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A circular zone partition method for identifying Duffing oscillator state transition and its application to BPSK signal demodulation

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Abstract When a Duffing oscillator is applied to signal detection, identifying its state transition is indispensable. Due to lack of an effective method for automatically distinguishing the state transition, phase analysis is extensively used. However, it needs ocular estimation to identify phase pattern corresponding to transition of Duffing oscillator. Hence it is not fit for communication signal demodulation. To solve the problem, this paper proposes a method, called circular zone partition (CZP), for partitioning trajectory on the phase plane of Duffing oscillator. First, a computing model for Duffing oscillator is described. Then, the fundamental principle and algorithm for the CZP method are discussed. Meanwhile the equation of a circular zone divider and its realization are presented. Thus, by way of the divider, the two-dimensional phase trajectory pattern of Duffing oscillator driven by an external signal can be transformed into the one-dimensional time signal, whose envelop after being filtered is able to indicate the state transition, i.e. the presence or the absence of external signal. Finally, to verify the effect of the CZP method on binary phase shifted keying (BPSK) signal demodulation, two examples are presented and simulation results show that this CZP method is accurate and valid for BPSK signal demodulation.

Keywords chaos, nonlinear oscillators, signal detection, digital communication, receivers

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1 Introduction

At present, chaotic oscillators have been successfully applied to weak signal detection [1–3]. Chaotic oscillator's phase transition functions as an important clue for indicating external signal existence; therefore observing phase transition of chaotic oscillator, namely observing whether the phase trajectory of chaotic oscillator, driven by the external signal, appears in great periodic state (or intermittent chaos state) is an indispensable task in detecting. Several methods including Lyapunov exponents, Poincare section, power spectrum, fractional dimension, and Melnikov function [4], have been developed to detect the state transition of chaotic oscillator, but they are only applicable to few chaotic systems. Therefore, phase analysis is today considered the most universal and simplest analytical method [5, 6]. Since this

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method needs ocular estimation to identify the phase pattern of chaotic oscillator on the waveform display, many problems arise when implementing them. For example, automatic recognition is difficult, and manual recognition is not fit for detecting the high speed transition of phase trajectory, especially for BPSK signal demodulation. Hence, the problem of how to automatically recognize the phase transition has drawn increasing attention. Ref. [7] presented a method of identifying chaotic nature based on image recognition, and used it to solve the problem of detecting signals from the phase plane. But this method required calculating Euler number of the image after the whole data of phase trajectory were obtained, and the calculation was too time-consuming to be used in real-time signal detection such as BPSK signal demodulation.

In order to solve the problem of transition identification, here a CZP method based on Duffing oscillator, which aims at BPSK signal demodulation, is proposed. The basic idea is to divide the two-dimensional phase plane into two regions, i.e. interior and exterior, by a circular zone partition line, and then to detect whether the state transition occurs, by means of the boundary of the interior. Specifically speaking, through the partition, the phase trajectory can be mapped into the one-dimensional time signal whose envelop contains all information about the phase transition. Further, after smoothing the time signal by a low pass filter (LPF), it can be used as a phase transition indicator to indicate whether the external signal exists or not. In this paper, first a computing model for Duffing oscillator is described. Then, the fundamental principle and algorithm for CZP are discussed. Meanwhile the equation of circular zone divider and its realization are presented. Finally, to verify the effect of the CZP method on BPSK signal demodulation, two examples are given. The simulation results show that this CZP method is accurate and valid for BPSK signal demodulation.

The rest part of this paper is organized as follows. In section 2, a brief introduction to the computing model of Duffing oscillator and its phase plane characteristic is given. In section 3, the CZP method is described, and the equation of circular zone divider and its realization structure are discussed in detail. In section 4, simulation results and performance analysis are presented. Finally, in section 5 some concluding remarks are given.

2 Model and phase trajectory features of Duffing oscillator

In principle, the proposed method is widely applicable to systems with plane phase trajectory. For simplicity, we choose the well-known Duffing oscillator as the chaotic oscillator model.

