非线性多输入多输出连续时间系统基于历史采样数据的不确定性因素补偿控制

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摘要 本文针对一类具有强耦合不确定性因素的连续时间多输入多输出非线性系统, 提出了基于历史采样数据的不确定性因素补偿控制策略 (sampled-data-based compensation control, SDBCC). 与基于观测器的不确定性因素补偿控制不同, 本文首先利用当前以及历史采样数据计算出由系统非线性耦合未建模动态和外部扰动构成的总扰动在前一采样周期内某时刻的精确值, 然后利用该精确值在反馈环节对总扰动进行补偿以消除它的不利影响. 连续时间系统在数据驱动的反馈控制作用下构成了一个混杂闭环控制系统, 这连同系统的强耦合非线性不确定性因素为控制闭环系统的稳定性收敛性分析带来了挑战. 为克服这一难题, 本文发展了基于特征值和迭代序列的分析方法, 证明了当系统跟踪目标为有界函数时, 跟踪误差可随采样周期的减小而任意小, 进而当跟踪目标为常数, 并且系统的非线性项为时不变连续可微函数时, 跟踪误差随时间趋于无穷大而趋近于零. 二自由度无人机姿态控制的仿真结果验证了本文所提出方法的有效性和优越性.

关键词 不确定性, 非线性, MIMO 系统, PID 控制, 数据驱动控制

1 引言

众所周知, 大量的实际控制系统都是非线性系统^[1,2], 通常难以建立精确的数学模型且常常受到外部干扰的作用^[3], 因此处理由系统非线性未建模动态以及外部干扰等构成的不确定性因素是控制理论研究的重要课题^[4]. 处理不确定性因素控制算法的设计及其基础理论被深入研究, 产出了大量的优秀成果, 如自适应控制^[5~8]、鲁棒控制^[9,10]、滑模变结构控制^[11,12]等. 这些控制方法在不同方面体现了各自的优点和特点, 但大部分方法或者在处理复杂不确定系统时的控制性能品质有限, 或者设计

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复杂难以物理实现, 或者控制器的设计需要利用系统的精确模型, 因此针对复杂非线性不确定系统发展易于物理实现的高性能控制方法与理论是自动控制领域亟待解决的重要课题。

大量的仿真实验与工程实践显示不确定性因素在线估计并补偿的控制策略具有优异的处理不确定性因素的能力。这种控制策略最早可以追溯到著名的 PID (proportion integration differentiation) 控制^[13], PID 控制中的积分项可以补偿常量干扰或者消除静差。当系统受到随时间持续变化的外部干扰作用, 且产生干扰的外系统的数学模型已知时, 可利用内模原理或输出调节原理消除这类干扰的影响^[14, 15]。类似的想法还可参见基于干扰观测器的控制^[16, 17]。文献^[16]提出了利用干扰观测器估计并补偿干扰的控制方法——基于干扰观测器的控制方法。更多基于干扰观测器控制的研究参见文献^[17]。现实中干扰通常难以精确建模且与系统非线性未建模动态相互耦合, 为处理这类不确定性因素, 文献^[18]提出了扩张状态观测器用于在线估计系统状态的同时估计由系统内外不确定性因素构成的总扰动。因不确定性因素在对系统造成不良影响之前已被及时补偿, 这类不确定性因素估计与补偿的方法具有主动抗扰的特点而被命名为自抗扰控制^[19]。类似的想法还可参见平坦系统的代数估计^[20]、不确定性因素与扰动估计器^[21]、等效输入扰动控制^[22]等。

自抗扰控制等不确定性因素在线估计与补偿的控制技术已被运用于大量的实际系统^[23~28], 其基础理论问题在连续时间框架内也被逐步完善^[29~34]。然而在实际运用中, 尽管受控对象是连续时间系统, 但反馈控制的实现总是通过采样数字化离散实现的。连续时间系统的自抗扰控制等不确定性因素补偿控制算法也经常被离散化实现^[19], 但由离散化算法导致的混杂闭环系统的稳定性收敛性还没有被系统研究。尽管当采样周期足够小时, 每一个采样周期内不确定因素的观测值与其采样值之间的误差充分小, 然而随着时间的无限增大, 观测值与采样误差之间的误差是否能够被及时消除还未被深入研究。众所周知小误差在无限时间的累积可能会造成控制性能品质的下降, 甚至导致系统不稳定。

受文献^[35]针对离散时间系统计算未建模动态在前一拍的精确值补偿控制思想的启发, 文献^[36]针对一类二阶连续时间非线性不确定牛顿力学系统, 利用历史采样数据计算了总扰动在前一采样周期内某时刻处的精确值对总扰动进行补偿。牛顿力学系统仅仅是控制实践中最基本的系统模型, 大量的实际控制问题由多输入多输出 (multi-input multi-output, MIMO) 系统来描述, 比如, 工业过程控制^[37]、无人机的姿态控制^[38]、导弹的拦截^[39]、永磁同步电机的电流控制^[40]、机器人控制^[41]等。迄今为止, 连续时间非线性不确定 MIMO 系统基于历史采样数据的不确定性因素补偿控制还没有被系统研究。MIMO 系统中的非线性不确定耦合因素对问题的研究带来了困难, 为克服这一难题, 本文提出了一类非线性不确定 MIMO 系统基于历史采样数据的不确定性因素补偿解耦控制设计方法 (sampled-data-based compensation control, SDBCC)。

连续时间系统的采样控制已经被广泛深入研究^[42~45], 然而现有的文献没有考虑不确定性因素的计算与补偿, 理论分析方法也无法直接运用于本文混杂闭环系统收敛性稳定性的研究。本文通过发展新的分析方法证明了混杂控制闭环系统的稳定性收敛性。通过历史采样数据可以直接计算出总扰动在前一采样周期内某时刻的精确值, 故与基于观测器估计值的补偿控制方法不同, 本文直接利用不确定性因素在前一采样周期内某时刻的精确值, 而非前一采样时刻的观测值对总扰动进行补偿, 从而进一步提升了控制精度。

本文的创新性体现在 3 个方面: 第一, 针对一类强耦合非线性不确定性连续时间 MIMO 系统, 提出了基于历史采样数据的不确定性因素主动补偿解耦控制策略。第二, 针对连续时间系统在离散时间控制作用下构成的混杂控制闭环系统, 本文发展了基于特征值和迭代序列的分析方法, 证明了闭环系统的收敛性稳定性。第三, 在跟踪目标为常数且系统的非线性项为时不变连续可微函数的假设下, 证明了控制闭环系统的渐近稳定性, 即跟踪误差随时间趋于无穷大而趋近于零。

本文剩余内容安排如下: 第 2 节是问题描述. 第 3 节给出了基于历史采样数据的不确定性因素补偿控制设计方法和主要理论结果. 第 4 节通过建立控制误差与采样周期之间的关系, 对闭环系统的稳定性收敛性进行了严格的证明. 第 5 节采用二自由度无人机姿态控制系统模型进行仿真并与其他控制方法进行对比. 最后是全文总结.

2 问题描述

本文考虑的系统是如下具有强耦合不确定性因素的连续时间 MIMO 非线性系统:

$$\begin{cases} \dot{p}_{11}(t) = p_{12}(t), \\ \dot{p}_{12}(t) = f_1(t, p(t), \omega_1(t)) + u_1(t), \\ \dot{p}_{21}(t) = p_{22}(t), \\ \dot{p}_{22}(t) = f_2(t, p(t), \omega_2(t)) + u_2(t), \\ y_i(t) = Cp_i(t), \quad i = 1, 2, \end{cases} \quad (1)$$

其中 $p(t) = \text{col}(p_1(t), p_2(t)) = (p_1^T(t), p_2^T(t))^T$ 是系统状态, $p_i(t) = (p_{i1}(t), p_{i2}(t))^T$, $u(t) = (u_1(t), u_2(t))^T$ 是控制输入, $y(t) = (y_1(t), y_2(t))^T$ 是量测输出, $\omega(t) = (\omega_1(t), \omega_2(t))^T$ 是外部干扰; 矩阵 $C = (1, 0)$, 非线性函数 $f(t, p(t), \omega(t)) = (f_1(t, p(t), \omega_1(t)), f_2(t, p(t), \omega_2(t)))^T$ 表示由内外不确定性因素构成的总扰动.

模型 (1) 可以用于描述大量的实际控制系统, 如二自由度无人机姿态控制等. 由文献 [38] 可知, 二自由度无人机姿态控制系统模型为

$$\begin{cases} \dot{\theta}(t) = \omega_{\theta}(t), \\ \dot{\omega}_{\theta}(t) = \frac{\tau_{pp}F_p(t)}{I_p + m_h l_{cm}^2} - \frac{\tau_{py}F_y(t)}{I_p + m_h l_{cm}^2} - \frac{D_p \omega_{\theta}(t)}{I_p + m_h l_{cm}^2} + \frac{m_h g \cos(\theta(t)) l_{cm}}{I_p + m_h l_{cm}^2} \\ \quad + \frac{m_h \omega_{\varphi}^2(t) \sin(\theta(t)) l_{cm}^2 \cos(\theta(t))}{I_p + m_h l_{cm}^2} + q_1(\theta(t), \omega_{\theta}(t), \varphi(t), \omega_{\varphi}(t), \omega_1(t)), \\ \dot{\varphi}(t) = \omega_{\varphi}(t), \\ \dot{\omega}_{\varphi}(t) = \frac{\tau_{yp}F_p(t)}{I_y + m_h l_{cm}^2} - \frac{\tau_{yy}F_y(t)}{I_y + m_h l_{cm}^2} - \frac{D_y \omega_{\varphi}(t)}{I_y + m_h l_{cm}^2} \\ \quad + \frac{2m_h \omega_{\varphi}(t) \sin(\theta(t)) l_{cm}^2 \cos(\theta(t)) \omega_{\theta}(t)}{I_y + m_h l_{cm}^2} + q_2(\theta(t), \omega_{\theta}(t), \varphi(t), \omega_{\varphi}(t), \omega_2(t)), \end{cases} \quad (2)$$

其中 $\theta(t)$, $\varphi(t)$ 分别表示偏航角和俯仰角, $F_p(t)$ 和 $F_y(t)$ 分别是俯仰电机/螺旋桨和偏航电机/螺旋桨产生的推力, $\omega_{\theta}(t)$ 和 $\omega_{\varphi}(t)$ 分别是俯仰和偏航方向角速度, $\omega(t) = (\omega_1(t), \omega_2(t))^T$ 是外部干扰, $q_1(\cdot)$ 和 $q_2(\cdot)$ 分别是俯仰和偏航方向由系统未建模动态和外部干扰等因素耦合而成的不确定性因素. 系统 (2) 中相关符号的物理意义见表 1.

令

$$\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} \tau_{pp}/(I_p + m_h l_{cm}^2) - \tau_{py}/(I_p + m_h l_{cm}^2) \\ \tau_{yp}/(I_y + m_h l_{cm}^2) - \tau_{yy}/(I_y + m_h l_{cm}^2) \end{bmatrix} \begin{bmatrix} F_p(t) \\ F_y(t) \end{bmatrix},$$

表 1 系统 (2) 中符号的物理意义
Table 1 The physical meanings of symbols in system (2)

Symbol	Unit	Description
τ_{pp}	$N \cdot m/V$	Thrust torque constant of the yaw propeller
τ_{py}	$N \cdot m/V$	Thrust torque constant acting on pitch axis from the yaw propeller
τ_{yy}	$N \cdot m/V$	Thrust torque constant acting on yaw axis from the yaw propeller
τ_{yp}	$N \cdot m/V$	Thrust torque constant acting on yaw axis from the pitch propeller
D_p	N/V	Equivalent viscous damping about pitch axis
D_y	N/V	Equivalent viscous damping about yaw axis
I_p	$kg \cdot m^2$	Total moment of inertia about pitch axis
I_y	$kg \cdot m^2$	Total moment of inertia about yaw axis
m_h	kg	Total mass of the helicopter
l_{cm}	m	Center of mass length along the helicopter body from pitch axis
g	m/s^2	Gravitational acceleration

$$\begin{aligned}
 f_1(\theta(t), \omega_\theta(t), \varphi(t), \omega_\varphi(t), \omega_1(t)) = & -\frac{D_p \omega_\theta(t)}{I_p + m_h l_{cm}^2} + \frac{m_h \omega_\varphi^2(t) \sin(\theta(t)) l_{cm}^2 \cos(\theta(t))}{I_p + m_h l_{cm}^2} \\
 & + \frac{m_h g \cos(\theta(t)) l_{cm}}{I_p + m_h l_{cm}^2} + q_1(\theta(t), \omega_\theta(t), \varphi(t), \omega_\varphi(t), \omega_1(t)), \\
 f_2(\theta(t), \omega_\theta(t), \varphi(t), \omega_\varphi(t), \omega_2(t)) = & -\frac{D_y \omega_\varphi(t)}{I_y + m_h l_{cm}^2} + \frac{2m_h \omega_\varphi(t) \sin(\theta(t)) l_{cm}^2 \cos(\theta(t)) \omega_\theta(t)}{I_y + m_h l_{cm}^2} \\
 & + q_2(\theta(t), \omega_\theta(t), \varphi(t), \omega_\varphi(t), \omega_2(t)),
 \end{aligned}$$

则二自由度无人机姿态控制系统 (2) 可转化为 (1).

