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THE ORIGINS AND LEGACY OF 'KEPLER'S GAP'

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Abstract: This paper shows that Johannes Kepler's work on defining the structure of the Solar System was closely linked to his knowledge and understanding of Lutheranism, specifically what Martin Luther wrote. His study of harmonic proportions in geometric figures led him to note the gap between Mars and Jupiter, where he proposed an unseen planet existed. This was 206 years before the discovery of the first asteroid. But there were many other gaps in Kepler's scheme of the Solar System, which he also suggested might be filled by unseen planets. The role of Providence is explored in his search for how geometrical forms fit the data, and how he yearned for still more data proving Mars and Venus had moons in order to fill bothersome gaps. Kepler's famous three-dimensional representation of the Solar System is put in the context of other sixteenth century diagrams; a study of its artistic style shows that Kepler redefined the epistemological status of pictures. Two unexpected ways in which a void arises in Kepler's work are explored, and his influence upon the writings of William Wordsworth and Samuel Taylor Coleridge is explained in terms of the Trinitarian approach adopted by Kepler.

Keywords: Johannes Kepler, 'Kepler's gaps', *Mysterium Cosmographicum*, Martin Luther, William Wordsworth, Samuel Taylor Coleridge, Solar System, geometry

1 INTRODUCTION

While it was not written to describe the search by Johannes Kepler (1571–1630) for a new world order of the cosmos, the following lines from *Paradise Lost* by John Milton (1608–1674) are descriptive of the only man who could traverse the palpable obscure and land safely:

> But first whom shall we send
> In search of this new world, whom shall we find
> Sufficient? Who shall tempt, with wand'ring feet
> The dark unbottomed infinite abyss
> And through the palpable obscure find out
> His uncouth way, or spread his aery flight
> Upborne with indefatigable wings
> Over the vast abrupt, ere he arrive
> The happy isle? (Milton, 1688, Book 2: 39).

Kepler certainly believed that he had been selected by God to plumb the abyss and thus reveal the workings of the cosmos to his fellow mortals. Standing astride the divine and the physical, he grasped for immortality and achieved it. But one discerns a certain degree of *Weltschmerz* in his glorious evocation upon reaching the happy isle. In the words of Kepler paraphrased by another immortal, Edgar Allan Poe (1809–1849),

> I care not whether my work be read now or by posterity. I can afford to wait a century for readers when God himself has waited 6000 years for an observer. I triumph. I have stolen the golden secret of the Egyptians. (cited in Stroe, 2019).

Thus, Kepler was reconciled to being misunderstood during his lifetime. Indeed, the full importance of his three laws of planetary motion only became accepted after his death (Humboldt: 1868: 711).

In this respect he resembles many artists, whose fame is only fully expressed with the hindsight of decades or centuries. In his attempt to sketch the plan of the cosmos, Kepler can be thought of in artistic terms, and that is how he was viewed by the art historian Erwin Panofsky (1892–1968; 1937). In his 1924 study of how perspective influences perception, he shows how Kepler initially denied that the objectively straight tail of a comet could be curved because of his training in linear perspective. Grootenboer (2005: 119) explains that Kepler

> … unwittingly used linear perspective as a paradigm for his vision. In Kepler's case, linear perspective *produced* his view of the comet's tail, and thus his perception of the world (and of the universe), instead of being an expression of a worldview.

Likewise, his adherence to the so-called Platonic solids [each of the five identified by Plato (429–347 BCE) in *Timaeus* ca. 360 BCE as a regular, convex polyhedron] produced his view of the Universe; furthermore, this was grounded in theology, as Kepler believed "… that the *Timaeus* is nothing but a Pythagorean commentary on Moses." (Mehl, 2016: 202). While Platonic philosophy agreed with Aristotle that mathematics was perfect, Platonists believed a better guide to truth and reality "… could only be found in the abstract perfection of forms, rather than their material manifestation." (Johnson, 2013: 140). An exploration of orbital gaps, and his fitting of these forms (solids and other basic geometrical shapes) to both orbits and gaps, is the subject of the first part of this study. Why he looked to geometry for answers is best encapsulated in his own words:

> … I sometimes wonder whether the whole of Nature and all the beauty of the Heavens is not symbolized in Geometry. (Kepler, 1610; quoted in Walker, 1978: 55).

The legacy of his vision that posited a planet in the gap between Mars and Jupiter is explored as a matter of inspiration for those astronomers who followed him up to the discovery of Ceres in 1801; however, Kepler's influence transcended astronomy in the late eighteenth and early nineteenth centuries. How this was expressed by England's great poets William Wordsworth (1770–1850) and Samuel Taylor Coleridge (1772–1834) will be examined. While this study will contend that Kepler's Lutheran beliefs were important in his intellectual pursuits, one must be wary of overreach. For example, the work of Barker and Goldstein (2001), who claimed theological factors were crucial for the derivation of the ellipse, has been soundly refuted by Blåsjö (2009). And Kepler was not averse to setting his own agenda. As Albert Einstein (1951) notes, "Kepler was a pious Protestant, who made no secret of the fact that he did not approve all decisions of the Church."

Figure 1: A larger-than-life-size seated statue of Copernicus, created in 1822 by Bertel Thorvaldsen. In the collection of the Thorvaldsen Museum, Copenhagen. (photograph: Clifford Cunningham).

2 KEPLER'S *MYSTERIUM COSMOGRAPHICUM*

Mehl (2016: 197) explains that

> In the introduction to the edition of Rheticus' *Narratio prima* that accompanies the first edition of *Mysterium cosmographicum* (1596), Michael Maestlin (professor at the University of Tübingen) uses the biblical metaphor suggesting that the new astronomy (the Copernican one) is to the old one as the New Testament Law is to the Old Testament Law.

This metaphor can best be understood in terms of an old penal code (Graham, 2021). Luther's close associate Philip Melanchthon (1497–1560), in referring to the Mosaic Law of the Old Testament with a view consonant with that of Martin Luther (1483–1546), wrote that "… the entire Law has been abolished." (Melanchthon, 2007: 158). Thus, the metaphor informs the reader the old Ptolemaic system, with the Earth at the centre of all, has likewise been abolished. The metaphor also established at the outset that the *Mysterium* book (Kepler, 1999) was, to a significant degree, a theological text—not an astronomical or mathematical one. Indeed, it was Kepler's original intent "… to show in the *Mysterium* that Copernicus [Figure 1] could not be refuted by Scripture …", although he was compelled to eliminate that section under the logical pressure (Voelkel, 2001: 63; Rosen, 1975). And at the time he was working on *Mysterium* and confessed to his astronomy teacher Michael Mästlin (1550–1631) in a letter of 3 October 1595 that

> I had the intention of becoming a theologian. For a long time I was restless: but now see how God is, by my endeavours, also glorified in astronomy. (Baumgardt, 1951: 31).

As Barker (2000: 86) has noted regarding the sixteenth century, "The earliest Lutheran humanist astronomers were Lutherans first, humanists second, and astronomers after that."

Kepler had the weight of historical precedence on his shoulders, stretching back some 1800 years to the cosmological artifice erected by Aristotle (384–322 BCE) and Ptolemy (100–170). As the German poet Friedrich Hölderlin (1770–1843) wrote in *Reif Sind* about bearing a burden,

> And as
> A load of logs upon
> The shoulders, there is much
> To bear in mind. (Berkowitz, 2017).

To throw those logs off his shoulders so that he could "… let go of the past and taste the ripeness of the present …" was Kepler's driven goal, one he approached with a harmonic theory outlined in 1599 in a letter to his patron, the Bavarian chancellor Johann Georg Herwart von Hohenburg (1553–1622). In this letter Kepler grounded

> … harmonic proportions in geometrical fig-

> ures rather than in the status of certain numbers, which means bucking a tradition that had lasted since the Pythagoreans. (Regier, 2016: 219).

While Kepler clearly applied reason to his formulation of the Platonic solids in a planetary context, his underlying faith-based understanding of reason has not been given its due in this regard. Luther made a strong case for reason in his 1536 disputation, *De Homine* (*Concerning the Human*). In his fourth thesis, Luther links reason with the divine; in this he was perhaps influenced by Aristotle, who wrote "… thought is, no doubt, something more divine and impassible." (Smith, 1984: 651). Luther writes

> And it is certainly true that reason is the most important and the highest in rank among all things and, in comparison with other things in this life, the best and something divine.

