

Effective depth-of-penetration range due to hardness variation for different lots of nominally identical target material

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Abstract

The effect of small variations of target hardness on the depth of penetration for nominally identical target material has not been addressed systematically in publications yet and is often neglected. An investigation of this issue for laboratory-scale long rod projectiles penetrating into semi-infinite rolled-homogeneous-armor steel targets was conducted. The tungsten-heavy-alloy penetrators were of length 90 mm and diameter 6 mm. Five lots of armor steel with a nominal hardness range of 280–330 BHN provided material for the targets. The pursued approach consisted of hardness testing of the targets, in total 17 ballistic experiments at velocities in between 1250 m/s and 1780 m/s and data analysis.

A linear regression analysis of penetration vs. hardness shows that a target hardness increase within the given range of 280–330 BHN may result in a reduction of penetration depth of about 5.8 mm at constant velocity. This is equal to a change of –12% at an impact velocity of 1250 m/s. A multiple linear regression analysis included also the influence of yaw angle and impact velocity. It shows that small yaw angles and slight variations of impact velocities provide a smaller variation of the semi-infinite penetration depths than a variation of target hardness within a typical specification span of 50 BHN. For such a span a change in penetration of approximately –4.8 mm due to hardness variation is found, whereas 1° of yaw angle or –10 m/s of velocity variation gives a change of about –1.0 mm respectively –0.9 mm. For the given example, the overwhelming part of the variation is to be attributed to hardness effects –4.8 mm out of 5.8 mm (83%). For nominally identical target material the target hardness thus influences the ballistic test results more severely than the typical scatter in impact conditions.

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1. Introduction

Many parameters determine the outcome of a ballistic test. Even for depth-of-penetration experiments with tungsten-heavy-alloy (WHA) rod penetrators against semi-infinite rolled-homogeneous-armor (RHA) steel targets, a variety of parameters come into play. Examples are impact velocity, yaw angle, target obliquity and material properties [1–6]. Some of those parameter variations are of statistical nature and controllable in an experiment within some scatter. Others are rather systematic types of error, such as variations between material lots that could occur, e.g. if the quality of a reference target material changes over time.

The target material hardness class is known to have a strong effect on penetration results. A number of investigations are

published addressing this issue. For example, the work done by Rapacki et al. [7] analyzes the coarse effects of hardness for large variations of the full relevant armor steel hardness range from below 200 BHN up to 600 BHN. Penetration formulae, e.g. by Lanz and Odermatt [8], take care of those dependencies, too.

We address the case of relatively small variations that are within the range of a single target material hardness class, i.e. that are compatible with the same specification. This is of relevance as often ballistic results obtained with material of different origin in different series are compared or combined rather than repeating expensive tests with same-grade material. From the data correction for hardness effects done by Rosenberg et al. in [9], the significance of this problem for such small hardness differences emerges implicitly. However, we are not aware of a publicly available data set allowing for a more systematic analysis of such effects within a narrow hardness regime.

While actual impact velocities and yaw angles are typically measured in each test, the hardness may not be measured for each target specimen. Rather values are often picked from

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were taken on a certified hardness reference block of nominally 290 BHN. The 6 measurements showed values of 288–292 BHN with a mean hardness of 289 BHN.

It turned out that material samples/targets from lot B could be grouped into 2 subsets B1 and B2, respectively, with significantly different hardness. For a consistent analysis, B1 and B2 were considered independently in the following.

Although there is scatter in all measured values, it can be concluded that hardness can roughly be ordered as B2, A, B1, D, C, i.e. B2 is the softest and C is the hardest material. The hardness of most samples of lots C and D exceeded the material specification.

3.2. Penetration tests

In total, 17 ballistic experiments were available for evaluation (Table 2). The table lists the target hardness *BHN*, impact velocity v , the total yaw angle γ , the penetration depth P , and the penetration depth P_{REL} normalized by the smallest measured value in each velocity regime.

