Chinese Annals of History of Science and Technology **2** (1), 001–033 (2018)
doi: 10.3724/SP.J.1461.2018.01001

The Early Stage of the Professionalization and Institutionalization of Mathematics in Late Nineteenth-Century China[1]

***Tian Miao* 田淼**[2]

(*Institute for the History of Natural Sciences*, *Chinese Academy of Sciences*, *Beijing* 100190, *China*)

Abstract: In the late nineteenth century, stimulated by the demand for modern military technology, mathematics research and education in China developed rapidly, resulting in the emergence of the first generation of professional mathematicians. This marks the starting point for the professionalization of mathematics in China, a process which this paper sets out to analyze. The author first provides a brief survey of the status of Chinese mathematicians prior to 1860. Then, the paper analyzes the social and political context of mathematics, focusing on the improvement of the civil service examination system. Finally, the paper studies the development of mathematics education in the late nineteenth century, leading to the conclusion that mathematics and Chinese mathematicians were professionalized in three major ways: (1) some mathematicians could earn a stable livelihood from their mathematical competence; (2) the social position of mathematicians improved; and (3) the social status of the specialty of mathematics was enhanced.

Keywords: professionalization of mathematician, mathematics education, late nineteenth century

Professionalization is a key factor in the formation of modern science; hence, the context of its emergence is an important issue in the history of science. A number of outstanding mathematicians feature in the long history of mathematics in China, but the first generation of professional mathematicians began to form in the second half of the nineteenth century. What was the context for this formation process? How did these mathematicians differ from their predecessors? These are the basic issues that this paper intends to address.

1 This paper has been copyedited by John Moffett and Charlie N. Zaharoff.

2 Research interests: History of Mathematics and history of mechanics. Email: miaotian17@hotmail.com

1 A survey of the status of mathematicians before the second half of the nineteenth century

To explore the process of the professionalization of Chinese mathematicians in the late nineteenth century, we need first to survey the status of mathematicians prior to this period.

Historically, there were few mathematicians making a living from mathematical research and education. With the exception of those working in the Imperial Astronomical Bureau 欽天監, most mathematicians were officials, such as Qin Jiushao 秦九韶 (1208–1261) and Yang Hui 楊輝 (the thirteenth century), or successful Confucian scholars, such as Li Ye 李冶 (1192–1279).[3] They were interested in mathematics, but had other occupations. So little documentation survives about some other mathematicians, such as Zhu Shijie 朱世傑 (the late thirteenth century to the early fourteenth century), that despite their great contributions to mathematics, we snatch only glimpses of their lives from limited information in the prefaces to their books. In general, mathematical research was insufficient to bring them a stable income, high social status, and general social respect. Even though some of them, such as Zhu Shijie, had a kind of mathematical education, their expertise and careers were not inherited by their students.[4]

In the seventeenth century, with the support of Emperor Kangxi (1654–1722),

3 Qin Jiushao held different kinds of official positions, such as county official in charge of security, prefecture official in charge of law, and prefecture chief. In 1247, he finished the *Shushu jiuzhang* 數書九章 (Book of mathematics in nine chapters), in which he developed the method of numerical solution of high equations with one unknown, and Dayan Method 大衍術 (Qian 1966; Yan 1987; Li 1987). Li Ye was a famous scholar in his time. He passed the imperial civil service examination in 1230 and was one of the first six Hanlin Academicians 翰林學士 appointed by Kubla Khan 忽必烈 (1215–1294) in the Yuan dynasty. For most of his life, he served as a Confucian teacher in the Fenglongshan Academy 封龍山書院. His two mathematical books, *Ceyuan haijing* 測圓海鏡 (Sea-mirror of the circle measurements) and *Yigu yanduan* 益古演段 (Calculation of pieces of figures for the development of ancient mathematics), were both posthumously published in 1282. He mentions that he worked on mathematics without paying mind to the ridicule of others (Mei 1966). In the *Ceyuan haijing*, Li Ye produced a systematic study on the numerical relationship among circle, right-angled-triangle, and squares using Tianyuan algebra. In *Yigu yanduan*, he explained Tianyuan algebra using geometrical methods.

4 Zhu Shijie published his *Siyua yujian* 四元玉鑑 (Jade mirror of the four elements) in 1303. In this he developed the numerical solution of high equations with four unknowns 四元術 and the sum of finite series 垛積術, which represented the peak of mathematics in China. These methods were not further developed, however, and could not be understood until the 1820s, hundreds of years later. There are no documents handed down to us about Zhu Shijie's life except a few sentences written by Mo Ruo 莫若 and Zu Yi 祖頤 in the preface and postscript to the *Siyuan yujian*. They both mention that Zhu Shijie was famous for mathematics and many students followed him, but since his mathematical works were neither mentioned nor understood for the next 200 years, it is safe to say that his study had not been carried on by his students (Zu 1303; Du 1966; Tian 1999b).

mathematics drew the attention of more officials and scholars (Jami 2012). The most famous mathematician, Mei Wending 梅文鼎 (1633–1721), was invited by officials to teach mathematics in their homes. But such positions were rare, and were influenced by the vagaries of the emperor's attitude toward mathematics and the interest of particular officials (Han 1996). Also in the seventeenth century, some Confucian scholars began to concentrate on the rediscovery of ancient works, and in order to understand the mathematical parts, they also advocated the study of mathematics. In the eighteenth century, mathematics was commonly accepted as part of Confucian scholarship among those scholars practicing evidential scholarship (考據學).[5] Thus, some scholars vigorously proposed research into and study of mathematics, offering a mathematics curriculum in the educational institutions they established. In this context, the first Chinese collection of biographies of mathematicians and astronomers, *Chouren zhuan* 疇人傳 (Biographies of astronomers and mathematicians) was compiled and published in 1810 (Tian 2005, chap. 3, 134–161). Such evidence shows that in the eighteenth and early nineteenth centuries Chinese mathematicians did in some ways achieve their highest position in Chinese history up to that time. Nevertheless, this does not mean that they were already professionalized. In fact, those educational institutions offering mathematics education did not employ mathematics specialists as teachers, thus failing to ensure a basic income for mathematicians at that time. For example, in 1801, the most influential supporter of mathematics, Ruan Yuan 阮元 (1764–1849), established Gujing Jingshe 詁經精舍 (The refined lodge for the exegesis of the classics) in Hangzhou. Besides the study of Confucian classics, Ruan Yuan included mathematics, astronomy, and geography in its curriculum. However, he followed the normal organizational model of traditional academies, which means that he invited a famous successful scholar, Sun Xingyan 孫星衍 (1753–1818), to be the sole master. Li Rui 李銳 (1769–1817), whose talent for mathematics was widely appreciated, was not invited, even though he had helped Ruan Yuan to compile the *Chouren zhuan* and edit mathematical texts (Yan 1990). The reason may be that even though Yuan Ruan supported mathematics, mathematics education was for him only a part of the training of an erudite Confucian. Thus, a specialist in mathematics who had failed at the civil service examinations, such as Li Rui, definitely did not meet his requirements.[6]

5 About evidential scholarship, see Elman 1984.

6 Sun Xingyan achieved great success in the Imperial examination, winning second place (*bangyan* 榜眼 in Chinese) in 1787. He got into the Imperial Academy, and later was appointed to official positions. He quit these positions in 1811 to concentrate on editing ancient books and teaching. His research covered almost all aspects of evidential scholarship, so he was an ideal figure for Ruan Yuan.

Generally, though mathematicians received more respect than their predecessors, they still had to turn to other occupations for their livelihood. For example, Jiao Xun 焦循 (1763–1820), Wang Lai 汪萊 (1768–1813), Shen Qinpei 沈欽裴, and Xiang Mingda 項名達 (1789–1850) all worked as Confucian teachers;[7] Zhang Dunren 張敦仁 and Xu Youren 徐有壬 (1800–1860) served as officials. Li Rui participated in the reconstruction of many ancient books and was involved with the rediscovery of almost all the early lost mathematical methods, but he was still unable to meet his basic living expenses from these endeavors, and had to rely on the support of Ruan Yuan and Zhang Dunren (Horng and Liu 1993); Dong Youcheng 董祐誠 (1791–1823) "traveled across a large part of country to make a living, even though he had the ability to serve society" (Li 1830).[8]

7 Jiao Xun was an erudite scholar. His research on the *Yi jing* 易經 (Book of changes) and *Mengzi* 孟子 was much appreciated among his contemporaries (Xu [1938] 1990, chap. 120, 1a). His mathematical works mainly focus on the reconstruction and transmission of thirteenth- and fourteenth-century Chinese mathematical achievements. In his *Jiajian chengchu shi* 加減乘除釋 (An explanation on the four arithmetical operations), he established an abstract system of arithmetical operations. Jiao Xun passed the provincial level examination, but never held an official position. From 1787 on, he mainly lived on teaching in different households and family-established schools, except the periods during 1795–1796 and 1800–1803, when he served as Ruan Yuan's assistant. From the preface to his *Jiajian chengchu shi*, we know that though he taught his students mathematics, it is certain that his main topic of teaching was still Confucian learning (Liu 2005; Tian 2005, 161–165). Wang Lai was a representative of Western mathematics at the end of the eighteenth century and the early nineteenth century in China. He not only understood the equation theories introduced into China, but also systematically developed them. In 1807, he began to teach Confucian learning in official schools until his death, except for a period around 1809, during which he served as an editor of the astronomical parts of the historical record of the Qing dynasty (Luo 1840; Li 1992; Tian 2005, 265–271). Xu Youren passed the civil service examination in 1829, and was then appointed to various official positions. He died during the Taiping Rebellion when he was the governor of Jiangsu Province. He made contributions to analytical series and the reconstruction of the fourteenth-century Chinese mathematical achievements concerning numerical solutions of high equations with four unknowns (Li 1995, 275–277; Tian 1999b).

8 "方立負經世才，衣食奔走，足跡半天下。"

There are no obvious changes in this pattern up to 1860. Mathematicians still aimed to pass the civil service examinations while they were young and had to shoulder other social responsibilities.[9] Some of our leading protagonists in the following sections had already appeared on the stage by this time. Li Shanlan 李善蘭 (1811–1882) had completed most of his mathematical publications (Wang 1990; Horng 1991a; Martzloff 1992).[10] He was helping the American Protestant A. Wylie (1815–1887) to translate the last nine chapters of Euclid's *Elements*, A. D. Morgan's *Elements of Algebra*, and E. Loomis' *Elements of Analytical Geometry and of the Differential and Integral Calculus* into Chinese (Wang 1990). At the same time, he served as Xu Youren's assistant, hoping that Xu might help him to obtain an official position by donating money to the government

9 Based on analyses of the *Chouren zhuan* and the community of mathematicians and astronomers in the eighteenth and early nineteenth centuries, Jonathan Porter concluded: "By the late seventeenth century, scientific activity in China evinced many characteristics of a continuous and systematic social activity. . . . The value of science—particularly mathematics and astronomy—was increasingly appreciated apart from its role in symbolic legitimation of the emperor. With the differentiation of scientific from humanistic knowledge, practitioners of mathematical sciences now increasingly perceived them as a distinct and autonomous activity based on independent and objective criteria of validity. . . . On the other hand, as a group of Chinese scientists constituted a cohesive community. Scientific activity exhibited a systematic pattern of interaction which reinforced the identity of its members with an intellectual community and promoted the development of scientific knowledge. The scientist's self-image was further enhanced by his sense of participation in a common enterprise. A strong sense of the continuity and cumulative nature of scientific knowledge prevailed throughout the scientific community" (Porter 1982, 542–543). Such arguments suggest that from the late seventeenth century on, science, especially mathematics, already bore some institutional characteristics. I partly agree with his summary. In my understanding, even the publication of a book such as *Chouren zhuan* shows that Chinese scholars already accepted the idea that mathematics could be an autonomous enterprise. Nevertheless, the argument of its main author, Ruan Yuan, is that mathematics is an important part of Confucian scholarship. Not only he, but most of the other scholars who produced so-called scientific publications in Porter's text, including mathematicians like Li Rui and Jiao Xun, all clearly declared such an idea. Mathematics may have been regarded as a branch of Confucian scholarship, yet books on intellectual history, such as the *Song Yuan xue'an* 宋元學案 (Biographies of scholars in the Song and Yuan dynasties) and *Mingru xue'an* 明儒學案 (Biographies of Ming Confucians), excluded mathematicians, suggesting that a work such as *Chouren zhuan* was regarded as a complement to intellectual history. This can be seen even more clearly in context: though scholarly study became more detailed and subdivided in the eighteenth century, at the same time all of those involved believed that their work was part of the same framework of Confucian scholarship. In this sense, we cannot conclude that mathematicians were self-consciously building their roles outside of the community of Confucian, or in Porter's term, humanist studies (Tian 2005, chap. 3, 134–161).

