



Perspective

Neutron stars as extreme laboratories for gravity tests

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Neutrons were discovered 90 years ago by James Chadwick. The concept of neutron stars was hypothesized around that time by Lev Landau, Walter Baade, and Fritz Zwicky, and it was further developed by Richard Tolman, Robert Oppenheimer, George Volkoff, and other physicists. Neutron stars are astrophysical compact objects formed after the death of massive stars (more massive than our own Sun). According to current understanding, the typical mass of neutron stars is comparable to that of the Sun, while the radius is only ~ 10 km. As its name implies, the main interior ingredients of such stars are neutrons, but they also consist of protons, electrons, muons, and presumably even more exotic particles, like hyperons, kaons, or quarks. The field of neutron stars was bolstered by the discovery of radio pulsars by Jocelyn Bell Burnell, Antony Hewish, and collaborators in 1967 [1]. A famous achievement brought forth by the first binary pulsar, the so-called Hulse–Taylor pulsar, is the first-ever empirical proof that gravitational waves exist in our Universe [2]. In 2017, the first binary neutron star merger was observed directly via gravitational waves (ripples of spacetime) and accompanied by enormous electromagnetic follow-up observations [3]. This event, known as GW170817, marked the dawn of multi-messenger astronomy and was chosen as “Science’s 2017 Breakthrough of the Year”.

Studies of neutron stars provide us with unique extreme laboratories for fundamental physical laws, including tests of gravitational theories in the strong field, superdense nuclear matter at low temperatures, and energetic astrophysical phenomena underpinning the evolution of stars. Interests in the research field of neutron stars range from gravity, nuclear and particle physics, plasma physics, and even condensed matter physics. From an observational point of view, pulsar-timing and gravitational-wave observations provide two widely used tools to achieve these rich scientific goals.

Pulsar timing provides the most accurate measurements in modern astronomy. It is usually done by large-area radio telescopes or arrays and sometimes done by high-energy satellites. These instruments record the times of arrival of pulses originating from rotating neutron stars that emit beams towards Earth. These times of arrival are calibrated with atomic clocks at telescope sites,

and they form the central data for a clean pulsar-timing experiment. Such an experiment can last for decades. For a neutron star in a binary system that is observed as a pulsar on Earth, the times of arrival, after subtracting the contribution from the interstellar medium and the telescopes’ motion, foremost reflect the movement of the pulsar. As per the gravitational dynamics of the binary, information about underlying gravity is encoded with the pulsar’s movements and hence with the times of arrival. By reverse engineering, we infer the underlying gravity theory and check if it is consistent with predictions from General Relativity, the *tour-de-force* of Albert Einstein’s masterpiece describing gravitation and spacetime.

Why do we need to test General Relativity? From a theoretical standpoint, gravity cannot be correctly quantized under General Relativity to describe microcosmic-scale gravitational physics and to formulate a theory to unify all four known forces in physics. From an observational standpoint, we only know about 5% of the energy content of matter in our Universe, and the remaining 95% consists of dark matter and dark energy. The former is responsible for the missing mass observed within galaxies and beyond, while the latter, having negative pressure, is the origin of the currently accelerating expansion of the Universe. Interestingly, one could explain these cosmological unsolved problems by going beyond General Relativity without introducing unknown dark matter or dark energy.

Binary pulsars are one of the best testbeds for gravitational experiments [4,5]. For the Hulse–Taylor pulsar PSR B1913+16, astronomers have measured the shrinkage of the orbit due to the emission of gravitational waves which takes away reservoir energy from the orbit [2]. This was the first indirect sign that gravitational waves exist and, at that time, greatly reinforced the scientific motivation to build gravitational wave detectors, which have proven to be extremely successful at the present time. However, nowadays the Hulse–Taylor pulsar is no longer competitive in measuring the orbital shrinkage as the uncertain contamination from its position and motion in the Galactic potential has dominated the error budget. Fortunately, we have several dozens of pulsar systems to continue performing tests of gravity, with the double pulsar PSR J0737–3039 being a prime example [6].

