

Theory and application of the transient injection well test

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Abstract Based on the theory of the pumping well test, the transient injection well test was suggested in this paper. The design method and the scope of application are discussed in detail. The mathematical models are developed for the short-time and long-time transient injection test respectively. A double logarithm type curve matching method was introduced for analyzing the field transient injection test data. A set of methods for the transient injection test design, experiment performance and data analysis were established. Some field tests were analyzed, and the results show that the test model and method are suitable for the transient injection test and can be used to deal with the real engineering problems.

Keywords: injection test, transient injection well test, seepage flow, permeability coefficient, analysis method.

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1 Introduction

One of the key problems in hydraulic power projects, mining drainage, water flooding of oil fields and geological hazard control projects is to determine the hydrologic characteristics of the water-bearing layer (such as seepage flow radius, permeability coefficient or transmissibility coefficient, storage coefficient or specific water yield and pressure conductivity coefficient) in geologic bodies. One of the effective methods is the pumping well test which has been widely applied in engineering projects to determine the characteristic parameters of the aquifer^[1]. For this method, a man-made water depth drop field (pumping cone) is formed to obtain the test data, and then the characteristic parameters of the test aquifer can be determined by analyzing the pumping flow rate and the spatio-temporal variation of water depth drop. The different pumping well tests vary according to the testing process, such as the artesian well and the gravity well test in terms of the type of ground water, the individual and multi-well test in terms of the number of test wells, the fully penetrating and partially penetrating well test in terms of the well completion manner, the layered and composite pumping well test in terms of the

number of tested layers in the aquifer, the constant and variable flow rate pumping well test in terms of the changing flow rate, and the steady and transient flow pumping well test in terms of the flow pattern of ground water.

The theory of the pumping well test based on well flow consists of the steady and transient flow theory. These two kinds of theories have made great progress in describing the water drop depth and enhancing the precision of the pumping well test in the past^[1-6]. As early as 1863, DuPuit proposed the first steady pumping test formula, namely the well-known DuPuit Formula, which established the theoretical basis of steady well flow. Subsequently, Thiem made an explanation of the investigation radius from the DuPuit Formula, and developed the field test method to determine the aquifer parameters through the individual pumping test. Forchheimer put forward the near river well pumping test formula by introducing the conformal mapping method, flow net structure and complex function theory. Glee presented the steady flow pumping test formula for the confined artesian aquifer. As a result, a complete DuPuit-Thiem-Forchheimer test theory for the pumping well test was taken into a new stage. After 1930's, almost no new progress was made on the theory of the steady pumping well test. On the contrary, the transient flow theory caught the scientists' eyes.

In 1935, Theis^[7] rendered the pumping well test formula for transient flow, the well-known Theis formula which established the theoretical basis of transient flow. Afterward, researches on the transient flow had rapid progress and numerous pumping theoretical formulae were developed. Among them, the achievements made by Boulton, Hanstush and Neuman are the representative work which founded a systematic transient flow pumping well test theory. Boulton^[8] developed the gravity dewatering and water supply theory, and published the first transient flow formula in 1954. Then, he rendered the water supply lagging theory in advance and published the second theory of transient flow formula of ground water. These two formulas are of importance for transient phreatic flow. Hanstush et al. presented some of their research results on cross flow^[9], constant drop depth^[10], partially penetrating well^[11], and weak permeable stratum^[12], which made very good progress in the pumping well test theory of transient flow. From the late 1960's to the middle 1970's, Neuman et al. presented the general theory of double confined aquifers and leakage aquifer flow^[13], the water lagging theory^[14], the field measuring method for a multiple layer aquifer system^[15], and the gravity lagging theory^[16] of a confined aquifer. So the theory and method of Theis-Boulton-Hanstush-Neuman on the transient flow of pumping well tests were formed.