10 Li Shanlan was regarded as the best Chinese mathematician of the late Qing period. His work covers almost all the most active areas of study in China. About him, see Horng 1991a and Wang 1990.

in the future (Tang 1987, 117).[11] Liu Yicheng 劉彝程, as the son of the governor in charge of education in Guangdong Province, assisted his father Liu Xizai 劉熙載 and concentrated on mathematics learning and research (Tian 1992). If the development mode of Chinese traditional society had been maintained, the lives of those people would not have been greatly different from those of Li Rui, Wang Lai, and Xia Luanxiang. However, unexpected changes in the social and political context of China in the late nineteenth century altered their careers.

2 The social and political context of mathematics in late nineteenth-century China

In 1842, China lost the Opium War (First Anglo-Chinese War) against Britain, and five Chinese ports were forced open for foreign trade. Besides the establishment of a new international commercial and political relationship, a new wave of the transmission of modern science and technology broke over China. Protestant missionaries arrived in greater numbers, and brought with them not only their religious propaganda, but also new knowledge. Symbolic algebra and calculus were transmitted to China in 1859. In search of new mathematical knowledge, Li Shanlan followed this transmission closely.[12]

From 1840 onwards, some officials and scholars, such as Lin Zexu 林則徐 (1785–1850) and Wei Yuan 魏源 (1794–1857), came to learn Western science, technology, politics, and culture, and advocated "beating foreign countries with skills learned from them."[13] For almost twenty years, however, this appeal showed no obvious sign of influencing the government, nor of being accepted by scholars more widely. Twenty years later, the foundations of the Qing government were shaken by the Second Opium War (Anglo-French expedition to China) and the massive Taiping Rebellion. In 1860, when the British-French army captured the capital, the Chinese Emperor fled to Rehe 熱河, dying there the following year. This event had far-reaching effects in China. In 1860, the so-called "Self-Strengthening Movement" began and

11 In his diary on May 14, 1859, Wang Tao 王韜 records: "Now, as Mr. Junqing 君青 (Xu Youren) is still here, I do not have any request. When he leaves this position, I will ask him to save some money and donate it (to the government). I would be pleased if I can have a position as county or prefectural official" (今君青先生在此，予絕不干求，待其任滿時，請其為予攢資報捐，得一州縣官足矣).

12 From 1859 onwards, modern mathematics was systematically transmitted to China. Before the 1910s, Protestant missionaries played a key role in this transmission. Apart from translating mathematics books, they taught mathematics in Protestant schools. From the 1920s, Chinese mathematicians came back from Japan, America, and Europe to undertake the transmission of mathematics independently. As this paper focuses on the professionalization of Chinese mathematicians, I will not dwell on this issue (Bennett 1967; Fisher 1911; Wang 1999).

13 "師夷之長技以制夷。"

lasted for about forty years. Attributing the inferiority of China to the backwardness of its military technology, leaders of the early Self-Strengthening Movement agreed that the most important way to achieve their aims was to learn advanced Western military technologies and cultivate scientific and technical talent. "Adopting Western knowledge requires the study of mathematics, as mathematics is the origin of all Western knowledge and is learned by all Western people from the age of ten" (Feng [1861] 2002, 56).[14] Since studying mathematics might serve the country, this idea exerted wide influence on Chinese Confucians, as one core ideology of Confucianism was that people should serve society. In 1860, a young man, Liu Guangfen 刘光蕡 (1843–1903), was attending the civil service examination. His son later described his experience as follows:

> In the year Gengshen (1860) of the reign of Emperor Xianfeng, when the Anglo-French Army had just entered Beijing, he attended the county level civil service examination. Later, a treaty was signed and was posted in all provinces, prefectures, and counties. My father saw it, and then knew that Western people were rich and strong because their manufactured goods were excellent, and [that this] was based on mathematics. Thus, he made a determined effort to learn mathematics. (Liu 1915, 1a)[15]

This is the context in which mathematics came to be highlighted throughout the country. During the Self-Strengthening period, there were two main ways in which mathematics was promoted. One was to make mathematics an independent subject in the civil service examination system. The other was the development of mathematics education. It is in the latter context that Chinese mathematicians became professionalized. Nevertheless, the former will be my starting point, as it can provide us with a broad understanding of the context of the early stage of professionalization of mathematicians in China.

Historically, gaining an official position through the civil service examination system was seen as the only orthodox career for those in pursuit of knowledge, which meant that any educational path failing to offer official rank held little attraction to students. Thus, if mathematics could be included in the examination system, it would be a powerful stimulus for the development of mathematics and mathematics education. During the Self-Strengthening period, appeals for this were repeated again and again by scholars and officials. As early as 1843, Qi Gong 祁埙 (1777–1844), governor-general of Guangdong and Guangxi provinces, presented a memorial requesting a change in the examination system. In this, he suggested that history, military strategy, instrument manufacturing and mathematics, divination, and maps

---

14 "一切西學皆從算學出，西人十歲外無人不學算。今欲采西學，自不可不學算。"

15 "應童子試時，當咸豐庚申英法聯軍初入京師。後訂和約，張示直省州縣。府君見之，始知西人富強以製造精奇，原本算術。乃發憤治算學。"

should be the main subjects of the examination. His request was rejected by the Ministry of Rites (Ministry of Rites [1877] 1985). In 1861, Feng Guifen 馮桂芬 (1809–1874) suggested reform of the system and proposed that mathematics should be attached to the exam of the Confucian classics (Feng [1861] 2002, 38). In 1870, Heseri Hala Yinggui 郝舍里・英桂 (1821–1879), governor-general of Fujian and Zhejiang provinces, and Shen Baozhen 沈葆桢 (1820–1879), minister for Administration of Shipbuilding Affairs (總理船政大臣), presented a memorial to the emperor: "The strength of the navy depends on cannons and ships; the skillfulness of [the manufacture] of cannon and ships is based on mathematics."[16] Thus, they proposed to establish a special section on mathematics in the examination system (Heseri Hala and Shen [1870] 1986). A year later, mathematics was added to the additional exam of general knowledge. However, this additional exam was not decisive in the final evaluation. In 1874, the grand secretary of the Hall of Literary Glory (文華殿大學士), Li Hongzhang 李鴻章 (1823–1901), proposed the establishment of a section on foreign affairs in the examination.[17] In the same year, Pan Yantong 潘衍桐 (1841–1899), the director of students in the Imperial Academy (翰林院司業), proposed an examination section on arts for which persons proficient in manufacturing, mathematics, and maps all qualified as candidates (Pan [1874] 1986). In the same period, some scholars also wrote articles and books calling for the inclusion of mathematics in the examination (Tian 1997).

Finally, in 1877, the Ministry of Rites and Ministry of Foreign Affairs (總理各國事務衙門) held a discussion and consequently established a rule for selecting mathematical talent in the examination. This stipulated that

> For the mathematics candidates, besides the normal examination on the *Si shu* 四書 (Four books), ancient classics, poems, and essays, mathematical problems should be tested in the examination on ancient classics. If a candidate really knows mathematics, his examination paper should be sent to the Ministry of Foreign Affairs for reexamination and registration. In the year of the provincial level examination, in accordance with regulations, they should come to the Ministry of Foreign Affairs and be tested with problems on physics and mathematics, weapons manufacture, military methods (on land and sea), ships, cannons, and sea mines, or international law and history of foreign countries. Any who proves to be brilliant and knowledgeable will be sent to attend the examination in Shuntian prefecture. . . . If more than twenty candidates are selected, their examination papers should be stamped with "mathematics," and they should attend the same examination on poems, essays, etc., with other candidates. . . . From every twenty candidates, one should be selected. . . . If there are a great number of such examination

16 "水師之強弱，以炮船為宗，炮船之巧拙，以算學為本。"

17 Li was one of the main leaders of the Self-Strengthening Movement. For his role in the late nineteenth century, see Fairbank and Liu 1980, 243–250.

> papers, the total number of mathematics candidates selected should not exceed three. This is to make restrictions. . . . Any *juren* 舉人 (a successful candidate in the provincial level examination) from the section on mathematics should rejoin the common examination, and attend the imperial level examination with candidates from all provinces. Their selection will depend only on their essays. (Ministry of Rites [1877] 1985, chap. 13, 18–19)[18]

Could mathematics really be stimulated with such strict selection rules? Let us examine the result. In 1888, "via the county level examination, the Ministry of Foreign Affairs sent candidates to the Imperial Academy and the Tongwenguan for mathematics examinations. Among them, thirty–two were selected and sent to [the examination in Shuntian prefecture]. . . . Among them, one was selected [as *juren*]" (Aisin Gioro [1889] 1958).[19] According to Qin Shouzhang 秦綬章 (1849–1925), "In the following years, [mathematics candidates] for the county level examination all numbered less than twenty. Thus, they joined the common examination" (Qin [1896] 1899).[20]

Such a result is definitely disappointing for thirty years of effort. It seems that the improvement of the examination system by adding mathematics as an independent subject had almost no positive effect in stimulating the development of mathematics. Was such a process of no significance at all, however? A brief discussion on the characteristics of the examination system and Chinese traditional statecraft may throw some light on this problem. The fundamental purpose of the civil service examinations was the selection of officials, and the first requirement for an official was not his efficiency, but rather moral perfection in the framework of Confucian ethics. Thus, the examinations and education for them can be regarded as training in Confucian ethics. Even though the characteristics of the examinations became dogmatized over time, with an emphasis on the formulation of essays that seems to bear little relation to improving morality, the basic idea never changed. I do not want to dwell on this subject further, but merely want to stress one thing—that criticism of the system concentrated on the problem of whether such a dogmatized system was efficient for the selection of moral men. Modern scientific and technological knowledge, especially mathematics, had nothing to do with the

---

18 "報考算學者，除正場仍試以《四書》、經文、詩、策外，其考試經古場內另出算學題目，果能通曉演算法，即將原卷諮送總理各國事務衙門複勘註冊。俟鄉試之年，按冊諮取赴總理衙門，試以格物測算及機器製造、水陸軍法、船炮水雷，或公法條約、各國史事諸題，擇其明通者錄送順天鄉試。……如人數在二十名以上，統於卷面加印'算學'字樣，與通場士子一同試以詩、文、策問，無庸另出算學題目。……每于二十名额外取中一名。……卷數最多亦不得過三名，以示限制。……凡由算學中式之舉人，應仍歸大號，與各省士子合試，憑文取中。"

19 "鄉試，總理各國事務衙門將各省送到生監及同語言館學生試以算學題目，共錄送三十二人……取中一名。"

20 "此後曆科鄉試均以不滿二十名，散入大號。"

perfection of morality. Liu Guangfen, whom I have mentioned above, argued, "It is certain that learning how to write the 'Eight–Legged Essay' is useless. If one learns mathematics but does not apply it to machine manufacture, it is also useless. Furthermore, the essay still relates to humanity, justice, morality, and benevolence, while mathematics has no relation either to one's body and mind or to one's morality. If one indulges oneself in it, how could one be an intellectual?" (Liu 1915).[21] Liu's argument was even representative of enthusiastic promoters and practitioners of mathematics education (I shall return to this point later). Thus, the main challenge facing the improvement of the civil service examination system was how to deal with the relationship between one or more specific branches of knowledge and Confucian ethics—in this way it touched the core of the Chinese cultural and political system.