Within the lower velocity impact regime, a clear systematic is observed. The smallest semi-infinite penetration is observed for one of the tests with material C (100%). The penetration depths measured in the other tests with materials C and D ranged between 102%–104% and 101%–102%, respectively. As for the hardness tests, there is no clear difference between the results for lots C and D observable. Those materials seem to be comparable. In the single test with material A, the relative penetration goes up to 113%. Even more, for the softest lot B2, values in between 111% and 119% were measured, i.e. under similar test conditions the penetration depth of the projectile into the target increases by 19% for the softest material in relation to the hardest one. For the intermediate and higher velocity regime, similar effects are visible. However, they are less pronounced.

The effects observed for the lower velocity regime appear to be significant. Of course, the absolute values of all penetration depths also depend on the actual impact velocities and the yaw

angles. The impact velocity is constant within the error of the velocity measurements. The total yaw angles are mainly in the range of 0.5°–2.6° for the lower regime. For the tests with material C with the highest hardness values, relatively strong yaw angles did occur. But still, due to direct comparison with the test done with material A, which is one of the tests with large yaw angle in the lower velocity regime, one can clearly see that the resulting small penetration values are still attributable to high hardness values.

In general, with respect to the yaw angles, it can be stated that for the case of $L/D = 15$, values of around 2°–3° are still in a range in which the influence of the yaw on the semi-infinite penetration is sufficiently small. More quantitatively, according to [11], the critical angle for an $L/D = 15$ penetrator is in an approximate range of 2.5°–3° for impact velocities of 1500–1750 m/s. Only for the 2 tests in the upper velocity regime against material C with values of 3.1° and 3.5°, the critical yaw angle effects are likely to have occurred. Further details about effects due to projectile yaw can be found in other publications. Some of those studies focused on large yaw angles, e.g. [3,4], or on the critical yaw and the penetration decrease for yaw angles exceeding those critical values, e.g. [5]. A more detailed analysis of the effects of the yaw angle regarding the present data, i.e. for small angles below or around the critical threshold, is given in Section 4.2.

4. Analysis

4.1. Curve fitting

In this section, the effects of impact velocity and target hardness on the semi-infinite penetration depth of the rod penetrator are further quantified based on curve fitting the data.

Fig. 3 shows the penetration depth P of the rod penetrator vs. the impact velocity v for the 5 material lots of different hardness. As to be expected, e.g. from [1], the penetration depth is strongly dependent on the velocity.

For the considered velocity range, there is a linear behavior like expected, i.e. the data can be parameterized as follows:

Table 2
Experimental data.

No.	Lot	BHN	$v/(m/s)$	$\gamma(^{\circ})$	P/mm	P_{REL}/mm
12618	B2	300	1240	1.2	43.6	111
12916	B2	295	1240	0.5	46.4	118
12924	B2	283	1250	0.9	46.8	119
12617	A	313	1250	2.6	44.5	113
12926	D	323	1240	3.7	39.8	101
12620	D	337	1240	2.4	40.0	102
12917	C	341	1240	1.7	40.2	102
12925	C	326	1250	2.6	39.4	100
12619	C	344	1250	2.3	41.1	104
12627	A	307	1560	1.0	73.4	105
12626	B1	323	1570	2.7	71.7	103
12628	D	337	1560	1.6	69.6	100
12621	A	311	1770	1.3	91.7	105
12622	B1	321	1770	2.1	90.3	103
12624	D	337	1780	0.6	90.6	104
12623	C	344	1780	3.1	87.5	100
12625	C	345	1790	3.5	88.0	101