Additionally, other scholars also made some important contributions to the transient pumping well test theory. Jacob^[17] suggested the pumping well test theory of an elastic confined aquifer. Witherspoon put forward the steady flow pumping formula by considering the effects of a partially penetrating well and the effective radius being too small to reach the layer above in a weak pervious bed. Papadobulos rendered the transient flow pumping formula by considering the pump well volume capacity (the water storage in

well bore). Lai and Chen-Wusu^[18] published the transient flow pumping formula of a penetrating well by considering the pump well capacity and the cross flow; Streltsova^[19] posed the transient flow pumping theory of a partially penetrating gravity well. And Boulton and Streltsova^[20] raised the transient flow pumping formula of a practically penetrating well by considering the pump well capacity. In practice, Chang Shibiao^[21], a Chinese scientist, found that the water heads are different between the bottom and the top of the water strainer in the pump well, that is, the water head at the bottom is higher than that at the top, and presented a method to acquire the water head drop variation and the computation formula of the integration approach. Nawroski^[22] conducted comparative results by using Theis' standard type curve method, Jacob's semi-log straight line method, Zhou Wende's tangent method and Hanstush's cross flow type curve method. The comparison results suggested that the parameters (hydraulic conductivity coefficient and storage coefficient) calculated from these four methods are rather reliable with an error less than 9%.

Generally, the theoretical research on the pumping well test has developed over a relatively long period, but little progress has been made on steady flow since 1930. The subsequent theories and the experimental and analytical methods pertaining to transient flow were gradually perfected and systematized until the middle 1970's. Standard type curves combined with the corresponding mathematical function tables were used to describe aquifer properties, weak permeable stratum, well penetration and the water drop process, which is convenient to determine aquifer parameters. Since the 1970's, the computer and programmable pocket calculator have been used in the pumping well test, so that the parameters can be determined more rapidly and precisely. Walton^[23,24] summarized the theories and parameter calculation methods of the pumping well test, but the recommended formulae and the methods for solving the current practical problems are still derived in the 1950's and 1960's. Although some certain aspects (for example, some formulas are only applicable to a small drop depth, and there exist heterogeneous, lagging storage and directional seepage velocity differences in the aquifers) are still not rather mature, the recommended formulae could basically meet the testing requirements of the hydrogeologic parameters.

Since the 1980's, some progress has been made in pumping well test theory, and consequently the theory and methods of Theis-Boulton-Hanstush-Neuman for transient flow are still adopted in pumping well tests. Although the pumping well test has some serious limitations in practice, the transient injection well test was hardly mentioned in the previous two decades. In some cases, there is too little water in the test layer or the pumping lift is too large, so it is impossible to perform the pumping well test, but the injection well test is suitable. However, the injection well test lacks the sufficient theory and method. Therefore, based on the theory and method of the pumping well test, this study developed the theory and the test design, data processing and analytical methods for the transient injection well test. The applicable scope of the transient injection well test was also suggested. The field experiments verified that the method developed in this paper is appropriate and applicable.

appropriate and applicable.

2 Principle of the injection well test

2.1 Fundamental theory and classification

The injection well test is a field test method that injects fresh water in the test well to raise the water level until a steady height is attained, and records the water level change in the well. The hydrogeologic parameters (such as permeability coefficients) of the test layer can be determined by analyzing the injection well test data. Its theoretical basis is that the water flow from the well to the stratum shall conform to the laws of seepage flow in a porous medium. According to the injection process, the test is either a steady or a transient injection well test. In a steady injection well test water is injected into the well to a steady height at a constant flow rate, while the flow rate in the transient well test is unsteady. Individual and multiple-well injection well tests may be conducted. The individual well injection well test is primarily used to check the filtration characteristics in the target layer and to judge the existence of the leakage zone, while the multiple-well injection well test is used to determine the directional properties of the test layer, such as the permeability and leakage.

The injection well test is mainly used when it is difficult to conduct a pumping well test or when the test aquifer contains no water. In those cases, the injection well test can replace the pumping well test to determine the aquifer parameters. By using the injection well test, the following aquifer characteristics can be determined clearly: (1) the permeability of the test layer (such as rock stratum, soil stratum or rock and soil mixed stratum) which will be the average value of the test layer, and the directional properties (the vertical or horizontal average values for two dimensions and the average values in the x , y , z direction for the three dimension problem), and the heterogeneous distribution of the test layer; (2) the position of a leaking segment in the test layer; (3) the flow direction or the leakage direction; (4) the boundary properties of stratum and the location of the simple boundary.

In order to perform the injection well test, the necessary conditions are the geological character of the test layer, an adequate and clean water source, and the well borehole. The injection well test can be conducted following the drilling. Advanced test instruments and facilities are also essential to improve the precision of the test.