Consider the following Duffing oscillator equation [2]:

$$\ddot{y}(t) + k\dot{y}(t) - y(t) + y^3(t) = \gamma\cos(t) + ax(t), \tag{1}$$

where k is the damping ratio, $\gamma\cos(t)$ is the internal driving force, $x(t)$ is the external signal composed of to-be-detected signal $s(t)$ and background noise $n(t)$, that is, $x(t) = s(t) + n(t)$, and a is a coefficient used to limit the energy intensity of $x(t)$ fed into Duffing oscillator.

In order to enable Duffing oscillator to have good signal detection ability, it needs to work at the critical state, i.e. chaos, but on the verge of changing to the great periodic motion. This critical state can be found out by experiment. Below is a sample procedure.

In eq. (1), let $a = 0$ and k be fixed, and then vary the value of γ from small to large until the system state varies from small periodic motion to chaotic motion, and at last to great periodic motion. Threshold value of the system in the critical state is denoted by γ_c. Hence, when γ is set to less than and near γ_c, the Duffing oscillator is put into the chaos state nearby the critical point.

Since Duffing oscillator represented by eq. (1) is normalized in frequency, before employing it to detect some external signal with non-normalized frequency, a corresponding frequency transformation must be accomplished.

Take $t = \omega\tau$. Then eq. (1) is expressed as

$$\frac{1}{\omega^2}\ddot{y}(\tau) + \frac{k}{\omega}\dot{y}(\tau) - y(\tau) + y^3(\tau) = \gamma\cos(\omega\tau) + ax(\tau), \tag{2}$$

where $x(\tau)$ is the representation of $x(t)$ on another time scale, $\omega = 2\pi f$ and f is frequency of the to-be-detected signal. Since eq. (2) is derived from eq. (1), the dynamic property and critical value are not changed.

To get the computing model of Duffing oscillator, eq. (2) is rewritten in the state equation form

$$\begin{cases} \dot{y}_1 = \omega y_2, \\ \dot{y}_2 = \omega[-ky_2 + y_1 - y_1^3 + \gamma\cos(\omega t) + ax(t)]. \end{cases} \tag{3}$$

Obviously, from eq. (3) it follows that

$$\begin{cases} \dot{Y} = \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} f_1(t, y_1, y_2) \\ f_2(t, y_1, y_2) \end{bmatrix} = f(t, Y), \\ Y_0 = \begin{bmatrix} y_1(0) \\ y_2(0) \end{bmatrix}, \end{cases} \tag{4}$$

where $Y = [\begin{smallmatrix} y_1 \\ y_2 \end{smallmatrix}]$ is a state vector to be solved and Y_0 is the initial state, namely the value of Y at time t=0.

If time step size is set at h, applying Runge-Kutta fourth-order algorithm [8] to eq. (4), we can get a set of recursive formulae

$$\begin{cases} K_1 = hf(t_n, Y_n), \\ K_2 = hf\left(t_n + \dfrac{h}{2}, Y_n + \dfrac{1}{2}K_1\right), \\ K_3 = hf\left(t_n + \dfrac{h}{2}, Y_n + \dfrac{1}{2}K_2\right), \\ K_4 = hf(t_n + h, Y_n + K_3), \\ Y_{n+1} = Y_n + \dfrac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4), \end{cases} \tag{5}$$

where K_i, i =1, 2, 3, 4 is the intermediate parameter. $t_n = n \times h, n = 1, 2, \ldots,$ and $Y_n, n = 1, 2, \ldots,$ are time and state vector at the nth step recursive calculation, respectively. h must be limited to the range $\frac{1}{10f} \leqslant h < \frac{1}{5f}$ so as to ensure the retrieval quality of the to-be-detected signal with frequency f, and efficiency and stability of recursive calculation.

Employing the above recursive model and the dynamic system analysis tool SystemView by ELANIX [9], the phase diagrams of Duffing oscillator are obtained, as shown in Figure 1.

