提供良好的技术支持.

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Summary for “基于晶格缺陷的谷极化三维弹性声子晶体的拓扑界面传播特性”

Topological interface propagation characteristics of valley-polarized three-dimensional elastic phononic crystals induced by lattice defects

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Lattice defects (including disclinations and dislocations) widely exist in various materials, showing excellent physical and mechanical properties. In classical wave systems, lattice defect states were first applied to the two-dimensional optical system. The valley-polarized topological phase evolution induced by lattice deformation results in the formation of a topological interface equivalent to the domain wall, which is distinct from the general bulk-edge correspondence principle. It is capable of realizing valley-polarized topological interface states. However, until now, the majority of the research on topological states induced by lattice defects has concentrated on two-dimensional classical wave systems, and more research on three-dimensional wave systems is needed.

Specifically, we present the design of a three-dimensional elastic phononic crystal in this paper. When the radii of the sublattices A and B are the same ($R_a=R_b=8.45$ mm), a two-fold nodal line along the K-H direction is formed. However, when the sublattices A and B have radii of 7.67 and 9.23 mm, respectively, the reflection-symmetry of the unit cell is broken, and the two-fold nodal line along the K-H direction is lifted, resulting in the formation of a complete band gap, which is accompanied by the valley-polarization. When the lattice disclination is introduced into the three-dimensional elastic phononic crystal with valley-polarization, the numerical results of band structures and eigenfrequencies show that the topologically protected interface state will be generated within the band gap. On the contrary, the hexagonal three-dimensional elastic phononic crystal without lattice disclinations has no interface state. At the same time, the pentagonal three-dimensional elastic phononic crystal with reflection-symmetry has only bulk modes near the degenerate line along the K-H direction. The generation of the topological interface of the pentagonal three-dimensional elastic phononic is due to the opposite topological phases of the unit cell crossing the interface.

Following that, we investigate the topological interface transmission properties of three-dimensional elastic phononic crystals induced by lattice defects. It is discovered that elastic wave energy can be transported steadily along an interface, regardless of whether the interface is a linear interface or a curve interface. Thus, the valley-polarized interface states induced by lattice disclinations exhibit an excellent immunity to the scattering and back-reflection of elastic waves, as demonstrated by this work. For valley-polarized interface states induced by lattice dislocations, even though the structure is severely deformed everywhere, elastic wave energy still efficiently propagates along the interface, and elastic wave energy diminishes exponentially away from the topological interface. On the contrary, in the three-dimensional elastic phononic crystal with a trivial waveguide, the elastic wave energy is only concentrated at the point of excitation and cannot be transported continuously along the interface. This demonstrates that the topological interface of valley-polarized three-dimensional elastic phononic crystals induced by lattice defects has excellent transmission efficiency.

The realization of valley-polarized topological interface states induced by lattice defects breaks through the limitation of classical wave systems in a single two-dimensional plane. And it realizes the robust transmission of elastic wave energy along the three-dimensional topological interface. The three-dimensional elastic topological phononic crystal proposed in this research is periodic along the vertical direction, which allows it to operate on multiple working planes. Therefore, the functional devices built on the basis of three-dimensional stacking structures can be suitable for multi-dimensional working conditions, which provide an excellent technological platform for the design of three-dimensional complex topological waveguide devices.

three-dimensional elastic phononic crystals, valley-polarized quantum Hall effect, lattice defects, topological interface states

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