Against this background, let us return briefly to the selection rules for mathematics talent in 1877. Establishing mathematics as an independent subject in the civil service examination system was not analogous to adding a new branch, say information engineering, to a university curriculum in modern times. The 1877 rules can be regarded as a model for adapting traditional statecraft to the new requirements of society.[22] Even though from a modern perspective it seems deeply conservative, it denoted a starting point for the departure from traditional statecraft. It provides us with a valuable historical case study for the analysis of the process of change of a very well-established political and cultural system, and for observing interactions between science, technology, politics, and culture.

After defeat in the Sino-Japanese War of 1894 and the capture of the Chinese capital by the Eight-Nation Alliance forces in 1900, the traditional political and cultural system was shaken to its foundations, bringing a quick response in the reform of the examination system. In 1901, the "Eight-Legged Essay" examination format was

---

21 "習八股，誠無用，學算而不能制器亦畫餅也，且八股尚言仁義道德，算術不言身心，不知品行。沈溺無用，烏足為士人乎。"

22 Another attempt to adapt traditional statecraft to the new requirements of society was made by Aisin Gioro Yixin 愛新覺羅・奕訢 (1833–1898), one of the leaders of the Self-Strengthening Movement, in 1866. He suggested that successful examination candidates learn mathematics in the Tongwenguan in Beijing, and that such candidates should be given priority for promotions. This was bitterly opposed by Confucian officials, represented by Wesin Woren 烏齊格里・倭仁 (1804–1871). Even though Yixin's suggestion seems to have been accepted by the government (Yixin himself was the Prince Regent at that time), the fact was that no scholars with titles had ever enrolled in the Tongwenguan. On the contrary, outstanding students took the civil service examinations after they graduated from the Tongwenguan. This is understandable in the context of our discussion of the process of improvement of the examination system in this paper. This controversy is famous in the history of modern China. Most studies, however, criticize Woren's conservative arguments. Within the political and culture context of the late nineteenth century, I have provided a new interpretation of this case (Tian 2005, 199–205).

abandoned; Chinese politics and history, politics in foreign countries, and various branches of scientific and technological knowledge became the new content for the examination. At the same time, all traditional academies were required to transform into schools. In 1904, the civil service examination was terminated. The modern phase of the history of education in China began.[23] I want to argue in this paper, however, that the emergence of the first generation of professional mathematicians had already begun forty years prior to that time, and that this process played out in the context of the development of mathematics education.

3 Mathematics education in late nineteenth-century China

Mathematics education was not a new occurrence in China even in the 1860s. As one of the Six Arts, mathematics had occupied a position in education since the Zhou dynasty (1046–256 BCE). It had been a discipline in the Imperial College and the Imperial Astronomical Bureau since the Sui dynasty (581–618), when a fixed mode of Chinese traditional education took shape. During the Tang (618–907) and Song (960–1279) dynasties a program known as "Understanding Mathematics" (明算) was periodically included within the structure of the civil service examination system (Li [1933] 1998). However, the view that mathematical education was a vital part of traditional education is not accurate. I have mentioned above that Chinese traditional education basically focused on ethics and morality, the Confucian classics being the primary teaching content. Students ultimately strove to cultivate themselves, regulate their families, serve the government, and make the whole world tranquil and happy, giving rise to the opinion that a moral intellectual could become a good official. Even those who did not aim at success in the examinations and becoming an official promoted the view that "to study is to learn how to be a Confucian. Being a Confucian, the only thing one needs to do is to investigate the Confucian classics" (Yu 1898).[24] In such a cultural context, the content of the civil service examinations constituted the major part of both official and private education. Therefore, mathematics did not see much improvement in the Tang and Song dynasties even with periodic mathematical programs.[25] The royal educational institutions, such as the Imperial Astronomical Bureau, mainly cultivated specialists of astronomical and

23 Japan exerted great influence upon the structure of the first modern Chinese education system (Abe 1987). After 1904, the modern system for education and research on science and technology was established, and the professionalization and institutionalization of mathematics, together with science and technology, advanced rapidly.

24 "學者，學為儒而已；為儒無它，治經而已。"

25 A successful candidate in "Understanding Mathematics" in the Tang dynasty could only be appointed to a position under the associate ninth rank (從九品), while a successful *jinshi* candidate was usually appointed to a position higher than the seventh rank.

astrological observation and calculation. Some local educational institutions did cover mathematics. For example, Hu Yuan 胡瑗 (993–1059) taught the course "Understanding Numbers through Mathematics and the Calendar" (算曆以明數是也) in the branch of "Administration of State Affairs" (治事齋) in the Huzhou state-run academy (湖州府學) (Huang [1838] 1990). But such a course was highly dependent upon the institution's organizer. Some excellent mathematicians, like Yang Hui and Zhu Shijie, also participated in mathematics education. However, mathematical research was unable to bring them a stable income, high social status, and general social respect, with the result that their careers were not inherited by their students. Besides, in existing historical materials, there is no record of any student in an official or private school producing outstanding achievements in mathematics.

Mathematics education in the Qing dynasty made some progress. It was strengthened in imperial institutions, but that did not bring about marked improvement. Minggantu 明安圖 (1692–1725), Luo Shilin 羅士琳 (1789–1853), and Chen Jie 陳傑 were the only astronomy students famous for mathematics who graduated from the Imperial Astronomical Bureau and the Imperial Academy. Mathematical education did exist in some academies and private schools, such as the Yongshang Zhengren Academy 甬上證人書院 established by Huang Zongxi 黃宗羲 (1610–1695) and Zhangnan Academy 漳南書院 led by Yan Yuan 顏元 (1635–1704). In the eighteenth and early nineteenth centuries, scholars of evidential research undertook mathematical education in some academies established and mainly taught by them. These academies included the Ziyang Academy 紫陽書院 led by Qian Daxin 錢大昕 (1728–1804), Youjiao Academy 友教書院 led by Wang Chang 王昶 (1724–1894), Gujing Jingshe 詁經精舍 created by Ruan Yuan (1764–1818), Daliang Academy 大梁書院 led by Qian Yiji 錢儀吉 (1783–1850), Xiangshui Jiaojing Tang 湘水校經堂 created by Wu Rongguang 吳榮光 (1773–1843), and Jingyang Academy 涇陽書院 led by Li Zhaoluo 李兆洛 (1769–1841). All the organizers and teachers in these institutions were erudite scholars interested in mathematics; few of them, however, had made contributions in mathematics. Few students, except Li Rui and Tan Tai 談泰 in the Ziyang Academy, developed into experts in the field of mathematics. Furthermore, all these institutions eventually ceased mathematics education, and indeed most of them closed, after the organizer left the institution or position (Tian 1997, chap. 1). Thus, in the 1860s, when Liu Guangfen made up his mind to learn mathematics, there was nowhere for him to make a start. By chance he found a copy of the *Siyuan yujian* from a friend. This book was too difficult for a beginner, and he overworked himself, developing hematemesis (Liu 1915). Before the late nineteenth century, the success of outstanding mathematicians was mainly the result of auto-didacticism and private teaching.

From the 1860s on, there were mainly three types of institutions that began to have mathematics in their curriculum. One was official educational institutions for foreign languages and learning; the second, schools attached to arsenals.[26] These were both established by leaders of the Self-Strengthening Movement. The third was academies. Some were of the traditional sort, that is, they aimed at producing erudite scholars, while others represented a new type, in which mathematics education held the same position as the Confucian classics. The first institution for foreign languages and learning to include mathematics in its curriculum was the Shanghai Tongwenguan 上海同文館 (The School of Combined Learning in Shanghai) (later renamed as Shanghai Guangfangyan Guan 上海廣方言館 [Shanghai Institute of Various Languages]). It was established by Li Hongzhang in 1863, and offered a mathematics curriculum from the outset. In 1906, it was modified into a military technology school. During these thirty-three years it witnessed the engagement of Chen Yang 陳暘 (1806–1863), Shi Yuechun 時曰醇 (1807–1880), and Liu Yicheng as mathematics teachers and assistant teachers, and the cultivation of several outstanding mathematics students, such as Xi Gan 席淦, Wang Fengzao 汪鳳藻 (1851–1918), Yang Zhaoyun 楊兆鋆 (1854–?), Gong Jie 龔傑, and Liu Xuncheng 劉循程. All of them produced or were involved with mathematical publications. Xi Gan was later moved to Jingshi Tongwenguan 京師同文館 (The School of Combined Learning in Beijing), and became the mathematics teacher there (Tian 1997, chap. 2).

The Qiuzhi Academy 求志書院 was representative of the new type of academy, and the level of mathematics education there was considerably higher than other academies (ibid.). In this paper, I will not discuss the content of its mathematics education, but its structure. The Qiuzhi Academy was established by Feng Junguang 馮焌光 (1830–1878), governor of Shanghai. Feng died two years later in the academy, but it continued for another twenty years. Only after 1894, when its education system turned into that of the old style, did it close. In fact, there was something of a fashion that an official, especially a governor or an official in charge of education, established

26 All the arsenals established in the nineteenth century had an affiliated school and all such schools provided mathematics education. The Jiangnan arsenal played an important role in the translation and publication of scientific and technological books (Meng 1999). The education in the Foochow Navy Yard was strongly influenced by the system of French technological education, and could be regarded as a compressed French model in adapting to China's demand (Li 2018, 51–81). In addition, those arsenal schools sent students abroad. In 1886, among fourteen students sent by the Foochow Navy Yard, two enrolled in the École normale supérieure: one in the Department of Mathematics, and the other in the Department of Physics (Li and Martzloff 1998). Generally, mathematics education in schools affiliated with arsenals concentrated more on practical purposes. For a comprehensive understanding of the development of arsenals and manufactures in the late nineteenth century and the first half of the twentieth century, see Zhang 1992.

an academy, and it was very common that such academies closed after they left their official position. This is an important characteristic of such institutions that I will return to later.

The structure of the Qiuzhi Academy was quite different from traditional ones. It consisted of six parallel sections (*zhai* 齋),[27] covering the classics, history, anecdotes, essays and poetry, geography, and mathematics. Each section was headed by a *zhaizhang* 齋長 (master), and the master of mathematics section was Liu Yicheng. Traditional academies usually had only one master, who was in charge of education throughout the academy.