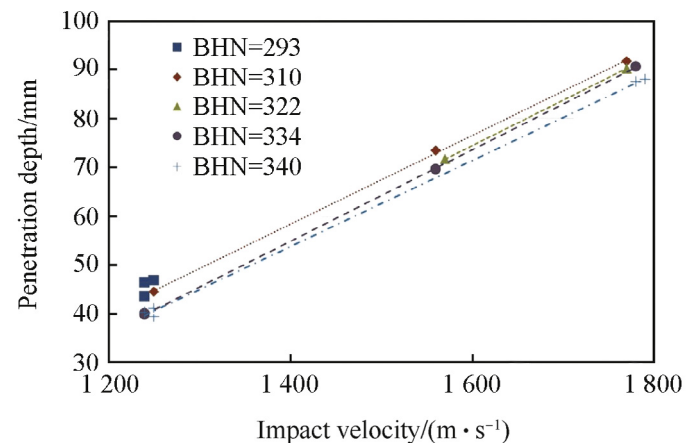


Fig. 3. Penetration depth in semi-infinite RHA target plotted vs. impact velocity for different lot hardness showing the well-known effect of velocity on the penetration depth. The influence of individual target hardness is shown in Fig. 4.

Table 3

Fit parameters α_1 and α_2 describing the penetration as a function of velocity for different lot hardness.

Lot	BHN (lot)	α_1 /ms	α_2 /mm
A	310	0.091	−69.0
B1	322	0.093	−74.3
B2	293	N/A	N/A
C	340	0.088	−69.8
D	334	0.094	−76.4
Average		0.092	–

$$P = \alpha_1 \cdot v + \alpha_2 \quad (1)$$

The fit parameters α_1 and α_2 are listed in Table 3. Obviously, the slopes α_1 of the curves obtained for different target hardness are identical within scatter, i.e. the influence of impact velocity is constant within the considered hardness range.

The prime interest of the present investigation is the effect of the hardness on the depth of penetration at constant velocity. The corresponding data are plotted in Fig. 4 for the 3 nominal velocities of 1250 m/s, 1560 m/s, and 1780 m/s.

As can be seen, for the considered data, the depth of penetration decreases linearly with target hardness at constant velocity and thus can be parameterized as follows

$$P = \beta_1 \cdot BHN + \beta_2 \quad (2)$$

The fit parameters β_1 and β_2 are listed in Table 4. Again, for the slope β_1 there is no clear systematics, i.e. this parameter has a similar value for all velocity regimes.

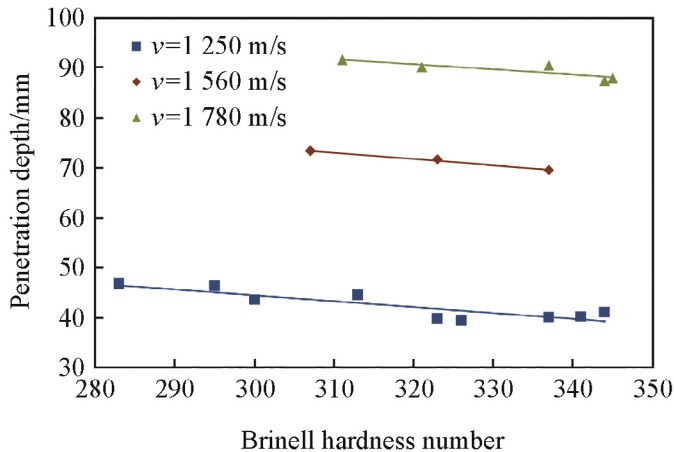


Fig. 4. Penetration depth in semi-infinite RHA target plotted vs. individual target hardness for 3 nominal impact velocities.

Table 4

Fit parameters β_1 and β_2 describing the penetration as a function of individual hardness for different nominal impact velocities.

v /(m/s)	β_1 /mm	β_2 /mm
1250	−0.118	80
1560	−0.126	112
1780	−0.103	124
Average	−0.116	–

Table 5

Reference penetration depth for hardness of 280 BHN and 330 BHN for 3 different impact velocities calculated with Eq. (2) and parameters of Table 4.

v /(m/s)	P_{280} /mm	P_{330} /mm	ΔP /mm	$\Delta P_{REL}/\%$
1250	46.7	41.1	−5.6	−12
1560	76.7	70.4	−6.3	−8
1780	95.2	90.0	−5.2	−5

Based on Eq. (2), conclusions about the influence of target hardness can be drawn. Based on the fit parameter according to Eq. (2) and the fit parameters in Table 5, reference penetration depths for the lower and upper hardness limit of the RHA class used as a target can be calculated (Table 5).