2.2 Short-time model

Assuming that the ambient strata have been saturated with water and the test stratum is a homogeneous, horizontal and tabular layer, the issue can be simplified as a two-dimensional problem of changing hydraulic head. According to Darcy's law and the mass conservation equation, the governing equation of fluid flow in the stratum can be derived from:

$$\frac{\partial}{\partial x} \left(\frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{\partial h}{\partial y} \right) = \frac{S}{kH} \frac{\partial h}{\partial t}, \quad (1)$$

where S is the storage coefficient, i.e., the released or absorbed water volume of unit cross sectional area from the aquifer as a function of unit head drop, or the ratio of the released or absorbed water volume from unit height to aqueous volume of the stratum; $H(m)$ is the thickness of the entire aquifer; $k(m/h)$ is the permeability coefficient; $h(m)$ is water head; x and y are the coordinates of the position.

The initial condition for eq. (1) is

$$h(t=0) = h_0. \quad (2)$$

The inner and outer boundary conditions for eq. (1) are

$$2\pi kH \left. \frac{\partial h}{\partial r} \right|_{r=r_w} = q, \quad (3)$$

$$h(r \rightarrow \infty) = h_0, \quad (4)$$

where $h_0(m)$ is the maximum water head; $r(m)$ is the distance from the borehole center.

To generalize the equation and to determine the stratum parameters by matching the actual test data with the dimensionless theoretical type curve, the following dimensionless parameters are selected for the dimensionless governing equation in stratum:

$$r_D = \frac{r}{r_w}, \quad h_D = \frac{2\pi kH(h_0 - h)}{q}, \quad t_D = \frac{kt}{Sr_w^2},$$

where r_D is the dimensionless distance; h_D is the dimensionless pressure head; t_D is the dimensionless time; $r_w(m)$ is the borehole radius; $q(m^3/h)$ is the injection flow rate.

Consequently, the dimensionless form of eq. (1) can be written as

$$\frac{\partial}{\partial x_D} \left(\frac{\partial h_D}{\partial x_D} \right) + \frac{\partial}{\partial y_D} \left(\frac{\partial h_D}{\partial y_D} \right) = \frac{\partial h_D}{\partial t_D}. \quad (5)$$

The dimensionless initial condition is

$$h_D(t_D = 0) = 0, \quad (6)$$

The dimensionless inner and outer boundary conditions are

$$\left. \frac{\partial h_D}{\partial r_D} \right|_{r_D=1} = -1, \quad (7)$$

$$h_D(r_D \rightarrow \infty) = 0. \quad (8)$$

By defining the matching parameters for the dimensionless type curve and the real test data, the average permeability coefficient and the average storage coefficient of the test stratum can be derived from the type curve matching method.

2.3 Long-time model

It is assumed that the ambient strata have been saturated by water during the drilling process, but the strata in the outer regions are unsaturated, so the properties of the test stratum change at the interface of the two regions. Therefore, a model must be adopted for a test layer with a radial composite stratum. Alternatively, if it is assumed that the test stratum is homogeneous over the radial range, the issue can be simplified as a two-dimensional problem of changing hydraulic head. According to Darcy's law and the mass conservation equation, the fluid flow governing equation of the different composite regions in the stratum can be derived as follows:

In the region near the borehole,

$$\frac{\partial}{\partial x} \left(\frac{\partial h_1}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{\partial h_1}{\partial y} \right) = \frac{S_1}{k_1 H} \frac{\partial h_1}{\partial t}; \quad (9)$$

in the outer region of the composite stratum,

$$\frac{\partial}{\partial x} \left(\frac{\partial h_2}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{\partial h_2}{\partial y} \right) = \frac{S_2}{k_2 H} \frac{\partial h_2}{\partial t}. \quad (10)$$

Where S_1 and S_2 are the storage coefficients of internal and external radial composite regions, respectively; k_1 and k_2 (m/h) are the filtration coefficients of internal and external regions, respectively.

The initial condition for the equation:

$$h_i(t=0) = h_0, \quad i=1, 2. \quad (11)$$

The inner and outer boundary conditions for the equation are

$$2\pi k_1 H \left. \frac{\partial h_1}{\partial r} \right|_{r=r_w} = q, \quad (12)$$

$$h_i(r \rightarrow \infty) = h_0, \quad i=1, 2. \quad (13)$$

The interface boundary conditions are

$$h_1|_{r=r_{\text{int}}} = h_2|_{r=r_{\text{int}}}, \quad (14)$$

$$2\pi k_1 H \left. \frac{\partial h_1}{\partial r} \right|_{r=r_{\text{int}}} = 2\pi k_2 H \left. \frac{\partial h_2}{\partial r} \right|_{r=r_{\text{int}}}, \quad (15)$$

where r_{int} (m) is the distance from the interface to the borehole center.