Luther goes on to praise reason as the inventor and mentor of wisdom. Grosshans (2009: 181) identifies science as being under reason's jurisdiction, with reason being "… capable of making sound decisions about economy, politics, and the sciences."

Kepler was keenly aware of the need to discern and reveal the divine plan of the planetary distances. His Lutheran background gave him the intellectual grounding to reason that this was not a problem of celestial mechanics that could be revealed by mathematics, but a divine mechanism that revealed itself as a series of nested Platonic solids. Thus, he was 'skating on thin ice' as Luther specifically warned against extending reason into the heavenly sphere. It highlights the difference between how Galileo Galilei (1564–1642) and Kepler got to grips with reality (Todorov, 2017):

> Galileo's mathematization of movement on earth and in the heavens leads to the development of mechanics, to which the new world picture owed its identity. Kepler's mathematics, on the other hand, rests fully in the Renaissance tradition, as does his search into the *causa formalis* of the universe. (van der Schoot, 2001: 59).

Proportion held a central position in Kepler's planetary work. Its importance was best defined by the Swedish philosopher Thomas Thorild (1759–1808) in 1799. The essence of his message is

> … that all science is reducible to measurement. Philosophy, the science of sciences, is therefore 'Archimetrie,' as it were, the 'doctrine of archmeasurement.' The essence of reason is accuracy; the essence of accuracy is proportion. (Adickes, 2013).

This Archimetrie is what Kepler applied in his study, where he employs measurement—defined by his reason—to adduce proportions in the Solar System (Cunningham, 2017b: 335). Measurement was at the heart of a complaint the Austrian astronomer Georg Joachim Rheticus (1514–1574) had about gaps. In his presentation of the Copernican theory, Rheticus (1540: 146) wrote

> … there has not yet been established the common measure (*mensura communis*) whereby each sphere may be geometrically confined to its place … [and where] they are all so arranged that no immense interval is left between one and the other.

Westman (1975: 184) writes that in these claims, one important assumption is that "… there are no gaps between the spheres." Kepler begins his analysis in *Mysterium*

> … with a recognition of the gaps in the Copernican system that seems to have been an embarrassment for Rheticus and of no concern to Copernicus. To emphasize these gaps, at the end of chapter 1 of the *Mysterium*, Kepler presented two plates, one illustrating the Copernican system and the other the Ptolemaic system, both drawn approximately to scale for the first time. (Owen and Manning, 2018).

These are shown here in Figures 2 and 3. Kepler dismisses the approach of Rheticus as the inverse of what *should* be done, namely giving sanctity not to perceived numbers, but to the framework of creation:

> The opinion advanced by Rheticus in his Narrative is improbable, where he reasons from the sanctity of the number six to the number of the six moveable heavens; for he who is inquiring of the frame of the world itself, must not derive reasons from these numbers, which have gained importance from things of later date. (Kepler, 1596: 7).

Once he had discerned the divine plan, Kepler admitted reason could have no further power over the divine, despite his utmost efforts. He writes of it in this striking passage that delimits the reach of reason:

> What is worthy of admiration (since I had then no proof of any prerogatives of the bodies with regard to their order) is, that employing a conjecture which was far from being subtle, derived from the distances of the planets, I should at once attain my end so happily in arranging them, that I was not able to change anything afterwards with the utmost exercise of my reasoning powers. (Kepler, 1596: 8).

According to nearly every major seventeenth century philosopher, there were 'truths above reason' and 'truths according to reason'. Kepler was accepting here that he had discerned

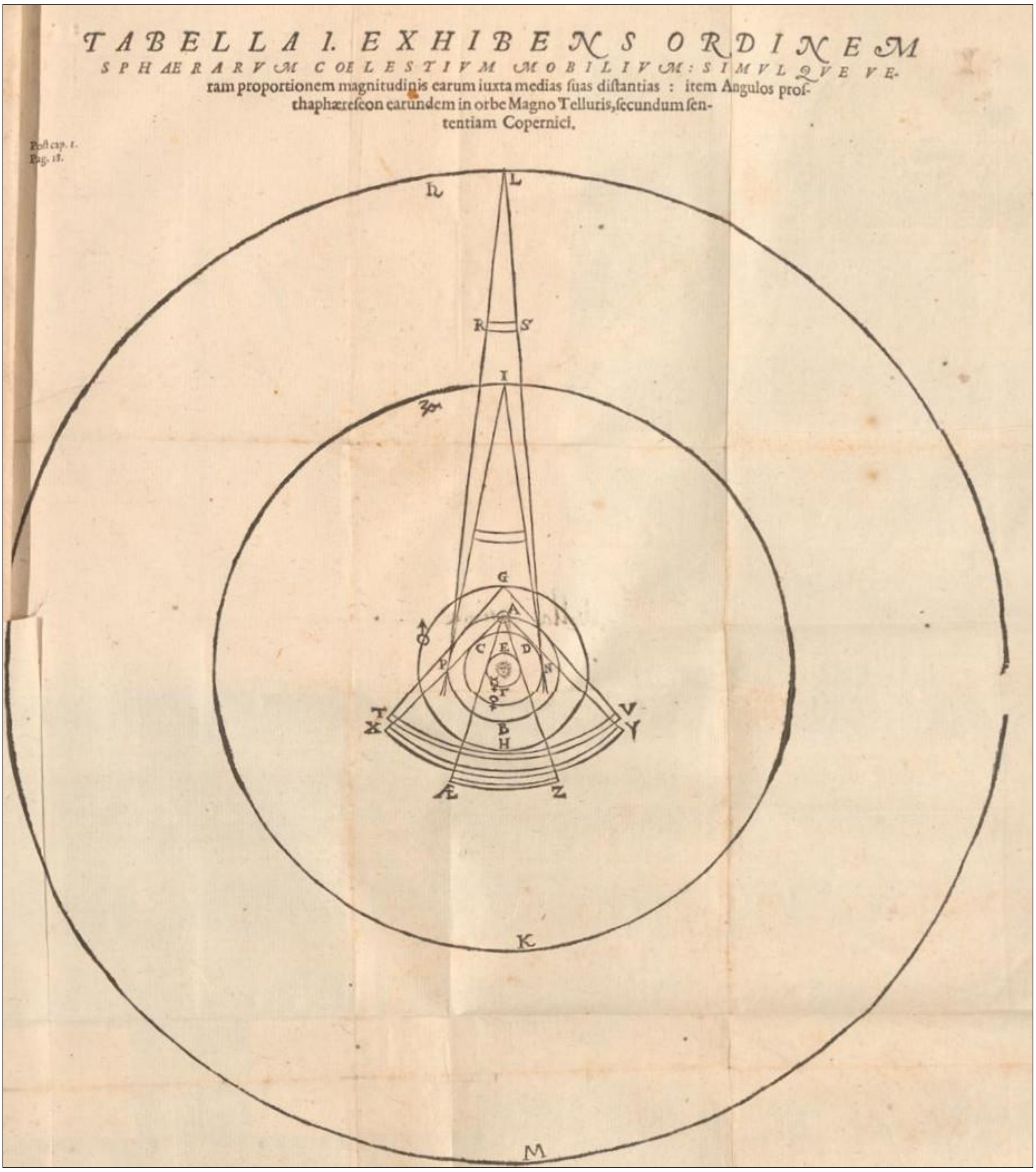


Figure 2: The Copernican Solar System, Table I from *Mysterium Cosmographicum* (1596; Tubingen: Georg Gruppenbach), inserted between pages 18 and 19 [courtesy: ETH-Bibliothek Zürich (https://www.e-rara.ch/i3f/v20/123207/manifest)].

'truths according to reason' to its utmost degree. Beyond that was what Luther had declared was the final standard of truth: "Scripture, which contains mysteries beyond the ken of our natural light." (Beiser, 2014: 4).

At the outset of the explanation of his study in *Mysterium*, Kepler couched his application of reason in terms that starkly displays the divide noted by van der Shoot. Here the 'adapted motions' he refers to means that orbital velocity declines with distance from the Sun.

> I reasoned, that if God had adapted motions to the orbits in some relation to the distances [of the planets], it was probable that he had also arrayed the distances themselves in relation to something else. (Kepler, 1596: 6).

This led him directly to the stunning supposition that gaps in the weave of the universe might be filled by unseen planets.

