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Star Point Energy Center Correction of Star Simulator Based on Polar Coordinates

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Abstract In this paper, various aberrations have been analyzed. Not only the effects of aberration on geometrical center position are taken into account, but also the deviation of displayed star position energy center caused by aberration is analyzed. These two aspects have been taken into comprehensive evaluation and star position correction. The correction method based on polar coordinates is proposed, and cumbersome partition correction and calculated quantity based on two-dimensional coordinates can be simplified. The experimental results show that the correction processing based on polar coordinates is simpler and easier compared with any other correction methods. In addition, the correction results are significantly more accurate.

Key words Aberration, Energy center, Correction, Polar coordinates

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0 Introduction

The calibration of star sensor is the main function of star simulator. However, with the development of star sensor and star simulator technology, the high requirements of specifications and accuracy have become the key of designing and manufacturing of star simulator.

For previous star simulator, except for the requirement of hardware with high accuracy, the improvement of correction and operating rate based on software is indispensable. But the correction of optical system aberration still relies on theodolite testing with human eyes, by recording the experimental data and correcting the data to realize the correction, or just correcting the displayed geometrical center of star point caused by distortion. Only the influence of distortion on geometrical center can be corrected and the energy center deviation caused by other aberrations is neglected, while the star sensor recognizes the star point by recognizing the energy center. Simultaneously, the former correction regards the image plane as a two-dimensional surface, and realizes the correction by establishing two-dimensional coordinates, and separating the regions of coordinates system according to different positions in correction. But the optical system of star simulator is coaxial system completely composed of spherical lens, and the

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aberration in the same field of view through the free directions of optical system is rotationally symmetric surrounding the optical system. The correction appears to be complicated under separated regions according to the two-dimensional system.

On the basis of the summaries of the former star point error correction, taking a star simulator with 22° field of view as example, a new correction method is proposed in this paper, and the method to ensure the position of energy center by analyzing the point distribution function of optical system and correcting the error based on polar coordinates is also proposed.

1 Working Principle of Star Simulator and Aberration Analysis

1.1 Working Principle of Star Simulator

As the calibration device of star sensor, the star simulator provides infinity star simulation for star sensor, and realizes the star point display by the displaying device. While the emergent rays of star point passing through the collimation optical system, they will be parallel and image at the entrance pupil of star sensor, which finally fulfill the simulation of infinity star point, as shown in Figure 1.

1.2 Aberration Analysis

The aberration of optical system is divided into monochromatic aberration and chromatic aberration. The monochromatic aberration includes spherical aberration, coma, astigmatism, field curvature and distortion, while the chromatic aberration includes

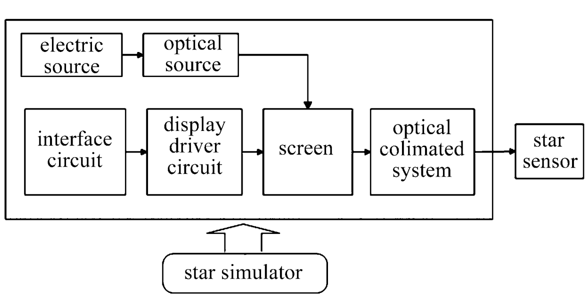

Fig. 1 Interrelation of star simulator compositions

axial chromatic aberration and lateral chromatic aberration. Due to the existence of various aberrations in collimation optical system, the actual energy center and theoretical energy center are different, which will result in the loss of simulation accuracy of star simulator.

(1) Spherical aberration. While the emergent light beam from on-axis object point pass through the optical system, the rays with different angels which is relative to optical axis intersect in different positions on axis, and the image plane presents a circular dispersion spot.

(2) Coma. The off-axis object point locates off the main axis, emits monochromatic cone-beam to optical system. The existed light beam is refracted by optical system and imaged unclearly on ideal image point, and the image point brings bright tail and forms a comet dispersion spot.

(3) Astigmatism and field curvature. As the luminescence object point is off the axis of optical system, the emitted beam has inclined angle with optical axis. The light beam is refracted by lens and the convergence of meridian beamlet and sagittal beamlet is not on the same point. That is the beam convergent in different point and the image is unclear, finally forms elliptical dispersion spot[1].