俞蔭甫先生評閱史學
鍾子勤先生評閱經學
高仲瀛先生評閱掌故之學
劉省庵先生評閱算學
張經甫先生評閱輿地之學
俞蔭甫先生評閱詞章之學

丙子夏課姓名錄
經學
超等第一名宗汝成江蘇蘇州府常熟縣學附生
第二名姚文枏江蘇松江府上海縣學附生
第三名朱逢甲江蘇松江府華亭縣學貢生
第四名錢國祥江蘇蘇州府學附生
第五名王履階江蘇松江府南匯縣學增生
特等第一名李慶恆江蘇太倉州鎮洋縣學廩生
第二名李經瑩江蘇蘇州府元和縣學優附生
求志書院夏課 姓名錄 一

第二名何四鋼江蘇松江府奉賢縣學附生
第三名黃致堯江蘇太倉州寶山縣學附生
算學
超等第一名沈善蒸浙江嘉興府桐鄉縣監生
特等第一名鄭興森浙江湖州府歸安縣學附生
壹等第一名方德華江蘇松江府青浦縣學附生
第二名宋慶雲江蘇松江府華亭縣學貢生
第三名郁晋培
第四名朱 鎮江蘇松江府華亭縣學貢生
求志書院夏課 姓名錄 三

Figure 1: Front page and page listing the authors of best papers of *Shanghai Qiuzhi shuyuan keyi* 上海求志書院課藝 (The collection of exam papers of the Qiuzhi Academy in summer of 1876) (Qing edition).

From the front page of *Shanghai Qiuzhi shuyuan keyi*, we can see that all the masters who reviewed the papers were listed parallel to one another. Among them, Yu Yue 俞樾 (courtesy name Yu Yinfu 俞蔭甫) (1821–1907) had successfully passed the imperial level civil service examination, and had held a position as the official in charge of education in Henan Province. The master of mathematics, Liu Yicheng, had never taken the civil service examination, however, and we have no further information about him,

27 *Zhai* 齋 means house or room. For example, a study in Chinese is called *shuzhai* 書齋, literally translated as book room. Some academies are also named *zhai*.

except that he had a famous father and published four mathematical books.[28] Yet in the Qiuzhi Academy, he held a position parallel to that of Yu Yue. Not only that, all the names of the successful attendees from each section had been marked in front of them with the name of the section they belonged to, were listed parallel to one another, and accepted honors and prizes at the same level. Thus, its structure bears some characteristics of modern departmental teaching.

Among the institutions providing mathematics education, the Jingshi Tongwenguan was definitely the most influential one. It was developed on the basis of the Russian language school established in 1757. In the process of negotiating with Western powers after the First Opium War, especially in reaching the Treaty of Tianjin after the Second Opium War, the chief Chinese leader for the negotiations in Tianjin, Prince Gong Aisin Gioro Yixin 愛新覺羅・奕訢 (1833–1898), realized that talents in Western languages were in urgent need. In response to his appeal, two branches were established, English and French. Hence, the Russian language school was expanded to the Jingshi

28 Liu Yicheng published four mathematical books, among which the most important is the *Jianyi'an suangao* 簡易庵算稿 (The mathematical text in Jianyi'an) (1898). This book is a collection of mathematical examination problems and answers written by Liu himself in the Qiuzhi Academy from 1876 to 1898. In this book, Liu included almost all the mathematical knowledge that Chinese mathematicians dealt with, including algebra, calculus, trigonometry, and logarithm, among others. An interesting point is that all his mathematical works were also presented in the book, including his two most significant achievements, the method for calculating the sum of finite series and the integral solution of right-angled-triangles, which related to the solution of second degree indefinite equations. The sum of finite series formed an independent branch in traditional mathematics in China, namely the Duoji method (垛積術, lit. method of the sum of piles). Until the very end of the nineteenth century, the method used in this branch remained traditional. It was in the *Jianyi'an suangao* that Liu developed the algebraic formulation of the Duoji method, which led to its modernization. Later, his students Zhang Xi 张熺 and Cui Chaoqing 崔朝慶 further developed Liu's algebraic formulation of the Duoji method, and provided algebraic proof of formulas in this branch (Tian 1992; Tian 2003). As a collection of answers to examination questions, the book unveils one characteristic of mathematics education in the second half of the nineteenth century: there remained no standard mathematics text books in China, and the content of the education was defined and limited by the capacity of mathematics teachers. Liu not only focused on his own research, but also served as the proof reviewer of the Chinese translation of William Wallace's "algebra" and "fluxions" in the eighth edition of *The Encyclopedia Britannica* as well as J. Hymens' *Treatise on Plane and Spherical Trigonometry*. Hua Hengfang 華蘅芳, the Chinese translator of these three books, mentioned that there were a great number of algebraic formulas in the *Weiji suyuan* 微積溯源 (Chinese version of "fluxions"), and Liu made great contributions to the verification of those formulas (Hua 1874) (All the three books were translated by John Fryer and Hua Hengfang, and published by the Jiangnan Arsenal. Concerning the translation of the three books and the publication by Jangnan Arsenal, see Bennett 1967 and Wright 1996). Thus, Liu acquired a wider scope in mathematics than most of his contemporaries. The content of his education encompassed traditional branches, such as the measurement of circles (relation between a circle and its inscribed and circumscribed right-angled-triangles), the method of Duoji, and the modern branches mentioned above.

Tongwenguan in 1862 (Aisin Gioro [1861] 1986).[29] In 1866, Yixin established an additional department of mathematics, the Suanxue Guan 算學館 (House of mathematics). In the regulations of this institution, Yixin considered in detail all kinds of administrative and educational problems, including sources of funding and the number and selection criteria for administrative leaders; the selection, promotion, and salary of teachers; the selection and nurturing of students and scholarships for them; the system of rewards and penalties; as well as examination methods (Zhu 1986). Such regulations guaranteed the continuity and coherence of the organization. From 1862 to 1901, when it was combined into Jingshi Daxuetang 京師大學堂 (Imperial University of Peking), there were nineteen principals (總管大臣, official for overall supervision), forty-two chief directors (提調), and three mathematics teachers. Despite some detailed changes, the general structure of the Jingshi Tongwengun remained the same (ibid.).

Figure 2: Li Shanlan and his mathematics class in Jingshi Tongwenguan (Thomson 1873).

Several sections concerning mathematics teachers and students are worth mentioning in more detail. In the regulations Yixin designed in 1862 and 1863, one finds the following statements:

29 The Shanghai Tongwenguan and the Guangdong Tongwenguan were both established following the model of Jingshi Tongwenguan in the two most important ports open to the West. For the negotiations and the treaties, see Fairbank 1978, 213–261.

> Every three years, officials in my ministry will carry out examinations to verify and select the students. As per previous regulations, the good students will be conferred official titles of the seventh, eighth, or ninth rank.[30] The inferior ones will be demoted and continue to study. . . . If an associate teacher is skillful in teaching and able to provide guidance, our ministry will also apply to promote him as secretary (主事) to wait in a ministry and be appointed to a position whenever there is a vacant one. (Aisin Gioro [1862] 1986, 9)[31]
>
> Each student in Tongwenguan will be given three *liang* silver as a scholarship every month. . . . Chinese teachers will have a salary of eight *liang* silver every month. (Aisin Gioro [1863] 1986, 10)[32]
>
> Every year, in the fourth month [of the lunar calendar] there will be an examination, called the seasonal examination; and in the tenth month, there will also be an examination, called the annual examination. . . . In the annual examination, for each department, there will be two first rank students and they will be given four *liang* as a prize; there will be three second rank students, each of whom will receive two *liang*. In the seasonal examination, [each department] will have two first rank students and three second rank students, and each will receive three *liang* and one *liang* five *qian* respectively. . . . The salary of an associate teacher is eighty *liang* annually; the salary of an official of the seventh rank is forty-five *liang* annually; the salary of an official of the eighth rank is forty *liang* annually; the salary of an official of the ninth rank is thirty-two *liang* five *qian* annually. (Aisin Gioro [1865] 1986, 11)[33]

For the students in the Department of Mathematics, Yixin had a further arrangement:

> In addition to their food prepared and provided by my ministry, they will be given a salary of ten *liang* each month as a subvention. This is so that they do not need to worry about their family and can be more absorbed in study.[34]

In sum, in the Jingshi Tongwenguan, a normal student could have a scholarship equivalent to 1/3 that of a county magistrate; an outstanding student might already have an official rank and salary. It is possible that a student in the Department of Mathematics could even earn a salary more than double that of a county magistrate. In

30 In China, the rank of a county magistrate was the seventh.

31 “每屆三年，由臣衙門堂官自行考試一次，核實甄別，按照舊例，優者授為七、八九品官等，劣者分別降革、留學。……同文館考取七品官複考一等，授為主事者，請仍准掣分各門行走，遇缺即補。至考試學生時，該助教等如果教導有方，亦應由臣衙門春天請以主事分部遇缺即補，仍兼館行走。”

32 “同文館學生，……每名每月撥給膏火銀三兩……漢教習每月薪水銀八兩。”

33 “每年四仲之月考試一次，謂之季考；每年十月考試一次，謂之歲考。……歲考一等，每館二名，每名四兩；二等三名，每名二兩。季考一等二名，每名三兩；二等三名，每名一兩五錢。……助教每年俸銀八十兩，七品官每年俸銀四十五兩，八品官每年俸銀四十兩，九品官每年俸銀三十二兩五錢。”

34 “除飯食由臣衙門備給外，每月仍各給薪水銀十兩，俾資津貼，庶內顧無憂而心益專一矣。” I am not sure whether this was put into effect. When Yixin made such an arrangement, he planned to enroll the successful candidates of the imperial level examination. Since no such candidates applied to study in the Tongwenguan, it is quite possible that this section was not put into effect. This has not been mentioned in the other materials I have seen.

addition to this income, they also had the opportunity to win a prize twice a year. A mathematics teacher could earn ninety-six *liang* per year, the equivalent of double a county magistrate's salary. With such incomes, mathematics teachers and students had sufficient financial support to concentrate on their mathematical study. The promotion of students and teachers in this way still bore the characteristics of the traditional idea that a good scholar should become an official. In fact, in 1898, mathematics teacher Xi Gan was promoted to become an official of the fourth rank as an alternate director in the Military Ministry. Nevertheless, such promotion opportunities were very attractive, meaning that through an education in the Jingshi Tongwenguan, a person could get a position much higher than a successful candidate in the imperial level civil service examination.[35] Returning to a case mentioned earlier, Li Rui, one of China's most important mathematicians, was unable to achieve success in the provincial level examination.

Here, I do not intend to argue that after 1860 all mathematicians throughout the country could earn a livelihood comparable to that of a teacher in the Jingshi Tongwenguan or Qiuzhi Academy, or that mathematics education was already popular all over China—this is not true. I have made a preliminary statistical study of the institutions that offered mathematical courses.[36] Before 1901, there were at least sixty-five institutions offering mathematics education, among which twenty-nine began their courses after 1894, the year China lost the Sino-Japanese War (Tian 1997, chap. 2). With the exception of the Jingshi Tongwenguan located in Beijing, most of these institutions were located in Shanghai (eight), Guangdong (seven), Hunan (twelve), Hubei (four), Jiangsu (ten), and Zhejiang (five) provinces, where the leaders of the Self-Strengthening Movement exerted strong influence. An interesting phenomenon is that among the five institutions providing mathematics education in Shaanxi Province, three were established by Liu Guangfen, whom I have mentioned previously and will mention again later. Thus, mathematics education was far from widespread across the whole country. To take another statistic, only forty-six mathematics teachers can be identified from that time (Tian 1998).[37]

In 1898, Emperor Guangxu decreed that all academies should be transformed into schools teaching both Chinese and Western knowledge. Even though this decree was soon rescinded by the Empress Dowager Cixi, it did exert influence on the modes of Chinese education, and mathematics gained more popularity as a result. With the

35 Normally, a successful candidate would be appointed to a position of the seventh rank.

36 It is impossible to carry out an absolutely comprehensive study. The information is scattered through all kind of materials, such as local gazetteers (地方志), prefaces to mathematical books, and other literature, biographies, officials' memorials, letters between scholars, and so on. I also think that there may have been some institutions never mentioned in sources at all.