文献 [38] 针对多输入多输出系统 (2) 的简化模型提出了基于采样数据的自适应最优输出反馈控制, 但并没有考虑耦合非线性不确定性因素的计算与补偿. 事实上多输入多输出系统 (1) 的解耦控制近年来被深入研究, 产出了大量的成果, 例如文献 [46~50], 然而上述文献均没有考虑基于历史采样数据的耦合不确定性因素补偿解耦控制. 本文提出了包括系统 (2) 在内的一大类非线性不确定多输入多输出系统基于历史采样数据的非线性耦合不确定性因素的计算与补偿控制方法. 控制目的是设计反馈控制器使得系统的输出 $y(t)$ 跟踪到参考信号 $v(t) = (v_1(t), v_2(t))^T$, 同时保证系统状态的有界性.

令 $x_{11}(t) = p_{11}(t) - v_1(t)$, $x_{12}(t) = p_{12}(t) - \dot{v}_1(t)$, $x_{21}(t) = p_{21}(t) - v_2(t)$, $x_{22}(t) = p_{22}(t) - \dot{v}_2(t)$, 其中 $x(t) = \text{col}(x_1(t), x_2(t))$, $x_i(t) = (x_{i1}(t), x_{i2}(t))^T$. 则跟踪误差系统为

$$\begin{cases} \dot{x}_{11}(t) = x_{12}(t), \\ \dot{x}_{12}(t) = f_1(t, p(t), \omega_1(t)) + u_1(t), \\ \dot{x}_{21}(t) = x_{22}(t), \\ \dot{x}_{22}(t) = f_2(t, p(t), \omega_2(t)) + u_2(t), \end{cases} \quad (3)$$

即

$$\dot{x}(t) = Ax(t) + B[f(t, x(t) + \hat{v}(t), \omega(t)) + u(t)], \quad (4)$$

其中

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix},$$

$$\hat{v}(t) = \text{col}(\hat{v}_1(t), \hat{v}_2(t)), \quad \hat{v}_i(t) = (v_i(t), \dot{v}_i(t))^T.$$

下面给出本文的假设条件.

假设1 存在常数 $\hat{M}_1, \hat{M}_2 > 0$, 使得外部扰动和参考信号满足

$$\sup_{t \in [0, \infty)} \|\hat{v}(t)\|_{\infty} \leq \hat{M}_1, \quad \sup_{t \in [0, \infty)} \|\hat{\omega}(t)\|_{\infty} \leq \hat{M}_2, \quad (5)$$

其中 $\|\cdot\|_{\infty}$ 代表向量的无穷范数或者矩阵的无穷范数, $\hat{\omega}(t) = (\omega_1(t), \dot{\omega}_1(t), \omega_2(t), \dot{\omega}_2(t))^T$.

对于系统 (3) 中的非线性函数, 本文假设如下式 (6) 成立.

假设2 存在正常数 M_{11}, M_{12}, M_2 , 使得对任意的 $z = (z_1, z_2, z_3, z_4)^T \in \mathbb{R}^4$ 都有

$$\begin{aligned} \max \left\{ \|f(t, z, \omega)\|_{\infty}, \left\| \frac{\partial f(t, z, \omega)}{\partial t} \right\|_{\infty}, \left| \frac{\partial f_1(t, z, \omega_1)}{\partial \omega_1} \right|, \left| \frac{\partial f_2(t, z, \omega_2)}{\partial \omega_2} \right| \right\} &\leq M_{11} + M_{12}\|z\|_{\infty}, \\ \max \left\{ \left\| \frac{\partial f_1(t, z, \omega_1)}{\partial z} \right\|_{\infty}, \left\| \frac{\partial f_2(t, z, \omega_2)}{\partial z} \right\|_{\infty} \right\} &\leq M_2. \end{aligned} \quad (6)$$

在假设 1 和 2 下, 可以证明随着采样步长的减小, 本文设计的控制算法可使得跟踪误差收敛到以零点为中心的任意小邻域. 为进一步证明控制闭环系统的渐近稳定性, 我们需要假设参考信号为常数设定值 $v(t) = (p_1^*, p_2^*)$, 且 $f(\cdot)$ 满足如下假设 3.

假设3 存在正常数 M_1, M_{11}, M_{12} , 使得对任意的 $z = (z_1, z_2, z_3, z_4)^T \in \mathbb{R}^4$, 有

$$\begin{aligned} \|f(t, z + \hat{v}, \omega)\|_{\infty} &\leq M_{11} + M_{12}\|z\|_{\infty}, \\ \max \left\{ \left\| \frac{\partial f(t, z + \hat{v}, \omega)}{\partial t} \right\|_{\infty}, \left| \frac{\partial f_1(t, z + \hat{v}, \omega_1)}{\partial \omega_1} \right|, \left| \frac{\partial f_2(t, z + \hat{v}, \omega_2)}{\partial \omega_2} \right| \right\} &\leq M_1\|z\|_{\infty}, \\ \max \left\{ \left\| \frac{\partial f_1(t, z, \omega_1)}{\partial z} \right\|_{\infty}, \left\| \frac{\partial f_2(t, z, \omega_2)}{\partial z} \right\|_{\infty} \right\} &\leq M_2. \end{aligned} \quad (7)$$

注1 假设 1 意味着外部干扰参考信号以及它们的一阶导数有界. 大量的非线性函数可以满足假设 2, 包括正弦函数、余弦函数以及如下的时变系数线性函数:

$$f_i(t, z_1, z_2, z_3, z_4, \omega_i) = a_0(t) + a_1(t)z_1 + a_2(t)z_2 + a_3(t)z_3 + a_4(t)z_4 + a_5(t)\omega_i, \quad (8)$$

其中系数 $a_j(t)$, $j = 1, 2, \dots, 5$ 及其导数有界. 当 $a_0(t) + a_5(t)\omega_i$ 恒等于常数时, 式 (8) 中的 $f_i(\cdot)$ 满足假设 3.

3 控制器的设计与理论结果

本节给出非线性连续时间 MIMO 系统 (4) 的不确定性补偿控制设计和主要理论结果. 本节包括两个小节, 第 1 小节是基于历史采样数据的不确定性因素补偿控制器的设计. 第 2 小节给出本文的主要理论结果.

3.1 控制器的设计

本文考虑在等周期采样下基于历史采样数据的不确定因素补偿控制设计, 用于反馈设计的信息是误差系统 (4) 的状态采样数据:

$$x((k-1)\tau) = (x_{11}((k-1)\tau), x_{12}((k-1)\tau), x_{21}((k-1)\tau), x_{22}((k-1)\tau))^T, \quad k = 1, 2, \dots, \quad (9)$$

其中 $\tau > 0$ 为采样步长.

受 PID 和扰动/不确定性补偿控制^[13, 19, 35, 36]启发, 本文反馈控制器由 PD 控制器和补偿器两部分构成:

$$u(t) = (u_1(t), u_2(t))^T = Kx((k-1)\tau) - \text{Com}_k, \quad \forall t \in [(k-1)\tau, k\tau), \quad k = 1, 2, \dots, \quad (10)$$

其中

$$u_i(t) = K_i x_i((k-1)\tau) - \text{Com}_{ki}, \quad i = 1, 2,$$

$$K = \begin{pmatrix} K_1 & 0_{1 \times 2} \\ 0_{1 \times 2} & K_2 \end{pmatrix} \in \mathbb{R}^{2 \times 4}, \quad \text{Com}_k = \begin{pmatrix} \text{Com}_{k1} \\ \text{Com}_{k2} \end{pmatrix} \in \mathbb{R}^2, \quad (11)$$

$K_i = (-\alpha_{pi}, -\alpha_{di}), i = 1, 2, \alpha_{pi}, \alpha_{di}$ 均为正常数. 为了简单起见, 我们假设 $\alpha_{di}^2 = 4\alpha_{pi}$. $\text{Com}_{ki} \in \mathbb{R}$ 是通过采样数据 $x((j-1)\tau), j = 1, 2, \dots, k$ 设计的补偿器, 用以补偿“总扰动” $f_i(\cdot)$.

控制器设计的关键是补偿器 Com_k 的设计. 由式 (3) 的第 2 个和第 4 个式子可得在采样区间 $((k-2)\tau, (k-1)\tau)$ 上,

$$x_{i2}((k-1)\tau) = x_{i2}((k-2)\tau) + \int_{(k-2)\tau}^{(k-1)\tau} f_i(s, p(s), \omega_i(s)) ds + \tau u_i((k-2)\tau). \quad (12)$$

由积分中值定理知, 存在 $\xi_{i,k-1} \in ((k-2)\tau, (k-1)\tau)$ 使

$$x_{i2}((k-1)\tau) - x_{i2}((k-2)\tau) = \tau f_i(\xi_{i,k-1}, p(\xi_{i,k-1}), \omega_i(\xi_{i,k-1})) + \tau u_i((k-2)\tau). \quad (13)$$

于是我们获得在上一采样区间 $((k-2)\tau, (k-1)\tau)$ 内 $\xi_{i,k-1}$ 时刻处“总扰动”的真实值:

$$f_i(\xi_{i,k-1}, p(\xi_{i,k-1}), \omega_i(\xi_{i,k-1})) = \frac{1}{\tau} [x_{i2}((k-1)\tau) - x_{i2}((k-2)\tau)] - u_i((k-2)\tau), \quad (14)$$

因此补偿器 Com_k 设计为

$$\text{Com}_k = (\text{Com}_{k1}, \text{Com}_{k2})^T, \quad (15)$$

其中

$$\text{Com}_{ki} = \begin{cases} 0, & k = 1, 2, \\ \frac{1}{\tau} [x_{i2}((k-1)\tau) - x_{i2}((k-2)\tau)] - u_i((k-2)\tau), & k = 3, \dots \end{cases} \quad (16)$$

3.2 主要结果

基于 3.1 小节中给出的补偿器和反馈控制器, 闭环控制系统可写为

$$\dot{x}(t) = Ax(t) + B[f(t, p(t), \omega(t)) + Kx((k-1)\tau) - \text{Com}_k], \quad i = 1, 2, \quad k = \lceil t/\tau \rceil + 1, \quad t \in [0, \infty), \quad (17)$$

其中 $\lceil t/\tau \rceil$ 为小于或等于 t/τ 的最大整数.

对于上述控制闭环系统, 如下结论成立.

引理 1 若假设 1 和 2 成立, 则存在正常数 τ^* 和 M_0^* , 对任意的采样周期 $\tau \in (0, \tau^*)$ 有

$$\{x(t) : t \in [0, \infty)\} \subset \mathcal{B}, \quad (18)$$

其中 $\mathcal{B} = \{z = (z_1, z_2, z_3, z_4)^T \in \mathbb{R}^4 : \|z\|_\infty \leq M_0^*\}$.

引理 1 控制闭环系统的一致有界性, 其证明在第 4 节给出. 在引理 1 的基础上我们可以证明本文的主要理论结果定理 1.

定理 1 (i) 若假设 1, 假设 2 和引理 1 成立, 则对任意的初始状态 $x(0) = \text{col}(x_1(0), x_2(0)) \in \mathbb{R}^4$, $x_i(0) = (x_{i10}, x_{i20})^T$, 存在 $\tau^* > 0$ 和 $T^* > 0$, 使得当采样周期 $\tau \in (0, \tau^*)$ 时系统 (17) 的状态满足

$$\|x(t)\|_\infty \leq \bar{M}\sqrt{\tau}, \quad \forall t \in (T^*, \infty), \quad (19)$$

其中 $\bar{M}$ 是一个与 τ 无关的常数.

(ii) 当假设 3 也成立且参考信号为常数设定值时, 对于任意的初始状态 $x(0) = \text{col}(x_1(0), x_2(0)) \in \mathbb{R}^4$, $x_i(0) = (x_{i10}, x_{i20})^T$, 存在 $\tau^* > 0$, 使得当采样周期 $\tau \in (0, \tau^*)$ 时, 系统 (17) 的状态满足

$$\lim_{t \rightarrow \infty} \|x(t)\|_\infty = 0. \quad (20)$$

上述定理表明当非线性函数 $f(\cdot)$ 满足假设 2 且采样周期足够小时, 跟踪误差可以任意小. 如果非线性函数 $f(\cdot)$ 还满足假设 3, 那么随着时间趋近于无穷时跟踪误差渐近收敛于 0.

4 主要结果的证明

本节由 3 个小节组成, 第 1 小节给出了反馈闭环系统的迭代序列和相关矩阵的范数, 第 2 小节是引理 1 的证明, 第 3 小节是定理 1 的证明.