> Finding no success by this method, I tried another, of singular audacity. I inserted a new planet between Mars and Jupiter, and

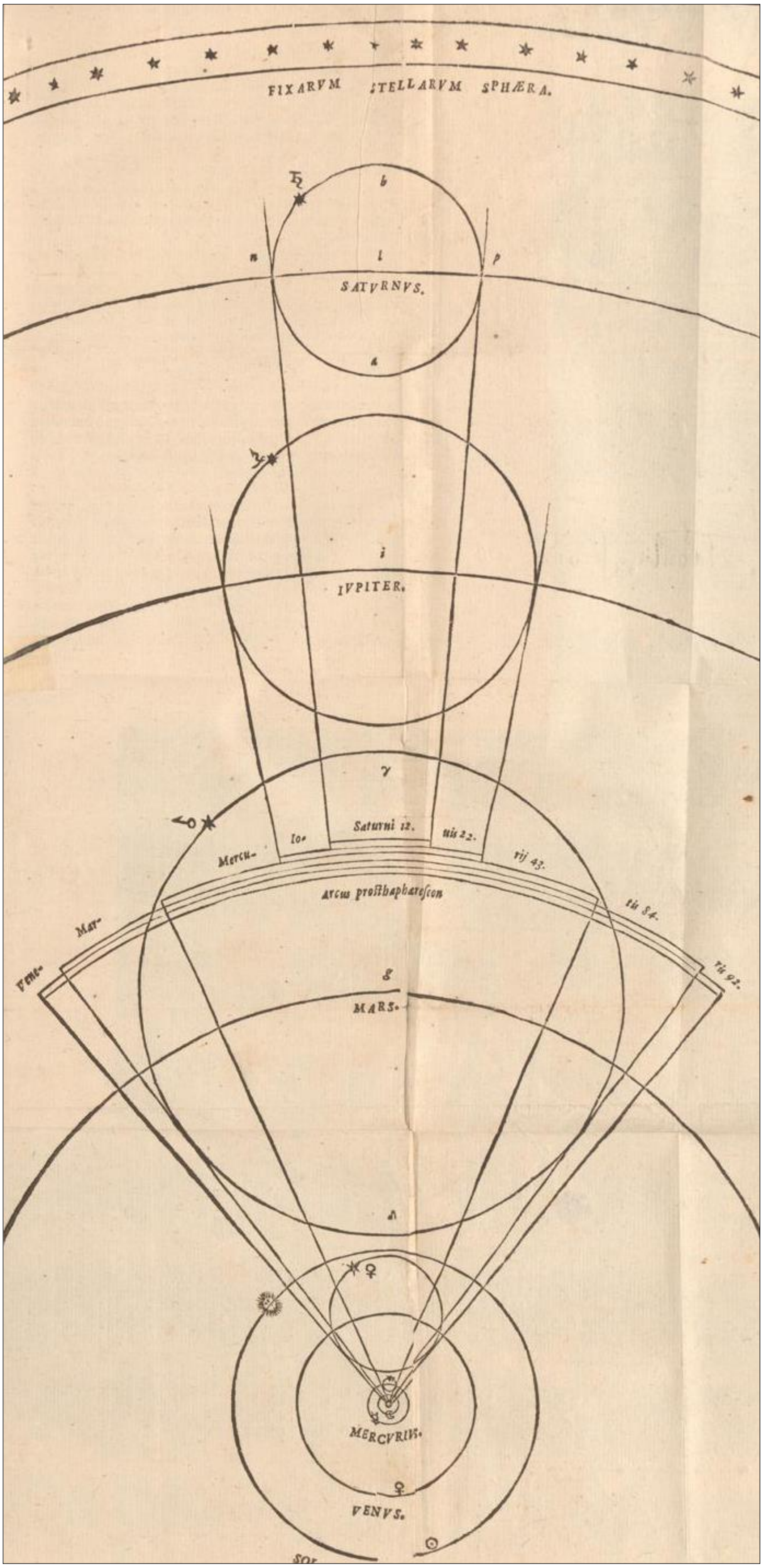


Figure 3: The Ptolemaic Solar System, Table II from *Mysterium Cosmographicum* (1596; Tubingen: Georg Gruppenbach), inserted between pages 18 and 19 [courtesy: ETH-Bibliothek Zürich (https://www.e-rara.ch/i3f/v20/123207/manifest)].

> another between Venus and Mercury, both of which I supposed invisible, perhaps on account on their smallness, and I attributed to each a certain period of revolution. I thought that I could thus contrive some equality of proportions, increasing between every two, from the sun to the fixed stars. For instance, the Earth is nearer Venus in parts of the terrestrial orbit, than Mars is to the Earth in parts of the orbit of Mars. But not even the interposition of a new planet sufficed for the enormous gap between Mars and Jupiter; for the proportion of Jupiter to the new planet was still greater than that of Saturn to Jupiter. And although, by this supposition, I got some sort of a proportion, yet there was no reasonable conclusion, no certain determination of the number of the planets either towards the fixed stars, till we should get as far as them, nor ever towards the Sun, because the division in this proportion of the residuary space within Mercury might be continued without end. Nor could I form any conjecture, from the mobility of particular numbers, why, among an infinite number, so few should be moveable. (Kepler, 1596: 7).

Perhaps unknowingly, Kepler was applying the infamous 'saving the appearances' strategy of Plato by proposing the existence of new planets to rescue his proportionality argument. After explaining that a trigonometrical approach failed, Kepler next explains that something akin to the divine happened when he was giving a lecture in July 1595:

> … by a trifling accident, I lighted more nearly on the truth. I looked on it as an interposition of Providence, that I should obtain by chance, what I had failed to discover with my utmost exertions; and I believed this the more, because I prayed constantly that I might succeed, if Copernicus had really spoken the truth. (Kepler, 1596: 8).

What did Kepler mean by an 'interposition of Providence'? Charles Mathewes from the University of Virginia has recently written on Luther's insistence

> … on the absolute governance of the world by a sovereign and providential deity … In his early work [of 1525], *On the Bondage of the Will*, Luther emphasizes that God's providential control is over all aspects of our lives. (Mathewes, 2021).

It was the belief in a providential deity that

> … motivated the special Lutheran interest in astronomy … and provided Kepler with the resources to give the strongest and most lasting defense of Copernican cosmology. (Barker, 2000: 62).

Kepler's text continues:

> It happened on the 9th or 19th day of July [Julian or Gregorian date], in the year 1595, that, having occasion to show, in my lecture-room, the passages of the great conjunctions through eight signs, and how they pass gradually from one trine aspect to another, I inscribed in a circle a great number of triangles, or quasi-triangles, so that the end of one was made the beginning of another. In this manner a smaller circle was shadowed out by the points in which the lines crossed each other. The radius of a circle inscribed in a triangle is half the radius of that described about it, therefore the proportion between these two circles struck the eye as almost identical with that between Saturn and Jupiter, and the triangle is the first figure, just as Saturn and Jupiter are the first planets.

In the following sentence, one can literally hear Kepler's heart thumping in that lecture room as, before what must have been a dumbstruck audience, he desperately tries to fit the gap between Mars and Jupiter with geometrical figures.

> On the spot I tried the second distance between Jupiter and Mars with a square, the third with a pentagon, the fourth with a hexagon. *And as the eye again cried out against the second distance between Jupiter and Mars, I combined the square with a triangle and a pentagon*. There would be no end of mentioning every trial. The failure of this fruitless attempt was the beginning of the last fortunate one; for I reflected, that in this way I should never reach the sun, if I wished to observe the same rule throughout; nor should I have any reason why there were six, rather than twenty or a hundred moveable orbits. And yet figures pleased me, as being quantities, and as having existed before the heavens; for quantity was created with matter, and the heavens afterwards. (My italics).

Notice Kepler invokes "… the eye …", which means 'the eye of reason', a subject we will return to in our discussion of Coleridge. In his 1604 publication *Paralipomena*, Kepler (2000: 109) made explicit how he regarded such excursions in geometry:

> For geometrical terms ought to be at our service for analogy. I love analogies most of all: they are my most faithful teachers, aware of all the hidden secrets of nature.

He revisited the topic in *Harmony*, writing

> To successfully produce natural knowledge we may follow the thread of analogy and pass through the labyrinths of the mysteries of nature. (Kepler, 1997: 495).

According to Heward (1912), Kepler first postulated the existence of an unseen planet while assisting Tycho Brahe (1546–1601) in preparing the Rudolphine Tables:

> Tycho's very exact observations of the places of the planets suggested to Kepler that Jupiter was very much farther away from Mars than accorded with his sense of just proportion of distances. All through his life Kepler had been dominated by a sense of analogy; he believed with unwavering faith that unity of design was an ordinance of the Creator's plan. Hence he concluded that, though invisible to the eyes now, a large planet existed in this region.