In simulating, parameters used for the Duffing oscillator model are a=0.2, k=0.5, γ=0.84, and $h = 1/(10f)$. In addition, the amplitude and frequency of $s(t)$ which is not modulated are 900 mV and 36.050 MHz, respectively, $n(t)$ is zero-mean additive white Gaussian noise, and signal power to noise power ratio (SNR) is set at 10 dB.

From Figure 1, we can see that the phase trajectory is inordinate when $s(t) = 0$, and on the contrary, it appears regularly orbicular when $s(t) \neq 0$. These characteristics have been applied to signal detection, but an effective method, able to automatically identify phase feature of Duffing oscillator and extract baseband information from the phase feature, has not been available as yet. This is because BPSK signal demodulation based on Duffing oscillator cannot be accomplished so far.

3 Circular zone partition method

We know from the above discussion that phase pattern of Duffing oscillator is explicitly dependent upon the existence of $s(t)$ and there is quite great difference in feature. Thus, if we place a close circular region inside the regularly orbicular track of Duffing oscillator, then the existence of $s(t)$ can be automatically recognized according to whether the trajectory traverses the boundary of the circular region or not, in

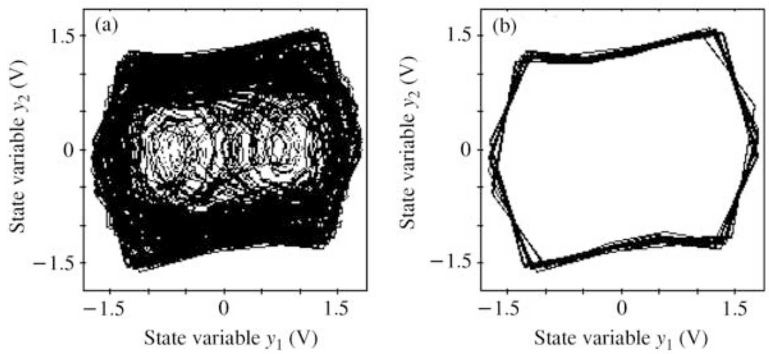

Figure 1 Phase diagrams of Duffing oscillator perturbed by $x(t)$ with noise. (a) $s(t) = 0$; (b) $s(t) \neq 0$.

other words, reporting the absence of $s(t)$ when the trajectory touches or traverses the boundary, conversely, and reporting the presence of $s(t)$ when the trajectory does not touch nor traverse the boundary. This is the operating rule for a CZP method. The specific implementation about the method can be described in detail as follows:

(1) Get the phase trajectory diagram of Duffing oscillator driven by $s(t)$ without modulation, then set a close circular region as large as possible within but not across the orbicular track of Duffing oscillator, and use R to denote the radius of the circular region. Figure 2 gives a sample explanation to the circular zone partition line.

(2) Let high level represent the case where phase trajectory does not touch nor enter into the circular region and let low level represent the case where phase trajectory touches or enters into the circular region. Here, high level and low level denote the fact that $s(t)$ occurs and the fact that $s(t)$ does not occur, respectively. Obviously, this is just a mapping of transforming the two-dimensional phase diagram of Duffing oscillator into the one-dimensional time signal. Hence, through it, information of $s(t)$, included in phase diagram, is shifted into envelop of the time signal.

(3) If we take high level as "+1" and low level as "−2", then using the boundary of the circular region as the decision criterion or threshold, a circular zone divider can be represented as

$$\bar{y}(t) = \begin{cases} +1, & \sqrt{y_1^2 + y_2^2} > R, \\ -2, & \sqrt{y_1^2 + y_2^2} \leqslant R, \end{cases} \tag{6}$$

where $\bar{y}(t)$ is the output of the divider, y_1 and y_2 are state variables from the computing model of Duffing oscillator, respectively, and R is the radius of the circular region. Moreover, taking low level as "−2" but not "−1" is to get the waveform symmetry after $\bar{y}(t)$ passes a low pass filter for smoothing and cancelling interference.