The double pulsar is a unique system that has two neutron stars, both of which were observed as active pulsars (see Fig. 1). It has a relativistic orbit with an orbital period

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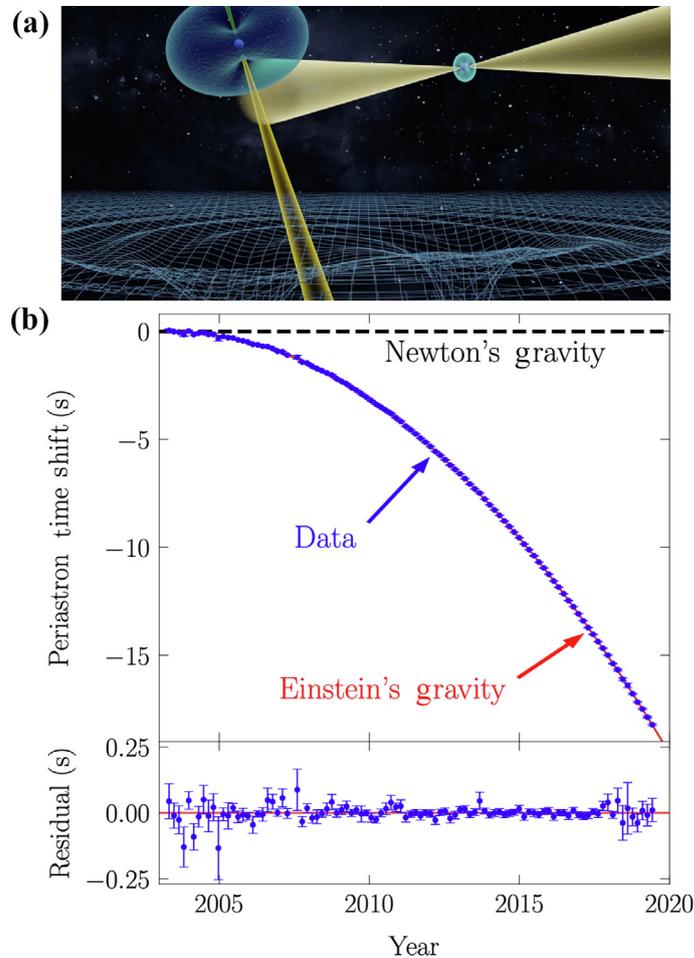


Fig. 1. (Color online) (a) Artistic illustration of the double pulsar (figure courtesy of M. Kramer). (b) The top panel presents the confrontation of the double pulsar data with the binary periastron time shift due to gravitational-wave radiation predicted in Newton's gravity and Einstein's gravity [6]. The bottom panel shows the deviation from Einstein's gravity, which is consistent with random noises, with a normalized $\chi^2 \simeq 0.76$.

$P_b = 0.1022515592973 \pm 0.000000000010$ day. Combined with its exquisite measurement accuracy, a number of relativistic gravity tests become available within this system, including higher-order effects in the orbital motion, light propagation in a strong gravitational field, and so on. The double pulsar also revealed the first-ever information about the moment of inertia of a single neutron star via its effects on the orbit. The moment of inertia is a quantity characterizing how easily one can spin an object (figure skaters change their moment of inertia by stretching and shrinking their arms to change the rate of their spin on ice) and is related to certain universal relations that we will explain later. The most valuable fundamental test comes from the measurement of the double pulsar's orbital decay, whose precision exceeds all existing results by orders of magnitude. As shown in Fig. 1, while Newton's gravity theory obviously fails the test, Einstein's General Relativity gives a precise prediction of the data and passes the test with flying colors [6]. It is worth mentioning that such a test of gravitational radiation in General Relativity has a much broader range of implications in fundamental theories. A few modified gravity theories are constructed to account for dark-matter and/or dark-energy phenomena by introducing extra degrees of freedom, which are very likely to result in new channels of gravitational radiation. Thus, the radiative test from the orbital decay of the double pulsar has strong implications for the above cosmological problems.

Gravity tests from radio pulsars are versatile [5]. The internal strong gravity of neutron stars additionally promotes these tests

to the strong gravity field, meaning that neutron stars also probe gravitational phenomena which are unique to spacetime regions where gravity is extreme. For example, one of the most well-studied alternative theories of gravity is scalar-tensor theory, where gravity is coupled to a scalar field. Just like how electric charges source electric fields, there are sources called scalar charges that generate scalar fields in these theories beyond General Relativity. A phase-transition-like phenomenon called "scalarization" may arise where a large number of scalar charges are spontaneously produced for suitable neutron stars within a certain mass range [4]. Neutron stars appear to be a natural laboratory for testing these strong-field predictions [7].