(4) Distortion. While the astigmatism and field curvature are corrected to be zero, the chief ray is not duplication with ideal image point, and this only causes the position error of star point and there will be no disperse spot.

(5) Axial chromatic aberration. Similar with spherical aberration, while various light passing through the lens, the beam cannot converge to be image point on image plane and form colorful circular dispersion spot.

(6) Lateral chromatic aberration. As the refraction of lens is different with different colorful light, this causes different colorful lights with different image size and forms colorful dispersion spot.

In conclusion, the aberrations of optical system almost all have influences on star point energy center. In order to induce the deviation of energy center caused by aberration, an optical system with 22° field of view and 56.0006 mm focal length is designed.

2 Correction Processing of Energy Center

In the star simulator optical system, the position and orientation of the actual incident ray is uncertain. In order to simulate the actual path of light propagation, non-sequential ray tracing method is used. Monte Carol algorithm is used to generate the stochastic rays emitted from the star point, and these rays are used to simulate the light path for ray tracing. Finally, the simulation produces the image point distribution function, and the computer calculation is used to analyze the data of this function, and to determine the offset relationship between the actual energy center position and the theoretical energy center position.

Monte Carlo ray tracing method selects random rays based on the probability function and the sampling algorithm, and the normalized probability function is

$$\int_{-\infty}^{+\infty} P(x)\mathrm{d}x = 1, \tag{1}$$

where x represents the random variable, and for any x value, $P(x) \geqslant 0$. The corresponding point

$$D(x) = \int_{-\infty}^{x} P(x')\mathrm{d}x'. \tag{2}$$

According to a given probability function $P(x)$, the algorithm to obtain a series of random number consists of two steps:

(1) To select a uniformly distributed random number p during the interval of $[0, 1]$;

(2) Using $p = D(x_p)$ in the cumulative distribution function curves to calculate the corresponding value x_p.

Repeating these two steps and you can get x_p according to $P(x)$ distribution[2−3].

According to ZEMAX correlation function, the differences between actual energy center and the theoretical energy center in each field can be figured out, which is shown in Table 1.

Through the position deviations between the theoretical and actual energy center calculated from the point distribution function, the curve and surfaces can be fitted along the polar direction.

With the least square method, the correspondence function between the actual energy center and the theoretical energy center is fitted, and theoretical energy center correction is made. So the amended actual energy center is displayed on the screen, which is consistent with the theoretical energy center.

According to the point distribution function analyzed from ZEMAX, the energy center's offset of any field of view can be obtained. These offsets can be fitted with a correction curve $\omega = \varphi(\alpha)$, which shows the actual theoretical energy center after correction. Using the principle that make the sum of squares minimum as

$$\sum_{i=1}^{m} \delta_i^2 = \sum_{i=1}^{m} [\varphi(\alpha_i) - \omega_i]^2 = \min,$$

also called least squares method, the fitting curve can be figured out as:

$$\begin{aligned}\omega = &-8.7318 \times 10^{-7}\alpha^5 + 1.9528 \times 10^{-5}\alpha^4 - \\ &2.4018 \times 10^{-4}\alpha^3 + 3.6368 \times 10^{-4}\alpha^2 + \\ &2.2193 \times 10^{-2}\alpha + 2.6272 \times 10^{-5}.\end{aligned} \tag{3}$$

Table 1 Energy center difference

theoretical field $\alpha_{\text{theory}}/(^\circ)$	actual field $\alpha_{\text{real}}/(^\circ)$
0	0
1	0.977 566 31
2	1.955 790 70
3	2.917 287 21
4	3.916 665 30
5	4.900 417 56
6	5.787 012 54
7	6.876 976 66
8	7.880 125 21
9	8.869 340 87
10	9.873 799 61
11	10.874 316 46

After correction, according to the direction of corrected point azimuth angle θ, the point coordinates is restored from the polar coordinates into the form of two-dimensional coordinate system like (x, y), the star point at the corresponding coordinate position is lit on the screen. The conversion formula is:

$$\begin{cases} x = f\tan w\cos\theta, \\ y = f\tan w\sin\theta. \end{cases} \tag{4}$$

Here θ is the azimuth angle of the star point needed to be lit before the correction.