What was Kepler trying to do when he tried first a square, then added a triangle and a pentagon, to deal with the recalcitrant distance between Jupiter and Mars? Klinger (2011) has identified seven dimensions to the act of judging; the third of these is measurement (German: *Anmessung*), in consonance with Thorild's application of measurement. In an English translation (Gumbrecht, 2021: 160), Klinger writes that

> Acts of judging not only add new forms to reality; in doing so they also and of course presuppose that reality exists. Trying to make good (and not only random) judgments, we want to take into account the existing reality, with as much of its complexity as we can possibly perceive and process. We may also be concerned about how the forms we produce will 'fit' or will change reality in ways that we are hoping for.

We can see Kepler adding new forms to reality—not randomly, but with care. His toolkit is comprised not of irregular shapes, but of shapes (forms) basic to the real world: the square, triangle and pentagon. He is further grounded in the existing reality of the planetary orbits as revealed by observations, first and foremost those of his mentor Tycho Brahe. How the forms fit the data is what this is all about.

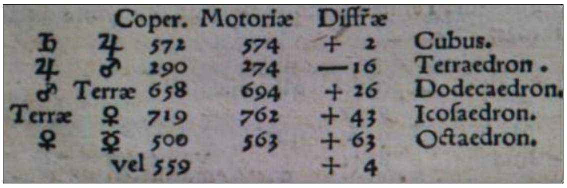
All of the above quotes are from the preface to his book. It is not until Chapter 21 (Kepler, 1596: 75) that he touches again on the sore point of the Mars–Jupiter distance:

> Kepler's attempt to give an account of the errors in the period-distance relation was at the same time an argument for the polyhedral hypothesis and for the compatibility of the two. He did this by preparing a table of the absolute differences of the distances derived from the period-distance relation from those taken from Copernicus, and noting the similarities in the differences to the solids that determine the same spacing in the polyhedral hypothesis. Thus, for example, only in the case of the Jupiter-Mars distance was the difference negative, and that corresponded to the tetrahedron (Voelkel, 2001: 55).

In his discussion immediately following the table (shown in Figure 4), Kepler mentions the cube, dodecahedron, icosahedron and octahedron, but sidesteps any mention of the tetrahedron. Kepler was also keenly aware that the separation of the planets from one another—the gaps between them—was a matter of great importance:

> Since the Copernican theory allowed the actual proportions of the planetary orbs to be calculated from observations, the gaps between them were also determinate. In explaining these gaps Kepler removed another apparently arbitrary feature from the Copernican description of the Universe. (Field, 1988: 71).

Yet there was still a problem for the astronomer who wanted everything to fit perfectly. After all, how could God create an imperfect system? He was quite candid about the issue in a letter to Galileo in April 1610:



		Coper.	Motoriæ	Diffræ	
♄	♃	572	574	+ 2	Cubus.
♃	♂	290	274	—16	Tetraedron.
♂	Terræ	658	694	+ 26	Dodecaedron.
Terræ	♀	719	762	+ 43	Icoſaedron.
♀	☿	500	563	+ 63	Octaedron.
		vel 559		+ 4	

Figure 4: Kepler (1595: 75) shows the differences in the distances between one planetary orbit and another, fitted to five Platonic solids. Only the distance between Mars and Jupiter exhibits a negative value.

> Recently, while recalculating the orbits and motions of Mars, the Earth and Venus from Brahe's observations, I noticed the spaces between the orbs are slightly too large, so that when the vertices of the dodecahedron are placed as far out as the perihelion of Mars the centres of the faces do not touch the Moon at its apogee when the Earth is at aphelion. Nor when the centres of the faces of the icosahedron are fitted to the aphelion of Venus do its vertices reach the Moon at its apogee when the Earth is at perihelion. This shows that there is extra space between the perihelion of Mars and the vertices of the dodecahedron, as between the centres of the faces of the icosahedron and the aphelion of Venus ... I hope that I shall easily get moons of Mars and Venus into these spaces, Galileo, if you find such moons. (Caspar, 1959: 310).

Kepler evinces here the rather desperate circumstance he has fallen into. In the *Mysterium* book of 1596 he was willing to at least consider adding more primary planets, notably in 'Kepler's Gap' between Mars and Jupiter. Here, fourteen years later, we see him hoping against hope that secondary planets will also be found to "… improve the agreement between the observed planetary orbs and those calculated from theory." (Field, 1988: 80). There appeared to be no end to Kepler's gap-filling.

2.1 Cosmological Representation

Before leaving the subject of the Platonic solids, a study of Kepler's famous three-dimensional representation is in order. I will contrast it here with the approach adopted by the Swiss music theorist Heinrich Glarean (1488–1563) whose most famous work was the *Dodecachordon* of 1547. This book, which Kepler was familiar with, embodies in its title a merging of the geometrical solid, the dodecahedron, with the musical term chord, thus promoting Glarean's idea that there are 12 modes of music instead of eight. What interests me here, however, is a comparison of Glarean's and Kepler's approach to cosmological representation. In another major work, *De Geographia*, Glarean (1527) included much of astronomical interest. But how to represent an Aristotelian Universe populated by circles that moved in perpetual harmony? In his draft manuscript, Glarean tried

> … to create a sense of depth by using darker colours to shade the back halves of the major and minor circles in his most elaborate diagram of the universe, but in the end he abandoned the effort, including no three-dimensional illustrations in the final version of *De geographia*. All the visual refinement in the world could not produce on the printed page a perfect model of the perfect universe. (Johnson, 2013: 149).

Kepler was not concerned by this consideration, boldly using shading on the inner right portions of his nested spheres, and also on the outer left portions of those spheres, in what he identified as Tabula III (the diagram between pages 24 and 25 in the 1596 edition, and reproduced here as Figure 5). The light source casting these shadows, which appears to be coming from the upper right, is a matter left unsaid. In engaging with this important diagram Field (1988: 38–41) wrote that

> Some disembodied dotted lines have been drawn to indicate the positions of the centres of faces of the polyhedra and the points where their vertices touch the inner surfaces of spheres … It is possible Kepler's illustrator made use of the published versions of Leonardo's pictures …

of the Platonic solids in a book for Luca Pacioli (1509), and

> The weakness of the plate as an astronomical diagram is underlined by the brevity of the key, which contains only twelve entries. In any case, the uncooperative nature of the astronomical facts ensures that the inner part of the picture is exceedingly difficult to read.

One wonders if this was deliberate, as zoomed-in detail of the inner Solar System would reveal the gaps in the scheme (see Figure 6). Andrews (2017: 293) notes that whoever created the image "… was well versed in the print genre of unbuilt, polyhedral models and used them as his primary source of inspiration."

In an analysis that was commissioned for this paper, Dr Elvira Bojilova (pers. comm. 2021) offers some unique insights into Tabula III:

> The first thing that struck me is the fact that the image on page 24 is quite different from the other ones, not only in terms of style but also size, etc. It is much more detailed and highly 'finished' as art historians would sometimes describe it. Stylistically, it is hard to say what the artist might have known, but one can only assume he was familiar with some of the great engravers of his time, for instance Hendrick Goltzius. Keeping in mind that Kepler's book was not a work of art, the quality and the size of this etching are remarkable. The comparison to Leonardo/Pacioli is certainly interesting, and reminds me of Leonardo's machine drawings.
>
> The hatching in Kepler's image does not, however, produce a perfectly natural three-dimensional effect, especially in comparison to Leonardo's drawings and the way he used hatching. First, it is difficult to define what 'natural' hatching in an art work would look like since it doesn't have an equivalent in the natural world. Second, in Kepler's image the illusion of 'perfect naturalism' is somewhat disrupted by the way the artist applied the hatchings inside the big sphere for instance. It almost appears to be flat due to the dense cross hatching and the way the lines change direction and formation. By the same token, if you look closely, you'll see that the parallel hatching on the outer left side of the sphere is a bit too straight instead of curvilinear. It does *not quite* follow the 'underlying' round form of the thing it represents. Given the importance of this image, the slightly awkward layout of the page in general is odd, especially in the left corner where the writing almost overlaps the engraving. Maybe all these considerations played into the decision to rework the illustration for the 1621 edition [see Figure 7]. In that later edition the hatching is much more 'organized' and less chaotic, if you will. In addition, the writing in the lower corners no longer clashes with the image. Note how the depth of the sphere is indicated by vertical parallel hatching that seem to be deliberately juxtapositioned