The following dimensionless parameters can be selected to conduct the dimensionless equations and boundary conditions of the above problem:

$$r_D = \frac{r}{r_w}, \quad h_{iD} = \frac{2\pi k_1 H (h_0 - h_i)}{q}, \quad t_D = \frac{k_1 t}{S_1 r_w^2}, \quad M = \frac{k_2}{k_1}, \quad N = \frac{S_2}{S_1}, \quad r_{\text{int}D} = \frac{r_{\text{int}}}{r_w},$$

where r_D is the dimensionless distance, h_{iD} is the dimensionless water pressure head, t_D is the dimensionless time, M is the permeability ratio, N is the storage coefficient ratio, $r_{\text{int}D}$ is the dimensionless distance of the interface of two regions.

The dimensionless governing equations of the fluid flow in the composite stratum are

$$\frac{\partial}{\partial x_D} \left(\frac{\partial h_{1D}}{\partial x_D} \right) + \frac{\partial}{\partial y_D} \left(\frac{\partial h_{1D}}{\partial y_D} \right) = \frac{\partial h_{1D}}{\partial t_D}, \quad (16)$$

$$\frac{\partial}{\partial x_D} \left(\frac{\partial h_{2D}}{\partial x_D} \right) + \frac{\partial}{\partial y_D} \left(\frac{\partial h_{2D}}{\partial y_D} \right) = \frac{N}{M} \frac{\partial h_{2D}}{\partial t_D}. \quad (17)$$

The dimensionless initial conditions:

$$h_{iD}(t_D = 0) = 0, \quad i = 1, 2. \quad (18)$$

The dimensionless inner and outer boundary conditions:

$$\left. \frac{\partial h_{1D}}{\partial r_D} \right|_{r_D=1} = -1, \quad i = 1, 2, \quad (19)$$

$$h_{iD}(r_D \rightarrow \infty) = 0. \quad (20)$$

The dimensionless interface boundary conditions:

$$h_{1D}|_{r=r_{\text{int}D}} = h_{2D}|_{r=r_{\text{int}D}}, \quad (21)$$

$$\left. \frac{\partial h_{1D}}{\partial r_D} \right|_{r=r_{\text{int}D}} = M \left. \frac{\partial h_{2D}}{\partial r_D} \right|_{r=r_{\text{int}D}}. \quad (22)$$

The theoretical type curve of dimensional head pressure vs. dimensionless time can be derived from the above mathematical model. By matching the type curve with the test data, the parameters of the test stratum can be determined.

3 Test and data analysis

3.1 Test apparatus and method

The high precision water level recorder is employed for the injection well test. The

water level automatic recorder used in this study is made in Holland by Eijkelpkamp Company (Fig. 1) with a length of 125 mm and a diameter of 22 mm, which can automatically record the water level and temperature. This kind of recorder is completely enclosed with stainless steel and can be used in any monitoring well. It consists of a pressure sensor, a temperature sensor, a data accumulator and a battery. The diver can be suspended with cables in the monitoring well to avoid the artificial damage.



Fig. 1. Water level automatic recording diver.



Fig. 2. Diver reader.

The diver can measure the pressure and temperature with temperature compensation. The recording modes are linear, logarithmic or based on event occurrence. The memory capacity of each parameter is up to 24,000 points. The precision is 0.1% of the full measurement range and the measurable temperature range is from -10°C to 40°C . If the data is recorded once per 10 min, the internal memory will suffice for about half a year. The life span of the battery is up to 10 years. By connecting the diver reader (see Fig. 2) with a computer, the stored data can be downloaded directly to the computer.

In the transient injection well test, water was rapidly injected to the borehole with a random flow rate. When the water level in the test well reached the required height, the injection was terminated. After that, the change of water level should be recorded. If the purpose of the transient injection well test is to check for the existence of a leakage zone or the filtration quality of the target stratum, the individual injection well test is suitable with a relative low testing expense.

