SCIENCE CHINA
Physics, Mechanics & Astronomy

• **Article** •

March 2014 Vol. 57 No. 3: 411–417
doi: 10.1007/s11433-014-5395-3

Fluctuation with dust of de Sitter ground state of scalar-tensor gravity

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Received May 20, 2013; accepted August 14, 2013; published online January 23, 2014

An exact de Sitter solution of scalar-tensor gravity is found in our recent work, in which the non-minimal coupling scalar is rolling along a non-constant potential. Based on this solution, a dust-filled FRW universe is explored in frame of scalar-tensor gravity in this article. The effective dark energy induced by the sole non-minimal scalar can be quintessence-like, phantom-like, and more significantly, can cross the phantom divide. The rich and varied properties of scalar-tensor gravity even with only one scalar is shown.

scalar tensor gravity, de Sitter space, dark energy, phantom divide

PACS number(s): 98.80.-k, 95.36.+x, 11.10.Lm

Citation: Tang Y K, Zhang H S, Chen C Y, et al. Fluctuation with dust of de Sitter ground state of scalar-tensor gravity. Sci China-Phys Mech Astron, 2014, 57: 411–417, doi: 10.1007/s11433-014-5395-3

1 Introduction

Dark energy is believed to drive the current cosmic speed-up [1]. A lot of models of dark energy have been suggested, including cosmological constant (vacuum energy), a scalar field dominated by potential (quintessence), a scalar field with false kinetic term (phantom), chameleon, Chaplygin gas etc., for a nice review see ref. [2]. However, although it determines the destiny of the universe, we are still far from completely understanding its physical nature.

Cosmological constant (CC)'s history, in fact, is much longer than dark energy, and is found to be an appropriate candidate for dark energy. However, it falls from famous theoretical problems: fine-tuning and coincidence problems. Furthermore, more and more abundant observation data illuminate a remarkable possibility of dark energy: it evolves with redshift. Usually, we introduce a scalar field, called quintessence, to simulate the evolution of the dark energy, for details see review article [2]. A quintessence field is minimally coupled to gravity, ie, there is no product term of Ricci scalar R and quintessence field ϕ in the Lagrangian. Contrarily to the ordinary lore, it is necessary under several conditions that a non-minimal coupled scalar is involved. The most competitive arguments have quantum origin. When we consider the quantum field theory in curved space, we find that we need a term $\phi^2 R$ to renormalize a minimally coupled scalar field in loop order [3]. We often get such terms when we reduce a higher dimensional theory, such as Kaluza-Klein theory, string/M theory, to a 4-dimensional one. The non-minimal coupling scalar has been investigated in cosmology, in context of inflation [4] and in dark energy [5].

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The Minkowskian ground state is an obvious solution of STG. For any physical theories, to find exact mathematical solutions is an important topic. Next comes the physical interpretations of the solution thus obtained. The spherical symmetric solution of STG has been found in ref. [6]. Mathematically, de Sitter as one of the three maximally spaces is undoubtedly important for any metric gravitational theories. From observational viewpoint, recent investigations illuminate that both the very early universe (inflation period) and the late-time universe (current cosmic acceleration) could be treated as fluctuations on a de Sitter background. So de Sitter takes a pivotal status in gravity, especially in modern cosmology. However, the de Sitter ground state of STG has not been found in literatures, though STG has been widely studied for several decades, and many "almost de Sitter" cases have been proposed [4]. We construct an exact de Sitter solution in STG, in which the scalar is rolling along a power-law potential [7]. At the same time, the energy density and pressure are variable, which seems impossible for the minimal coupling case. The primordial spectrum of this solution is studied. For some early works for the primordial perturbation in STG, see ref. [8]. In this article we shall explore the case in which dust matter presents to simulate the late acceleration.

This paper is organized as follows: In the next section we review the de Sitter solution for the STG. The dust fluctuation upon the de Sitter background, which is corresponding to the late time universe, is investigated in sect. 3. We conclude this paper and present some discussions in sect. 4.