Pulsar timing is quite complementary to the novel strong-field tests of gravity with gravitational waves and black hole shadows that have become available in the last several years. First, compared with short gravitational-wave bursts from stellar compact binary coalescences, pulsar timing is a long-term experiment. Therefore, while the former probes highly dynamical spacetimes and has a greater sensitivity to higher-order effects beyond Newtonian gravity, the latter probes quasi-stationary spacetimes and has a higher sensitivity to effects that accumulate over time. Both aspects are equally important to scrutinize a gravitational theory. Second, gravitational-wave observations and black hole shadows are superb means to study black hole spacetimes, while pulsars are especially powerful to investigate a few gravitational theories where the coupling between matter and spacetime is important.

The aforementioned neutron star scalarization is a vivid example along this line.² Third, pulsar timing and black hole shadows not only deal with the dynamics under gravity, but also study the propagation of light in curved spacetimes. In contrast, gravitational waves handle nonlinear back reactions of, in quantum parlance, gravitons in spacetime. Overall, concerning the complementarities of pulsar timing, gravitational waves, and black hole shadows, combinations of these three kinds of strong-gravity experiments were shown to be powerful and would allow us to understand gravitation and spacetime more thoroughly [7,8].

Due to their strong gravitational field and extreme density, neutron stars are ideal astrophysical laboratories to probe not only gravitational physics but also nuclear physics. For example, the relation between the neutron star mass and radius depends very sensitively on the underlying equations of state of nuclear matter, which are relations between pressure and density (see the inset in Fig. 2). Heavy pulsars with masses $\sim 2M_{\odot}$ have ruled out some of the “soft” equations of state whose maximum mass of a neutron star is smaller than $2M_{\odot}$. Recent X-ray observations with the NICER satellite have measured the mass and radius of neutron stars independently, which allowed us to constrain the equations of state. The LIGO/Virgo Collaboration has measured the tidal deformability of neutron stars in a merging binary, which has further constrained the equations of state. The aforementioned measurements are not precise enough at present, and we still have sizable uncertainties in the equations of state.

This means we face a challenge that if one wants to test strong-field gravity, there will be systematic errors due to uncertainties in nuclear physics. For example, the mass-radius relation of neutron stars depends not only on the nuclear equations of state but also on gravitational physics. Therefore, if we can measure the mass and radius of a neutron star simultaneously, we can probe gravitational theory, provided that we know the underlying equations of state. If, on the other hand, we do not have a complete understanding of the equations of state, there will be systematic uncertainties in the probes of gravitational physics.

One way to break such a degeneracy between uncertainties in gravitational and nuclear physics is to use universal relations among neutron star observables that do not depend sensitively on the equations of state. One example of such relations is the “ I -Love- Q ” relation between the moment of inertia (typically denoted as I), tidal Love number (or tidal deformability, which tells us how easily an object can tidally deform), and quadrupole moment (characterizing a quadrupolar deformation of an object in its shape, typically represented by Q) [10] (see Fig. 2). Although these relations are almost independent of the choice of the equations of state (with an equation-of-state variation of $\sim 1\%$ or even smaller), they do depend on the underlying gravitational physics. Therefore, if we can independently measure any two of the I -Love- Q trio, we can test gravity theories without the contamination from uncertainties in nuclear physics. In this regard, the I -Love relation may be the most interesting because the Love number has been measured by gravitational waves from colliding neutron stars [3] while radio observations of the double pulsar binary have started to constrain the moment of inertia of the primary pulsar as mentioned earlier [6]. Yagi and Yunes [10] applied this idea to test a specific theory of gravity that breaks parity (mirror symmetry) and showed that multi-messenger observations of neutron stars can be probed with an accuracy that is several orders of magnitude higher than the current bound from Solar System experiments. This was recently confirmed by Silva et al. [11] who used the direct measurement of the Love number with gravitational waves from