4.1 序列和矩阵范数

容易证明系统 (4) 等价于以下积分系统:

$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-s)}Bf(s, p(s), \omega(s))ds + \int_0^t e^{A(t-s)}Bu(s)ds, \quad (21)$$

其中 $x(0) = \text{col}(x_1(0), x_2(0)) \in \mathbb{R}^4$, $x_i(0) = (x_{i10}, x_{i20})^T$. 于是在每个采样时刻我们有

$$x(k\tau) = e^{A\tau}x((k-1)\tau) + \int_{(k-1)\tau}^{k\tau} e^{A(k\tau-s)}Bu(s)ds + \int_{(k-1)\tau}^{k\tau} e^{A(k\tau-s)}Bf(s, p(s), \omega(s))ds. \quad (22)$$

令 I 为 4×4 阶单位矩阵. 由 $A^2 = 0$ 可知

$$e^{A\tau} = I + A\tau,$$

$$e^{A(k\tau-s)}B = (I + A(k\tau-s))B = \begin{pmatrix} k\tau-s & 0 \\ 1 & 0 \\ 0 & k\tau-s \\ 0 & 1 \end{pmatrix}. \quad (23)$$

由式 (10) 和 (22), 我们有离散化的闭环反馈控制系统:

$$\begin{aligned} x(k\tau) &= e^{A\tau}x((k-1)\tau) + \int_{(k-1)\tau}^{k\tau} e^{A(k\tau-s)}Bu(s)ds + \int_{(k-1)\tau}^{k\tau} e^{A(k\tau-s)}Bf(s, p(s), \omega(s))ds \\ &= \left(I + (A+BK)\tau + \frac{\tau^2}{2}ABK \right) x((k-1)\tau) \\ &\quad + \int_{(k-1)\tau}^{k\tau} \begin{pmatrix} k\tau-s & 0 \\ 1 & 0 \\ 0 & k\tau-s \\ 0 & 1 \end{pmatrix} (f(s, p(s), \omega(s)) - \text{Com}_k) ds. \end{aligned} \quad (24)$$

与文献 [35] 中的离散控制系统不同, 上式含有非线性不确定性因素的积分项. 本文控制器设计与理论证明的关键是处理这些非线性不确定性因素的积分项.

设

$$H_i = \begin{pmatrix} 1 - \frac{\alpha_{pi}\tau^2}{2} & \tau - \frac{\alpha_{di}\tau^2}{2} \\ -\alpha_{pi}\tau & 1 - \alpha_{di}\tau \end{pmatrix}, \quad i = 1, 2, \quad (25)$$

$$H = I + (A+BK)\tau + \frac{\tau^2}{2}ABK = \begin{pmatrix} H_1 & 0_{2 \times 2} \\ 0_{2 \times 2} & H_2 \end{pmatrix} = \begin{pmatrix} 1 - \frac{\alpha_{p1}\tau^2}{2} & \tau - \frac{\alpha_{d1}\tau^2}{2} & 0 & 0 \\ -\alpha_{p1}\tau & 1 - \alpha_{d1}\tau & 0 & 0 \\ 0 & 0 & 1 - \frac{\alpha_{p2}\tau^2}{2} & \tau - \frac{\alpha_{d2}\tau^2}{2} \\ 0 & 0 & -\alpha_{p2}\tau & 1 - \alpha_{d2}\tau \end{pmatrix}, \quad (26)$$

$$\Delta_k = \int_{(k-1)\tau}^{k\tau} \begin{pmatrix} k\tau-s & 0 \\ 1 & 0 \\ 0 & k\tau-s \\ 0 & 1 \end{pmatrix} (f(s, p(s), \omega(s)) - \text{Com}_k) ds. \quad (27)$$

由式 (2), (16) 和 (24), 反馈闭环控制系统 (17) 可以写成

$$x(k\tau) = Hx((k-1)\tau) + \Delta_k, \quad k = 1, 2, \dots \quad (28)$$

因为对任意的 $t \in ((k-1)\tau, k\tau)$, $k = 1, 2, \dots$, 有

$$\begin{aligned} x(t) &= x((k-1)\tau) + \int_{(k-1)\tau}^t (Ax(s) + Bf(s, p(s), \omega(s)) + Bu(s))ds \\ &= x((k-1)\tau) + \int_{(k-1)\tau}^t (Ax(s) + Bf(s, p(s), \omega(s)) + B(Kx((k-1)\tau) - \text{Com}_k))ds. \end{aligned} \quad (29)$$

所以对于 $k = 3, 4, \dots$, 由式 (28) 可得如下的迭代序列:

$$x(k\tau) = H^{k-2}x(2\tau) + H^{k-3}\Delta_3 + H^{k-4}\Delta_4 + \dots + H\Delta_{k-1} + \Delta_k. \quad (30)$$

对于迭代序列 (30) 中的矩阵 H , 我们在附录 A 中证明存在正常数 $\tau^* > 0$, 使得对于任意的 $\tau \in (0, \tau^*)$, 有

$$\begin{aligned} \|H^j\|_\infty &\leq \frac{4\alpha_{p\max}^2 + 8\alpha_{p\max}\alpha_{d\max} + 3\alpha_{d\max}^2}{2\alpha_{p\min}\sqrt{\alpha_{p\min}\alpha_{d\min}}} \frac{1}{\sqrt{\tau}} \lambda_\tau^j, \quad j = 1, 2, \dots, \\ \lambda_\tau &= 1 - \frac{2\alpha_{d\min}\tau + \alpha_{p\min}\tau^2}{8} \in (0, 1), \end{aligned} \quad (31)$$

并且

$$\frac{1}{1 - \lambda_\tau} < \frac{4}{\alpha_{d\min}\tau}, \quad (32)$$

其中 $\alpha_{p\max} = \max\{\alpha_{p1}, \alpha_{p2}\}$, $\alpha_{p\min} = \min\{\alpha_{p1}, \alpha_{p2}\}$, $\alpha_{d\max} = \max\{\alpha_{d1}, \alpha_{d2}\}$, $\alpha_{d\min} = \min\{\alpha_{d1}, \alpha_{d2}\}$.

4.2 引理 1 的证明

本小节将证明存在正常数 $\tau^* > 0$ (τ^* 的表达式在附录式 (A3) 中给出), 对于任意的 $\tau \in (0, \tau^*)$, 通过 3 步证明式 (19) 成立, 即反馈闭环系统 (17) (或 (28)) 状态是一致有界的.

第 1 步 对任意的 $\tau \in (0, \tau^*)$,

$$\max_{t \in [0, 2\tau]} \|x(t)\|_\infty < \|x(0)\|_\infty + \frac{1}{2}, \quad (33)$$

其中 $x(0) = \text{col}(x_1(0), x_2(0)) \in \mathbb{R}^4$, $x_i(0) = (x_{i10}, x_{i20})^T$ 是任意给定的初始状态.

现在用反证法来证明以上结论. 如果式 (33) 不成立, 由系统状态的连续性可知, 存在 $t_1 \in [0, 2\tau]$, 使得对于任意 $t \in [0, t_1]$ 有

$$\|x(t)\|_\infty < \|x(0)\|_\infty + \frac{1}{2}, \quad (34)$$

且

$$\|x(t_1)\|_\infty = \|x(0)\|_\infty + \frac{1}{2}. \quad (35)$$

结合假设 1 和 2, 对于任意的 $t \in [0, t_1]$ 有

$$\|f(t, p(t), \omega(t))\|_\infty \leq M_{11} + M_{12}\|p(t)\|_\infty \leq M_{11} + M_{12}\left(\|x(0)\|_\infty + \frac{1}{2} + \hat{M}_1\right). \quad (36)$$

由于

$$\|K\|_\infty \leq \alpha_{p\max} + \alpha_{d\max}, \quad (37)$$

并且 $k = 1, 2$ 时, $\text{Com}_k = (0, 0)^T$, 故

$$\|u(t)\|_\infty = \|Kx((k-1)\tau) - \text{Com}_k\|_\infty \leq (\alpha_{p\max} + \alpha_{d\max})\left(\|x(0)\|_\infty + \frac{1}{2} + \hat{M}_1\right). \quad (38)$$

再结合式 (A3) 可得

$$\|x(t_1)\|_\infty \leq \|x(0)\|_\infty + \left\| \int_0^{t_1} (Ax(s) + B(f(s, p(s), \omega(s)) + u(s))) ds \right\|$$

$$\begin{aligned} &\leq \|x(0)\|_{\infty} + 2\tau \left(M_{11} + (1 + \alpha_{p \max} + \alpha_{d \max} + M_{12}) \left(\|x(0)\|_{\infty} + \frac{1}{2} + \hat{M}_1 \right) \right) \\ &\leq \|x(0)\|_{\infty} + \frac{1}{2}. \end{aligned} \quad (39)$$

这与式 (35) 矛盾. 因此式 (33) 成立, 且

$$\|x((k-1)\tau)\|_{\infty} < \|x(0)\|_{\infty} + \frac{1}{2}, \quad k = 1, 2, 3. \quad (40)$$

第 2 步 在这里我们将证明对于任意 $\tau \in (0, \tau^*)$, 系统的解满足

$$\{(x_{11}(t), x_{12}(t), x_{21}(t), x_{22}(t)) : t \in [0, \infty)\} \subset \mathcal{A}, \quad (41)$$

其中

$$\mathcal{A} = \left\{ z \in \mathbb{R}^4 : \|z\|_{\infty} \leq \frac{M_0}{\sqrt{\tau}} \right\}, \quad (42)$$

这里 τ^* 的表达式由附录式 (A3) 给出, M_0 是不依赖于 τ 的正常数, 其表达式由式 (A6) 给出.

由第 1 步所得结果可知

$$\{(x_{11}(t), x_{12}(t), x_{21}(t), x_{22}(t)) : t \in [0, 2\tau]\} \subset \mathcal{A}^{\circ}, \quad (43)$$

其中 $\mathcal{A}^{\circ}$ 是集 $\mathcal{A}$ 的内部.

如果式 (41) 不成立, 则存在 $t_0 \in (2\tau, \infty)$ 使得

$$\{x(t) : t \in [0, t_0]\} \subset \mathcal{A}^{\circ}, \quad (44)$$

并且

$$x(t_0) \in \partial \mathcal{A}. \quad (45)$$

不失一般性, 我们假设 $t_0 \in [k_0\tau, (k_0+1)\tau)$ 且 $k_0 \geq 2$. 由式 (44) 知对于任意的 $t \in [0, t_0]$, $\|p(t)\|_{\infty} \leq \|x(t)\|_{\infty} + \|\hat{v}(t)\|_{\infty} \leq M_0/\sqrt{\tau} + \hat{M}_1$. 由假设 2 可得

$$\max \left\{ \|f(t, z, \omega)\|_{\infty}, \left\| \frac{\partial f(t, z, \omega)}{\partial t} \right\|_{\infty}, \left| \frac{\partial f_1(t, z, \omega_1)}{\partial \omega_1} \right|, \left| \frac{\partial f_2(t, z, \omega_2)}{\partial \omega_2} \right| \right\} \leq M_{11} + M_{12}\hat{M}_1 + \frac{M_0 M_{12}}{\sqrt{\tau}}, \quad (46)$$

$$\max \left\{ \left\| \frac{\partial f_1(t, z, \omega_1)}{\partial z} \right\|_{\infty}, \left\| \frac{\partial f_2(t, z, \omega_2)}{\partial z} \right\|_{\infty} \right\} \leq M_2, \quad (47)$$

设

$$\begin{aligned} \tilde{\Delta}_k = & \sup_{t_1, t_2 \in [(k-2)\tau, k\tau)} \|f(t_2, p_{11}(t_2), p_{12}(t_2), p_{21}(t_2), p_{22}(t_2), \omega(t_2)) \\ & - f(t_1, p_{11}(t_1), p_{12}(t_1), p_{21}(t_1), p_{22}(t_1), \omega(t_1))\|_{\infty}. \end{aligned} \quad (48)$$

由式 (44) 和 (45) 可得存在不依赖于 τ 的正常数 $\tilde{M} > 0$ (其表达式由式 (A7) 给出) 使得

$$\max_{2 \leq k \leq k_0} \tilde{\Delta}_k \leq \tilde{M}\sqrt{\tau}, \quad \max_{2 \leq k \leq k_0} \|\Delta_k\|_{\infty} \leq \tilde{M}\tau^{3/2}. \quad (49)$$

式 (49) 的证明在附录 B 中给出.