2 de Sitter ground for STG

We find the exact de Sitter space in ref. [7],

$$\phi = c_2\left[-\mathrm{e}^{2c_1 b\xi} + \mathrm{e}^{bt}(4\xi - 1)\right]^{\frac{2\xi}{4\xi-1}}, \tag{1}$$

$$\begin{aligned} V = {} & \frac{3b^2}{\kappa} - \frac{\xi b^2}{\phi^2(1-4\xi)^2}\Big[(3 - 34\xi + 96\xi^2)\phi^4 \\ & + 8\xi c_2^{2-\frac{1}{2\xi}}\mathrm{e}^{2c_1 b\xi}(6\xi - 1)\phi^{2+\frac{1}{2\xi}} + 2\xi c_2^{4-\frac{1}{\xi}}\mathrm{e}^{4c_1 b\xi}\phi^{\frac{1}{\xi}}\Big], \end{aligned} \tag{2}$$

$$\rho = -p = \frac{3b^2}{\kappa}(1 - \kappa\xi\phi^2), \tag{3}$$

$$a = c_3\mathrm{e}^{bt}, \tag{4}$$

$$k = 0, \tag{5}$$

where $c_1,\ c_2,\ c_3$ denote integration constants. It is a solution of the following action of STG,

$$\begin{aligned} S = {} & \int_M \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2\kappa}R - \frac{\xi}{2}\phi^2 R + L_{\text{scalar}}(\phi) + L_{\text{matter}}\right] \\ & + \frac{1}{\kappa}\int_{\partial M}\mathrm{d}^3x\sqrt{-h}K. \end{aligned} \tag{6}$$

Here κ is the Newton constant, M and K are the spacetime and the extrinsic curvature of its boundary, R, ξ, h, g are the Ricci scalar, a constant, the determinants of the 4-metric $g_{\mu\nu}$ and its induced 3-metric $h_{\mu\nu}$ on the boundary, respectively. L_{scalar} and L_{matter} denote the Lagrangians of the non-minimally coupled scalar and other matters minimally coupling to gravity, respectively. L_{scalar} has the same form as an ordinary scalar,

$$L_{\text{scalar}} = -\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi). \tag{7}$$

According to this action of the non-minimal coupled scalar, the corresponding equation of motion reads

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \xi R\phi - \frac{\mathrm{d}V}{\mathrm{d}\phi} = 0. \tag{8}$$

The field equation according to the action (6) reads

$$(1 - \kappa\xi\phi^2)G^{\mu\nu} = \kappa\left[T^{\mu\nu}(\phi) + T^{\mu\nu}(\text{matter})\right], \tag{9}$$

where $G^{\mu\nu}$ is Einstein tensor, $T^{\mu\nu}$(matter) represents the energy-momentum corresponding to L_{matter} in eq. (6), and $T^{\mu\nu}(\phi)$ denotes the energy-momentum of the non-minimal coupling scalar,

$$\begin{aligned} T^{\mu\nu}(\phi) = {} & \nabla^\mu\phi\nabla^\nu\phi - \frac{1}{2}g^{\mu\nu}g^{\alpha\beta}\nabla_\alpha\phi\nabla_\beta\phi - Vg^{\mu\nu} \\ & + \xi\left[g^{\mu\nu}g^{\alpha\beta}\nabla_\alpha\nabla_\beta(\phi^2) - \nabla^\mu\nabla^\nu(\phi^2)\right]. \end{aligned} \tag{10}$$

The above solution (1)–(5) is the vacuum de Sitter solution, i.e., $T^{\mu\nu}(\text{matter}) = 0$.

If we define an effective gravity constant κ_{eff},

$$\kappa_{\text{eff}} = \left(1 - \kappa\xi\phi^2\right)^{-1}\kappa, \tag{11}$$

then the field equation (9) degenerates to Einstein's form with a variable Newton "constant", which is the essential idea of Brans-Dicke proposal.

An exact de Sitter phase is a very good approximation in the inflation stage. However, for the present acceleration, we must consider the contribution from dust. Then in the next section we will investigate the dust fluctuations based on this exact de Sitter of STG.