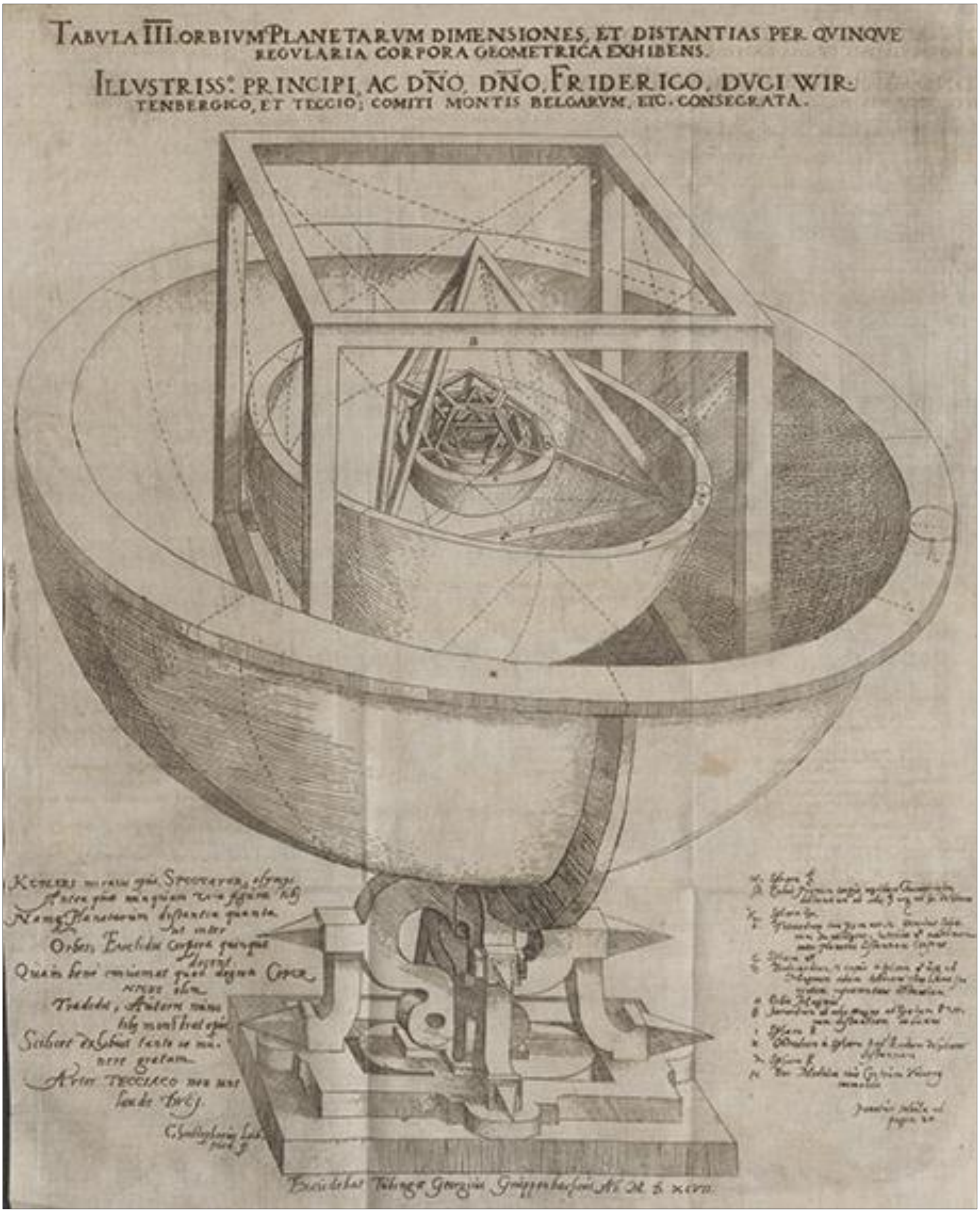


Figure 5: Table III from *Mysterium Cosmographicum* (1596; Tubingen: Georg Gruppenbach) inserted between pages 24 and 25 (Wikimedia Commons).

against the quasi-horizontal parallel lines of the outside sphere in order to draw the beholder in. This is not to say that the artist attempted to apply hatching in a semiotic sense, meaning that there was a direct 'meaning' attached to a specific hatching style (which is an approach I am skeptical of). But there was clearly an effort to improve the illustration.

See Bojilova (2021) for further relevant analysis of contemporary engravings, and Field (1997) for a discussion of perspective and the representation of the geometrical solids in the fifteenth century.

Throughout his career, Kepler

> … addressed the gap between knowable universals and concrete physical events.

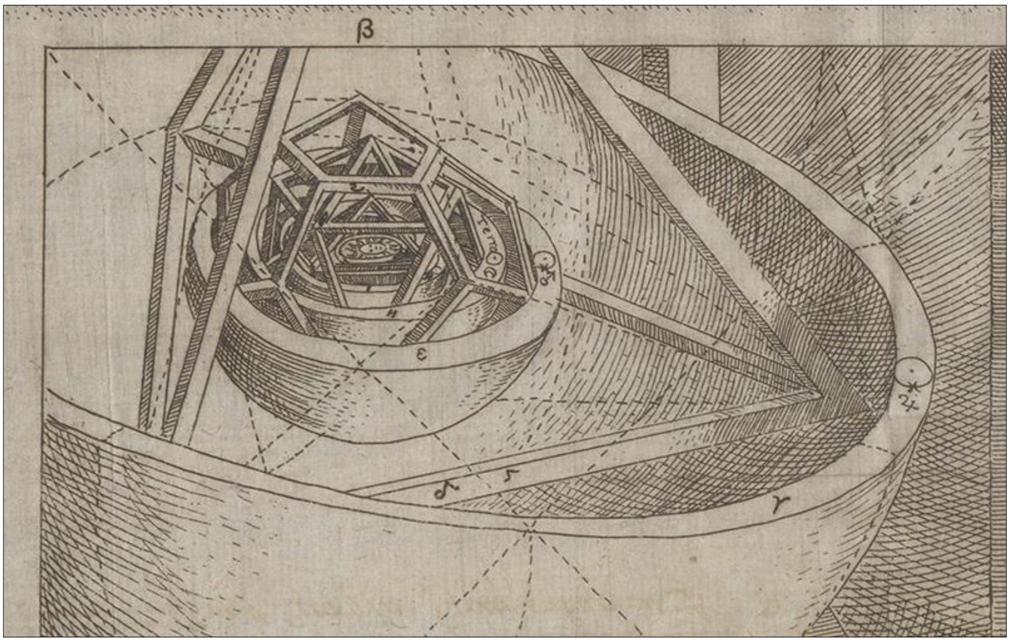


Figure 6: Detail of Table III from *Mysterium Cosmographicum* (1596; Tubingen: Georg Gruppenbach) (courtesy: Linda Hall Library).