The testing preparations include: (1) knowing clearly the hydrogeological properties of the test layer, which includes the burial depth, stratum distribution, water supplement from ground water in other strata or surface water, boundary conditions, water quality and the flow direction of ground water in the test stratum; (2) knowing clearly the position, distance, structure, depth and water seal of the water injection well, the layout of the filter and the hydrogeologic profile in the direction of the connecting line of the wells; (3) inspecting the installation and operating conditions of the injection equipment, power set and other facilities in the well test site; (4) examining whether the various tools and recording books are all ready; (5) installing, constructing and checking the drainage appliances.

The conduction of the injection well test includes: (1) Measuring the borehole depths before and after the injection well test. The purpose is to check the depth and position of the injection segment and whether the well-bore has collapsed, or retained deposits and choked. (2) Observing the natural water level and judging whether there exists water accumulation. (3) Initializing the water gauge and other measuring devices and putting them in the well. (4) Recording the injection flow rate, i.e., the injection process and injection time. (5) Recording the air temperature and water temperature over 2 - 4 h in common conditions.

3.2 Test data analysis methods

Until now, there have been no special methods to analyze injection well test data, but the analysis method in the pumping well test can be used for reference. The conventional analysis method for the pumping well test data is primarily the semi-logarithmic linear analysis method. Semi-logarithmic coordinates are used to plot the water level variation or flow rate vs. the logarithm of time to obtain the test curve. Then the parameters of the test stratum (mainly the permeability factor) can be determined using the slope of the straight-line portion of the test curve. The method is widely used in practical projects. However, the method encounters some difficulties in the analysis of the injection well test. For instance, it is extremely difficult to draw the semi-logarithmic graph. And it is also difficult to determine the required straight-line portion because two or more straight-line portions usually appear in a long-term injection well test. For this reason, two special analysis methods for the transient injection test data are presented in this paper; they are the direct observation method and the double logarithm type curve matching method of the inverse problem.

The direct observation method is used to qualitatively judge the stratum permeability by observing the change of water level in the injection well test. The method will be more powerful in judging whether the stratum leaks, especially for the case where the stratum leaks and the semi-logarithmic linear analysis method and the type curve matching method cannot be used to deal with the test data.

The type curve matching method of the inverse problem is mainly used to analyze the changing process of water level/flow rate in a log-log plot by matching the test data with the dimensionless type curve to determine the relevant parameters of the stratum. The key of the method is to select a suitable dimensionless model. All test data can be used with this kind of method.

The general analysis steps are: (1) draw the test data and the theoretical simulated type curve of the selected model on the same log-log plot using a computer; (2) match the test data with the theoretical type curve by adjusting the time match parameter t_M , water level match parameter h_M and the parameters of the theoretical simulated model; (3) check whether the semi-logarithmic plot and the history plot are matched well. If the fitting is bad, step 2 shall be repeated until all curves match perfectly; (4) calculate the

stratum parameters of the test stratum by using the obtained matching parameters.

4 Examples

Maoping landslide close to Geheyan Reservoir in the Qingjiang River is an active ancient congeries type slope. This kind of slope is widely distributed in the Three Gorge Reservoir area. Maoping landslide has deformed for more than 10 years after the reservoir started to operate. There are rather abundant geological investigations and hydro-geological monitoring data to analyze the mechanism of Maoping landslide. In order to verify the method presented in this paper, and analyze the effect of water seepage on the landslide stability of Maoping slope, the injection well tests were performed with three boreholes (No. 1, No. 2 and No. 3) in Maoping slope from December 2003 to January 2004. More than 20 injection well tests were conducted along with drilling. The analysis method of the injection well test presented in this paper was used to analyze partial test data, and to determine the stratum permeability.

4.1 Leakage phenomena analysis

The first type analysis method was adopted to analyze the leakage loss of the test stratum in Maoping landslide. Fig. 3 shows the test results of the water level, when borehole No.1 was drilled to a depth of 12 m. During the test, the maximum depth of injection water in borehole No.1 was 1.25 m. When the injection ended, the water level in the borehole dropped rapidly to 60 cm and remained constant. Since the real depth of borehole No.1 is 12 m, it can be concluded that there is a leakage segment at a depth of 11.4 m. After the injection has proceeded for some time, the water level in the borehole still remained at 60 cm, which suggests that the permeability of the stratum between 11.4 m and 12 m is relatively poor and the permeability coefficient is very low.