3 Error Analysis

According to the system requirements for precision, the single star position error $\leqslant 22''$ shows that for the field of view ω, its permissible error $\delta\omega$ must satisfy that the error is less than a single star position error, *i.e.*, $\delta\omega \leqslant 22''$. According to the fitting equation, the inferred maximum truncation error is $0.254\,13''$, which is much smaller than the requirements that the single star position error should be $\leqslant 22''$.

For static star simulator, bias based on the star point energy center displayed on the star check board is the correction error[4], which can meet the accuracy requirements.

However, due to the dynamic star simulator display device restrictions, the star point movement on LCOS per unit is 0.008 mm. The maximum error of the impact of display device is

$$\nabla\omega = \arctan\frac{0.008/2}{56.0006} \approx 14.733''.$$

Sub-pixel display technology is utilized in dynamic star simulator, and interpolation algorithm is used to refine a pixel into smaller pixel unit, such as 1/2 pixel, 1/5 pixel, 1/10 pixel. Sub-pixel is a method to make a pixel divided into smaller units. Taking the 8 bit system as an example, the gray level of the pixel value is 255, then in such systems a pixel could be divided into small units of 255[5] using the interpolation algorithm cooperated with surrounding point's gray value, to make the star point position display precision reach decimal range.

Mass center is the star theoretical position on the map. To make sure of the star position, method of back stepping each pixel's gray value in the display area from the mass center can be used in order to increase the display accuracy up to tenths.

To ensure the accuracy of the simulator, the Gaussian distribution function is chosen to simulate the light spot grey value distribution:

$$p = A\exp\left(\frac{-r^2}{b}\right), \tag{5}$$

where $r^2 = (x_p - x_0)^2 + (y_p - y_0)^2$; x_0, y_0 are the theoretical center coordinates; x_p, y_p are the pixel coordinates on the display device; A, B are the Gaussian distribution parameters.

Both sides of the equation can be transformed by logarithmic computation:

$$\ln p = \ln A + \left(-\frac{1}{B}\right)r^2. \tag{6}$$

When $y = \ln p$, $x = r^2$, $a = -1/B$, $b = \ln A$, further linear function relation is shown as $y = ax + b$.

Using the least squares fitting, the fitting coefficient can be calculated out:

$$\begin{cases} a = \dfrac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 (\sum x_i)^2}, \\ b = \dfrac{n\sum x_i^2 \sum y_i - \sum x_i \sum y_i x_i}{n\sum x_i^2 (\sum x_i)^2}. \end{cases} \tag{7}$$

Here $y = \ln p$; x_i, y_i, $i = 1, 2, \cdots, n$ are date points. The specific value can be calculated according to the pixels value inside the star light point. Finally, the real pixels value can be obtained by back-stepping the fitting equations. Taking a 6×6 pixels array for example as shown in Figure 2, a star point centroid accuracy of 0.1 is shown in Figure 3, where p_{ij} shows the corresponding pixel value of LCOS display device.

Through experimental analysis, after subpixel display, the precision can be improved to a higher level, and the energy distribution effect is improved substantially and is more closed to real star image, as shown in Figure 4.

p_{11}	p_{12}	p_{13}	p_{14}	p_{15}	p_{16}
p_{21}	p_{22}	p_{23}	p_{24}	p_{25}	p_{26}
p_{31}	p_{32}	p_{33}	p_{34}	p_{35}	p_{36}
p_{41}	p_{42}	p_{43}	p_{44}	p_{45}	p_{46}
p_{51}	p_{52}	p_{53}	p_{54}	p_{55}	p_{56}
p_{61}	p_{62}	p_{63}	p_{64}	p_{65}	p_{66}