3 Dust fluctuation on the de Sitter background

In an FRW universe, the field equation becomes Friedmann equations,

$$(1 - \kappa\xi\phi^2)\left(H^2 + \frac{k}{a^2}\right) = \frac{\kappa}{3}\rho, \tag{12}$$

$$(1 - \kappa\xi\phi^2)\frac{\ddot{a}}{a} = -\frac{\kappa}{6}(\rho + 3p), \tag{13}$$

where as usual, a, ρ and p are the scale factor, the total density and pressure, respectively. The extra term compared with the standard model $(1 - \kappa\xi\phi^2)$ displays that ϕ takes part in gravity interaction,

$$\rho = \frac{1}{2}\dot{\phi}^2 + V + 6\xi\phi\dot{\phi}H + \rho_{\text{dust}}, \tag{14}$$

$$p = \frac{1-4\xi}{2}\dot{\phi}^2 - V - 2\xi\phi\ddot{\phi} - 4\xi\phi\dot{\phi}H. \tag{15}$$

ϕ is a quantity with scale of energy. It should be much lower than Planck scale in late universe. Thus $(1-\kappa\xi\phi^2)^{-1} > 0$ in late universe and the acceleration condition is still

$$\rho + 3p < 0, \tag{16}$$

from eq. (13). Substituting eqs. (14) and (15) into the above equation, we derive

$$\dot{\phi}^2 - 2V - \dot{\phi}\phi H + \rho_{\rm dust} - \phi\ddot{\phi} < 0. \tag{17}$$

In the following context, we use a subscript 0 to denote the present value of a quantity. Under the condition that the present universe just inhabits on the stagnation point of the potential, eq. (17) reduces to

$$\phi\ddot{\phi} < 2V_0. \tag{18}$$

This is equivalent to ΛCDM. However, it is only equivalent at the stagnation point of the potential.

In the present article, we constrain ourself in the conformal coupling case $\xi = 1/6$, under which the solution becomes

$$V = \frac{3b^2}{\kappa} - \frac{{\rm e}^{2c_1 b/3}b^2}{2c_2^2}\phi^4, \tag{19}$$

while the other quantities in the above set-up become

$$\phi = c_2\left({\rm e}^{c_1 b/3} + \frac{1}{3}{\rm e}^{bt}\right)^{-1}, \tag{20}$$

$$\rho = -p = \frac{3b^2}{\kappa}\left(1 - \frac{1}{6}\kappa\phi^2\right), \tag{21}$$

$$a = c_3{\rm e}^{bt}, \tag{22}$$

$$k = 0. \tag{23}$$

When dust matter is included, the above solution is broken. According to observations, our present universe is a quasi-de Sitter space. So the exact de Sitter solution should offer some key information, ie, the potential (19), to the evolution of the universe. We inherit the potential from the exact solution,

$$V = A - B\phi^4, \tag{24}$$

where A and B are two constants. We take the Friedmann equation (12) and the equation of motion of ϕ (8) as the fundamental set,

$$\left(1 - \frac{x^2}{6}\right)y^2 = \frac{1}{6}y^2x'^2 + n - lx^4 + \frac{1}{3}y^2xx' + \Omega_{m0}{\rm e}^{-3s}, \tag{25}$$

$$yy'x' + y^2x'' + 3y^2x' + \frac{1}{6}N_t - 12lx^3 = 0. \tag{26}$$

Here we have set dimensionless parameters and variables,

$$x = \sqrt{\kappa}\phi, \tag{27}$$

$$y = \frac{H}{H_0}, \tag{28}$$

$$n = \frac{A\kappa}{3H_0^2}, \tag{29}$$

$$l = \frac{B}{3H_0^2\kappa}, \tag{30}$$

$$\Omega_{m0} = \frac{\kappa\rho_0}{3H_0^2}, \tag{31}$$

and N_t corresponds to the non-minimal coupling term,

$$N_t = x\left(1 - \frac{1}{6}x^2\right)^{-1}(12n - 12lx^4 + 3y^2xx' + xx'yy' + xy^2x''), \tag{32}$$

where $s \triangleq \ln a$, a prime denotes derivation with respect to s. It is difficult to find the analytical solution the set (25) and (26), we solve it by using numerical method. We shall see that even with a very simple potential (24), the property of dark energy in this model is very rich and varied due to the non-minimal coupling effects.