37 For some institutions, I was only able to ascertain that mathematics education was conducted in them, but no other information. And several schools in arsenals hired Western teachers.

termination of the civil service examination system in 1905, the institutionalization of mathematical education was basically fulfilled.

Even though mathematics education before 1901 was very limited in size, we can still find three characteristics that distinguish it from what came before.

First of all, mathematics education in some institutions became independent in the late nineteenth century. In all the institutions mentioned above, mathematics was included in their curriculum when they were established. Thus, unlike previous institutions where mathematics was taught, in which its continued instruction relied on a master or the founder's personal interest, these institutes intentionally chose mathematicians to teach mathematics. In this format, teachers and students in the section of mathematics could all concentrate on mathematics.

Secondly, mathematics education was relatively stable. In such institutions, mathematics education continued despite the comings and goings of their establishers and teachers. This was the case not just in those institutions mentioned above, but in others as well, such as the Nanjing Academy 南菁書院. It was a completely traditional academy to begin with; its early masters, such as Zhang Wenhu 張文虎 (1808–1885) and Huang Yizhou 黄以周 (1828–1899), were all erudite scholars, and mathematics education was just a side subject both for the master and the academy. Nevertheless, their departure did not bring about the cessation of mathematics education in the institution. In fact, it was strengthened and became more independent when a former student from Qiuzhi Academy, Cui Chaoqing 崔朝慶, was hired as mathematics master in 1884 (Tian 1997, chap. 2).

Thirdly, mathematics education in the late nineteenth century became specialized. This characteristic was directly connected to the first. As mathematics education became more independent, it also became more specialized. It is undeniable that the advocacy of mathematical education by the leaders of the Self–Strengthening Movement, such as Yixin, Zeng Guofan, and Li Hongzhang, was mainly for the practical purpose of learning Western advanced military and civil science and technology on the basis of mathematical knowledge. This idea was especially influential in the mathematical schools attached to arsenals. Nevertheless, even education at the Shanghai Guangfangyan Guan and Jingshi Tongwenguan, which were directly established by leaders of the Self–Strengthening Movement, inevitably became specialized when mathematics education became independent and mathematicians acted as teachers. The examination papers of these institutions demonstrate such a trend clearly (ibid., chap. 3).

These characteristics show that mathematics education started to be institutionalized to a certain degree and in limited areas in late nineteenth-century China. Below I argue that the first generation of professional mathematicians, mathematics teachers, emerged in such a context in China.

4 The emergence of the first generation of professional mathematicians in China

4.1 Some mathematicians achieved economic self-sufficiency and stability

In the previous section, we discussed the income of mathematics teachers and students in the Jingshi Tongweguan. Now, we shall have a quick look at the salary of teachers and students in other institutions. In the Shanghai Guangfangyan Guan, the salary of a mathematics teacher was twenty *liang* silver per month, and that of an associate teacher eighteen *liang* per month (Li [1863] 1986, 218; Nie [1894] 1986, 242). In the Guangdong Tongwenguan, the salary of a general master was forty *liang* per month (Mao [1864] 1986). These institutions also provided a certain quota of allowances for students. Students in the Shanghai Guangfangyan Guan got four to six *liang* every month. We can gain a sense of their economic situation by comparing their income with the salary of officials in Table 1 (Huang and Chen 2012, 541):

Table 1: The annual salary of officials in the Qing dynasty

Official's salary	First rank, associate first rank	Second rank, associate second rank	Third rank, associate third rank	Fourth rank, associate fourth rank	Fifth rank, associate fifth rank	Sixth rank, associate sixth rank	Seventh rank, associate seventh rank	Eighth rank, associate eighth rank
	180 *liang*	155 *liang*	130 *liang*	105 *liang*	80 *liang*	60 *liang*	45 *liang*	40 *liang*

For comparison: a mathematics teacher in the Jingshi Tongwenguan earned annually 96 *liang*, and an associate teacher, 80 *liang*; a mathematics teacher in the Shanghai Guangfangyan Guan, 240 *liang*, and an associate teacher, 216 *liang*; and a master in the Guangdong Tongwenguan, 480 *liang*.[38] A normal student in the Jingshi Tongwenguan earned 36 *liang*; a normal student in the Shanghai Guangfangyan Guan, 48 *liang*; and a normal student in the Guangdong Tongwenguan, 36 *liang*.

As the founding of academies was greatly reliant on donations from founders and local people, the salary of teachers in academies varied across time and institutions, and is not well documented. It is difficult for us to analyze the standard of their income, but the fact that most masters in academies stayed in their posts for long periods suggests

38 Beginning in 1723, apart from their official salaries, officials received extra income called *yanglian yin* 養廉銀, meaning money for cultivating the virtue of incorruptibility among officials. The amount varied in terms of different positions. Thus, the income of mathematics teachers was not as high as shown in the main text relative to other positions. For example, the governor-general of Jiangsu Province, where the Shanghai Guangfangyan Guan was located, earned an extra 2,500 to 3,000 *liang* silver annually, and the governor-general in Guangdong Province, where the Guangdong Tongwenguan was located, earned an extra 1,500 to 2,000 *liang* annually (Huang and Chen 2012, 552).

that their income was sufficient at least to support them. As mathematics education in these institutions usually was stable, mathematicians in these institutions, at least forty-six in number according to my incomplete study, could make a reasonable living based on their study of mathematics.

Achieving stability of employment and a living wage for practitioners by solely working in one field is the basic and most important factor in the professionalization of a field of study. Thus, the base for the professionalization of mathematicians began to be established to a certain degree in the late nineteenth century.

4.2 The social status of mathematicians was enhanced in the late nineteenth century

With the inclusion of mathematics into Confucian Learning starting in the eighteenth century, there had already been an improvement in the social status of mathematicians. Those involved in evidential scholarship accepted that mathematicians were Confucian scholars specialized in a particular field. From the late nineteenth century on, Chinese mathematicians could already claim social status based on their mathematical competence alone—that is independent from the status of Confucian scholars.

In the institutes offering mathematics education, mathematics teachers obtained status equivalent to that of scholars of the Confucian classics and history. When introducing the Qiuzhi Academy, we mentioned that the mathematics teacher there, Liu Yicheng, a mathematician without any official rank or experience in the civil service examinations, had the same status as well-known Confucian scholars like Yu Yue, Sun Yirang 孫詒讓, and Sun Kengming 孫鏗鳴, teacher of two influential leaders of the Self-Strengthening Movement, Li Hongzhang and Shen Baozhen. Liu was not the only case. In 1879, Huang Binghou 黄炳垕 (1815–1893) was invited to be master of mathematics at the Bianzhi wenhui 辨志文會, becoming a colleague of the famous scholar Huang Yizhou, who was the master of Confucian classics there.

As recorded in *Shizaitang zayi* 世載堂雜憶 (Recollections of Shizaitang), at the opening ceremony of the Lianghu Academy 兩湖書院, Liang Dingfen 梁鼎芬 (1859–1919), the eastern supervisor of the institution, stood in the northern position with masters of the different sections, while Tan Jixun 譚繼恂 (1823–1901), governor-general of Hubei Province, led students in performing the kowtow to them in the south. Then Liang and the masters led the students to invite Tan to assume the northern position and returned the courtesy.[39] This vividly portrays the social

39 Kowtow physically means the performance of kneeling down and bowing one's head to the ground. This performance signifies the highest reverence for one's elders, superiors, and cultural and religious objects of worship in China. As for the meaning of direction in China, standing (or sitting) in north and facing the south signifies that one holds a higher position.

tendency of honoring teachers in Chinese tradition, since an official of such high rank as Tan Jixun kowtowed to teachers in an academy. A mathematics teacher at the Lianghu Academy named Hua Hengfang 華蘅芳 (1833–1902) received the same courtesy (Liu 1960, 48). It was the first time that mathematicians had been treated with such dignity in Chinese history. That mathematicians with no official title or success in Confucian studies like Liu Yicheng and Hua Hengfang could stand on the same level as famous Confucians demonstrates the fact of the acceptance of mathematics experts in the learned society of that era.

After the Sino-Japanese War, with the rapid development of new-style schools, especially in Jiangsu, Zhejiang, Hunan, Hubei, and Guangzhou, and with the inclusion of mathematics curricula in most academies and private educational institutions, mathematics teachers became urgently needed. Therefore, the status of mathematicians was further consolidated.

The development of mathematical education was not the sole engine for the rise of their social position. With the deepening of the Self-Strengthening Movement, mathematics, as the basis of Western science and technology, attracted attention from those in power and society as a whole, which naturally enhanced the social position of mathematicians.

I would like to conclude this section with a story that presents a picture of the situation at that time. A person named Zou Daoji 鄒道濟 described his experience in a letter to Wang Kangnian 汪康年 in 1897:[40]

> I am from Xijiang Anfu County. I learned nothing when I was a child, but was interested in science when I was young. . . . Two years ago, I arrived in the Guanzhong plain with totally empty pockets, and I begged by the road and slept in the Temple of Guanyu at night. On the first day of the tenth month in the lunar calendar,[41] governor of Shaanxi Province Yao Xinpu was burning incense in the Temple. Seeing a wretch in ragged cloths lying in the temple, he ordered his servant to drive me away.[42]

Zou then chanted a poem to describe his situation.

> The governor heard and sighed, saying: "This is a scholar, how could he be in such dire straits?" So, he employed me in his official residence. He asked me: "What skills do you

40 It was mentioned that the letter reached Wang Kangnian on the eighth day of the fifth lunar month, but there was no information showing the year in which this letter was written. In another document by Zou Daoji, he mentioned that he begged in 1895 in Shaanxi Province (Zou [n.d.] 1958). According to the content of this letter, it was written two years after that. Thus, I assume that it was written in 1897.

41 The first day of the tenth month in the lunar calendar is a special festival in China. On that day, people make offerings to their dead relatives by sending them clothing for the winter.

42 "鄙人籍錄西江安福縣，幼識之無，壯好格致。……前年行至關中，資斧斷絕，丐食於途，夜宿關帝廟。時逢十月朔，西儲憲姚馨圃焚香廟中，見有籃褸者伏臥於廊下，命役呵逐之。"

> have?" And I answered: "None. I am only good at dusting." So, he sent me to his son's study as a servant to dust. One day, the governor gave his son some mathematical problems. His son began learning mathematics with the *Jiuzhang suanshu* 九章算術 (Nine chapters of mathematical procedures), and the method for extraction of roots [in it] is cumbersome. I told him: "If you solve this problem using the algebraic method, seeking the right answer by supposing an unknown, you can obtain the equation, and use the Tianyuan method to extract the root. That would be simpler and more convenient." His son was surprised and said: "How can a dusting servant know mathematics?" Then he reported this to the governor. He tested my mathematics, and I give the right answers. The governor asked: "You know mathematics, do you also know science and technology?" I answered: "That is what I learned when I was young, but I only know a little, and am not proficient in it."[43]

In fact, this official had just begun to establish a section of science and technology in the Guanzhong Academy 關中書院, and Zou was invited directly to revise the students' scientific examination papers (Zou [1897] 1987). Moving from a beggar and servant with some mathematical and scientific knowledge to a teacher at the best-known academy in Shaanxi Province, Zou Daoji's experience dramatically shows the improvement of mathematicians' and scientists' social status around the end of the nineteenth century.

4.3 Chinese mathematicians sought to conduct specialized mathematics research in the late nineteenth century

In his article "Tongwenguan suanxue jiaoxi Li Shanlan" 同文館算學教習李善蘭 (Li Shanlan: Mathematics teacher in Tongwenguan), Horng Wann-sheng 洪萬生 traced the specialization of mathematicians to the eighteenth- and early nineteenth-century evidential scholarship tradition. He argued that it is in the context of the specialization of various branches of Confucian learning that mathematics became specialized to a certain standard (Horng 1991b). This argument is reasonable and convincing.