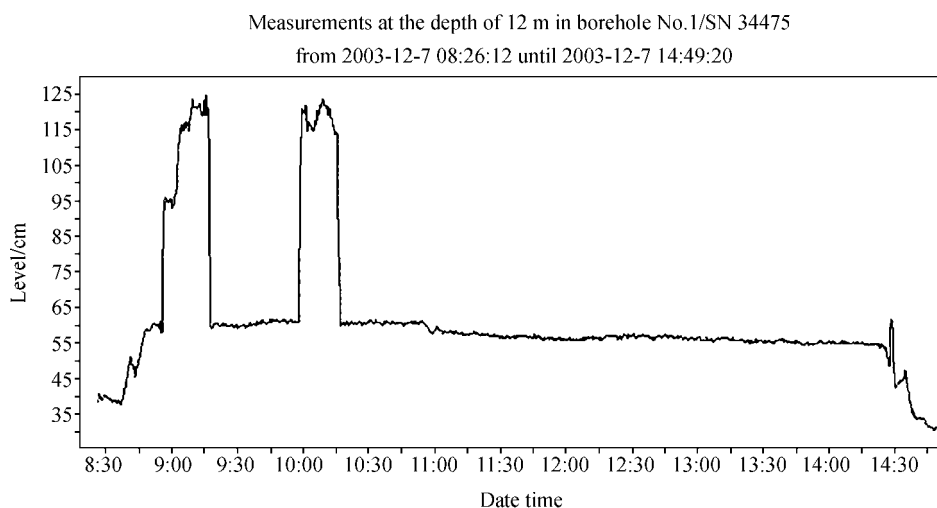


Fig. 3. Water level change at 12 m depth in borehole No. 1.

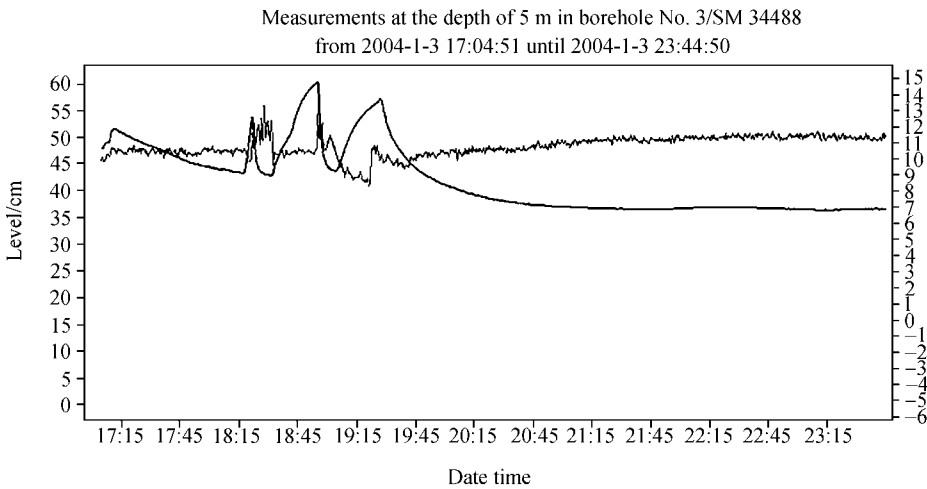


Fig. 4. Water level change at 5 m depth in borehole No. 3.

The water level test data of borehole No. 3 at 5 m are shown in Fig. 4. In the process of the test, the water level no longer rises with the injection after it reached 48 cm and then the water level remained at this depth. Therefore, it can be judged that there is a serious leakage segment at a depth of 4.5 m in borehole No. 3.