First we make a note about the dark energy in extended gravity theory. In the STG theory, it is an extra term $(1-\frac{1}{6}\kappa\phi^2)$ in the modified Friedmann equation (12). However, most properties of dark energy obtained via observations are in fact, in the background of general relativity. To interpret the evolving EOS of the effective dark energy, we suggest a conception "equivalent dark energy" or "virtual dark energy". For more motivations and examples in modified gravity models, see ref. [9]. We obtain the density of virtual dark energy yielded by the non-minimal coupled scalar by comparing the modified Friedmann equation with the Friedmann equation in general relativity. The general Friedmann equation in the general relativity reads

$$H^2 + \frac{k}{a^2} = \frac{\kappa}{3}(\rho_{\rm dm} + \rho_{\rm de}), \tag{33}$$

where the first term of RHS in eq. (33) denotes the dust and the second term is just the dark energy. Comparing eq. (33) with eq. (12), one gets the virtual dark energy's density in STG,

$$\begin{aligned}\rho_{\rm de} &= \frac{3}{\kappa}\left(H^2 + \frac{k}{a^2}\right) - \rho_{\rm dm}\\ &= \left(1 - \frac{1}{6}\kappa\phi^2\right)^{-1}\left(\frac{1}{2}\dot{\phi}^2 + V + 6\xi\phi\dot{\phi}H + \rho_{\rm dust}\right) - \rho_{\rm dust}.\end{aligned} \tag{34}$$

Note that we consider the case that dark matter is just the dust matter, i.e., $\rho_{\rm dust} = \rho_{\rm de}$. For convenience, we introduce dimensionless dark energy,

$$\begin{aligned}u &= \frac{\kappa\rho_{\rm de}}{3H_0^2} = \left(1 - \frac{x^2}{6}\right)^{-1}\left(\frac{1}{6}y^2x'^2 + n\right.\\ &\quad \left. - lx^4 + \frac{1}{3}y^2xx' + \Omega_{m0}{\rm e}^{-3s}\right) - \widetilde{\Omega}_{m0}{\rm e}^{-3s}.\end{aligned} \tag{35}$$

The initial condition at present requires $H = H_0$, i.e.,

$$1 = y_0^2 = \left(1 - \frac{x_0^2}{6}\right)^{-1} \left(\frac{1}{6} y_0^2 x_0'^2 + n - l x_0^4 + \frac{1}{3} y_0^2 x_0 x_0' + \Omega_{m0}\right). \tag{36}$$

We see that the unique reasonable $\widetilde{\Omega}_{m0}$ should be

$$\widetilde{\Omega}_{m0} = \left(1 - \frac{x_0^2}{6}\right)^{-1} \Omega_{m0}, \tag{37}$$

if we require dark energy is completely yielded by ϕ at present epoch.

Under the condition that the dust matter obeys the continuity equation ,the Bianchi identity requires that dark energy satisfies the continuity equation,

$$\frac{d\rho_{de}}{dt} + 3H(\rho_{de} + p_{eff}) = 0, \tag{38}$$

where p_{eff} is the pressure of the effective dark energy. And then the equation of state for the dark energy becomes

$$w_{de} = \frac{p_{eff}}{\rho_{de}} = -1 - \frac{1}{3\rho_{de}} \frac{d\rho_{de}}{ds}. \tag{39}$$

According to eq. (39) we see that the evolution of w_{de} is controlled by the term $\frac{d\rho_{de}}{ds}$. The cosmological constant $\frac{d\rho_{de}}{ds} = 0$ (vacuum energy) bounds quintessence and phantom. The most important reason why we adopt the density of dark energy to describe the evolution of dark energy is that the density is more directly adhered to observables, thus is more tightly constrained for the same observation data [10]. More and more data are released recently. Importantly, observations which favor dynamical dark energy become more and accurate. Usually a quintessence, i.e., a canonical scalar field dominated by potential, can satisfy the observation. Furthermore, some data analysis implies that the present EOS of dark energy is less than −1. A phantom field, i.e., a scalar field with false kinetic term, can describe such an evolution. An interesting possibility appears from recent observations: the EOS of dark energy may cross −1 (phantom divide) [11], which deliver an embarrassing challenge for theoretical physics. This interesting problem is under intensively researches very recently [12].

We find that the effective dark energy (34) can evolve as quintessence, phantom, or even cross the phantom divide. In the following text, we show our numerical results. In Figures 1–3 we show the quintessence-like evolution of dark energy in STG. Figure 1 displays the evolution of the density of dark energy, Figure 2 illuminate the corresponding EOS. Figure 3 displays the corresponding deceleration parameter q. The deceleration parameter is a most important parameter from the observation side, which carries the total effects of the fluids in the universe,

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \tag{40}$$

In Figures 4–6 we show the phantom-like evolution of dark energy. Figure 4 displays the evolution of the density of dark energy, Figures 5 and 6 illuminate the corresponding EOS and deceleration parameter. In Figures 7–9 we show the crossing −1 behavior of dark energy. Figure 7 displays the evolution of the density of dark energy, Figures 8 and 9 illuminate the corresponding EOS and deceleration parameter.