A leading scholar in evidential scholarship, Qian Daxin, argued that mathematics was a branch of solid learning in Confucianism; another, Ruan Yuan, argued a similar point. In the prefaces of mathematical books from around the turn of the nineteenth century, statements about the magical nature of mathematics—that it can help one to divine the future—are usually missing. In fact, the practical usage of mathematics was not commonly mentioned either. Instead, detailed introductions for a certain

43 "觀察聞而歎曰：此文士也。胡一寒至此？乃招入署中，問鄙人有何長技。鄙人答以一無所長，惟善掃除塵垢。遂命入公子書房，供洒掃役。一日，觀察以算學題課公子，公子系從《九章算術》入門，開方繁瑣。鄙人進言曰：此題若用代數術，以假求真，可得布算之法，以天元一術開方，則簡捷矣。公子駭異之曰：掃地夫亦諳算術耶？乃稟聞觀察。觀察面詢算術，對答不謬。觀察曰：既知算學，亦知格致之學否？鄙人對曰：幼所習也，但得其皮毛，未能精能耳。"

mathematical task or algorithm form the main content of such prefaces. As a mathematician aiming for new mathematical achievements, Wang Lai always "made clear what others had never said and what others could not say" (Luo 1840).[44] If we define mathematicians as those committing themselves to extend mathematical knowledge, some scholars in eighteenth- and early nineteenth-century China certainly fulfill such a definition. However, even if scholars focusing on mathematics could already achieve a certain respect for deepening this branch of Confucian learning, they could not secure stable employment or a livelihood from it. Take the case of Li Rui (mentioned above). It shows that pre-1860 mathematicians were not regarded as professional mathematicians. This is worth mentioning here because the first generation of professional mathematicians, including Li Shanlan, Hua Hengfang, Zou Daoji, and Liu Yicheng, all lived in such an environment, and they all had the inclination to be specialists in mathematics.

I have mentioned above several times that the leaders of the Self-Strengthening Movement supported mathematics for practical purposes, that is, they aimed to develop modern military and civil technology to achieve the ultimate goal of defending the empire and enriching the nation. However, once furnished with a stable livelihood, mathematicians were able to concentrate solely on mathematics. Several facts concerning Li Shanlan need to be mentioned here. One is that in 1859, he hoped that Xu Youren might help him to obtain an official position by donating money to the government. Secondly, in 1862, he made two cannons for Li Hongzhang (Li [1862] 1989, 184). Thirdly, he wrote *Qunjing suanxue kao* 群經算學考 (Evidential study on mathematics in the Confucian classics) in his youth. Thus, Li Shanlan's situation and role are not clear-cut, and we can understand his early career as that of a scholar who, beyond his interest in mathematics, was seeking all possible ways to make a living. Only later in his career did he become a mathematics teacher in the Tongwenguan, where it was easy for him to obtain an official position—in fact, one much higher than he had hoped for—in 1859.[45] In 1868, he came to Beijing and continued as a mathematics teacher. Subsequently, all his activities were limited to mathematics teaching and research, and his communications also became more limited to mathematical affairs (Horng 1991b). According to the chief master of the Tongwenguan, W. A. P. Martin (Ding Weiliang 丁韙良), Li Shanlan "taught his method [of mathematics] in the capital for eight years. He worked assiduously in the field, not even noticing old age creeping on. He really devoted all his heart and soul to this field" (Ding 1877).[46] While carrying out his educational duties, he published his *Kaoshu genfa*

44 "言人所未言，與人所不能言。"

45 See footnote 9 and the section in the regulations of the Tongwenguan concerning the promotion of teachers.

46 "在京授法於茲八載，維日孜孜求忘倦，不知老之將至。於斯道可謂殫心致志矣。"

考數根法 (Study on prime numbers), in which he developed the method for finding prime numbers.

Another leading mathematician of the time, Hua Hengfang, divided mathematics learners into two categories: “One aims at expounding the theory of mathematics and to compose books, and the other just calculates and applies methods for practical use” (Hua [1882] 1897, chap. 5, 14a–22b).[47] He argued, “The one aiming at calculation can only use the methods, while the one aiming at expounding theory can create methods” (ibid.).[48] He goes on to argue that “Mathematics is finer today but was crude in ancient times, is ingenious now but was clumsy in ancient times. Without the ones who aim at expounding theory, how could mathematics have been developed to this state? All other things are finite; only mathematics is infinite” (ibid.).[49] Clearly, what Hua Hengfang valued most were mathematicians who sought to extend research on mathematics. In addition, almost all the mathematics exercises in the book *Jianyi'an suangao* 簡易庵算稿, composed by Liu Yicheng, were about mathematics rather than military and technological practical use. And his proudest achievement was his research on the solution concerning the sum of finite series, which is a further development of Li Shanlan's method. The formula for the sum of the multiplication of two finite series offered by Liu Yicheng was of no practical value at that time (Tian 2003). Besides, as a mathematics teacher at Guangfangyan Guan, Liu Yicheng was supposed to maintain connections with officials of the Self-Strengthening Movement and some Western missionaries. Because of the status of his father, Liu Xizai, a famous scholar in the late Qing dynasty who once served as a governor in charge of education in Guangdong and interacted with a number of leaders of the Self-Strengthening Movement, Liu Yicheng was able to meet some active and powerful people. Yet, there is no material demonstrating that Liu participated in other activities beyond research on mathematics and served as a mathematics teacher.

Nurtured by mathematicians with a specialized mindset, some early mathematics graduates of the institutions offering mathematics education sought out more specialized work in the field. Hua Hengfang mentioned that he “met a person who just began to learn mathematics. He was young and impetuous, and every day indulged in mathematics. He was often lost in thought trying to deal with difficult parts, even forgetting to eat and sleep” (Hua [1882] 1897, chap. 5, 14b–15a).[50] Hua Shifang 華世芳 (1854–1906), an early student from the Qiuzhi Academy and Nanjing Academy, did outstandingly well in his study of the Confucian classics, history, and geography. In

47 “一為闡明數理以成著作，一為推演各數施之實用。”

48 “演數者只能用法，而明理者則能創法。”

49 “演算法古疏今密，古拙今巧，苟非明理而精益求精，安能至此乎？他事皆有止境，而算學無止境也。”

50 “嘗見有初學演算法之人，年少氣盛，日究心算學。遇有難通之處，積思致廢寢食。”

1896, he was invited to teach Confucian classics, history, maps, and mathematics at the Longcheng Academy 龍城書院. Although he gained high praise from local learned people, Hua Shifang was not satisfied with his teaching there. In a letter, he expressed his feelings:

> The Longcheng Academy once appointed me to be the teacher in the section of mathematics. As for the section of Confucian classics, since I had not studied hard at it, I would not have dared to hold the post. However, there was no suitable person for the position at that time; thus I had no choice but to take charge of it with my little talent. . . . Recently, the academy increased the mathematics examinations for the students living in the academy held on the first day and mid-month each month. I was busy reading and commenting on their work and was too exhausted to manage tasks for the other section. Therefore, I decide to quit the section of Confucian classics and history in order to be relieved [of the stress]. (Hua [n.d.] 1980, 802)[51]

When choosing between teaching positions in mathematics and the Confucian classics, without hesitation Hua Shifang chose mathematics.

In sum, the inclination towards specialization in mathematics was intensified in the late nineteenth century. With a stable livelihood, Chinese mathematicians could lay claim to their specialty and be independent from the group of Confucians.

4.4 The professionalization of mathematics in the nineteenth century bears the characteristic of transition

In the above discussion, I conclude that some Chinese mathematicians became professionalized largely in the context of mathematics education in the late nineteenth century. But I have no intention of arguing that the professionalization of mathematics in China was finished at that time. In fact, this was just the start of a period of transition.

First of all, as I mentioned above, the institutions offering mathematics education were limited both in number and location. The full popularization of mathematics across the whole of China began in 1905.

Secondly, the route to becoming mathematics teachers was complicated in the late nineteenth century. In fact, scholars with different inclinations were attracted to mathematics in the prevailing political and cultural contexts. By analyzing information about forty-six mathematics teachers in the late nineteenth century, I found that they can mainly be divided into three categories. The first were traditionally trained

51 “郡中龍城書院，……曾以算學一齋屬世芳校閱。至於經古一齋，自揣用力甚疏，本不敢謬膺斯席，無如彼時倉猝未得其人，遂以不才承乏。近……算學住院諸生，朔望加課，日記累累。披閱紛煩，精神不能兼顧，故決意辭去經古一席，以稍紓心力。”

Confucian scholars who were interested in mathematics. Their research and teaching mainly concentrated on mathematical content in the Confucian classics and other ancient works. The second were scholars who aimed at serving society. They were attracted to mathematics because it was useful for the nation. An interesting point here is that they were especially active in promoting the study of mathematics and establishing institutions offering mathematics education. The representative figure is Liu Guangfen, whom I have mentioned several times in this paper. When he found that his students were too absorbed in mathematics, showing little interest in technology, he became worried. In the end, he left mathematics education himself, and turned to work on the promotion of industry in Shaanxi Province (Liu 1915). The last group have constituted the main figures of this paper, the ones who already had an inclination towards the specialty of mathematics and seized the opportunity to become the first generation of professional mathematicians. With the development and popularization of mathematics, the numbers of this group expanded (Tian 1998).

5 Conclusion

The professionalization of mathematics in late nineteenth-century China was not a natural process in China's development. It was stimulated and demanded by the urgent need for military technology. Before the systematic changes in cultural and political structures in China, the development of mathematics education was limited both in quantity and geographical range. This is the context in which the first group of professional mathematicians emerged, and denotes the starting point of the professionalization of mathematics in China more generally. As political reform accelerated, the group of professional mathematicians expanded and institutionalization and professionalization were finally realized in the context of mathematics education at the university level in the first part of the twentieth century.[52]

Acknowledgments

Part of this paper's results were published in 1998 in "Qingmo shuxue jiaoyu dui Zhongguo shuxuejia de zhiyehua yingxiang" 清末数学教育对中国数学家的职业化影响

52 This paper focuses on the starting period of the professionalization of mathematics in China. For a comprehensive understanding of the history of mathematics in the late nineteenth and twentieth centuries, see Guo 2010, 753–843, Zhang 2000, and Tian 2005, 182–359. For the community of mathematicians in the nineteenth and twentieth centuries, see Xu 2002 and Dauben 2002; for the content of mathematics education in the late nineteenth century, see Li 2005; for a general understanding of the context of science in the nineteenth century, see Elman 2005; for mathematical and scientific terminology in the nineteenth century, see Bréard 2001 and Lackner, Amelung, and Kurtz 2001.

(The Impact of the Development of Mathematical Education on the Professionalization of Chinese Mathematicians in Late Qing Dynasty) in *Ziran kexueshi yanjiu* 自然科学史研究 (Studies in the History of Natural Sciences) 17 (2): 119–128. Heartfelt gratitude is extended to *CAHST*'s anonymous reviewers for their valuable comments, to Prof. Liu Qiang 刘蔷 of Tsinghua University, Associate Professor Chen Pu 陈朴, and the librarian Gao Feng 高峰 of the Institute for the History of Natural Sciences, CAS for their assistance in seeking literature, and to Liu Shuo 刘硕 for translating part of this paper.

References

Abe, Hiroshi. 1987. "Borrowing from Japan: China's First Modern Education System." In *China's Education and the Industrialized World: Studies in Cultural Transfer*, edited by Ruth Hayhoe and Marianne Bastid, 57–80. New York: M. E. Sharpe.