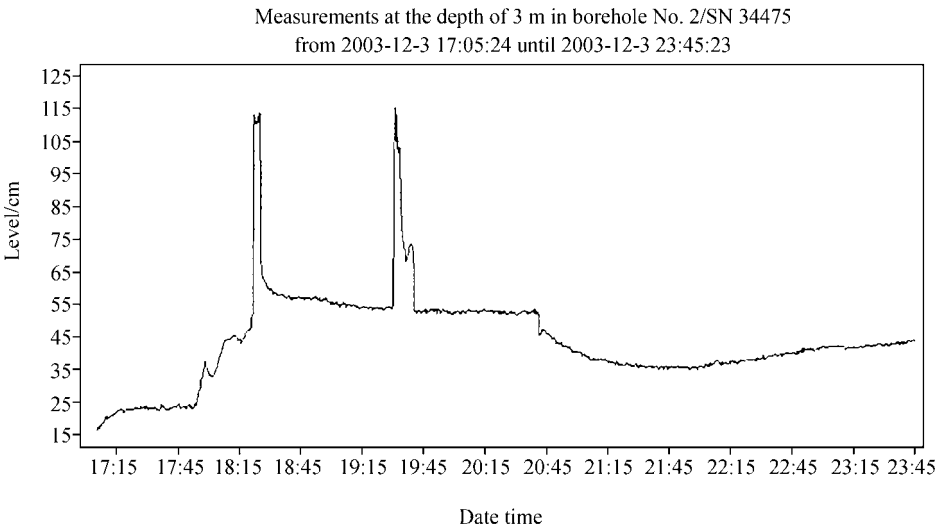


Fig. 5. Water level changing at 3 m in borehole No. 2.

The water level test data in borehole No. 2 at a depth of 3 m are shown in Fig. 5. It can be seen from the figure that two plateaus in water level were formed after the beginning of the injection well test. After the first water level plateau was formed, the water level dropped at a certain velocity. But the water level dropped severely after the second water level plateau was formed. This phenomenon indicates that the first injection got through the leakage segment in the borehole, i.e., there is a leakage segment within

1.5 2.5 m in borehole No. 2.

4.2 Quantitative analysis of the normal test data

The injection well test data in borehole No. 2 at a depth of 12 m was selected for analysis using the double logarithmic type curve matching method.

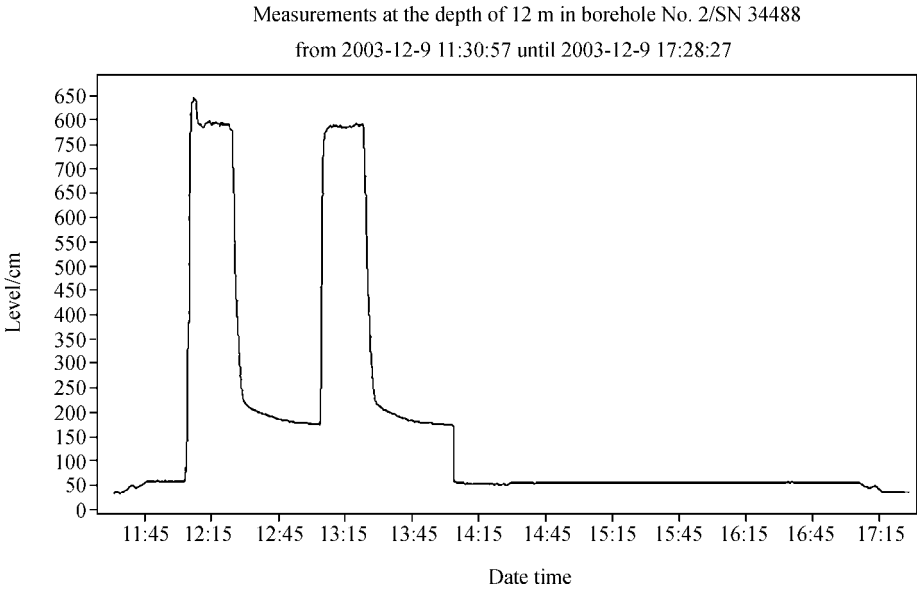


Fig. 6. Water level change at 12 m in borehole No. 2.

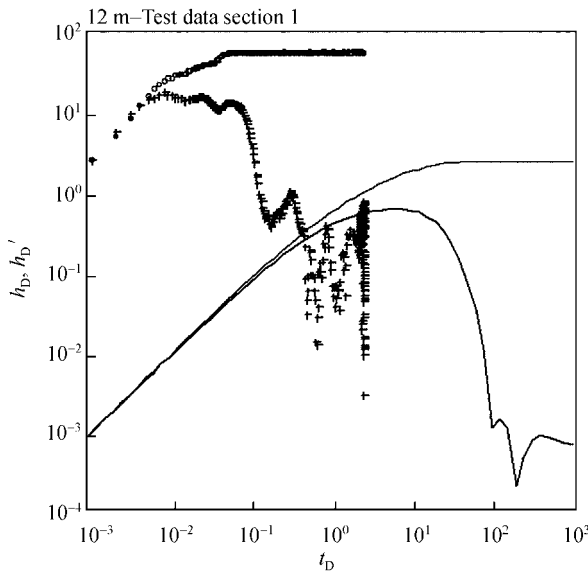


Fig. 7. Theoretical and real test data before log-log matching.