They show an absolute change of −5.6 mm, −6.3 mm, and −5.2 mm for the penetration depths at the different velocity regimes. In relation to the penetration depths achieved at a hardness of 280 BHN, the value for a hardness of 330 BHN is thus reduced by 12% at 1250 m/s, by 8% at 1560 m/s, and by 5% at 1780 m/s. The relative influence diminishes with higher velocities but the absolute influence of hardness remains the same.

In general, the total change in penetration dP as a function of velocity variation dv and hardness variation $dBHN$ is given by the total differential

$$dP = \frac{\partial P}{\partial v} \cdot dv + \frac{\partial P}{\partial BHN} \cdot dBHN \quad (3)$$

As discussed above, the penetration–velocity data are well approximated by straight lines of identical slope, independent of hardness. Furthermore, the penetration–hardness data also show a linear decrease of almost identical slope, independent of the velocity regime. Thus, the partial derivatives in Eq. (3) can be expressed by constants $\langle \alpha_1 \rangle$ and $\langle \beta_1 \rangle$ that are averages over data with different hardness and velocity, respectively. Thus, the absolute change in penetration ΔP as a function of velocity variation Δv and hardness variation ΔBHN can be approximated as follows

$$\Delta P \approx \langle \alpha_1 \rangle \cdot \Delta v + \langle \beta_1 \rangle \cdot \Delta BHN \quad (4)$$

The averaged constants $\langle \alpha_1 \rangle = 0.092$ ms (see Table 3) and $\langle \beta_1 \rangle = -0.116$ mm (see Table 4) thus describe the overall behavior. The opposite signs of the constants reflect that velocity and hardness have opposite effects. Both effects cancel for

$$\Delta v = -\frac{\langle \beta_1 \rangle}{\langle \alpha_1 \rangle} \cdot \Delta BHN \approx 1.3 \text{ m/s} \cdot \Delta BHN \quad (5)$$

Equation (4) implies that for a hardness change by 50 BHN, i.e. from 280 BHN to 330 BHN, the depth of penetration will change by −5.8 mm (equal to −6%, respectively −12% at 1780 m/s and 1250 m/s referring to the P_{280} -values in Table 5). This is independent of any change in velocity in the considered velocity range and reflects the numbers already discussed in Table 5. According to Eq. (5), an increase of the hardness by 50 BHN has the same effect on penetration depth as a decrease of impact velocity by 65 m/s at constant hardness. On the other

hand, a hardness range of 50 BHN is representative of the allowed range within a typical material specification, i.e. material compatible with the same technical standard might show such variations.

Two additional remarks are necessary here. First, it must not be expected that the given approximations are valid outside the considered velocity regime, hardness range and material used for penetrators and targets. Second, at least some minor effects of the nonzero yaw angles are to be expected that overlay the described phenomena. This is further investigated in the next section.

4.2. Multiple linear regression

A multiple linear regression of the experimental data was performed to better assess the combined effects of impact velocity, individual target hardness, and total yaw angle of the projectile.

In Section 4.1, the question of a possible influence of the total yaw angle was discussed but neglected in the curve fitting. The analysis of the experimental data is in the following continued by using multiple linear regression fits excluding and including the total yaw angle.

A multiple linear regression for a function y of n variables $\{x_i\}$ follows the general expression

$$y = a_0 + \sum_{i=1}^n a_i x_i \quad (6)$$

The parameters $\{a_i\}$ are determined numerically. This kind of linear fit is basically a generalization of the above performed curve fitting to multiple dimensions. For the discussed data, the depth of penetration P is analyzed as a function of hardness BHN , penetrator velocity v , and total yaw angle γ

$$P = a_0 + a_1 \cdot BHN + a_2 \cdot v + a_3 \cdot \gamma \quad (7)$$

The determined parameters are given in Table 6.