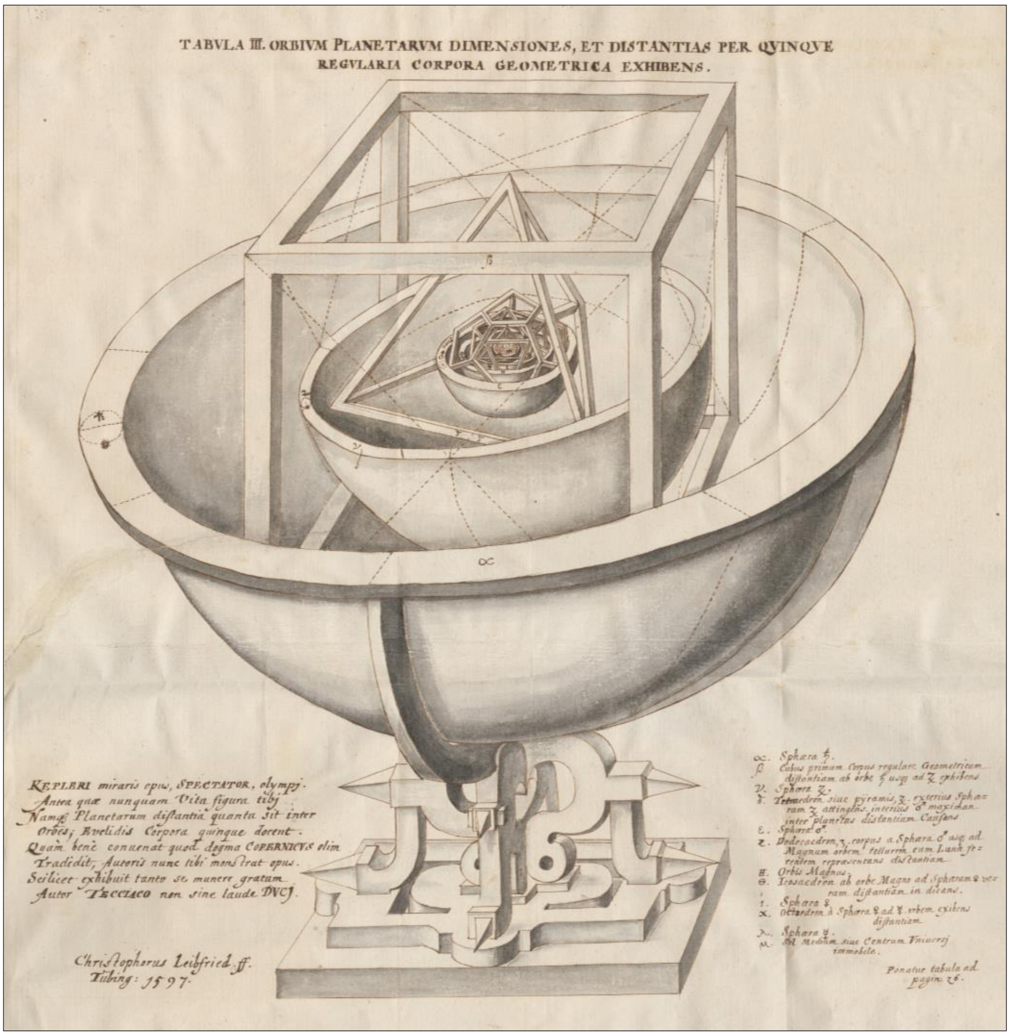


Figure 7: Table III from *Mysterium Cosmographicum* (1621; Frankfurt am Main: Godefridi Tampach) (courtesy: Linda Hall Library).

> In order to surmount this gap and to disentangle his science from the multilevelled interpretations of emblematic representations of the secrets of nature, Kepler had to redefine the epistemological status of pictures. (Chen-Morris, 2009: 152).

This he did with the famous 'picture' of the Solar System in *Mysterium Cosmographicum*; as the next section explores, addressing this 'gap' was just one of several that challenged Kepler.

3 OF GAPS, NOTHING, AND THE VOID

> Beyond the Universe NOTHING finds place,
> And NOTHING fills the mighty void of Space:
> On NOTHING turn the lucid orbs above,
> And all the Stars in mystic order move.

In this extract from a poem by the Reverend Belsham (1857), an English Unitarian minister, the psychic void left by modern science in man's view of the cosmos verges on the nihilistic, saved only by some 'mystic order'. Anttila (2017: 73) writes of an early sermon by Luther, where he defined a threefold *nihil* (nothing):

> The first is nothing in the literal sense, that is, related to being (*ens*), the second is nothing as an equivalent of being false, which is in opposition to truth (*verum*), and finally, nothing in the sense of evil, which is in opposition to good (*bonum*).

One aspect of nothing in nature (the literal sense of *nihil*) is a vacuum, famously derived from the *fuga vacui*. This has variously been translated as 'nature's flight from the void' and 'nature abhors a vacuum.' Inventive strategies were employed "… to avoid or justify or 'fill' the vacuum in natural philosophy." (Blum, 2017: 427). Robert Boyle (1627–1691), for example avoided

> … the persistent philosophical problems surrounding the concept of void by referring to it as the absence of matter rather than the presence of a new entity. (Jenkins, 2000: 160).

Kepler, by contrast, did invoke the presence of a new entity; when his most famous gap (Kepler's Gap) finally became filled, the asteroid/dwarf planet Ceres was revealed.

The first engagement with Kepler's "… a priori derivation of the relative distances [of the planets] from the ratios given by the inscribed and circumscribed polyhedral …" (Westman, 2011: 348) was made by the German physician, astrologer and anti-Copernican Helisaeus Roeslin (1545–1616). He too had correspondence with Hohenburg, and in May 1597 expressed doubts about Kepler's choice of Platonic solids. Although "… the cube gives the distance of the spheres of Saturn and Mars …" he did not know if perhaps

> … another of the five regular solids could not also give such [a result] … And although there may be five such distances of five planets, with every body especially arranged for specific planets, still I will not be of Copernicus' opinion for that reason. (quoted in Westman, 2011: 347).

Two months later Roeslin believed he had cracked the problem with what Nicholas Reimers Ursus (1551–1600) soon derisively termed a "… gappy hypothesis." (Westman, 2011: 348). Roeslin

> … asserted that the cube would give the size of Saturn's sphere, the tetrahedron would fit Jupiter, and the dodecahedron would fill the space between Mars and the Sun.

He further posited the 20-sided icosahedron would subsume three gaps, giving the space between the Sun and the Moon, in which space his geoheliocentric view of the Solar System placed both Mercury and Venus. Roeslin also noted that Kepler had also overlooked a gap, namely "The almost infinite distance between the sphere of Saturn and the fixed stars." Roeslin pondered the implications: "With which geometrical figure does he want to account for such an infinite empty space?" (Westman, 2011: 347). A year later, Tycho also criticized Kepler for ignoring that gap. In a letter to Kepler dated 1 April 1598, Tycho

> … claims the heliocentric hypothesis is incompatible with the basic premise that the cosmos has to be well proportioned: the empty space between Saturn and the fixed stars violates the principle of continuity and produced 'assymmetria'. (Mehl, 2016: 206).

"If this gap filling was not decisive," states Westman, "Roeslin believed that Herwart would agree his arrangement far better accommodated the polyhedra because of the last gap." Roeslin also wrote:

> I do not require a further geometrical demonstration, because I put the uppermost part of the sphere of Saturn to be contiguous to the eighth sphere. Thus Your Grace sees how his [Kepler's] invention confirms my system far better than his." (quoted in Westman, 2011: 348).

What was left unsaid by either Ursus or Roeslin was

> … the metaphysical relevance of the polyhedral as a new criterion for comparatively evaluating (and eliminating) multiple hypotheses of cosmic order. (Westman, 2011: 348).

Kepler was acutely aware of the fitting issue. Armed with the distances and excentricities given by Copernicus, how well did the dimensions of the spheres fit the Platonic sol-lids computed from them in such a way that the inner surface of a sphere coincided with the sphere circumscribed to the next solid below it, and the outer surface with the inscribed sphere of the solid right above it? He ident-ified two such gaps: "The orbit of the Earth does not even touch the sides of the proposed dodecahedron; neither does Venus with the corresponding icosahedron." In what must have been an act of some desperation that sacrificed the elegance of his system, he plac-ed a star-shape between Mars and Venus

> … constructed from five equilateral tri-angles laid down outward and raised from the edges of each of the pentagons that constitute a dodecahedron. (Cardona, 2016).

Somewhat like Ptolemy adding epicycles to make the observations fit the theory, the un-reality of his geometric system was staring Kepler in the face, but he doggedly persisted in believing it, even after he discovered plan-etary orbits are elliptical. This may have been the inspiration for some lines in the famous 1679 poem *On Nothing* by the Earl of Roch-ester (1647–1680; 2002) who writes "Nothing, who dwellest with fools in grave dispute/For whom they reverend shapes and forms de-vise."

The matter of a void arises in Kepler's work in two unexpected ways. As Barbour (2001) has described it, a void point is a point in space where no object, no matter, is located but which plays an essential role in describing the motion of a planet. Such is the case with Ptolemaic epicycles and equants. "The dy-namic void points in models of planetary orbits were exactly what drew Johannes Kepler's attention and objection." (Kosso, 2013: 385). The circular orbits that were thought to exist contained an empty point in space that direct-ed the planet's circular orbit. Kepler rejected this notion, writing

> A mathematical point, whether or not it is the centre of the world, can neither affect the motion of heavy bodies nor act as an object towards which they tend. (Kepler, 1992: 54).

As part of the abolition of Ptolemaic concepts noted by Melanchthon, "… the Keplerian mod-el of planetary orbits has no void points what-soever." (Kosso, 2013: 385).