(4) The realization structure of the circular zone divider can be given by (6), as is shown in Figure 3.

It must be noted that the divider's algorithm is highly dependent upon Duffing oscillator's computing model, because within each recursive calculation period, whose length is equal to the step size h, it is carried out only one time. Thus, the algorithm for the divider can be specified as follows: the first step, solve the value of $y_1^2 + y_2^2$ using data from Duffing oscillator's computing model; the second step, compare the solved result with R^2, i.e. square of threshold. If it is greater than R^2, then output "+1", otherwise output "−2"; the third step, check if next recursive calculation period starts; if it is true, then go to the first step, otherwise go to entry of this step and repeat again.

From the above one sees that the CZP method needs two multiplications, one summation and one comparison operations in total for each Duffing oscillator output, whose computational complexity is much less than that of the typical methods like Lyapunov exponents, Poincare section, power spectrum, fractional dimension and Melnikov function [10] (because chaotic signal detection methods commonly include the chaotic oscillator, its computational complexity is not taken into account). Thereby the CZP method is with good real-time performance and the ability of automatically recognizing phase transition.

Different from the existing chaotic weak signal detection methods, the CZP method creatively utilizes the image features of Duffing oscillator's trajectory, i.e. orbicular track of periodic motion and inordinate

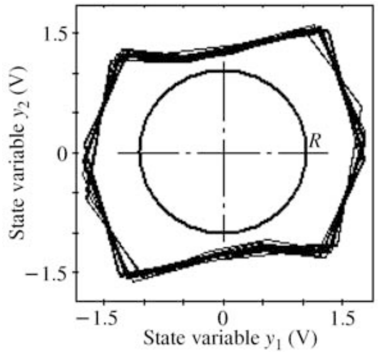

Figure 2 A sample explanation to circular zone partition line.

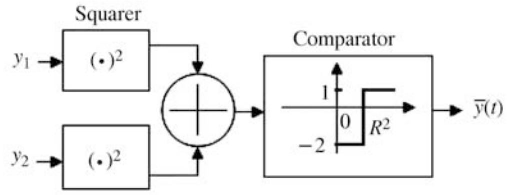

Figure 3 Structure of circular zone divider.

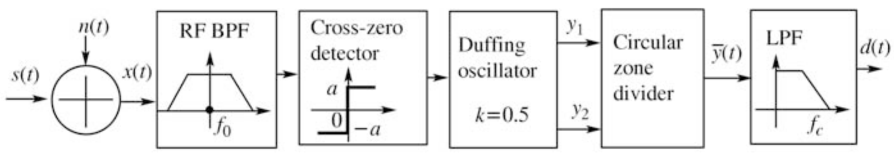

Figure 4 Evaluation scheme for CZP method.

track of chaotic motion besprinkled within the orbicular track, and transforms information of to-be-detected signal hidden in the two-dimensional phase plane into a one-dimensional time signal by means of the circular zone partition idea. This is the reason why the CZP method is able to detect the to-be-detected signal at smaller computing cost and with better effect than typical chaotic methods such as Lyapunov exponents, Poincare section, power spectrum, fractional dimension and Melnikov function.

4 Simulation results and performance analysis

The proposed CZP method has been evaluated using the dynamic system analysis tool SystemView by ELANIX. The evaluation scheme is given in Figure 4.

In Figure 4, RF BPF is a radio frequency band-pass filter whose central frequency and bandwidth are 36.050 and 2 MHz, respectively, LPF is a baseband filter used for cancelling carrier and noise in output of the circular zone divider, whose bandwidth is 2 MHz, and the cross-zero detector functions as a limiter, by which the maximal amplitude of $x(t)$ fed into Duffing oscillator is limited to less than 200 mV, corresponding to $a = 0.2$, aiming to avoiding the critical point offset of Duffing oscillator.