可证明对任意 $s \in [k_0\tau, t_0]$, 有 $\|Ax(s)\|_\infty \leq \|x(s)\|_\infty \leq M_0/\sqrt{\tau}$, $\|Kx(k_0\tau)\|_\infty \leq \frac{(\alpha_{p\max} + \alpha_{d\max})M_0}{\sqrt{\tau}}$. 从而

$$\|x(t_0)\|_\infty \leq \|x(k_0\tau)\|_\infty + \int_{k_0\tau}^{t_0} \|f(s, p(s), \omega(s)) - \text{Com}_{k_0+1}\|_\infty + \frac{(1 + \alpha_{p\max} + \alpha_{d\max})M_0}{\sqrt{\tau}} ds. \quad (50)$$

与式 (49) 的证明类似, 可得对于任意的 $s \in [k_0\tau, t_0]$,

$$\|f(s, p(s), \omega(s)) - \text{Com}_{k_0+1}\|_\infty \leq \tilde{M}\sqrt{\tau}. \quad (51)$$

因此

$$\|x(t_0)\|_\infty \leq \|x(k_0\tau)\|_\infty + \tilde{M}\tau^{3/2} + (1 + \alpha_{p\max} + \alpha_{d\max})M_0\sqrt{\tau}. \quad (52)$$

由式 (30)~(32) 以及式 (49) 可得

$$\begin{aligned} \|x(k_0\tau)\|_\infty &\leq \|H^{k_0-2}x(2\tau)\|_\infty + \|H^{k_0-3}\Delta_3\|_\infty + \cdots + \|H\Delta_{k_0-1}\|_\infty + \|\Delta_{k_0}\|_\infty \\ &\leq \frac{(4\alpha_{p\max}^2 + 8\alpha_{p\max}\alpha_{d\max} + 3\alpha_{d\max}^2)(\|x(0)\|_\infty + \frac{1}{2})}{2\alpha_{p\min}\sqrt{\alpha_{p\min}\alpha_{d\min}}} \frac{1}{\sqrt{\tau}} \\ &\quad + \frac{8\alpha_{p\max}^2 + 16\alpha_{p\max}\alpha_{d\max} + 6\alpha_{d\max}^2}{\alpha_{p\min}\alpha_{d\min}\sqrt{\alpha_{p\min}\alpha_{d\min}}} \tilde{M} + \tilde{M}\tau^{3/2}. \end{aligned} \quad (53)$$

对于任意的 $\tau \in (0, \tau^*)$, 结合式 (A3) 与 (A4), 我们有

$$\begin{aligned} \|x(t_0)\|_\infty &\leq \|x(k_0\tau)\|_\infty + \tilde{M}\tau^{3/2} + (1 + \alpha_{p\max} + \alpha_{d\max})M_0\sqrt{\tau} \\ &\leq \frac{(4\alpha_{p\max}^2 + 8\alpha_{p\max}\alpha_{d\max} + 3\alpha_{d\max}^2)(\|x(0)\|_\infty + \frac{1}{2})}{2\alpha_{p\min}\sqrt{\alpha_{p\min}\alpha_{d\min}}} \frac{1}{\sqrt{\tau}} + 2\tilde{M}\tau^{3/2} \\ &\quad + \frac{8\alpha_{p\max}^2 + 16\alpha_{p\max}\alpha_{d\max} + 6\alpha_{d\max}^2}{\alpha_{p\min}\alpha_{d\min}\sqrt{\alpha_{p\min}\alpha_{d\min}}} \tilde{M} + (1 + \alpha_{p\max} + \alpha_{d\max})M_0\sqrt{\tau} \\ &\leq \frac{(4\alpha_{p\max}^2 + 8\alpha_{p\max}\alpha_{d\max} + 3\alpha_{d\max}^2)(\|x(0)\|_\infty + \frac{1}{2})}{2\alpha_{p\min}\sqrt{\alpha_{p\min}\alpha_{d\min}}} \frac{1}{\sqrt{\tau}} + \frac{3}{4\sqrt{\tau}} < \frac{M_0}{\sqrt{\tau}}. \end{aligned} \quad (54)$$

这与式 (45) 矛盾, 因此式 (41) 成立.

第 3 步 对于任意 $\tau \in (0, \tau^*)$, 存在 $k_1 \in N$ 使得

$$\{x(t) : t \in [k_1\tau, \infty)\} \subset \mathcal{B}, \quad (55)$$

其中

$$\mathcal{B} = \{z \in \mathbb{R}^4 : \|z\|_\infty \leq M_0^*\}, \quad (56)$$

$$M_0^* = 1 + \frac{(1 + \alpha_{p\max} + \alpha_{d\max})M_0}{2} + \left(\frac{1}{6} + \frac{8\alpha_{p\max}^2 + 16\alpha_{p\max}\alpha_{d\max} + 6\alpha_{d\max}^2}{\alpha_{p\min}\alpha_{d\min}\sqrt{\alpha_{p\min}\alpha_{d\min}}} \right) \tilde{M}. \quad (57)$$

由第 2 步的结果可知, 对任意的 $\tau \in [0, \infty)$, 有

$$\|Ax(t)\|_\infty \leq \frac{M_0}{\sqrt{\tau}}, \quad \|Kx([t/\tau]\tau)\|_\infty \leq \frac{(\alpha_{p\max} + \alpha_{d\max})M_0}{\sqrt{\tau}}. \quad (58)$$

与式 (49) 的证明类似, 对任意的 $s \in [(k-1)\tau, k\tau]$, $k = 3, 4, \dots$, 有

$$\|f(s, p(s), \omega(s)) - \text{Com}_{k_0+1}\|_\infty \leq \tilde{M}\sqrt{\tau}, \quad \|\Delta_k\|_\infty \leq \tilde{M}\tau^{3/2}. \quad (59)$$

由 $\lambda_\tau \in (0, 1)$ 可知 $\lim_{k \rightarrow \infty} \lambda_\tau^k = 0$. 因此存在自然数 $k_1 > 2$ 使得对于任意的 $t > k_1 \tau$,

$$\lambda_\tau^{[t/\tau]} < \frac{2\alpha_{p \min} \sqrt{\alpha_{p \min} \alpha_{d \min}} \sqrt{\tau}}{(4\alpha_{p \max}^2 + 8\alpha_{p \max} \alpha_{d \max} + 3\alpha_{d \max}^2) (\|x(0)\|_\infty + \frac{1}{2})}. \quad (60)$$

考虑到式 (29) 和 (60), 对任意的 $\tau \in (0, \tau^*)$ 和 $t > k_1 \tau$, 我们有

$$\begin{aligned} \|x(t)\|_\infty &\leq \|x([t/\tau]\tau)\|_\infty + \tilde{M}\tau^{3/2} + (1 + \alpha_{p \max} + \alpha_{d \max}) M_0 \sqrt{\tau} \\ &\leq \frac{(4\alpha_{p \max}^2 + 8\alpha_{p \max} \alpha_{d \max} + 3\alpha_{d \max}^2) (\|x(0)\|_\infty + \frac{1}{2})}{2\alpha_{p \min} \sqrt{\alpha_{p \min} \alpha_{d \min}} \sqrt{\tau}} \lambda_\tau^{[t/\tau]} \\ &\quad + (1 + \alpha_{p \max} + \alpha_{d \max}) M_0 \sqrt{\tau} + 2\tilde{M}\tau^{3/2} + \tilde{M}\tau^{3/2} \left(\|H\|_\infty + \dots + \|H^{[t/\tau]-3}\|_\infty \right) \\ &\leq 1 + \frac{(1 + \alpha_{p \max} + \alpha_{d \max}) M_0}{\sqrt{6}} + \frac{\tilde{M}}{3\sqrt{6}} + \frac{8\alpha_{p \max}^2 + 16\alpha_{p \max} \alpha_{d \max} + 6\alpha_{d \max}^2}{\alpha_{p \min} \alpha_{d \min} \sqrt{\alpha_{p \min} \alpha_{d \min}}} \tilde{M} \leq M_0^*. \end{aligned} \quad (61)$$

因此对于任意的 $\tau \in (0, \tau^*)$, 式 (55) 成立. 故引理 1 得证.

4.3 定理 1 的证明

首先证明定理 1 之 (i). 由假设 2 和式 (55) 知对于任意的 $t > t_1$,

$$\begin{aligned} \max \left\{ \|f(t, z, \omega)\|_\infty, \left\| \frac{\partial f(t, z, \omega)}{\partial t} \right\|_\infty, \left\| \frac{\partial f_1(t, z, \omega_1)}{\partial \omega_1} \right\|_\infty, \left\| \frac{\partial f_2(t, z, \omega_2)}{\partial \omega_2} \right\|_\infty \right\} &\leq M_{11} + M_{12} (M_0^* + \hat{M}_1), \\ \max \left\{ \left\| \frac{\partial f_1(t, z, \omega_1)}{\partial z} \right\|_\infty, \left\| \frac{\partial f_2(t, z, \omega_2)}{\partial z} \right\|_\infty \right\} &\leq M_2, \end{aligned} \quad (62)$$

且

$$\|Ax(t)\|_\infty \leq M_0^*, \quad \|Kx([t/\tau]\tau)\|_\infty \leq (\alpha_{p \max} + \alpha_{d \max}) M_0^*. \quad (63)$$

类似式 (49), 我们可以证明存在与 τ 无关的常数 $\tilde{M}_1 > 0$, 使得对于任意的 $t > k_1 \tau$ 和 $k \geq k_1 + 1$,

$$\|f(s, p(s), \omega(s)) - \text{Com}_{k_0+1}\| \leq \tilde{M}_1 \tau, \quad \|\Delta_k\|_\infty \leq \tilde{M}_1 \tau^2. \quad (64)$$

由于 $\lim_{k \rightarrow \infty} \lambda_\tau^k = 0$, 故存在自然数 $k^* > k_1$, 使得对于任意的 $t > (k^* + 1) \tau$,

$$\lambda_\tau^{[t/\tau]} < \frac{2\alpha_{p \min} \sqrt{\alpha_{p \min} \alpha_{d \min}}}{(4\alpha_{p \max}^2 + 8\alpha_{p \max} \alpha_{d \max} + 3\alpha_{d \max}^2) (\|x(0)\|_\infty + \frac{1}{2})} \tau. \quad (65)$$

对于任意的 $t > \max\{t_1, k^* \tau\}$, 与式 (61) 同理, 可得

$$\begin{aligned} \|x(t)\|_\infty &\leq \|x([t/\tau]\tau)\|_\infty + \tilde{M}\tau^{3/2} + (1 + \alpha_{p \max} + \alpha_{d \max}) M_0 \sqrt{\tau} \\ &\leq \sqrt{\tau} + \frac{(1 + \alpha_{p \max} + \alpha_{d \max}) M_0^*}{\sqrt{6}} \sqrt{\tau} + \frac{\tilde{M}_1}{3\sqrt{6}} \sqrt{\tau} \\ &\quad + \frac{8\alpha_{p \max}^2 + 16\alpha_{p \max} \alpha_{d \max} + 6\alpha_{d \max}^2}{\alpha_{p \min} \alpha_{d \min} \sqrt{\alpha_{p \min} \alpha_{d \min}}} \tilde{M}_1 \sqrt{\tau} \leq \bar{M} \sqrt{\tau}, \end{aligned} \quad (66)$$

其中

$$\bar{M} = 1 + \frac{(1 + \alpha_{p \max} + \alpha_{d \max}) M_0^*}{2} + \left(\frac{1}{6} + \frac{8\alpha_{p \max}^2 + 16\alpha_{p \max} \alpha_{d \max} + 6\alpha_{d \max}^2}{\alpha_{p \min} \alpha_{d \min} \sqrt{\alpha_{p \min} \alpha_{d \min}}} \right) \tilde{M}_1. \quad (67)$$

接下来我们证明定理 1 之 (ii). 当假设 3 成立时, 假设 2 自然满足. 由定理 1 之 (i) 知, 存在一个与 τ 无关的常数 $\bar{M}_1 > 0$, 使得对于任意的 $\tau \in (0, \tau^*)$, $t > t_0$, 有 $\|x(t)\|_\infty \leq \bar{M}_1 \sqrt{\tau}$, 其中 $t_0 > 0$ 是一个与 τ 无关的常数. 结合假设 3, 对于任意的 $t \in (t_0, \infty)$,

$$\begin{aligned} \|f(t, z + \hat{v}, \omega)\|_\infty &\leq M_{11} + M_{12} \bar{M}_1 \sqrt{\tau}, \\ \max \left\{ \left\| \frac{\partial f(t, z + \hat{v}, \omega)}{\partial t} \right\|_\infty, \left| \frac{\partial f_1(t, z + \hat{v}, \omega_1)}{\partial \omega_1} \right|, \left| \frac{\partial f_2(t, z + \hat{v}, \omega_2)}{\partial \omega_2} \right| \right\} &\leq M_1 \bar{M}_1 \sqrt{\tau}, \\ \max \left\{ \left\| \frac{\partial f_1(t, z, \omega_1)}{\partial z} \right\|_\infty, \left\| \frac{\partial f_2(t, z, \omega_2)}{\partial z} \right\|_\infty \right\} &\leq M_2. \end{aligned} \quad (68)$$

因此对于任意的 $\tau \in (0, \tau^*)$, $t_1, t_2 \in [(k-2)\tau, k\tau]$, 以及每一个 $k > [t_0] + 2$, 由附录 B 中式 (B2)~(B7) 可得

$$\begin{aligned} &\|f(t_2, p_{11}(t_2), p_{12}(t_2), p_{21}(t_2), p_{22}(t_2), \omega(t_2)) - f(t_1, p_{11}(t_2), p_{12}(t_2), p_{21}(t_2), p_{22}(t_2), \omega(t_2))\|_\infty \\ &\leq 2M_1 \bar{M}_1 \tau^{3/2}, \end{aligned} \quad (69)$$

$$\begin{aligned} &\|f(t_1, p_{11}(t_2), p_{12}(t_2), p_{21}(t_2), p_{22}(t_2), \omega(t_2)) - f(t_1, p_{11}(t_1), p_{12}(t_2), p_{21}(t_2), p_{22}(t_2), \omega(t_2))\|_\infty \\ &\leq 2M_2 \bar{M}_1 \tau^{3/2}, \end{aligned} \quad (70)$$

$$\begin{aligned} &\|f(t_1, p_{11}(t_1), p_{12}(t_2), p_{21}(t_2), p_{22}(t_2), \omega(t_2)) - f(t_1, p_{11}(t_1), p_{12}(t_1), p_{21}(t_2), p_{22}(t_2), \omega(t_2))\|_\infty \\ &\leq 2M_2 \tau \left[\tilde{\Delta}_k + \tilde{\Delta}_{k-1} + 2(\alpha_{p \max} + \alpha_{d \max}) \bar{M}_1 \sqrt{\tau} \right], \end{aligned} \quad (71)$$