Note that in Figures 7–9, we set $l = 0$, which means the potential is a constant. This is a very interesting case in which the scalar field rolls on a flat potential, which generates an effective dark energy crossing the phantom divide.

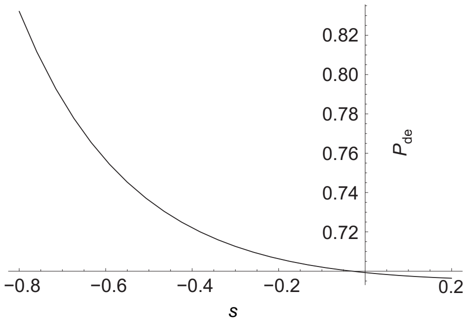

Figure 1 The evolution of dark energy density with respect to $s = \ln a$. In this figure, the parameters are taken as follows: $l = 1, n = 0.7$, Ω_{m0}=0.29, $\widetilde{\Omega}_{m0} = 0.3$. We see that the dark energy evolves as quintessence.

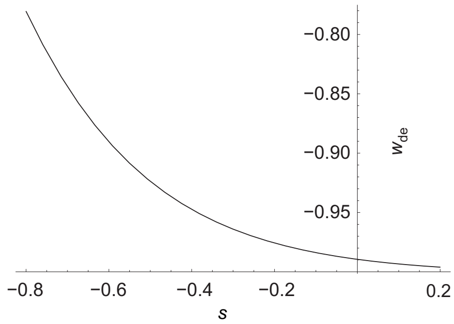

Figure 2 w_{de} evolves as a function of s. We set the same parameter set as of Figures 1. It is clear that w_{de} is always larger than −1.

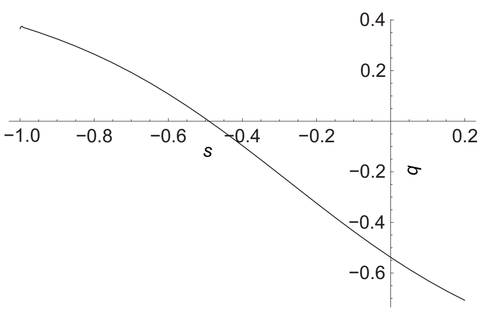

Figure 3 q evolves as a function of s. We set the same parameter set as of Figures 1. One sees that $q \sim -0.6$ at present epoch and becomes positive at high redshift, which is consistent with observations.

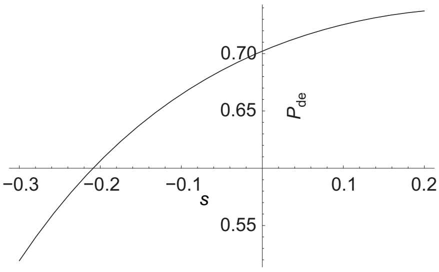

Figure 4 The evolution of dark energy density with respect to $s = \ln a$. In this figure, the parameters are taken as follows: $l = 1, n = 0.8$, Ω_{m0}=0.27, $\widetilde{\Omega}_{m0} = 0.3$. We see that the dark energy evolves as phantom.

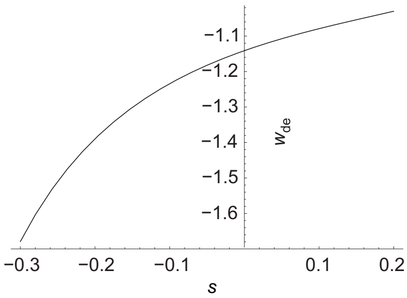

Figure 5 $w_{\rm de}$ evolves as a function of s. We set the same parameter set as of Figures 4. It is clear that $w_{\rm de}$ is always less than −1.

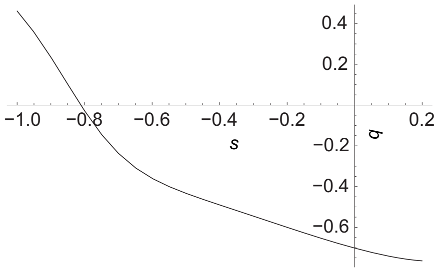

Figure 6 q evolves as a function of s. We set the same parameter set as of Figures 4. One sees that $q \sim -0.6$ at present epoch and becomes positive at high redshift, which is consistent with observations.