Duffing oscillator used for this simulation is the same as in Figure 1, and $s(t)$ is BPSK-modulated by baseband data with rate 1 Mbps, i.e. one-bit-interval $T = 1$ μs. The radius of the circular zone divider, or threshold is determined as 1.0 V by way of the experimental curve in Figure 2. n(t) is zero-mean additive white Gaussian noise, with power spectral density N_0=40.5×10^{-9} W/Hz.

Using the given simulation parameters, bit energy to noise power spectral density ratio (E_b/N_0), in the case of input impedance being one Ohm, is calculated below:

$$\frac{E_b}{N_0} = 10\text{lg}\frac{\frac{A^2}{2} \times T}{N_0} = 10\text{lg}\frac{\frac{0.9^2}{2} \times 1 \times 10^{-6}}{40.5 \times 10^{-9}} = 10 \text{ dB}. \tag{7}$$

Simulation results within time length of 90 μs are presented in Figure 5.

Referring to Figure 5(c), we can see that our method accurately extracts the baseband information from Duffing oscillator's phase transition caused by $s(t)$, and meanwhile Figure 5(d) indicates that the baseband data recovered by LPF is in good state, and well identical with original baseband data, as shown in Figure 5(a).

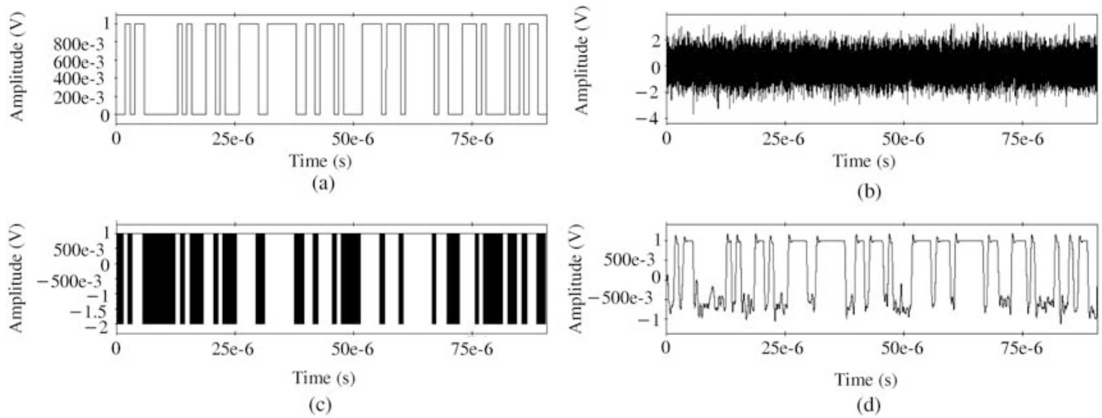

Figure 5 Evaluation results for CZP method. (a) Original baseband data; (b) external signal $x(t)$ in the case of E_b/N_0=10 db; (c) output of circular zone divider; (d) output of LPF.

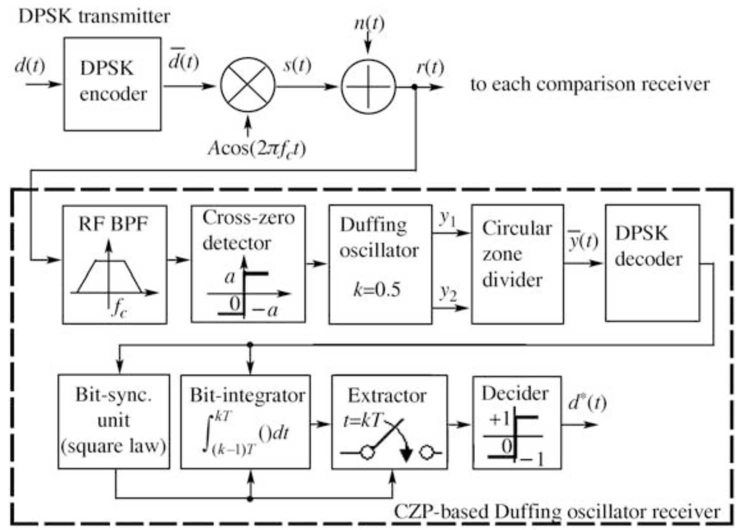

Figure 6 CZP-based DPSK Duffing oscillator receiver.