Note that the analysis has been done including and excluding the yaw angles, i.e. in the latter case, a_3 was forced to be zero. The adjusted R^2 , which is a measure for the fit quality, is close to 1 in both cases, i.e. the data set can be described by a linearization in all variables. In the case in which the yaw angles are not considered, the parameters a_2 and a_1 are in good agreement with the above discussed parameters $\langle\alpha_1\rangle$ and $\langle\beta_1\rangle$, respectively, as to be expected for a consistent analysis. If the yaw angle is included in the analysis, the parameter a_1 changes by -23% . This means that the relative weight of the hardness is partly lowered because of the added yaw angle dependency, but remains the dominant factor (see below).

Table 6
Parameters determined by multi-linear-regression.

	Yaw included	Yaw neglected
a_0	−39.0 mm	−32.2 mm
a_1	−0.095 mm	−0.124 mm
a_2	0.091 ms	0.092 ms
a_3	−0.951 mm/°	0 by definition
Adjusted R^2	0.9986	0.9968

Quantitatively, from Eq. (7), and in the spirit of Eq. (5), it can be seen that for constant velocity and penetration the following relation holds

$$\Delta BHN = -\frac{a_3}{a_1} \cdot \Delta \gamma \approx 10/^\circ \cdot \Delta \gamma \quad (8)$$

The effect of 1° of total yaw is thus approximately equivalent to a hardness increase of the target by 10 BHN and shows that there is of course also some dependency of the penetration on the yaw angle. The relative weights of those effects due to total yaw angle and hardness at constant velocity are further shown in Table 7. At constant hardness, total yaw angles of 1.5° and 3° change the penetration depths by about -1.5 mm and -3 mm in relation to the zero yaw case. If large yaw angle of 3° overlay the hardness effect, the total change in penetration can be more than -7 mm in the worst case. This corresponds to about -8% relative change at 1780 m/s impact velocity and to more than -15% at 1250 m/s impact velocity.

It is thus clearly visible from Table 7 that within a narrow range of impact velocities, the hardness of the target governs the penetration results. The yaw angle comes in second. Both hardness and yaw angle have their major impact on the relative change of penetration at lower velocities, whereas the overall driving force behind penetration depth remains the velocity.

5. Conclusions

Variations of armor steel hardness have a significant effect on depth-of-penetration values even when testing with target specimen of single target material hardness class, i.e. that are compatible with the same specification.

For material class hardness variations from 280 BHN to 330 BHN, the semi-infinite penetration depths of a length $L = 90$ mm rod penetrator, a decrease of penetration depth of about -6 mm can be attributed to hardness variation (neglecting yaw influence). At 1250 m/s this is equal to around -12% of the reference penetration. At 1780 m/s the relative change diminishes to -6% .

For comparison such a change is to be expected if the impact velocity is decreased by about 65 m/s at constant hardness.

A multiple linear regression analysis clarifies that the observed effects are not due to the occurrence of nonzero yaw

Table 7

Typical variations of penetration depth to be expected for the limiting cases of hardness of 280 BHN and 330 BHN and total yaw angles of 0° , 1.5° and 3° , calculation based on Eq. (7) and parameters of Table 6 (total yaw included).

v /(m/s)	γ (°)	P_{280} /mm	P_{330} /mm
1250	0	48.2	43.4
	1.5	46.7	42.0
	3	45.3	40.5
1560	0	76.4	71.6
	1.5	74.9	70.2
	3	73.5	68.8
1780	0	96.4	91.6
	1.5	95.0	90.2
	3	93.5	88.8

angles or due to a scatter of the nominally constant impact velocity.

The presented results quantify how crucial hardness measurement of individual target specimen is for sensitive trend analysis in ballistic testing besides the measurement of impact velocity and yaw.

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