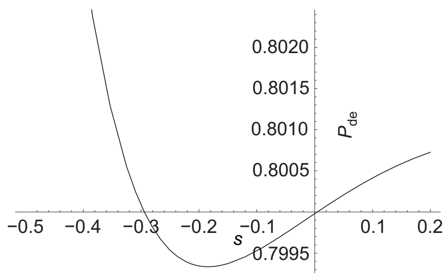

Figure 7 The evolution of dark energy density with respect to $s = \ln a$. In this figure, the parameters are taken as follows: $l = 0, n = 0.8$, Ω_{m0}=0.18, $\widetilde{\Omega}_{m0} = 0.2$. We see that the dark energy evolves from quintessence to phantom.

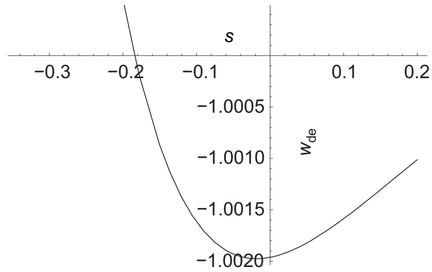

Figure 8 $w_{\rm de}$ evolves as a function of s. We set the same parameter set as of Figures 7. It is clear that $w_{\rm de}$ crosses phantom divide at about $s = -0.2$.

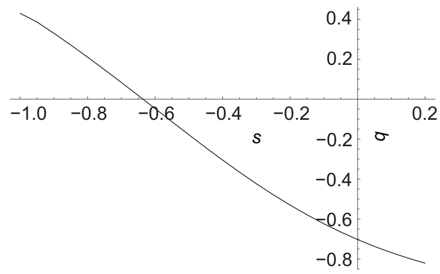

Figure 9 q evolves as a function of s. We set the same parameter set as of Figures 7. One sees that $q \sim -0.6$ at present epoch and becomes positive at high redshift, which is consistent with observations.

From Figures 1–9, we see that the dark energy in STG has rich properties with a very simple potential (24). Qualitatively, the extra term $(1 - \kappa\xi\phi^2)$ plays a critical role to yield the differences from general relativity. For example, though $\rho_{\rm de}$ decreases, the variation of $(1 - \kappa\xi\phi^2)$ can weaken, counteract, or even turn over this trend.

Scalar-tensor gravity is a nature and simple extension of general relativity. The universe is a quasi-de Sitter both in the early inflation stage and in late time cosmic acceleration. An significant problem is to find the exact de Sitter solution and hence inflation and cosmic acceleration can be treated as a fluctuation of this solution. In a previous study we find the exact de Sitter state for scalar tensor gravity. In present universe the partition of dust matter is about 30 percent, which cannot be omitted in a realistic model. In this work we introduce the dust sector in the original exact de Sitter solution to get a realistic model. The above figures show the significant effects of the non-minimal coupling term. The virtual dark energy can be quintessence, phantom, and more interestingly, crossing the phantom divide due to different parameters in the concrete models. At the same time, the deceleration parameter, which carries the total information of the cosmic fluids, is well consistent with observation data.

4 Conclusion and discussion

Though numerous works have been done in the area of approximate de Sitter space for STG [4], the exact de Sitter is not found till our previous works [7]. Once we have such a solution, our further works in the early universe and in late-time universe will be founded on solid rocks, since de Sitter space is a proper approximation both for the early universe and the late-time universe.

We first find a de Sitter solution of STG. In this solution, the effective gravity constant is variable with respect to time, which exactly counteracts the effects of varies of density and pressure. Thus the spacetime holds maximally symmetric. Based on this solution, we explore cosmology in frame of STG. We find the single non-minimal scalar can simulate quintessence, phantom, and can cross the phantom divide.

This work was supported by the Program for Professor of Special Appointment (Eastern Scholar) at Shanghai Institutions of Higher Learning, the National Education Foundation of China (Grant No. 200931271104), the National Natural Science Foundation of China (Grant Nos. 11075106, 11275128, 11203006 and 11303003), Shandong Provincial Natural Science Foundation of China (Grant Nos. ZR2012AQ029 and ZR2013AM002), and Nature Science Foundation of Zhejiang Province (Grant No. Y6110778).

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