To verify the effect of the CZP method on BPSK signal demodulation, a CZP-based Duffing oscillator differentially binary phase shifted keying (DPSK) receiver is designed and its specific realization is shown in Figure 6.

In Figure 6, $d(t)$ denotes original baseband data produced by a pseudo-random number (PN) sequence with rate 1 Mbps, i.e. one-bit-interval $T = 1$ μs, $\bar{d}(t)$ corresponding to DPSK coding of $d(t)$ is bipolar, $d^*(t)$ is a retrieval version of $d(t)$, $s(t)$ is either a BPSK-modulated signal related to $\bar{d}(t)$ or a DPSK-modulated signal related to $d(t)$, and $n(t)$ is zero-mean additive white Gaussian noise with power spectral density N_0. $r(t)$ is a received signal not only at the receiver, but also at each comparison receiver and can be expressed as

$$r(t) = s(t) + n(t) = \bar{d}(t)A\cos(2\pi f_c t) + n(t), \tag{8}$$

where f_c is the carrier frequency, equal to 36.05 MHz, and A is the amplitude of cosine carrier, equal to 900 mV. Parameters of RF BPF, cross-zero detector, Duffing oscillator, and circular zone divider in the receiver are the same as in Figure 4. In addition, DPSK decoder is used to cancel differentially encoding in $s(t)$. Bit-sync is designed based on square-law principle and used to produce synchronization clock for bit-integrator and extractor, which jointly serve as a baseband data matched filter, and the decider is used to retrieve the waveform of baseband data.

Table 1 Simulation parameters

E_b/N_0 (dB)	N_0 (W/Hz)	Simulation time (ms)
0	405e−9	10
1	321.702935e−9	20
2	255.537724e−9	20
3	202.98083e−9	40
4	161.233404e−9	40
5	128.072245e−9	40
6	101.7314e−9	80
7	80.808123e−9	100
8	64.188174e−9	100
9	50.986479e−9	500
10	40.5e−9	1000

In this receiver, Duffing oscillator and circular zone divider act jointly as a BPSK demodulator. Exactly speaking, Duffing oscillator accomplishes the task of mapping existence of $s(t)$ into phase pattern change, and circular zone divider does the task of mapping the phase pattern change of Duffing oscillator into one-dimensional time signal whose amplitude is highly dependent upon phase pattern. Since the rest part of receiver is in principle similar to the typical digital BPSK receiver, no more statements are presented.

The simulation has been run in a PC computer under the support of SystemView by ELANIX, with a system sampling rate of 360.5 MHz. Other main simulation parameters such as noise power spectral density N_0, bit-energy to noise power spectral density ratio E_b/N_0, and simulation time under every E_b/N_0 of interest are listed in Table 1.

Through simulation, the bit-error-rate (BER) curve of the CZP-based Duffing oscillator receiver has been obtained, as is shown in Figure 7.

For performance comparison, a differentially encoded coherent BPSK receiver and a coherent DPSK receiver [11], which both use carrier-recovery based on square-law-principle, are selected for comparison, and simulated under the same E_b/N_0 as the Duffing oscillator receiver. Their BER curves obtained and the corresponding theoretic performance limit are also dipicted in Figure 7.

From Figure 7, we noticed that BER for the CZP-based Duffing oscillator receiver is not able to reach both theoretical limit of ideal BPSK receiver and theoretical limit of ideal DPSK receiver. But, the curves tell us that the CZP-based Duffing oscillator receiver obviously outperforms the coherent DPSK receiver in BER under any E_b/N_0, and also outperforms the differentially encoded coherent BPSK receiver when E_b/N_0 is greater than 6 dB. Although under E_b/N_0 less than 6 dB it is not better than the differentially encoded coherent BPSK receiver in BER, its performance is very close to and nearly the same as that of the latter.