且

$$\begin{aligned} &\|f(t_1, p_{11}(t_1), p_{12}(t_1), p_{21}(t_1), p_{22}(t_1), \omega(t_2)) - f(t_1, p_{11}(t_1), p_{12}(t_1), p_{21}(t_1), p_{22}(t_1), \omega(t_1))\|_\infty \\ &\leq 2M_1 \hat{M}_1 \hat{M}_2 \tau^{3/2}. \end{aligned} \quad (72)$$

故对于任意的 $\tau \in (0, \tau^*)$, $t > t_0 + 3\tau$, 并且每一个 $k > [t_0] + 3$,

$$\begin{aligned} \tilde{\Delta}_k &\leq 2M_1(1 + \hat{M}_2) \bar{M}_1 \tau^{3/2} + 4M_2 \bar{M}_1 \tau^{3/2} + 4M_2 \tau \left[\tilde{\Delta}_k + \tilde{\Delta}_{k-1} + 2(\alpha_{p \max} + \alpha_{d \max}) \bar{M}_1 \sqrt{\tau} \right] \\ &\leq 2M_1(1 + \hat{M}_2) \bar{M}_1 \tau^{3/2} + 4M_2 \bar{M}_1 \tau^{3/2} + 4M_2 \tau \left[2M_1(1 + \hat{M}_2) \bar{M}_1 \tau^{3/2} + 4M_2 \bar{M}_1 \tau^{3/2} \right. \\ &\quad \left. + 4M_2 \tau \left(\tilde{\Delta}_k + \tilde{\Delta}_{k-1} + 2(\alpha_{p \max} + \alpha_{d \max}) \bar{M}_1 \sqrt{\tau} \right) + 2M_1(1 + \hat{M}_2) \bar{M}_1 \tau^{3/2} + 4M_2 \bar{M}_1 \tau^{3/2} \right. \\ &\quad \left. + 4M_2 \tau \left(\tilde{\Delta}_{k-1} + \tilde{\Delta}_{k-2} + 2(\alpha_{p \max} + \alpha_{d \max}) \bar{M}_1 \sqrt{\tau} \right) + 2(\alpha_{p \max} + \alpha_{d \max}) \bar{M}_1 \sqrt{\tau} \right] \\ &\leq \beta \bar{M}_1 \tau^{3/2}, \end{aligned} \quad (73)$$

其中

$$\beta = (2 + 16M_2)[M_1(1 + \hat{M}_2) + 2M_2(1 + 2\alpha_{p \max} + 2\alpha_{d \max})] + 128M_2^2(M_{11} + M_{12}), \quad (74)$$

所以

$$\|\Delta_k\|_\infty \leq \beta \bar{M}_1 \tau^{5/2}. \quad (75)$$

类似可得

$$\|f(t, p(t), \omega(t)) - \text{Com}_{[t/\tau]+1}\| \leq \beta \bar{M}_1 \tau^{3/2}, \quad (76)$$

与式 (66) 类似, 对于任意的 $t > t_0 + 1$, 有

$$\|x(t)\|_{\infty} \leq \|x([t/\tau]\tau)\|_{\infty} + 2\beta\bar{M}_1\tau^{5/2} + (1 + \alpha_{p\max} + \alpha_{d\max})\bar{M}_1\tau^{3/2} \leq \|H^{[t/\tau]-2}\|_{\infty} \|x(2\tau)\|_{\infty} + \frac{8\alpha_{p\max}^2 + 16\alpha_{p\max}\alpha_{d\max} + 6\alpha_{d\max}^2}{\alpha_{p\min}\alpha_{d\min}\sqrt{\alpha_{p\min}\alpha_{d\min}}}\beta\bar{M}_1\tau + 2\beta\bar{M}_1\tau^{5/2} + (1 + \alpha_{p\max} + \alpha_{d\max})\bar{M}_1\tau^{3/2}, \quad (77)$$

令

$$\tau^{**} = \min \left\{ \tau^*, \left(\frac{\beta}{4(1 + \alpha_{p\max} + \alpha_{d\max})\bar{M}_1} \right)^2, \frac{1}{4\bar{M}_1^{3/2}}, \left(\frac{1}{\beta\gamma} \right)^2 \right\}, \quad (78)$$

其中

$$\gamma = 1 + \frac{8\alpha_{p\max}^2 + 16\alpha_{p\max}\alpha_{d\max} + 6\alpha_{d\max}^2}{\alpha_{p\min}\alpha_{d\min}\sqrt{\alpha_{p\min}\alpha_{d\min}}}. \quad (79)$$

因为 $\lim_{k \rightarrow \infty} \lambda_{\tau}^k = 0$, $\lim_{t \rightarrow \infty} \|H^{[t/\tau]-2}\|_{\infty} \|x(2\tau)\|_{\infty} = 0$, 所以存在 $t_2 > t_1$, 对任意 $t > t_2$, 有

$$\|H^{[t/\tau]-2}\|_{\infty} \cdot \|x(2\tau)\|_{\infty} < \beta\gamma/2.$$

由此可知对于任意的 $t > t_2$ 和 $\tau \in (0, \tau^{**})$,

$$\|x(t)\|_{\infty} \leq \beta\gamma\bar{M}_1\tau, \quad (80)$$

同理可证存在 $t_3 > t_2$, 对于任意的 $t > t_3$ 和 $\tau \in (0, \tau^{**})$,

$$\|x(t)\|_{\infty} \leq \bar{M}_1(\beta\gamma)^2\tau^{3/2}. \quad (81)$$

一般地, 存在 $t_j > t_{j-1}$, 对于任意的 $t > t_j$ 和 $\tau \in (0, \tau^{**})$,

$$\|x(t)\|_{\infty} \leq \bar{M}_1\tau^{1/2} (\beta\gamma\tau^{1/2})^j. \quad (82)$$

又由于对任意的 $\tau \in (0, \tau^{**})$, $\beta\gamma\tau^{1/2} < 1$, 故 $\lim_{j \rightarrow \infty} (\beta\gamma\tau^{1/2})^j = 0$. 因此定理 1 之 (ii) 得证.

5 仿真

本节以二自由度无人直升机控制系统 (2) 为例, 通过仿真说明本文所提出控制方法的有效性并与 PID 控制以及线性自抗扰控制 (linear active disturbance rejection control, LADRC) 进行比较. 控制目的是设计反馈控制器 $u(t) = (u_1(t), u_2(t))^T$, 使得系统的输出即俯仰角 θ , 偏航角 φ 跟踪到设定值 θ^*, φ^* . 在仿真中选取采样周期 τ 为 0.001, 系统初值为 (1,1). 系统参数分别为 $\tau_{pp} = 20.4 \text{ N} \cdot \text{m/V}$, $\tau_{py} = 0.68 \text{ N} \cdot \text{m/V}$, $\tau_{yy} = 7.2 \text{ N} \cdot \text{m/V}$, $\tau_{yp} = 2.19 \text{ N} \cdot \text{m/V}$, $D_p = 80 \text{ N/V}$, $D_y = 31.8 \text{ N/V}$, $I_p = 3.84 \text{ kg} \cdot \text{m}^2$, $I_y = 4.32 \text{ kg} \cdot \text{m}^2$, $m_h = 138.72 \text{ kg}$, $l_{cm} = 18.6 \text{ m}$, $g = 9.81 \text{ m/s}^2$.

由式 (10), (15) 和 (16), 基于历史采样数据的不确定性因素补偿 (SDBCC) 反馈控制器设计为

$$u_k = \begin{cases} \text{Upd}_k, & k = 1, 2, \\ \text{Upd}_k - \text{Com}_k, & k = 3, \dots, \end{cases} \quad (83)$$

其中 PD 控制器 Upd_k 设计为

$$\text{Upd}_k = (-x_{11}((k-1)\tau) - 2x_{12}((k-1)\tau), -x_{21}((k-1)\tau) - 2x_{22}((k-1)\tau))^T, \quad (84)$$

图 1 (网络版彩图) PID 控制与 SDBCC 控制下的系统输出. (a) 俯仰角数值结果; (b) 偏航角数值结果

Figure 1 (Color online) System outputs under PID and SDBCC. (a) Numerical results of the pitch angle; (b) numerical results of the yaw angle

补偿器 Com_{ki} 设计为

$$\text{Com}_{ki} = \frac{1}{\tau} [x_{i2}((k-1)\tau) - x_{i2}((k-2)\tau)] - u_i((k-2)\tau). \quad (85)$$

在仿真中选取设定值 $\theta^* = \varphi^* = -1$. 首先为便于和 PID 控制比较, 系统 (2) 中不确定性因素 $q_i(\cdot)$ 选取为常值干扰 2. PID 控制器设计为 PD 控制器 (84) 加上积分和项:

$$\text{U}_{\text{pid}_k} = \begin{pmatrix} -x_{11}((k-1)\tau) - 2x_{12}((k-1)\tau) - 0.2 \sum_{j=1}^k \tau x_{11}((j-1)\tau) \\ -x_{21}((k-1)\tau) - 2x_{22}((k-1)\tau) - 0.2 \sum_{j=1}^k \tau x_{21}((j-1)\tau) \end{pmatrix}. \quad (86)$$

数值结果如图 1 所示. 由图 1 可见与 PID 控制相比较, 本文的设计方法具有快速无超调收敛的优点.

接下来考虑不确定性因素 $q_i(\cdot)$ 为如下时变非线性函数的情况:

$$\begin{aligned} q_1(\theta(t), \omega_\theta(t), \varphi(t), \omega_\varphi(t), \omega_1(t)) &= \sin(\theta(t) + \omega_\varphi(t)) + \sin(2t) + 2\cos(3t), \\ q_2(\theta(t), \omega_\theta(t), \varphi(t), \omega_\varphi(t), \omega_2(t)) &= \cos(\varphi(t) + \omega_\theta(t)) + \sin(t + \pi/4) + 2\cos t. \end{aligned}$$

在这种情况下 PID 控制中的积分器失效. 此时我们利用基于线性扩张状态观测器 (linear extended state observer, LESO) 的 LADRC 和本文的控制方法进行仿真比较. 根据单参数整定 LESO 的常用设计方法^[34, 51], LESO 设计如下:

$$\begin{cases} \dot{\hat{x}}_{i1}(t; r) = \hat{x}_{i2}(t; r) + 3r(x_{i1}(t) - \hat{x}_{i1}(t; r)), \\ \dot{\hat{x}}_{i2}(t; r) = \hat{f}_i(t; r) + 3r^2(x_{i1}(t) - \hat{x}_{i1}(t; r)) + u_i(t), \\ \dot{\hat{f}}_i(t; r) = r^3(x_{i1}(t) - \hat{x}_{i1}(t; r)), \end{cases} \quad (87)$$

其中 $r > 0$ 是增益参数, $\hat{x}_{ij}(t; r)$, $\hat{f}_i(t; r)$ 分别是对 $x_{ij}(t)$, $f_i(\cdot)$ 的估计, $i = 1, 2$, $j = 1, 2$. 则基于 LESO 的补偿控制的控制器 LADRC^[34, 51] 设计为

$$u_i(t) = -x_{i1}(t) - 2\hat{x}_{i2}(t; r) - \hat{f}_i(t; r), \quad i = 1, 2. \quad (88)$$

图 2 (网络版彩图) LADRC 控制和本文 SDBCC 控制下的系统输出. (a) 俯仰角数值结果; (b) 偏航角数值结果
Figure 2 (Color online) System outputs under LADRC and SDBCC. (a) Numerical results of the pitch angle; (b) numerical results of the yaw angle

图 3 (网络版彩图) LADRC 和 SDBCC 中的补偿信号与总扰动之差. (a) 俯仰角数值结果; (b) 偏航角数值结果
Figure 3 (Color online) Estimation errors of total disturbance between the compensation values of LADRC and SDBCC. (a) Numerical results of the pitch angle; (b) numerical results of the yaw angle

在仿真中选取采样周期 $\tau = 0.001$, LESO (87) 中的增益参数 r 分别为 50 和 200. 数值结果如图 2 和 3 所示.

图 2 中的曲线分别为 LADRC 和 SDBCC 控制方法下的系统输出, 图 3 中的曲线分别是 LADRC 以及本文 SDBCC 中补偿器与总扰动之差. 从图 2 和 3 可以看出, 当 ESO 中增益参数较小 ($r = 50$) 时 LADRC 控制下跟踪误差的收敛速度较慢且精度较差, 随着增益参数的增大可以实现快速高精度跟踪, 但会出现比较严重的峰化现象 (见图 3). 而本文提出的 SDBCC 不存在上述问题, 在跟踪精度和收敛速度方面都有较好的表现.

需要指出的是本文假设 2 与 3 仅仅是保证闭环控制系统稳定的充分条件. 在接下来的仿真中令 $q_i(\cdot)$ 为如下具有高阶非线性项的非线性函数 (不满足假设 2 或者假设 3):

$$q_1(\theta(t), \omega_\theta(t), \varphi(t), \omega_\varphi(t), \omega_1(t)) = \sin(2t) + \theta^2(t), \quad q_2(\theta(t), \omega_\theta(t), \varphi(t), \omega_\varphi(t), \omega_2(t)) = 2 \cos t + \varphi^2(t),$$

其他参数保持不变. 在这种情况下的仿真结果如图 4 所示.