Secondly, in his effort to "… prove that a vast portion of the universe should be partic-ularly different …", Kepler makes a bold pro-posal about a spatial void that ultimately fails to persuade as it is based on a circular argu-ment as "… the same measurements used to propose the distance to the stars are employ-ed to predict the cavity without any further proof." (Luna, 2021: 79). In his book on the supernova of 1604, Kepler (1606: 689) writes

> For let it be admitted that the fixed stars are extended outward to infinity. Never-theless, it is a fact that in this inner bosom there shall be an immense cavity, distinct from the spaces among the fixed stars and vastly different in proportion. This, if it oc-curred to somebody to examine only this cavity, from the sole comparison of this void with the surrounding spherical region filled with stars, he would utterly conclude that this is a certain particular place and, in fact, the main cavity of the world.

One can see in this passage Kepler attempts to apply the concept of proportionality once more, in keeping with his unwavering stance "… that the geometry of the universe amounts to a complete and physically fused astronomi-cal landscape." (Luna, 2021: 79). In proposing this cosmic void within the landscape, Kepler was attempting to prove his model which in-cluded the heliocentric Copernican view, but his application of proportionality was over-shadowed by a gap Copernicus himself ident-ified (in Chapter 10 in his 1543 book *De Revo-lutionibus Orbium Coelestium*): "From Saturn, the highest of the planets, to the sphere of the fixed stars there is an additional gap of the largest size." (Dobrzycki, 1978: 22). The pro-portion problem posed by the stars remained unresolved for Kepler, as it had for Copernic-us.

Reflecting on the beguiling influence of Kepler's Gap between Mars and Jupiter as seen from the beginning of the nineteenth cen-tury, the man who co-discovered Ceres in 1801, Giuseppe Piazzi (1746–1826), wrote this about Kepler:

> The first we can mention to have an idea about a planet between Mars and Jupiter was Kepler's thought as the father of mod-ern astronomy. Living at the time of the Renaissance, he was overwhelmed by the fascination, common at that time, of the ancient philosophy made majestic by the names of Pythagoras and Ptolemy. He believed in the mysterious property of numbers: he thought that in the multiplicity of their relationship was the seed of human knowledge "so I looked in their order and structure in the sky." But being a great genius more worthy of the title of divine than Ptolemy, submersed by the most absurd extravagance of a dream of ce-lestial harmony and by a myriad of com-binations, he pointed out an emptiness between Mars and Jupiter that could only be explained through a dissonance and

> lack of harmony. This dissonance was not felt by him about the other planets, which combined in direct or inverse order to create a beautiful concert. (Piazzi, 1802).

In the same year, the mathematician Carl Gauss (1777–1855) accorded Kepler a very high accolade: he believed that as a man of science, Kepler was quite capable of discarding even his most cherished belief in the face of new observational evidence:

> I do not want to disapprove of the fact that one seeks in nature such approximate correspondences. The greatest men subscribed to such lusus ingenii [games of nature]. But as proud as Kepler was of his regular bodies reconciled with the distances of planets (as he said he did not want to give his find to the Electorate of Saxony) he certainly would not have used it to challenge Uranus' planetism (if this discovery had been made in his times) simply because it did not match his ideas. Most likely he would have abandoned them immediately. (Gauss, 1802).

Gauss alludes in this missive to the 'game of nature' popularly known as Bode's Law, which noted the famous gap between Mars and Jupiter. In 1681, Jacob Bernoulli (1655–1705) was the first to assign numerical values to the orbital properties of the 'missing planet' in this gap. After a century of general speculation about the supposed planet (Beswick, 1851), Baron Franz von Zach followed with his own calculation of orbital properties in 1785. He became one of the first persons to see that long-sought planet, Ceres, in December 1801. The American astronomer Ormsby Mitchel (1810–1862) celebrated the discovery of Ceres in extravagant Victorian prose:

> The vast interplanetary space between Mars and Jupiter was the real locality of a discovered world, whose existence had been conjectured by Kepler 200 years before, and whose discovery, by combined systematic and scientific examination, constituted the crowning glory of the age. (Mitchel, 1860: 98).

For the astronomical and philosophical details of the grand venture begun by Kepler in 1595, see Cunningham (2017a).

4 THE POETIC LEGACY

It was Plato whose name was given to the solids Kepler employed, but Plato filled another role not as clearly appreciated in Kepler's work. In the following quote, Roberto Calasso refers to the eighth century BCE Greek poet Homer, and the Chaldeans. In the second century CE *Chaldean Oracles*, their god "… can be considered to be Nous, whose function is to 'think' the world of Platonic forms into being." (Dietz, 2014). One great fault of Homer, writes Calasso (1993: 274),

> … for which Plato never forgave the poet, was that he omitted any serious comment on the structure of the cosmos ... But with the Orphics, followers of the Book, and later with Plato, Chaldean wisdom took its revenge on Homer. The roving islands of celestial bodies, the frayed progress of the Milky Way, the soft sounds of the spheres all regained their privileges.

The "… soft sounds of the spheres found a new maestro in the person of Europe's greatest astronomer …" (Cunningham, 2017b: 336), and found poetic expression in the person of the English amateur astronomer Capel Lofft (1751–1824). After lines that enumerate the orbital periods of the planets, he wrote:

> Nor Poetry these numbers will disdain,
> Since Harmony, her sister, these approves;
> In perfect scale most musically true:
> So sweet a concert regulates the spheres.
> Justly, O KEPLER! Are the Ides of May
> Rever'd, which taught to thee this wondrous truth. (Lofft, 1781: 38).

While a full exploration of musical harmony and the cosmos is beyond the remit of this study (see Haase, 1975), it must be noted that in Chapter 12 of *Mysterium*, Kepler

> … tested some arguments that established relations among planets and the lengths of strings that determine harmonious combinations. (Cardona, 2016).

Kepler's 1619 book *Harmonice Mundi* (*Harmony of the Spheres*) opens with a presentation of the Platonic Solids he had employed in the *Mysterium* book, but goes further "… in alluding to the relationship between the harmonic proportions and the five regular solids." (Haase, 1989: 117). For a study of Kepler's use of geometry and music in his astrological writings, see Linde and Greenbaum (2010).

Kepler's methodology "… was heavily influenced by the spirit of Pythagoras." (Cardona, 2016). Kepler was imbued by this tradition through the work of the fifth century Greek philosopher Proclus (412–485): three books of *The Harmony of the Spheres* begin with epigraphs from Proclus, one of which reads:

> Thus Plato teaches us many wonderful doctrines about the gods by means of mathematical forms, and the philosophy of Pythagoreans clothes its secret theological teaching in such draperies. (Proclus, 1970: 19).

This directly links forms, such as the Platonic solids, with divinity—Kepler's animating principle. By invoking Proclus, Kepler himself relates how his ability to know the plan of the cosmos came to be:

> For to know is to compare that which is externally perceived with inner ideas and to judge that it agrees with them, a process which Proclus expressed very beautifully by the word 'awakening' as from sleep. (Kepler, 1619).

In Book V of *Harmony*, Kepler admits his polyhedral hypothesis (published 23 years earlier) does not perfectly account for planetary distances. Abandoning his effort to fill the gaps between the nested solids, he explains they are necessary for the harmonies of the cosmos to emerge: "The forces that cause the ellipse also bring about the harmonies." (Regier, 2016: 234). By explaining all this in anthropomorphic terms (the original bulk of the world, determined by polyhedral forms, was fine-tuned by harmonies so that the world's body could take on the "… organs necessary to life …", Kepler (1619: 490) opened the door to connect cosmic harmony with humans.

Three brief examples will suffice to show the intertwined nature of humanity and musical harmony from the sixteenth to eighteenth centuries. In a sermon of 1538, Luther

> … comments that Pythagoras claims that the movement of the stars begets a sweet harmony, but people are unable to perceive it because they are accustomed to it. (Anttila, 2013: 86).

An early version of Shakespeare's play *King Lear*

> … has Cordelia bring Lear back to sanity partly through the force of music which, operating alongside medicines, retunes him to the order of the cosmos. (Davis, 2011).

According to Newton's nephew, John Conduitt (1688–1737),

> Sir Isaac used to say he believed Pythaoras had some notion of gravity, and meant by that what is vulgarly called the Musick of the Spheres. (Conduitt, 1732).

In reality, what Pythagoras believed or did not believe is highly speculative, but for the purposes of this study the reality is irrelevant—the *perception* of what was believed about Pythagoras, the harmony of the spheres, and its relationship to actual planetary distances and musical tones, is expressed in the Keplerian legacy. Nowhere was this legacy more profoundly felt than in the poetry of William Wordsworth and Samuel Taylor Coleridge.