Aisin Gioro, Yikuang 愛新覺羅•奕劻. [1889] 1958. "Guangxu shiwunian qiyue ershijiuri zongli geguo shiwu Yikuang deng zou" 光緒十五年七月二十九日總理各國事務奕劻等奏 (Memorial on the twenty-ninth day of the seventh lunar month in the fifteenth year of the reign of Emperor Guangxu by Foreign Minister Yikuang et al.). In *Yangwu yundong* 洋務運動 (Self-Strengthening Movement), edited by Institute of Modern History, Chinese Academy of Sciences, 2:212. Shanghai: Shanghai People's Publishing House.

Aisin Gioro, Yixin 愛新覺羅•奕訢. [1861] 1986. "Xianfeng shinian shieryue chusanri Gongqinwang Yixin deng zou" 咸豐十年十二月初三日恭親王奕訢等奏 (Memorial on the third day of the twelfth lunar month in the tenth year of the reign of Emperor Xianfeng by Prince Gong Yixin et al.) In *Zhongguo jindai xuezhi shiliao* 中國近代學制史料 (Historical records of the Chinese modern education system), edited by Zhu Youhuan 朱有瓛, book 1:4–5. Shanghai: East China Normal University Press.

Aisin Gioro, Yixin. [1862] 1986. "Tongzhi yuannian qiyue ershiwuri zongli geguo shiwu Yixin deng zhe" 同治元年七月二十五日總理各國事務奕訢等折 (Memorial on the twenty-fifth day of the seventh lunar month in the first year of the reign of Emperor Tongzhi by Foreign Minister Yixin et al.) In *Zhongguo jindai xuezhi shiliao*, edited by Zhu Youhuan, book 1:6–9. Shanghai: East China Normal University Press.

Aisin Gioro, Yixin. [1863] 1986. "Tongzhi ernian sanyue shijiuri zongli geguo shiwu Yixin deng zhou" 同治二年三月十九日總理各國事務奕訢等奏 (Memorial on the nineteenth day of the third lunar month in the second year of the reign of Emperor Tongzhi by Foreign Minister Yixin et al.) In *Zhongguo jindai xuezhi shiliao*, edited by Zhu Youhuan, book 1:10. Shanghai: East China Normal University Press.

Aisin Gioro, Yixin. [1865] 1986. "Tongzhi sinian shiyiyue chuwuri zongli geguo shiwu Yixin deng pian" 同治四年十一月初五日總理各國事務奕訢等片 (Memorial on the fifth day the eleventh lunar month in the fourth year of the reign of Emperor Tongzhi by Foreign Minister Yixin et al.) In *Zhongguo jindai xuezhi shiliao*, edited by Zhu Youhuan, book 1:11–13. Shanghai: East China Normal University Press.

Bennett, Adrian Arthur. 1967. "John Fryer: The Introduction of Western Science and Technology into Nineteenth-Century China." In *Harvard East Asian Monographs* no. 24. Cambridge, Mass.: Harvard University Press.

Bréard, Andrea. 2001. "On Mathematical Terminology: Culture Crossing the Nineteenth-century China." In *New Terms for New Ideas: Western Knowledge and Lexical Change in Late Imperial China*, edited by Michael Lackner, Iwo Amelung and Joachim Kurtz, 305–326. Leiden: E. J. Brill.

Dauben, Joseph W. 2002. "Internationalizing Mathematics East and West: Individuals and Institutions in the Emergence of a Modern Mathematical Community in China." In *Mathematics Unbound: The Evolution of an International Mathematical Community*, edited by K. H. Parshall and A. Rice, 253–286. Providence and London: The American Mathematical Society and the London Mathematical Society.

Ding, Weiliang 丁韙良 (William Alexander Parsons Martin). 1877. "Li Renshu xiansheng xu" 李壬叔先生序 (Preface by Mister Li Renshu). In *Gezhi huibian* 格致彙編 (Collection on science and technology), edited by Fu Lanya 傅蘭雅 (J. Fryer), volume in winter:1a. Shanghai: Shanghai gezhi shushi 上海格致書室.

Du, Shiran 杜石然. 1966. "Zhu Shijie yanjiu" 朱世傑研究 (Study on Zhu Shijie). In *Song Yuan shuxueshi lunwenji* 宋元數學史論文集 (Collection of papers on history of mathematics in the Song and Yuan dynasties), edited by Qian Baocong, 166–209. Beijing: Science Press.

Elman, Benjamin A. 1984. *From Philosophy to Philology: Intellectual and Social Aspects of Change in Late Imperial China*. Cambridge, MA: Council on East Asian Studies, Harvard University.

Elman, Benjamin A. 2005. *On their Own Terms. Science in China, 1550–1900.* Cambridge, MA: Harvard University Press.

Fairbank, John K. 1978. *The Cambridge History of China: Late Ch'ing, 1800–1911*, Part I. Cambridge: Cambridge University Press.

Fairbank, John K., and Liu Kwang-Ching. 1980. *The Cambridge History of China: Late Ch'ing, 1800–1911* vol. 2. Cambridge: Cambridge University Press.

Feng, Guifen 馮桂芬. [1861] 2002. *Jiaobinlu kangyi* 校邠廬抗議 (Protests from Jiaobin study). Shanghai: Shanghai Ancient Books Publishing House.

Fisher, Daniel W. 1911. *Clavin Wilson Mateer. Forty-Five Years a Missionary in Shantung, China.* Philadelphia: The Westminster Press.

Guo, Shuchun 郭书春, ed. 2010. *Zhongguo kexue jishushi shuxue juan* 中国科学技术史・数学卷 (History of science and technology in China: Mathematics). Beijing: Science Press.

Han, Qi 韓琦. 1996. "Junzhu he buyi zhijian Li Guangdi zai Kangxi shidai de huodong ji qi dui kexue de yingxiang" 君主和布衣之間：李光地在康熙時代的活動及其對科學的影響 (Between the Emperor and Mathematician: Li Guangdi's Activity during the Kangxi Reign and Its Influence on Science). *Qinghua xuebao* 清華學報 (Tsing Hua Journal of Chinese Studies) 26 (4): 421–445.

Heseri Hala, Yinggui 郝舍里・英桂, and Shen Baozhen 沈葆桢. [1870] 1986. "Qing kaoshi suanxue zhe" 請考试算學折 (Memorial for establishing a mathematics section in the civil service examination). In "Libu kaoshi suanxue zhe" 禮部考試算學折 (Memorial about a mathematics examination by the Ministry of Rites). In *Zhongguo jindai xuezhi shiliao*, edited by Zhu Youhuan, book 2:18–19. Shanghai: East China Normal University Press.

Horng, W. S. 洪萬生. 1991a. "Li Shan-lan: The Impact of Western Mathematics in China during the Late nineteenth Century." PhD dissertation, The City University of New York.

Horng, Wann-sheng. 1991b. "Tongwenguan suanxue jiaoxi Li Shanlan" 同文館算學教習李善蘭 (Li Shanlan: Mathematics teacher in Tongwenguan). In *Jindai Zhongguo kejishi lunji* 近代中國科技史論集 (Science and technology in modern China), edited by Yang Tsui-hua 楊翠華 and Huang Yi-long 黃一農, 215–259. Taipei: Institute of Modern History, Academia Sinica and Institute of History, Tsing Hua University.

Horng, Wann-sheng, and Liu Dun 劉鈍. 1993. "Li Rui yu qianjia xuepai" 李銳與乾嘉學派 (Li Rui and Qianjia Scholarship). In *Tantian sanyou* 談天三友 (Three friends discussing the heavens), edited by Horng Wansheng, 9–36. Taipei: Mingwen Bookstore.

Hua, Hengfang 華蘅芳. 1874. "Xu" 序 (Preface). In *Weiji suyuan* 微積溯源 (The origin of calculus). Shanghai: Jiangnan Arsenal.

Hua, Hengfang. [1882] 1897. "Xuesuan bitan" 學算筆談 (Discussion on the learning of mathematics). Shanxi: Weijing Kanshuchu 味經刊書處.

Hua, Shifang 華世芳. [n.d.] 1980. "Zhi Miu Quansun" 致繆荃孫 (Letter to Miu Quansun). In *Yifengtang youpeng shuzha* 藝風堂友朋書札 (Correspondence of friends of Yifengtang), edited by Gu Tinglong 顧廷龍. Shanghai: Shanghai Ancient Books Publishing House.

Huang, Huixian 黄惠贤, and Chen Feng 陈锋. 2012. *Zhongguo fenglu zhidushi* 中国俸禄制度史 (The history of the system of Chinese officials' salary). Wuhan: Wuhan University Press.

Huang, Zongxi 黃宗羲. [1838] 1990. "Anding xue'an" 安定學案 (Intellectual biography of Anding). In *Song Yuan xue'an* 宋元學案 (Biographies of scholars in the Song and Yuan dynasties) chap. 1:56. Hangzhou: Zhejiang Ancient Books Publishing House.

Jami, Catherine. 2012. *The Emperor's New Mathematics: Western Learning and Imperial Authority during the Kangxi Reign (1622–1722)*. Oxford: Oxford University Press.

Lackner, Michael, Iwo Amelung, and Joachim Kurtz, eds. 2001. *New Terms for New Ideas: Western Knowledge and Lexical Change in Late Imperial China*. Leiden: E. J. Brill.

Li, Di 李迪. 1987. "Qin Jiushao zhuanlüe" 秦九韶传略 (Short Biology of Qin Jiushao). In *Qin Jiushao yu Shushu jiuzhang* 秦九韶与数书九章 (Qin Jiushao and the *Book of Mathematics in Nine Chapters*), edited by Wu Wenjun 吴文俊, 25–42. Beijing: Beijing Normal University Press.

Li, Hongzhang 李鴻章. [1862] 1989. "Zhi Zeng Guofan" 致曾國藩 (Letter to Zeng Guofan). In *Jindai Shanghai da shiji* 近代上海大事記 (A chronology of events in modern Shanghai), edited by Tang Zhijun 汤志钧, 184. Shanghai: Shanghai Lexicographical Publishing House.

Li, Hongzhang. [1863] 1986. "Tongzhi ernian siyue ershijiuri jianghai guandao xiang Nanyang tongshang dachen Li Hongzhang" 同治二年四月二十九日江海關道詳南洋通商大臣李鴻章 (Memorial on the twenty-ninth day of the fourth lunar month in the second year of the reign of Emperor Tongzhi by the governor of the Jiangsu Maritime Customs and Minister of Commercial Affairs of the Southern Sea Li Hongzhang). In *Jindai Zhongguo kejishi lunji*, edited by Zhu Youhuan, book 1:216–219. Shanghai: East China Normal University Press.

Li, Mingyang 李明洋. 2018. "Zaojian yu yucai: Fuzhou Chuanzheng jiqi Faguo jishu laiyuan (1866–1895)" 造舰与育才：福州船政及其法国技术来源（1866–1895）(Constructing warships and training engineers in Foochow Navy Yard: The technology transfer from France to China). PhD Dissertation, University of Chinese Academy of Sciences and Institute for the History of Natural Sciences, CAS, Beijing.

Li, Wenlin, and Jean-Claude Martzloff. 1998. "Aperçu sur les éxchanges mathématiques entre la

Chine et la France (1800–1949).” In *Archive for History of Exact Sciences* 53:181–200.