图 4 (网络版彩图) 二次非线性系统的 SDBCC. (a) 俯仰角数值结果; (b) 偏航角数值结果

Figure 4 (Color online) SDBCC for the system with a quadratic nonlinearity. (a) Numerical results of the pitch angle; (b) Numerical results of the yaw angle

由图 4 可见此时本文所提出的 SDBCC 控制仍然有效, 而且输出曲线与图 1 和 2 对应的输出曲线几乎相同. 这意味着本文的设计方法不仅具有很强的鲁棒性而且还适用于更一般的高阶非线性系统. 在今后的研究中我们将证明具有高阶非线性不确定因素系统在 SDBCC 控制下闭环系统的收敛性稳定性.

6 结论

本文提出了一类非线性不确定 MIMO 系统基于历史采样数据的不确定性因素补偿控制的设计方法 (SDBCC), 并发展了基于特征值和迭代序列的分析方法, 建立了控制误差与采样周期之间的依赖关系, 证明了混杂控制闭环系统的收敛性稳定性. 与基于观测器的不确定性因素补偿控制不同, 本文首先利用当前以及历史采样数据计算出由系统未建模非线性耦合动态和外部扰动构成的总扰动在前一采样周期内某时刻的精确值, 然后利用该精确值在反馈环节对总扰动进行补偿, 因而具有更好的控制效果. 二自由度无人直升机姿态控制的仿真结果见证该方法的有效性和优越性. 数值结果显示本文提出的 SDBCC 方法对于具有高阶非线性项的不确定系统仍然有效. 在今后的工作中我们将研究具有高阶非线性项的不确定系统在 SDBCC 控制下闭环系统的收敛性稳定性.

参考文献

- 1 Khalil H K. Nonlinear Systems. Upper Saddle River: Prentice-Hall, 1996
- 2 Isidori A. Nonlinear Control Systems. Dordrecht: Springer Science & Business Media, 2013
- 3 Tsien H S. Engineering Cybernetics. New York: McGraw-Hill, 1954
- 4 Xie L L, Guo L. How much uncertainty can be dealt with by feedback? IEEE Trans Automat Contr, 2000, 45: 2203–2217
- 5 Krstic M, Kanellakopoulos I, Kokotovic P. Nonlinear and Adaptive Control Design. Piscataway: Wiley, 1995
- 6 Chen H F, Zhang J F. Adaptive pole-placement of controllable systems. Sci China Ser E-Tech Sci, 1996, 26: 39–46 [陈翰馥, 张纪峰. 能控系统的适应极点配置. 中国科学 E 辑: 技术科学, 1996, 26: 39–46]
- 7 Gao W N, Jiang Y, Jiang Z P, et al. Output-feedback adaptive optimal control of interconnected systems based on robust adaptive dynamic programming. Automatica, 2016, 72: 37–45

- 8 Chen C Y, Gui W H, Guan Z H, et al. Adaptive neural control for a class of stochastic nonlinear systems with unknown parameters, unknown nonlinear functions and stochastic disturbances. *Neurocomputing*, 2017, 226: 101–108
- 9 Wu M, Gui W H, He Y. *Modern Robust Control*. 2nd ed. Changsha: Central South University Press, 2006 [吴敏, 桂卫华, 何勇. 现代鲁棒控制. 第二版. 长沙: 中南大学出版社, 2006]
- 10 Zhou K, Doyle J C, Glover K. *Robust and Optimal Control*. Upper Saddle River: Prentice-Hall, 1996
- 11 Gao W B. *Theory and Design Method of Variable Structure Control*. Beijing: Science Press, 1996 [高维炳. 变结构控制的理论及设计方法. 北京: 科学出版社, 1996]
- 12 Utkin V I. *Sliding Modes in Control and Optimization*. Dordrecht: Springer Science & Business Media, 2013
- 13 Astrom K J, Hangglund T. *PID Controllers: Theory, Design and Tuning*. 2nd ed. Research Triangle Park: Instrument Society of America, 2006
- 14 Francis B A, Wonham W M. The internal model principle of control theory. *Automatica*, 1976, 12: 457–465
- 15 Huang J. *Nonlinear Output Regulation: Theory and Applications*. Philadelphia: SIAM, 2004
- 16 Johnson C. Further study of the linear regulator with disturbances—the case of vector disturbances satisfying a linear differential equation. *IEEE Trans Automat Contr*, 1970, 15: 222–228
- 17 Chen W H, Yang J, Guo L, et al. Disturbance-observer-based control and related methods—an overview. *IEEE Trans Ind Electron*, 2016, 63: 1083–1095
- 18 Han J Q. The “Extended State Observer” of a class of uncertain systems. *Control Decision*, 1995, 10: 85–88 [韩京清. 一类不确定对象的扩张状态观测器. 控制与决策, 1995, 10: 85–88]
- 19 Han J Q. From PID to active disturbance rejection control. *IEEE Trans Ind Electron*, 2009, 56: 900–906
- 20 Fliess M, Levine J, Martin P, et al. A Lie-Bäcklund approach to equivalence and flatness of nonlinear systems. *IEEE Trans Automat Contr*, 1999, 44: 922–937
- 21 Zhong Q C, Kuperman A, Stobart R K. Design of UDE-based controllers from their two-degree-of-freedom nature. *Int J Robust Nonlin Control*, 2011, 21: 1994–2008
- 22 She J H, Fang M X, Ohyama Y, et al. Improving disturbance-rejection performance based on an equivalent-input-disturbance approach. *IEEE Trans Ind Electron*, 2008, 55: 380–389
- 23 Eom M, Chwa D. Robust swing-up and balancing control using a nonlinear disturbance observer for the pendubot system with dynamic friction. *IEEE Trans Robot*, 2015, 31: 331–343
- 24 Li J, Xia Y Q, Qi X H, et al. On the necessity, scheme, and basis of the linear-nonlinear switching in active disturbance rejection control. *IEEE Trans Ind Electron*, 2017, 64: 1425–1435
- 25 Kandula P, Dong L L. Robust voltage control for an electrostatic micro-actuator. *J Dynamic Syst Measurement Control*, 2018, 140: 061012
- 26 Sun J L, Pu Z Q, Yi J Q. Conditional disturbance negation based active disturbance rejection control for hypersonic vehicles. *Control Eng Practice*, 2019, 84: 159–171
- 27 Yuan Y, Wang Z D, Yu Y, et al. Active disturbance rejection control for a pneumatic motion platform subject to actuator saturation: an extended state observer approach. *Automatica*, 2019, 107: 353–361
- 28 Sun L, Jin Y H, You F Q. Active disturbance rejection temperature control of open-cathode proton exchange membrane fuel cell. *Appl Energy*, 2020, 261: 114381
- 29 Li S, Yang J, Chen W H, et al. *Disturbance Observer-based Control: Methods and Applications*. Boca Raton: CRC Press, 2014
- 30 Xue W C, Madonski R, Lakomy K, et al. Add-on module of active disturbance rejection for set-point tracking of motion control systems. *IEEE Trans Ind Appl*, 2017, 53: 4028–4040
- 31 Chen S, Xue W C, Zhong S, et al. On comparison of modified ADRCs for nonlinear uncertain systems with time delay. *Sci China Inf Sci*, 2018, 61: 070223
- 32 Zhao Z L, Guo B Z. A novel extended state observer for output tracking of MIMO systems with mismatched uncertainty. *IEEE Trans Automat Contr*, 2018, 63: 211–218
- 33 Wu Z H, Zhou H C, Guo B Z, et al. Review and new theoretical perspectives on active disturbance rejection control for uncertain finite-dimensional and infinite-dimensional systems. *Nonlin Dyn*, 2020, 101: 935–959
- 34 Zhao Z L. *Active Disturbance Rejection Control Design and Theoretical Analysis*. Beijing: Science Press, 2019 [赵志良. 自抗扰控制设计与理论分析. 北京: 科学出版社, 2019]
- 35 Chai T Y, Zhang Y J, Wang H, et al. Data-based virtual unmodeled dynamics driven multivariable nonlinear adaptive

- switching control. *IEEE Trans Neural Netw*, 2011, 22: 2154–2172
- 36 Zhao Z L, Chai T Y, Wei C, et al. Compensation-signal-driven control for a class of nonlinear uncertain systems. *Automatica*, 2021, 125: 109423
 - 37 Katal N, Narayan S. Automated synthesis of multivariate QFT controller and pre-filter for a distillation column with multiple time delays. *J Process Control*, 2021, 99: 79–106
 - 38 Gao W N, Huang M Z, Jiang Z P, et al. Sampled-data-based adaptive optimal output-feedback control of a 2-degree-of-freedom helicopter. *IET Control Theor Appl*, 2016, 10: 1440–1447
 - 39 Kim M, Jung B, Han B, et al. Lyapunov-based impact time control guidance laws against stationary targets. *IEEE Trans Aerosp Electron Syst*, 2015, 51: 1111–1122
 - 40 Guo B Z, Zhao Z L. On convergence of non-linear extended state observer for multi-input multi-output systems with uncertainty. *IET Control Theor Appl*, 2012, 6: 2375–2386
 - 41 Cui R X, Chen L P, Yang C G, et al. Extended state observer-based integral sliding mode control for an underwater robot with unknown disturbances and uncertain nonlinearities. *IEEE Trans Ind Electron*, 2017, 64: 6785–6795
 - 42 Nešić D, Teel A R, Sontag E D. Formulas relating stability estimates of discrete-time and sampled-data nonlinear systems. *Syst Control Lett*, 1999, 38: 49–60
 - 43 Qian C J, Du H B. Global output feedback stabilization of a class of nonlinear systems via linear sampled-data control. *IEEE Trans Automat Contr*, 2012, 57: 2934–2939
 - 44 Selivanov A, Fridman E. Sampled-data implementation of derivative-dependent control using artificial delays. *IEEE Trans Automat Contr*, 2018, 63: 3594–3600
 - 45 Jiang T T, Wu H X. Sampled-data feedback and stability for a class of uncertain nonlinear systems based on characteristic modeling method. *Sci China Inf Sci*, 2016, 59: 092205
 - 46 Pereira R D O, Veronesi M, Visioli A, et al. Implementation and test of a new autotuning method for PID controllers of TITO processes. *Control Eng Practice*, 2017, 58: 171–185
 - 47 Jin Q B, Cai W, Du X H. Disturbance observer-based control with partially coupled inverse model for stable TITO processes with time delays. *ISA Trans*, 2021, 110: 148–159
 - 48 Pawar S N, Chile R H, Patre B M. Modified reduced order observer based linear active disturbance rejection control for TITO systems. *ISA Trans*, 2017, 71: 480–494
 - 49 Srikanth M V, Yadaiah N. Optimal parameter tuning of modified active disturbance rejection control for unstable time-delay systems using an AHP combined multi-objective quasi-oppositional Jaya algorithm. *Appl Soft Comput*, 2020, 86: 105881
 - 50 Jin Q B, Wang Q, Liu L Y. Design of decentralized proportional-integral-derivative controller based on decoupler matrix for two-input/two-output process with active disturbance rejection structure. *Adv Mech Eng*, 2016, 8: 1–18
 - 51 Gao Z Q. Scaling and bandwidth-parameterization based controller tuning. In: *Proceedings of the American Control Conference*, 2003. 4989–4996

附录 A 不等式 (31) 和 (32) 的证明

设 λ 是矩阵 H 的特征值, 则 $\det(\lambda I - H) = 0$, 根据矩阵 H 的定义有

$$\det \begin{pmatrix} \lambda - \left(1 - \frac{\alpha_{p1}\tau^2}{2}\right) & -\tau + \frac{\alpha_{d1}\tau^2}{2} & 0 & 0 \\ \alpha_{p1}\tau & \lambda - (1 - \alpha_{d1}\tau) & 0 & 0 \\ 0 & 0 & \lambda - \left(1 - \frac{\alpha_{p2}\tau^2}{2}\right) & -\tau + \frac{\alpha_{d2}\tau^2}{2} \\ 0 & 0 & \alpha_{p2}\tau & \lambda - (1 - \alpha_{d2}\tau) \end{pmatrix} = 0, \quad (A1)$$

考虑到 $\alpha_{di}^2 = 4\alpha_{pi}$, 求解可得矩阵 H 的特征值为

$$\lambda_{1,2} = 1 - \left(\frac{\alpha_{d1}}{2}\tau + \frac{\alpha_{p1}}{4}\tau^2\right) \pm \tau \sqrt{\frac{\alpha_{p1}\alpha_{d1}}{4}\tau + \frac{\alpha_{p1}^2}{16}\tau^2}, \quad \lambda_{3,4} = 1 - \left(\frac{\alpha_{d2}}{2}\tau + \frac{\alpha_{p2}}{4}\tau^2\right) \pm \tau \sqrt{\frac{\alpha_{p2}\alpha_{d2}}{4}\tau + \frac{\alpha_{p2}^2}{16}\tau^2}. \quad (A2)$$

令

$$\tau^* = \left\{ \tau_0^*, \tau_1^*, \tau_2^*, \tau_3^*, \frac{1}{6}, \frac{1}{16M_2} \right\}, \quad (A3)$$