The injection well test data of borehole No. 2 at a depth of 12 m are shown in Fig. 6.

The graph shows a normal injection well test curve which can be analyzed by using the double logarithmic type curve matching method. Fig. 7 is the graphical chart before double logarithmic type curve matching. Major stratum characteristic parameters can be determined by matching the first water drop of the test data with the dimensionless theoretical type curve. The permeability coefficient of the inner region is about 0.00327 m/s, the interface of the composite stratum is at 2.3 m, and the permeability coefficient of the outer region is about 0.01308 m/s. The permeability coefficient obtained from the test agreed with that of a laboratory core flow test. According to the physical process of permeation, the distance of the interface in the composite stratum of 2.3 m is also reasonable. Additionally, it can be seen that the permeation rates of both the inner and outer regions are rather high, and the permeability coefficient of the outer region is higher than that of the inner region, which is the same as the observations of the permeation process. The double logarithm, semi-logarithm and history matching plots in the test are all shown in Fig. 8.

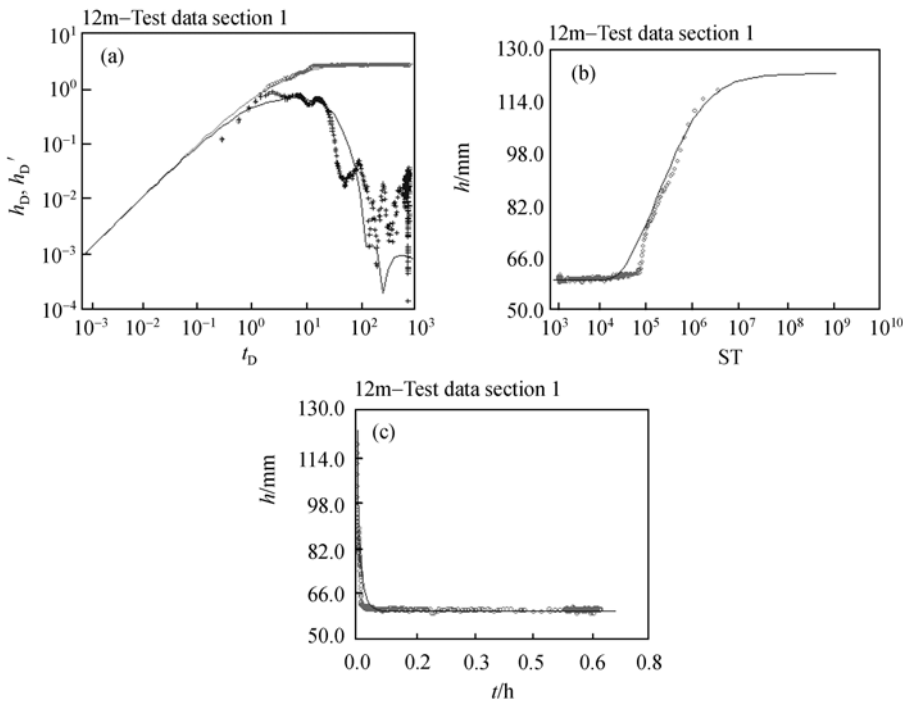


Fig. 8. The matching results of the type curve for the water test level. (a) Double logarithmic matching curve for the injection test; (b) semi-logarithmic matching curve for the injection test; (c) test history matching curve.

5 Conclusion

Owing to the high precision requirements of instruments, the difficulty of analysis method and the lack of appropriate software, the transient injection well test is not widely used and deeply developed. Based on the theory of the pumping well test, a new transient injection well test method is established in this paper. The mathematical model

for the short-time injection and long-time injection of the transient injection well test are derived and used to analyze the field test data. The testing data processing method of the transient injection well test was developed and the double logarithmic type curve matching method for the transient injection well test data is programmed as software. A test system set including the design, execution and data analysis from theory to practical test has been created. Some field test examples were analyzed to verify the applicability of the transient injection well testing method. The leakage in the test process was analyzed qualitatively and the permeability coefficient of the test stratum was determined using the double logarithmic type curve matching method. All these suggested that the methods presented in the paper are suitable for solving the practical engineering problems.

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