Li, Yan 李俨. [1933] 1998. “Tang Song Yuan Ming shuxue jiaoyu zhidu” 唐宋元明数学教育制度 (The system of education in the Tang, Song, Yuan, and Ming dynasties). In *Zhongsuanshi luncong* 中算史论丛 (A discussion on the history of mathematics in China). In *Li Yan Qian Baocong kexueshi quanji* 李俨钱宝琮科学史全集 (The complete collection of Li Yan and Qian Baocong’s studies on history of science), edited by Guo Shuchun and Liu Dun, book 8:223–266. Dalian: Liaoning Education Press.

Li, Zhaohua 李兆华. 1992. “Wang Lai fangchenglun yanjiu” 汪莱方程论研究 (A study on Wang Lai’s equation theory). *Ziran kexueshi yanjiu* 自然科学史研究 (Studies in the History of Natural Sciences) 11 (3): 193–208.

Li, Zhaohua. 1995. *Zhongguo shuxueshi* 中國數學史 (History of mathematics in China*)*. Taipei: Wenchin Press.

Li, Zhaohua, ed. 2005. *Zhongguo jindai shuxue jiaoyu shigao* 中国近代数学教育史稿 (A draft history of mathematics education in the late Qing dynasty). Jinan: Shandong Education Press.

Li, Zhaoluo 李兆洛. 1830. “Dong Fangli zhuan” 董方立傳 (Biography of Dong Fangli). In *Dong Fangli yishu* 董方立遺書 (Posthumous works of Dong Fangli). Shanghai: Jiangnan Manufacturing Bureau.

Liu, Jinhui 刘谨辉. 2005. “Jiao Xun nianpu” 焦循年谱 (Chronology of Jiao Xun). In *Jiao Xun pingzhuan* 焦循评传 (Biography of Jiao Xun): 177–208. Yangzhou: Guangling Press.

Liu, Ruijiao 劉瑞驕. 1915. “Xing zhuang” 行狀 (Biography). In *Yanxia Caotang wenji* 煙霞草堂文集 (Collection of papers of Yanxia Caotang).

Liu, Yusheng 劉禺生. 1960. *Shizaitang zayi* 世載堂雜憶 (Recollections of Shizaitang). Beijing: Zhonghua Book Company.

Luo, Shilin 羅士琳. 1840. “Wang Lai” 汪莱. In *Chouren zhuan xubian* 畴人传续编 (Sequel to the *Biographies of astronomers and mathematicians*). In *Guanwosheng shi huigao* 觀我生室匯稿 (Collected works of Guanwosheng shi) chap. 50:24b–25a.

Mao, Hongbin 毛鴻賓. [1864] 1986. “Tongzhi sannian liuyue chushiri Liangguang zongdu Mao Hongbin zhe” 同治三年六月初十日兩廣總督毛鴻賓折 (Memorial on the tenth day of the sixth lunar month in the third year of the reign of Emperor Tongzhi by the governor of Guangdong and Guangxi Mao Hongbin). In *Zhongguo jindai xuezhi shiliao*, edited by Zhu Youhuan, book 1:258–261. Shanghai: East China Normal University Press.

Martzloff, Jean-Claude. 1992. “Li Shanlan (1811–1882) and Chinese Traditional Mathematics.” *The Mathematical Intelligencer* 14 (4): 32–37.

Mei, Rongzhao 梅荣照. 1966. “Li Ye jiqi shuxue zhuzuo” 李冶及其數學著作 (Li Ye and his mathematical works). In *Song Yuan shuxueshi lunwenji*, edited by Qian Baocong, 104–148. Beijing: Science Press.

Meng, Yue. 1999. “Hybrid Science versus Modernity: The Practice of the Jiangnan Arsenal, 1864–1897.” *EASTM* (16): 13–52.

Ministry of Rites 禮部. [1877] 1985. “Yi kai suanxue ke zhe” 議開算學科摺 (Memorial for the discussion on establishing the section on Mathematics [in the civil service examination]). In *Guangxu zhengyao* 光緒政要 (Essence of policy in the rein of Emperor Guangxu), edited by Shen Tongsheng, chap. 13:18–19. Taipei: Wenhai Press Company.

Nie, Jigui 聂缉椝. [1894] 1986. "Guangxu shijiunian shieryue ershiqiri zhizaozongju yi xiaoqiaolinglong haiguandao Nie" 光緒十九年十二月二十七日製造總局移小巧玲瓏海關道聶 (Memorial on the twenty-seventh day of the twelfth lunar month in the nineteenth year of the reign of Emperor Guangxu by the Director of Jiangnan Arsenal and governor of Jiangsu Maritime Customs Nie). In *Zhongguo jindai xuezhi shiliao*, edited by Zhu Youhuan, book 1:239–243. Shanghai: East China Normal University Press.

Pan, Yantong 潘衍桐. [1874] 1986. "Qing kai yixueke zhe" 請開藝學科折 (Memorial for establishing the section on art in the civil service examination). In *Zhongguo jindai xuezhi shiliao*, edited by Zhu Youhuan, book 2:21. Shanghai: East China Normal University Press.

Porter, J. 1982. "Scientific Community in Early Modern China." *Isis* 73 (4): 529–544.

Qian, Baocong. 1966. "Qin Jiushao Shushu jiuzhang yanjiu" 秦九韶《數書九章》研究 (Study on Qin Jiushao's *Book of mathematics in nine chapters*). In *Song Yuan shuxueshi lunwenji*, edited by Qian Baocong, 36–59. Beijing: Science Press.

Qin, Shouzhang 秦綬章. [1896] 1899. "Zouqing zhengdun gesheng shuyuan yuchu rencai zhe" 奏請整頓各省書院預儲人才折 (Memorial for reorganizing traditional academies in each province and storing talents for the future). In *Huangchao jingshiwen tongbian* 皇朝經世文統編 (Comprehensive compilation of writings on statecraft in the Qing dynasty), edited by He Changling 賀長齡, chap. 9.

Tang, Zhijun, ed. 1987. *Wang Tao riji* 王韬日记 (Diary of Wang Tao). Beijing: Zhonghua Book Company.

Thomson, J. 1873. *Illustrations of China and Its people: A Series of Two Hundred Photographs, with Letterpress Descriptive of the Places and People Represented* vol. 4:Illustration 37. London: Sampson Low, Marston, Low, and Searle.

Tian, Miao 田淼. 1992. "Qingmo shuxuejia yu shuxue jiaoyujia Liu Yicheng" 清末数学家与数学教育家刘彝程 (Liu Yicheng, a mathematician and mathematics educator in the late Qing). In *Zhongguo shuxueshi lunwenji* 中国数学史论文集 (Collection of papers on the history of mathematics in China) (3): 117–122. Hohhot: Inner Mongolia Normal University Press.

Tian, Miao. 1997. "Qingmo shuyuan de shuxue jiaoyu" 清末书院的数学教育 (Mathematics education in traditional academies in the late Qing). PhD Dissertation, Institute for the History of Natural Sciences, CAS, Beijing.

Tian, Miao. 1998. "Qingmo shuxue jiaoshi de goucheng tedian" 清末数学教师的构成特点 (The characteristics of the formation of mathematics teachers at the end of the Qing dynasty). *Zhongguo keji shiliao* 中国科技史料 (China Historical Materials of Science and Technology) 19 (4): 19–24.

Tian, Miao. 1999a. "*Jiegenfang*, *Tianyuan*, and *Daishu*: Algebra in Qing China." *Historia Scientiarum* 9 (1) 101–119.

Tian, Miao. 1999b. "Siyuan yujian de Qingdai banben ji Jialing sicao de jiaokan yanjiu" 《四元玉鉴》的清代版本及《假令四草》的校勘研究 (On the different versions of *Jade mirror of the four elements* and the proofs for *Jialing sicao* in the Qing dynasty). *Ziran kexueshi yanjiu* 18 (1): 36–47.

Tian, Miao. 2003. "The Westernization of Chinese Mathematics—A Case Study on the Development of the *Duoji* Method." *EASTM* (20): 45–72.

Tian, Miao. 2005. *Zhongguo shuxue de xihua lichen* 中国数学的西化历程 (The Westernization of mathematics in China). Jinan: Shandong Education Press.

Xu, Shichang 徐世昌. [1938] 1990. "Litang xue'an" 里堂學案 (Biography of Litang [Jiao Xun]). In *Qing ru xue'an* 清儒學案 (Biographies of Qing Confucians). Beijing: China Bookstore.

Xu, Yibao 徐义保. 2002. "Chinese-U. S. Mathematical Relations, 1859–1849." In *Mathematics Unbound: The Evolution of an International Mathematical Community*: 287–309.

Wang, Xiaoqin 汪晓勤. 1999. "Weilie yali de xueshu shengya" 伟烈亚力的学术生涯 (The academic career of Alexander Wylie as a sinologist). *China Historical Materials of Science and Technology* 20 (1): 17–34.

Wang, Yusheng 王渝生. 1990. "Li Shanlan yanjiu" 李善蘭研究 (Study on Li Shanlan). In *Ming Qing shuxueshi lunwenji* 明清數學史論文集 (Collected papers on the history of mathematics in the Ming and Qing dynasties), edited by Mei Rongzhao, 334–408. Nanjing: Jiangshu Education Press.

Wright, David. 1996. "John Fryer and the Shanghai Polytechnic: Making Space for Science in Nineteenth-Century China." *The British Journal for the History of Science* 29 (1): 1–16.

Yan, Dunjie 严敦杰. 1987. "Qin Jiushao nianpu chugao" 秦九韶年谱初稿 (Primary chronology of Qin Jiushao). In *Qin Jiushao yu Shushu jiuzhang*, edited by Wu Wenjun, 12–24. Beijing: Beijing Normal University Press.

Yan, Dunjie. 1990. "Li Shangzhi nianpu" 李尚之年谱 (Chronology of Li Shangzhi). In *Ming Qing shuxueshi lunwenji*, edited by Mei Rongzhao, 445–472. Nanjing: Jiangsu Education Press.

Yu, Xiang 裕祥. 1898. "Preface". In *Jingzheng shuyuan keyi* 經正書院課藝 (Collection of examination essays of Jingzheng Academy).

Zhang, Baichun 张柏春. 1992. *Zhongguo jindai jixie jianshi* 中国近代机械简史 (A brief history of modern machinery in China). Beijing: Beijing Institute of Technology Press.

Zhang, Dianzhou 张奠宙. 2000. *Zhongguo jinxiandai shuxue de fazhan* 中国近现代数学的发展 (The development of modern and contemporary mathematics in China). Shijiazhuang: Hebei Science and Technology Press.

Zhu, Youhuan, ed. 1986. "Jingshi Tongwenguan" 京師同文館. In *Zhongguo jindai xuezhi shiliao* book 1:1–83. Shanghai: East China Normal University Press.

Zou, Daoji 鄒道濟. [n.d.] 1958. "Gedi yikuan shuo" 割地議款說 (Discussion on the ceding territory and paying indemnities). In *Jiawu Zhongri zhanzheng wenxueji* 甲午中日戰爭文學集 (Collection of literature concerning the Sino-Japan War in 1894), edited by Aying 阿英, 498–499. Beijing: Zhonghua Book Company.

Zou, Daoji. [1897] 1987. "Zhi Wang Kangnian" 致汪康年 (Letter to Wang Kangnian). In *Wang Kangnian shiyou shuzha* 汪康年師友書札 (Correspondence from teachers and friends to Wang Kangnian), edited by Shanghai Library, 2829. Shanghai: Shanghai Ancient Books Publishing House.

Zu, Yi 祖頤. 1303. "Hou xu" 後序 (Postscript). In *Siyuan yujian xicao* 四元玉鑑細草 (Detailed explanation of *Jade mirror of the four elements*) by Luo Shilin. Manuscript copy at Li Yan Library, Institute for the History of Natural Sciences, CAS, Beijing.