其中

$$\begin{aligned}\tau_0^* &= \min \left\{ \sqrt{\frac{6}{\alpha_{p \max}}} - \frac{\alpha_{d \max}}{\alpha_{p \max}}, \frac{(4\sqrt{3}-6)\alpha_{d \max}}{3\alpha_{p \max}} \right\}, \\ \tau_1^* &= \min \left\{ \frac{\alpha_{d \max}^2}{4\alpha_{p \max}\alpha_{d \max} + \alpha_{p \max}^2}, \frac{\alpha_{d \max}}{\alpha_{p \max}} \right\}, \quad \tau_2^* = \frac{1}{4M_1^*}, \\ \tau_3^* &= \min \left\{ \frac{1}{2\sqrt{2M}}, \frac{1}{16M_2^{*2}}, \frac{1}{4M_0(1 + \alpha_{p \max} + \alpha_{d \max})} \right\},\end{aligned}\quad (A4)$$

$$\begin{aligned}M_1^* &= M_{11} + (1 + \alpha_{p \max} + \alpha_{d \max} + M_{12}) \left(\|x(0)\|_\infty + \frac{1}{2} + \hat{M}_1 \right), \\ M_2^* &= \frac{\tilde{M}(8\alpha_{p \max}^2 + 16\alpha_{p \max}\alpha_{d \max} + 6\alpha_{d \max}^2)}{\alpha_{p \min}\alpha_{d \min}\sqrt{\alpha_{p \min}\alpha_{d \min}}},\end{aligned}\quad (A5)$$

$$M_0 = \frac{(4\alpha_{p \max}^2 + 8\alpha_{p \max}\alpha_{d \max} + 3\alpha_{d \max}^2)(\|x(0)\|_\infty + \frac{1}{2})}{2\alpha_{p \min}\sqrt{\alpha_{p \min}\alpha_{d \min}}} + 1, \quad (A6)$$

和

$$\begin{aligned}\tilde{M} &= \max \left\{ (1 + \hat{M}_2)(M_{11} + M_{12}\hat{M}_1 + M_0M_{12}) + 2M_0M_2(1 + \alpha_{p \max} + \alpha_{d \max}) + 2M_2\hat{M}_1, \right. \\ &\quad \left. M_{11}(1 + \hat{M}_2 + 4M_2) + [M_{12}(1 + \hat{M}_2 + 4M_2) + 2M_2(1 + \alpha_{p \max} + \alpha_{d \max})] \left(\|x(t)\|_\infty + \frac{1}{2} + \hat{M}_1 \right) \right\}.\end{aligned}\quad (A7)$$

对于任意的 $\tau \in (0, \tau^*)$, 有

$$0 \leq \frac{\alpha_{di}}{2}\tau + \frac{\alpha_{pi}}{4}\tau^2 \leq \frac{1}{2}, \quad 0 \leq \tau \sqrt{\frac{\alpha_{pi}\alpha_{di}}{4}\tau + \frac{\alpha_{pi}^2}{16}\tau^2} < \frac{1}{2} \left(\frac{\alpha_{di}}{2}\tau + \frac{\alpha_{pi}}{4}\tau^2 \right) < \frac{1}{4}, \quad (A8)$$

因此任意的 $\tau \in (0, \tau^*)$, 矩阵 H 的最大特征值 $\lambda_{\max}(H)$ 满足

$$0 < \lambda_{\max}(H) = \max\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\} < \lambda_\tau < 1, \quad (A9)$$

并且

$$\lambda_\tau = 1 - \frac{1}{2} \left(\frac{\alpha_{d \min}}{2} + \frac{\alpha_{p \min}}{4}\tau^2 \right) = 1 - \frac{2\alpha_{d \min}\tau + \alpha_{p \min}\tau^2}{8} \in (0, 1), \quad \frac{1}{1 - \lambda_\tau} = \frac{8}{2\alpha_{d \min}\tau + \alpha_{p \min}\tau^2} < \frac{4}{\alpha_{d \min}\tau}. \quad (A10)$$

计算可得特征值 $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ 对应的特征向量为

$$\begin{aligned}P_{11} &= \begin{pmatrix} \frac{\alpha_{d1}}{2} - \frac{\alpha_{p1}}{4}\tau + \sqrt{\frac{\alpha_{p1}\alpha_{d1}}{4}\tau + \frac{\alpha_{p1}^2}{16}\tau^2} \\ -\alpha_{p1} \\ 0 \\ 0 \end{pmatrix}, \quad P_{12} = \begin{pmatrix} \frac{\alpha_{d1}}{2} - \frac{\alpha_{p1}}{4}\tau - \sqrt{\frac{\alpha_{p1}\alpha_{d1}}{4}\tau + \frac{\alpha_{p1}^2}{16}\tau^2} \\ -\alpha_{p1} \\ 0 \\ 0 \end{pmatrix}, \\ P_{21} &= \begin{pmatrix} 0 \\ 0 \\ \frac{\alpha_{d2}}{2} - \frac{\alpha_{p2}}{4}\tau + \sqrt{\frac{\alpha_{p2}\alpha_{d2}}{4}\tau + \frac{\alpha_{p2}^2}{16}\tau^2} \\ -\alpha_{p2} \end{pmatrix}, \quad P_{22} = \begin{pmatrix} 0 \\ 0 \\ \frac{\alpha_{d2}}{2} - \frac{\alpha_{p2}}{4}\tau - \sqrt{\frac{\alpha_{p2}\alpha_{d2}}{4}\tau + \frac{\alpha_{p2}^2}{16}\tau^2} \\ -\alpha_{p2} \end{pmatrix}.\end{aligned}\quad (A11)$$

令

$$P = (P_{11}, P_{12}, P_{21}, P_{22}) = \begin{pmatrix} P_1 & 0_{2 \times 2} \\ 0_{2 \times 2} & P_2 \end{pmatrix}.$$

矩阵 H 可对角化为

$$H = P \begin{pmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \lambda_3 & \\ & & & \lambda_4 \end{pmatrix} P^{-1}, \quad (A12)$$

其中

$$P_i^{-1} = -\frac{2}{\alpha_{pi}\sqrt{4\alpha_{pi}\alpha_{di}\tau + \alpha_{pi}^2\tau^2}} \begin{pmatrix} -\alpha_{pi} & -\frac{\alpha_{di}}{2} + \frac{\alpha_{pi}}{4}\tau + \sqrt{\frac{\alpha_{pi}\alpha_{di}}{4}\tau + \frac{\alpha_{pi}^2}{16}\tau^2} \\ \alpha_{pi} & \frac{\alpha_{di}}{2} - \frac{\alpha_{pi}}{4}\tau + \sqrt{\frac{\alpha_{pi}\alpha_{di}}{4}\tau + \frac{\alpha_{pi}^2}{16}\tau^2} \end{pmatrix}, \quad P^{-1} = \begin{pmatrix} P_1^{-1} & 0_{2 \times 2} \\ 0_{2 \times 2} & P_2^{-1} \end{pmatrix}. \quad (A13)$$

对于任意的 $\tau \in (0, \tau^*)$ 有

$$\begin{aligned} \|P\|_{\infty} = \max & \left\{ \left| \frac{\alpha_{d1}}{2} - \frac{\alpha_{p1}}{4}\tau + \sqrt{\frac{\alpha_{p1}\alpha_{d1}}{4}\tau + \frac{\alpha_{p1}^2}{16}\tau^2} \right| + \left| \frac{\alpha_{d1}}{2} - \frac{\alpha_{p1}}{4}\tau - \sqrt{\frac{\alpha_{p1}\alpha_{d1}}{4}\tau + \frac{\alpha_{p1}^2}{16}\tau^2} \right|, 2\alpha_{p1}, \right. \\ & \left. \left| \frac{\alpha_{d1}}{2} - \frac{\alpha_{p1}}{4}\tau + \sqrt{\frac{\alpha_{p1}\alpha_{d1}}{4}\tau + \frac{\alpha_{p1}^2}{16}\tau^2} \right| + \left| \frac{\alpha_{d1}}{2} - \frac{\alpha_{p1}}{4}\tau - \sqrt{\frac{\alpha_{p1}\alpha_{d1}}{4}\tau + \frac{\alpha_{p1}^2}{16}\tau^2} \right|, 2\alpha_{p2} \right\} \\ & \leq 2\alpha_{p \max} + \alpha_{d \max}, \end{aligned} \quad (A14)$$

并且

$$\begin{aligned} \|P_i^{-1}\|_{\infty} & \leq \frac{2}{\alpha_{pi}\sqrt{4\alpha_{pi}\alpha_{di}\tau + \alpha_{pi}^2\tau^2}} \left(\alpha_{pi} + \frac{3\alpha_{di}}{2} \right) \leq \frac{2\alpha_{pi} + 3\alpha_{di}}{2\alpha_{pi}\sqrt{\alpha_{pi}\alpha_{di}}} \frac{1}{\sqrt{\tau}}, \\ \|P^{-1}\|_{\infty} = \max & \{ \|P_1^{-1}\|_{\infty}, \|P_2^{-1}\|_{\infty} \} \leq \frac{2\alpha_{p \max} + 3\alpha_{d \max}}{2\alpha_{p \min}\sqrt{\alpha_{p \min}\alpha_{d \min}}} \frac{1}{\sqrt{\tau}}. \end{aligned} \quad (A15)$$

对于任意的 $j = 1, 2, 3, \dots$, 由式 (A12) 和 (A9) 我们有

$$\|H^j\|_{\infty} = \left\| P \begin{pmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \lambda_3 & \\ & & & \lambda_4 \end{pmatrix}^j P^{-1} \right\|_{\infty} \leq \frac{4\alpha_{p \max}^2 + 8\alpha_{p \max}\alpha_{d \max} + 3\alpha_{d \max}^2}{2\alpha_{p \min}\sqrt{\alpha_{p \min}\alpha_{d \min}}} \frac{1}{\sqrt{\tau}} \lambda_{\tau}^j. \quad (A16)$$

因此不等式 (31) 是正确的.

附录 B 式 (49) 的证明

易知

$$\begin{aligned} & \|f(t_2, p_{11}(t_2), p_{12}(t_2), p_{21}(t_2), p_{22}(t_2), \omega(t_2)) - f(t_1, p_{11}(t_1), p_{12}(t_1), p_{21}(t_1), p_{22}(t_1), \omega(t_1))\|_{\infty} \\ & \leq \|f(t_2, p_{11}(t_2), p_{12}(t_2), p_{21}(t_2), p_{22}(t_2), \omega(t_2)) - f(t_1, p_{11}(t_2), p_{12}(t_2), p_{21}(t_2), p_{22}(t_2), \omega(t_2))\|_{\infty} \\ & \quad + \|f(t_1, p_{11}(t_2), p_{12}(t_2), p_{21}(t_2), p_{22}(t_2), \omega(t_2)) - f(t_1, p_{11}(t_1), p_{12}(t_2), p_{21}(t_2), p_{22}(t_2), \omega(t_2))\|_{\infty} \\ & \quad + \|f(t_1, p_{11}(t_1), p_{12}(t_2), p_{21}(t_2), p_{22}(t_2), \omega(t_2)) - f(t_1, p_{11}(t_1), p_{12}(t_1), p_{21}(t_2), p_{22}(t_2), \omega(t_2))\|_{\infty} \\ & \quad + \|f(t_1, p_{11}(t_1), p_{12}(t_1), p_{21}(t_2), p_{22}(t_2), \omega(t_2)) - f(t_1, p_{11}(t_1), p_{12}(t_1), p_{21}(t_1), p_{22}(t_2), \omega(t_2))\|_{\infty} \\ & \quad + \|f(t_1, p_{11}(t_1), p_{12}(t_1), p_{21}(t_1), p_{22}(t_2), \omega(t_2)) - f(t_1, p_{11}(t_1), p_{12}(t_1), p_{21}(t_1), p_{22}(t_1), \omega(t_2))\|_{\infty} \\ & \quad + \|f(t_1, p_{11}(t_1), p_{12}(t_1), p_{21}(t_1), p_{22}(t_1), \omega(t_2)) - f(t_1, p_{11}(t_1), p_{12}(t_1), p_{21}(t_1), p_{22}(t_1), \omega(t_1))\|_{\infty}. \end{aligned} \quad (B1)$$

由微分中值定理知, 存在常数 $\eta_{ij,k} \in ((k-2)\tau, k\tau)$, $i = 1, 2; j = 1, \dots, 6$, 对任意的 $t_1, t_2 \in [(k-2)\tau, k\tau]$, 有

$$\begin{aligned} & f_i(t_2, p_{11}(t_2), p_{12}(t_2), p_{21}(t_2), p_{22}(t_2), \omega_i(t_2)) - f_i(t_1, p_{11}(t_2), p_{12}(t_2), p_{21}(t_2), p_{22}(t_2), \omega_i(t_2)) \\ & = (t_2 - t_1) \frac{\partial f_i}{\partial t}(\eta_{i1,k}, p_{11}(t_2), p_{12}(t_2), p_{21}(t_2), p_{22}(t_2), \omega_i(t_2)), \end{aligned} \quad (B2)$$

$$\begin{aligned} & f_i(t_1, p_{11}(t_2), p_{12}(t_2), p_{21}(t_2), p_{22}(t_2), \omega_i(t_2)) - f_i(t_1, p_{11}(t_1), p_{12}(t_2), p_{21}(t_2), p_{22}(t_2), \omega_i(t_2)) \\ & = (t_2 - t_1) \dot{p}_{11}(\eta_{i2,k}) \frac{\partial f_i}{\partial p_{11}}(t_1, p_{11}(\eta_{i2,k}), p_{12}(t_2), p_{21}(t_2), p_{22}(t_2), \omega_i(t_2)), \end{aligned} \quad (B3)$$