In a lecture delivered in 1819, Coleridge singled out Kepler as marking the beginning of

> … truly scientific astronomy … [because] by laws demonstrably drawn out of his own mind he has, in that mind, not only, but as far as his own purposes require it, controlled the mighty orbs of nature. (quoted in Owens, 2019: 168).

Coleridge also

> … found it impossible not to admire the celestial harmony he found in Kepler which relied on a congruence between geometrical and physical phenomena. (Owens, 2019: 19).

Coleridge took the concept of forms—so central to the work of Kepler—and overlaid it on William Herschel's discoveries. "It was Herschelian 'forms & schemes of motion' of the stars and planets which excited Coleridge." (Owens, 2019: 168). To enable this rather dubious association, he appropriated the instrument Herschel used—the telescope—as the means by which to link reason and faith:

> Now that the telescope is to the eye, faith, that is the energies of our moral feelings, are to the reason. Reason is the eye, and faith the telescope. (Coleridge, 2020: 377).

The poet went much further than this however, adopting the very method of Kepler to design a unifying system that—in Coleridge's words—was "… based upon the reconciliation of faith, and consistent with human nature and experience." This was manifested by Coleridge in an utterly bizarre fashion, even employing the analogical approach of Kepler. As Owens (2019: 168) writes,

> It took a mind like Coleridge's to galvanize physics with metaphysics and to see in Herschel 'actual analogies' for a blazing variety of Trinitarian and broader epistemological proofs.

This Trinitarian approach is the very one that was adopted by Kepler! He wrote: "Before the universe was created there were no numbers except the Trinity, which is God himself." This was grounded in Martin Luther's belief "… in God's ubiquity, namely, the presence of the triune God in all of creation." (Raunio, 2009: 220).

In *Harmonices Mundi*, Kepler used the circle as an aid for measuring and constructing polygons, such as triangles, and he used that to

> … signify the human capacity for comprehension … Kepler compares the relation between human intelligence and God's knowledge to the relationship which exists between a circle and sphere: thus the human mind forms a two-dimensional copy cut from the three-dimensional Trinity. (Powrie, 2006: 215).

Here we see the intersection of Kepler's theology and science that so influenced Coleridge. The use of triangles by Kepler, which was explored earlier in this paper, was imbued with a special meaning. "The Trinity thus made triplicity or triangularity central to the shape of Coleridge's philosophy." (Owens, 2019: 16).

Kepler also

> … explained the stationary aspects of the heavens – Sun, fixed stars, and space – archetypally by drawing an analogy with the Holy Trinity. (Martens, 2009: 40).

Kepler expressed it thus in *Mysterium* (see page 15):

> The image of the Triune God is a spherical surface: the Father is in the centre, the Son in the outer surface, and the Holy Ghost in the equality of the relation between centre and circumference.

Just as Kepler tried to reconcile reason and religion, Coleridge was driven to do the same, but he did so by regarding the telescope as 'the Organ of Theology' via faith in Trinitarian Christianity.

Like Kepler, Coleridge used analogies, writing that

> … they present a far more perfect, both a fuller and a more precise & accurate language than that of abstract or general words. (quoted in Owens 2019: 164).

By using the telescope as an analogy, he found 'actual analogies' to bridge the gap between Man and God (Jackson, 2016). He was then able to invoke the very 'eye of reason' used by Kepler who was keenly concerned, as we have seen, with the distance (aweful depth) of the stars:

> Religion passes out of the ken of Reason only where the eye of Reason has reached its own Horizon; and that Faith is then but its continuation … the upraised Eye views only the starry Heaven which manifests itself alone: and the outward Beholding is fixed on the sparks twinkling in the aweful depth, though Suns of other Worlds. (Coleridge, 1817: 309).

Wordsworth was equally in the thrall of the Keplerian legacy, but neither poet was entranced by the use of mathematics by Isaac Newton (1643–1727):

> … geometric shapes gave the poets a unique capacity to perceive, interact, and respond to the world around them. They never relinquished this way of seeing and it was instrumental to Wordsworth's pronouncement in the Preface to Lyrical Ballads (1800) to gauge how 'the passions of men are incorporated with the beautiful and permanent forms of nature'. (Owens, 2019: 19).

In Wordsworth's Manuscript B of his poem *The Ruined Cottage* (1798), he introduces a character named The Pedlar. He must have been thinking of Kepler when he wrote such lines as "In all shapes | He found a secret and mysterious soul, | A fragrance and a spirit of strange meaning." (B.83–85). The Pedlar had "… an eye which evermore | looked deep into the shades of difference | As they lie hid in all exterior forms" (B.94–96), and which "Could find no surface where its power might sleep, | Which spake perpetual logic to his soul." (B.100–101). The Pedlar's ability to understand the world derived from a set of geometrical principles which "… lived to him | And to the God who looked into his mind." (B.88–89). To make matters even more clear, Book Six of *The Prelude* states that geometric science "… is | And hath the name of God." (Owens, 2019: 20). In these lines the investigations of Kepler are writ large, a literal description of what he accomplished in *Mysterium*, with even the word 'mysterious' inserted as a calculated effect to direct the reader's attuned mind to Kepler's book. The alliterative assonance of 'forms,' 'surfaces,' and 'perpetual logic' (as a synonym for reason) are striking, as is the mention of 'shades of difference,' which evokes a form of measurement. The use of the word 'soul' is also strategic:

> Kepler postulates for creation … and for the Sun and the planets in particular, not only an external dimension, but also a soul with a mind. (Gerdes, 1975: 345).

That Kepler believed God had literally looked into his mind can hardly be doubted; in the 1621 Second Edition of *Mysterium* he wrote: "It is as if the heavens had dictated to me an oracle." (Beer, 1975: 402). The identification of God as a geometer can be traced back to Plato (Burnyeat, 2000), originator of the forms that captivated him, and

> … the idea of the geometer God assumed special importance with the Lutheran emphasis on the providential plan. A geometer God would have a geometrical plan for his providentially ordered universe. (Barker, 2000: 82).

Wordsworth achieved through his Pedlar character a form of apotheosis of Kepler, one that has not hitherto been fully realized, and only becomes apparent when one performs the conformal mapping of *Mysterium* onto his poetry quoted here. Wordsworth's encounter with the void is described in *The Prelude* of 1799:

> And after I had seen
> That spectacle, for many days my brain
> Worked with a dim and undetermined sense
> Of unknown modes of being. In my thoughts
> There was a darkness—call it solitude,
> Or blank desertion—no familiar shapes
> Of hourly objects, images of trees,
> Of sea or sky, no colours of green fields,
> But huge and mighty forms that do not live
> Like living men moved slowly through my mind
> By day, and were the trouble of my dreams.
> (Wordsworth, 1991: 59).

Was the spectacle the very cosmos itself, the cosmos Kepler perceived as being composed of 'mighty forms' in the guise of the Platonic solids? As Gibson (2006: 19) writes,

> ... the 'darkness' of which the poet tells us, the experience of 'solitude' or 'desertion,' of the falling away of the familiar: all these suggest an experience of a void, the *tabula rasa*, an event which is not to be interpreted, understood or reasoned away.

As an evocation of the boundary between the physical and the unknowable Kepler encountered through his application of reason, these lines of Wordsworth are striking. "For Wordsworth, poetry and science differed only in degree and not in kind …" (Owens, 2019: 61), a way of thinking that gave us some of the finest poetry in the English language.

5 CONCLUDING REMARKS

Even though Kepler's use of Platonic solids, his belief that there are exactly six planets, and his insistence that the angular speeds of the planets must agree with musical intervals have all been relegated to the dustbin of history (Abramowicz, 2011: 287), his key belief in harmony is central to our modern understanding of Solar System dynamics. This was noted recently by Peter Lynch (2018), Emeritus Professor at University College Dublin, School of Mathematics & Statistics:

> Harmony was at the core of Kepler's cosmic model. This idea was not warmly supported by his contemporaries and never gained widespread support. Yet, harmonic relations are known today to be crucial, through the mechanism of dynamic resonance, which is of central importance in our current view of the solar system.

The orbital/harmonic gap he identified between Mars and Jupiter exercised the imagination of many astronomers and led to the search for a 'missing planet' there; the 1801 discovery of Ceres in the gap was a stunning vindication, followed in subsequent centuries by observations of a huge population of asteroids in what is now termed the 'main belt.' While written in another context, no words could better encapsulate the intellectual life of Kepler than this line in the poem *Mnemosyne* by Hölderlin: "Prophetically, dreaming on the hills of heaven." (Mitchell, 2007: 95).

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