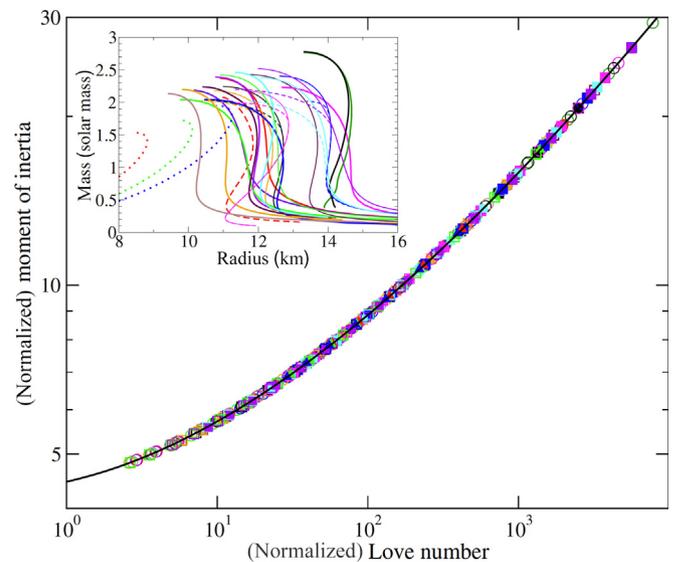


Fig. 2. (Color online) Universal relation between the moment of inertia and Love number for neutron stars (I -Love relation). Each quantity has been normalized to be dimensionless using the mass, speed of light c , and gravitational constant G . Each marker corresponds to one equation of state and the solid curve is the fit. Inset: the relation between the mass and radius that strongly depends on the equations of state (shown by different colors). This figure is taken and modified from Ref. [9].

LIGO/Virgo detectors and inferred measurement of the moment of inertia from X-ray observations with the NICER satellite. The equation-of-state variation in the I -Love- Q relations is small enough to perform the above tests of gravity given that typical measurement errors of I -Love- Q are of $\mathcal{O}(10\%)$.

This avenue of using universal relations to test strong-field gravity seems promising, though there are still some issues (or questions) that need to be investigated further. First, universal relations are typically studied for a single neutron star. This means that they can be applied to observations of two neutron stars with similar masses, but may fail for neutron stars with different masses. Therefore, one needs to study whether universal relations hold for observables of two neutron stars with different masses. A first attempt was made by Saffer and Yagi [12], but a more thorough analysis is necessary.

Second, there are many other universal relations known within General Relativity, so one needs to study their applications on strong-field tests of gravity. One example is the relation between the Love number and an oscillation frequency of a neutron star (called the fundamental mode oscillation, also known as the f -mode oscillation) [13]. The Love number has already been measured through static tidal effects, while the f -mode oscillation is expected to be detected with future gravitational-wave observations through dynamical tides. The advantage of using this relation is that both of the quantities can be observed with gravitational waves from a colliding binary neutron star. Therefore, we do not suffer from the issue mentioned in the previous paragraph of using universal relations for neutron stars with different masses. On the other hand, when the masses of neutron stars are similar, say $\sim 1.4M_{\odot}$, then we can Taylor-expand each observable (that is a function of the mass) relevant for the universal relations (like the Love number or f -mode oscillation frequency) about the fiducial mass, like $1.4M_{\odot}$, and we can measure the leading quantity in the expansion that corresponds to each quantity evaluated at $1.4M_{\odot}$. Since this quantity is common to all neutron stars with masses similar to the fiducial mass, we can combine different observations to improve the measurement accuracy of such observable (by a factor of \sqrt{N} where N is the number of events).

² Gravitational waves from binary neutron star mergers are also useful to probe the dynamical aspect of neutron star scalarization.

Third, most previous studies on tests of gravity with universal relations focused on probing specific gravitational theories. However, given that there are numerous modified theories of gravity being proposed, testing each theory one after another would be extremely time-consuming. A more efficient approach would be to first perform a model-independent test without specifying a theory and then map the information to a specific theory. One way of performing such a model-independent test is to construct parameterized universal relations that are characterized by parameters representing non-General-Relativity effects. If a known mapping exists between such parameters and theoretical constants in each theory beyond General Relativity, one can easily probe/constrain the theory. This approach was proposed and studied by Silva et al. [11] for the I -Love relation, taking the parity-breaking theory mentioned earlier as a base theory for constructing the parameterized relation. This is a very important attempt, and a more systematic analysis needs to be carried out for other universal relations and with other theories to find the mapping.

Since the inception of the concept of neutron stars about 90 years ago, they have been playing important roles in various aspects of fundamental physics, particularly in testing gravitational theories and probing dense nuclear matter. Realizing that neutron stars are very tiny celestial objects (with a size smaller than a typical city) and they are very distant from Earth (with pulsars being thousands of light-years away and binary neutron star mergers being more than millions of light-years away), it is truly remarkable that tons of precious information has been obtained from them. The future of this field is extremely bright. For pulsar observations, the next-generation radio telescope, the Square Kilometre Array [14], will probe the Southern sky with unprecedented sensitivity. A census of radio pulsars in the Milky Way is to be made, probably resulting in discoveries like pulsar-black hole binaries. For gravitational waves, the third generation of detectors are under construction [15], and they will detect almost all binary neutron star mergers in the Universe. With useful tools like the universal relations at hand, astrophysicists will disentangle uncertainties in tests of gravity and nuclear matter and eventually advance knowledge greatly in both subjects.

Conflict of interest

The authors declare that they have no conflict of interest.

Acknowledgments

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