Fig.2 Pixel distribution

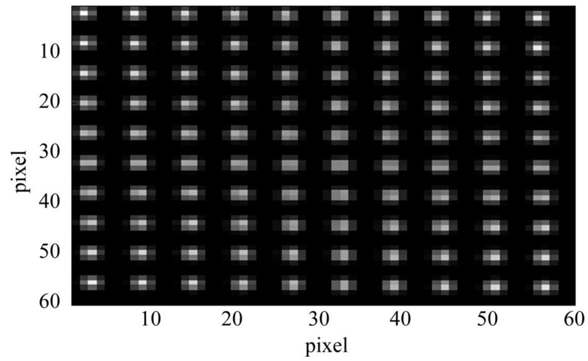

Fig. 3 Migration pseudo phenomenon

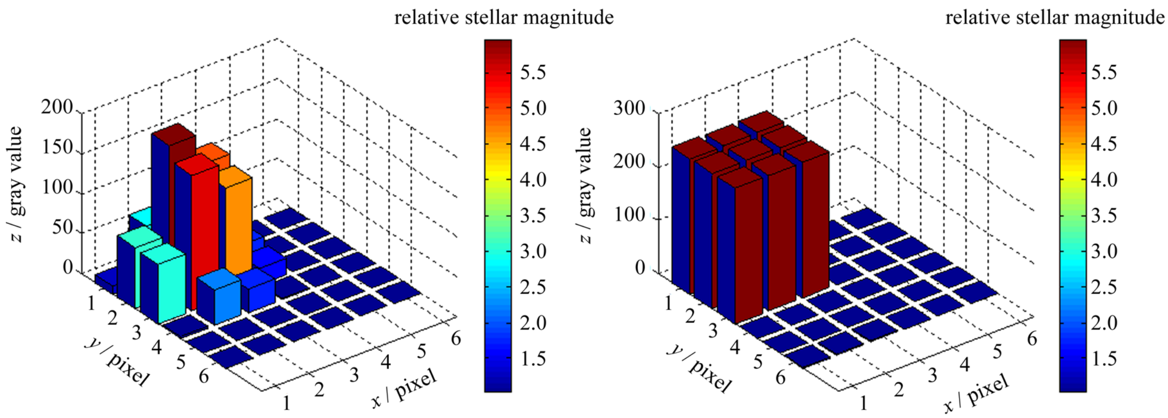

Fig. 4 Energy distribution after and before subpixel display technology

Table 2 Single star position error before and after correction

theoretical value/(°)	before		after	
	actual value/(°)	error/(″)	actual value/(°)	error/(″)
11	10.869 045 69	471.435 516	10.999 981	0.068 4
10	9.887 623 54	404.555 256	10.000 021	−0.075 6
9	8.879 524 75	433.710 9	8.999 914 6	0.307 44
8	7.880 568 12	429.954 768	8.000 012 1	−0.043 56
7	6.895 014 52	377.947 728	6.999 991 4	0.030 96
6	5.897 521 48	368.922 672	5.999 911 7	0.317 88
5	4.912 587 42	314.685 288	5.000 014 1	−0.050 76
4	3.908 874 51	328.051 764	4.000 011 2	−0.040 32
3	2.895 201 47	377.274 708	3.000 039 8	−0.143 28
2	1.915 784 12	303.177 168	2.000 015 4	−0.055 44
1	0.998 541 24	5.251 536	1.000 009 8	−0.035 28
0	0	0	0	0

The usage of sub-pixel star point display method will increase the energy center position display precision to 0.1 pixel, and the influence error is

$$\nabla\omega = \arctan\frac{0.0008/2}{56.0006} \approx 1.4733''.$$

After error accumulation, the error still can satisfy the system required precision.

4 In conclusion

The experimental result shows that the method by analyzing the point distribution function of the optical system to ensure the offset between actual energy center position and theoretical energy center position, and correcting in polar coordinates, simplify the cumbersome and computation based on the two-dimensional coordinate axes partition correction. The test results are shown in Table 2. This improves the accuracy of the star point simulation, and reduces the correction time significantly. It can also save huge human resources and costs.

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