其中 $\dot{p}_{11}(t) = p_{12}(t) = x_{12}(t) + \dot{v}_1(t)$.

$$f_i(t_1, p_{11}(t_1), p_{12}(t_2), p_{21}(t_2), p_{22}(t_2), \omega_i(t_2)) - f_i(t_1, p_{11}(t_1), p_{12}(t_1), p_{21}(t_2), p_{22}(t_2), \omega_i(t_2))$$

$$= (t_2 - t_1) \left. \frac{df_i(t_1, p_{11}(t_1), p_{12}(s), p_{21}(t_2), p_{22}(t_2), \omega_i(t_2))}{ds} \right|_{s=\eta_{i3,k}}, \quad (\text{B4})$$

$$\begin{aligned} & f_i(t_1, p_{11}(t_1), p_{12}(t_1), p_{21}(t_1), p_{22}(t_1), \omega_i(t_1)) - f_i(t_1, p_{11}(t_1), p_{12}(t_1), p_{21}(t_1), p_{22}(t_1), \omega_i(t_1)) \\ &= (t_2 - t_1) \dot{\omega}_i(\eta_{i6,k}) \frac{\partial f_i}{\partial \omega_i}(t_1, p(t_1), \omega_i(\eta_{i6,k})), \end{aligned} \quad (\text{B5})$$

如果 $\eta_{i3,k} \in ((k-1)\tau, k\tau)$, 则式 (B4) 写成

$$\begin{aligned} & f_i(t_1, p_{11}(t_1), p_{12}(t_2), p_{21}(t_2), p_{22}(t_2), \omega_i(t_2)) - f_i(t_1, p_{11}(t_1), p_{12}(t_1), p_{21}(t_2), p_{22}(t_2), \omega_i(t_2)) \\ &= (t_2 - t_1) (f_i(\eta_{i3,k}, p(\eta_{i3,k}), \omega_i(\eta_{i3,k})) + u_k) \frac{\partial f_i(t_1, p_{11}(t_1), p_{12}(\eta_{i3,k}), p_{21}(t_2), p_{22}(t_2), \omega_i(t_2))}{\partial p_{12}} \\ &= (t_2 - t_1) (f_i(\eta_{i3,k}, p(\eta_{i3,k}), \omega_i(\eta_{i3,k})) - f_i(\xi_{i,k-1}, p(\xi_{i,k-1}), \omega_i(\eta_{i3,k})) + Kx_k) \\ &\quad \cdot \frac{\partial f_i(t_1, p_{11}(t_1), p_{12}(\eta_{i3,k}), p_{21}(t_2), p_{22}(t_2), \omega_i(t_2))}{\partial p_{12}}, \end{aligned} \quad (\text{B6})$$

另外, 如果 $\eta_{i3,k} \in ((k-2)\tau, (k-1)\tau)$, 则式 (B4) 写成

$$\begin{aligned} & f_i(t_1, p_{11}(t_1), p_{12}(t_2), p_{21}(t_2), p_{22}(t_2), \omega_i(t_2)) - f_i(t_1, p_{11}(t_1), p_{12}(t_1), p_{21}(t_2), p_{22}(t_2), \omega_i(t_2)) \\ &= (t_2 - t_1) (f_i(\eta_{i3,k}, p(\eta_{i3,k}), \omega_i(\eta_{i3,k})) + u_{k-1}) \frac{\partial f_i(t_1, p_{11}(t_1), p_{12}(\eta_{i3,k}), p_{21}(t_2), p_{22}(t_2), \omega_i(t_2))}{\partial p_{12}} \\ &= (t_2 - t_1) (f_i(\eta_{i3,k}, p(\eta_{i3,k}), \omega_i(\eta_{i3,k})) - f_i(\xi_{i,k-1}, p(\xi_{i,k-1}), \omega_i(\eta_{i3,k})) + Kx_{k-1}) \\ &\quad \cdot \frac{\partial f_i(t_1, p_{11}(t_1), p_{12}(\eta_{i3,k}), p_{21}(t_2), p_{22}(t_2), \omega_i(t_2))}{\partial p_{12}}. \end{aligned} \quad (\text{B7})$$

从式 (B1) 可以看出, 两个不同时刻的 f 相减的范数小于等于 6 个范数之和, 其中第 2 个和第 4 个情况类似, 第 3 个和第 5 个情况类似, 在此不再赘述. 对于任意的 $t_1, t_2 \in [0, 2\tau]$ 满足式 (33), 因此有

$$\begin{aligned} & \|f(t_2, p_{11}(t_2), p_{12}(t_2), p_{21}(t_2), p_{22}(t_2), \omega(t_2)) - f(t_1, p_{11}(t_2), p_{12}(t_2), p_{21}(t_2), p_{22}(t_2), \omega(t_2))\|_\infty \\ & \leq 2\tau(M_{11} + M_{12}(\|x(0)\|_\infty + 1/2 + \hat{M}_1)), \end{aligned} \quad (\text{B8})$$

$$\begin{aligned} & \|f(t_1, p_{11}(t_2), p_{12}(t_2), p_{21}(t_2), p_{22}(t_2), \omega(t_2)) - f(t_1, p_{11}(t_1), p_{12}(t_2), p_{21}(t_2), p_{22}(t_2), \omega(t_2))\|_\infty \\ & \leq 2\tau M_2(\|x(0)\|_\infty + 1/2 + \hat{M}_1), \end{aligned} \quad (\text{B9})$$

$$\begin{aligned} & \|f(t_1, p_{11}(t_1), p_{12}(t_2), p_{21}(t_2), p_{22}(t_2), \omega(t_2)) - f(t_1, p_{11}(t_1), p_{12}(t_1), p_{21}(t_2), p_{22}(t_2), \omega(t_2))\|_\infty \\ & \leq 2\tau M_2[2(M_{11} + M_{12}(\|x(0)\|_\infty + 1/2 + \hat{M}_1)) + (\alpha_{p\max} + \alpha_{d\max})(\|x(0)\|_\infty + 1/2 + \hat{M}_1)], \end{aligned} \quad (\text{B10})$$

$$\begin{aligned} & \|f(t_1, p_{11}(t_1), p_{12}(t_1), p_{21}(t_1), p_{22}(t_1), \omega(t_2)) - f(t_1, p_{11}(t_1), p_{12}(t_1), p_{21}(t_1), p_{22}(t_1), \omega(t_1))\|_\infty \\ & \leq 2\tau \hat{M}_2(M_{11} + M_{12}(\|x(0)\|_\infty + 1/2 + \hat{M}_1)). \end{aligned} \quad (\text{B11})$$

由式 (B8)~(B11), 我们可以得到 $\bar{\Delta}_2 \leq \bar{M}\sqrt{\tau}$, 其中式 (48) 中定义了 $\bar{\Delta}_k$, 式 (A7) 中定义了 $\bar{M}$.

如果式 (41) 不成立, 则由式 (46), (47) 和 (B2), (B3), (B5) 得, 对于每个整数 $k = 3, \dots, k_0$ 和任意 $t_1, t_2 \in [(k-2)\tau, k\tau)$, 有

$$\begin{aligned} & \|f(t_2, p_{11}(t_2), p_{12}(t_2), p_{21}(t_2), p_{22}(t_2), \omega(t_2)) - f(t_1, p_{11}(t_2), p_{12}(t_2), p_{21}(t_2), p_{22}(t_2), \omega(t_2))\|_\infty \\ & \leq 2\tau(M_{11} + M_{12}\hat{M}_1) + 2\sqrt{\tau}M_0M_{12}, \end{aligned} \quad (\text{B12})$$

$$\begin{aligned} & \|f(t_1, p_{11}(t_2), p_{12}(t_2), p_{21}(t_2), p_{22}(t_2), \omega(t_2)) - f(t_1, p_{11}(t_1), p_{12}(t_2), p_{21}(t_2), p_{22}(t_2), \omega(t_2))\|_\infty \\ & \leq 2\tau M_2 \left(\frac{M_0}{\sqrt{\tau}} + \hat{M}_1 \right) \leq 2\sqrt{\tau}M_0M_2 + 2\tau M_2\hat{M}_1, \end{aligned} \quad (\text{B13})$$

$$\begin{aligned} & \|f(t_1, p_{11}(t_1), p_{12}(t_1), p_{21}(t_1), p_{22}(t_1), \omega(t_2)) - f(t_1, p_{11}(t_1), p_{12}(t_1), p_{21}(t_1), p_{22}(t_1), \omega(t_1))\|_\infty \\ & \leq 2\tau \hat{M}_2(M_{11} + M_{12}\hat{M}_1) + 2\sqrt{\tau}\hat{M}_2M_0M_{12}, \end{aligned} \quad (\text{B14})$$

由式 (46), (47) 和 (B6), (B7) 知, 对于每个整数 $k = 3, \dots, k_0$ 和任意 $t_1, t_2 \in [(k-2)\tau, k\tau]$, 如果 $\eta_{i3,k} \in ((k-1)\tau, k\tau)$, 则

$$\begin{aligned} & \|f(t_1, p_{11}(t_1), p_{12}(t_2), p_{21}(t_2), p_{22}(t_2), \omega(t_2)) - f(t_1, p_{11}(t_1), p_{12}(t_1), p_{21}(t_2), p_{22}(t_2), \omega(t_2))\|_\infty \\ & \leq 2\tau M_2 \tilde{\Delta}_k + 2\sqrt{\tau}(\alpha_{p\max} + \alpha_{d\max}) M_0 M_2, \end{aligned} \quad (\text{B15})$$

如果 $\eta_{i3,k} \in ((k-2)\tau, (k-1)\tau)$, 则

$$\begin{aligned} & \|f(t_1, p_{11}(t_1), p_{12}(t_2), p_{21}(t_2), p_{22}(t_2), \omega(t_2)) - f(t_1, p_{11}(t_1), p_{12}(t_1), p_{21}(t_2), p_{22}(t_2), \omega(t_2))\|_\infty \\ & \leq 2\tau M_2 \tilde{\Delta}_{k-1} + 2\sqrt{\tau}(\alpha_{p\max} + \alpha_{d\max}) M_0 M_2, \end{aligned} \quad (\text{B16})$$

其中式 (48) 中定义了 $\tilde{\Delta}_k$, 式 (A7) 中定义了 $\tilde{M}$. 对于任意的 $\tau \in (0, \tau^*)$, 由式 (A3) 知 $\tau \leq 1/16M_2$. 若 $\tilde{\Delta}_{k-1} \leq \tilde{M}\sqrt{\tau}$ 成立, 则它与式 (B12)~(B16) 结合得

$$\begin{aligned} \tilde{\Delta}_k & \leq 4M_2\tau(\tilde{\Delta}_k + \tilde{\Delta}_{k-1}) + 2\tau[(M_{11} + M_{12}\hat{M}_1) + \hat{M}_2(M_{11} + M_{12}\hat{M}_1) + 2M_2\hat{M}_1] \\ & \quad + 2\sqrt{\tau}[M_0M_{12} + \hat{M}_2M_0M_{12} + 2M_0M_2 + 2M_0M_2(\alpha_{p\max} + \alpha_{d\max})], \end{aligned} \quad (\text{B17})$$

因此 $\tilde{\Delta}_k \leq \tilde{M}\sqrt{\tau}$. 又因为

$$\|\Delta_k\|_\infty = \left\| \int_{(k-1)\tau}^{k\tau} \begin{pmatrix} k\tau - s & 0 \\ 1 & 0 \\ 0 & k\tau - s \\ 0 & 1 \end{pmatrix} (f(s, p(s), \omega(s)) - \text{Com}) ds \right\|_\infty \leq \tilde{M}\tau^{\frac{3}{2}}, \quad (\text{B18})$$

我们可以得到式 (49) 成立.

Sampled-data-based uncertainty compensation control for a class of continuous-time nonlinear MIMO systems

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Abstract This paper proposes a sampled-data-based uncertainty compensation control strategy for a class of continuous-time multi-input multi-output nonlinear systems with strong coupling uncertainties. Unlike observer-based uncertainty compensation control methods, this paper utilizes the true value of the total disturbance at an instant in the previous sampling interval rather than its observed value in the last sampling time to compensate for the total disturbance. The total disturbance considered in this paper is composed of the unmodeled nonlinear coupling dynamics and external disturbance. The continuous-time systems and the sampled-data feedback control make up a hybrid closed-loop control system, which, together with the strongly coupled nonlinear uncertainties of the system, brings a significant hurdle to verifying the stability and convergence of the closed-loop control system. A new eigenvalue-and-series-based analysis method is proposed to overcome this hurdle. It is proved that when the tracking target is a bounded function, the tracking error can be arbitrarily small when the sampling period is small enough. Furthermore, when the tracking target is a constant and the nonlinear term is time-invariant, the tracking error converges to zero as time tends to infinity. Numerical results on the attitude control for a 2-DOF UAV validate the effectiveness and merits of the proposed method.

Keywords uncertainty, nonlinearity, MIMO system, PID control, data-driven control