It is noted that our receiver is non-coherent in principle. The reason why the chaotic non-coherent receiver can be beyond the typical coherent receiver such as BPSK and DPSK receivers in the case of E_b/N_0 being greater than 6 dB is mainly due to chaotic oscillator's dependence upon frequency and phase of the to-be-detected signal. In addition, the mapping from two-dimensional phase pattern to one-dimensional time signal, accomplished by circular zone divider, maybe is another reason. Therefore, further analysis will be needed in the future.

Two extra advantages of the CZP-based Duffing oscillator receiver are that it has a non-coherent structure and there is no need for analog to digital (A to D) converter in hardware. Hence, as it is free of carrier-recovery and is no longer limited to A/D conversion speed, the receiver has potential applications in burst communications and wide-band communication signal processing.

At last, let it be noted that as the Duffing oscillator responds to phase transition of BPSK signals only at great periodic motion or chaotic motion, the CZP method can only be applied to demodulate signals with two-state transitions, such as BPSK signal. Generally speaking, the proposed CZP method can be extended to detect any modulation signal with multi-state transitions. However, before carrying out the related work we have to find a new-type chaotic oscillator able to respond to the modulation signal with

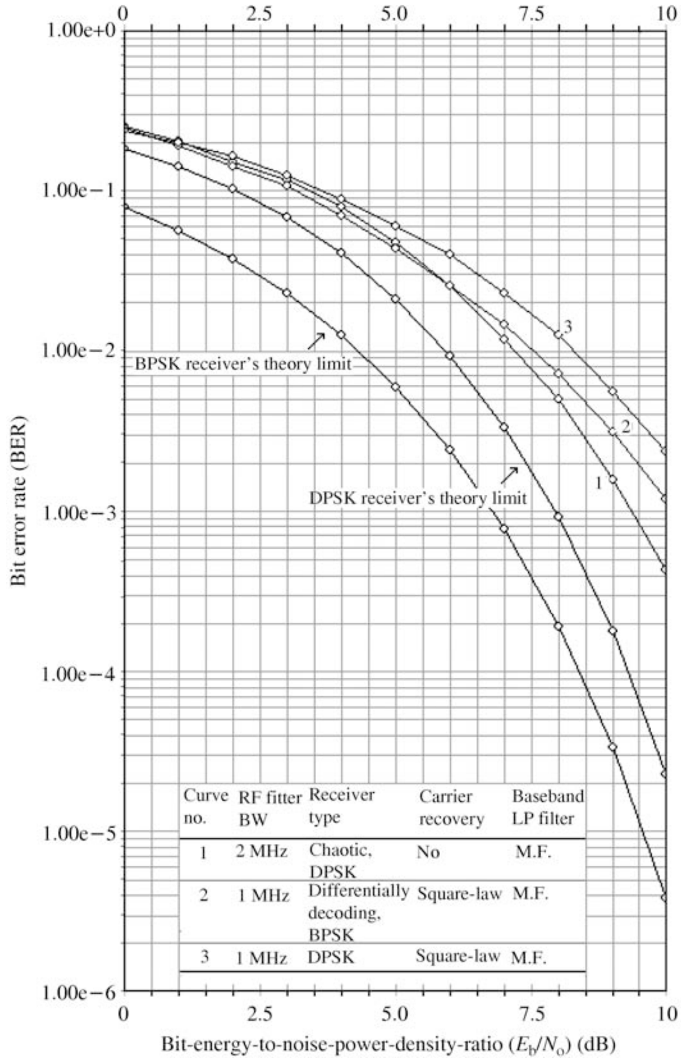

Figure 7 BER curves of CZP-based receiver versus typical receivers.

multi-state transitions at multi-different-periodic-motion states. The research is currently in progress and the related results will be reported elsewhere.

5 Conclusions

In this paper, we propose a Duffing oscillator CZP method, and give the equation, realization algorithm, and evaluate the performance of the circular zone divider. The simulation results show that this CZP method is accurate and valid for BPSK signal demodulation, and of good performance.

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