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Manufacturer Rebates in a Green Supply Chain with Hybrid Production

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Abstract This paper studies two manufacturer rebate strategies in a green supply chain. The results show that: 1) Irrespective of the type of rebate strategy used, channel members benefit; 2) Rebates for green products improve the green level of green products; 3) The more significant the slippage effects are, the more beneficial the rebate strategies are, for channel members. On this basis, the model is expanded to consider asymmetric potential market demands and asymmetric product substitution rates, respectively. The results show that the product substitution rate does not affect the strategic preference of channel members, but the potential market demand does.

Keywords Stackelberg game; manufacturer rebates; supply chain management; green level

1 Introduction

With the rapid growth of the global economy, people's living standards worldwide have greatly improved. However, environmental problems such as pollution and global warming are becoming increasingly grave. According to a report released by the International Energy Agency in September 2019, global carbon dioxide emissions reached 32.8 billion tons in 2017, and environmental pollution has become one of the most urgent issues facing the world[1]. The United Nations Intergovernmental Panel on Climate Change estimates that by 2099, the surface temperature of the earth will be eight degrees Celsius higher than it was in 1999[2]. To reduce environmental pollution, the Society of Manufacturing Engineers proposed the concept of green manufacturing in 1996[3]. Since then, many of the world's largest economies and companies have also participated in green manufacturing. For example, General Motors Company achieved full-cycle production at its Detroit tooling plant in 2007, in which 100% of the plant's waste

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materials and emissions can now be recycled in the production process[4]. At the same time, with the concept of environmental protection gradually creeping into the public consciousness, consumers are increasingly choosing green products. It has been reported that 75% of European consumers are willing to pay higher prices for environmentally friendly products[5]. However, as new products, green products have no absolute advantage in competing with conventional products and are even at a disadvantage in some aspects. For example, compared with fuel-powered vehicles, electric vehicles are often at a disadvantage in terms of mileage and price, which severely hinders their sales. As of January 17, 2020, the Gasgoo listed on its website that, from July to September 2019, monthly sales of new energy vehicles in China fell by 4.7%, 15.8%, and 18%, respectively.

To improve the market competitiveness of products, enterprises often offer promotions (advertisements, rebates, etc.). In the 1970s, Procter & Gamble Company (P & G) took the lead in sales promotions by issuing rebate coupons to consumers. Since then, an increasing number of manufacturers and retailers have used rebates as a strategy to attract and retain customers. For example, as of October 20, 2015, the Amazon(Z.CN) listed on its website that Amazon(Z.CN) used "F coin rebates", in which consumers place orders at the original price on the rebate network, and then are issued rebates approximately two months after the order is received and confirmed, with the rebates being in the form of an F coin, forcing consumers to go through a similar platform for their next purchase to redeem the F coin. Rebate strategies that reduce the price in the short term to improve product sales without changing the label price are widely used by companies. In fact, Chen, et al.[6] found that if not all consumers redeem the rebates, manufacturer rebates are beneficial to channel members. The slow extension of green products and the effectiveness of manufacturer rebates in the conventional products market motive us to introduce the manufacturer rebates in the green products market and examine the effects of manufacturer rebates.

2 Literature Review

Rebate strategies can be divided into two categories, depending on the objective, with one for enterprises and one for consumers. The rebates for enterprises, also known as channel rebates, are transfer payments offered from the manufacturer to the retailer based on the final sales or order quantities. For example, Taylor[7] studied the impacts of linear rebates and target rebates on supply chain decisions. Ha, et al.[8] constructed a model with two competing manufacturers and a common retailer and found that intense competition makes manufacturers reduce the rebate value or even stop offering rebates. Muzaffar, et al.[9] studied two cases of rebate strategies (mail-in rebates and channel rebates) in a two-level supply chain. By comparing them, the authors found that channel rebates are an effective tool for manufacturers to control retail prices. Considering retailers' risk aversion, Demirag, et al.[10] analyzed manufacturer's rebate decisions and the retailer's joint inventory and pricing decisions. Li, et al.[11] introduced the CVaR criterion to study channel rebates and penalty contracts with different degrees of risk aversion under stochastic demand.

Rebates for consumers include manufacturer rebates and retailer rebates. Manufacturer rebates refer to promotional offers where manufacturers provide rebates to consumers. This strategy has been widely studied and scholars have obtained many meaningful insights. For example, Khouja and Zhou[12] assumed that the estimated value of the rebate at the time of purchase decision is independent of the probability of redemption at the time of the redemption decision, and found that the consumer surplus decreases when a manufacturer offers a rebate. Yang, et al.[13] combined the manufacturer rebate with the manufacturer suggested retail price and proved that the optimal strategy of the manufacturer is determined by the slippage behavior of consumers and their loss aversion. Herrán and Sigue[14], however, found that manufacturers make higher profits when the coupon is redeemed than when it is not redeemed. Yang, et al.[15] studied two types of promotional models (mail-in rebates and retailer promotions), and showed that quantity discount contracts and buy-back contracts to coordinate the supply chain. Zhang[16] investigated the impacts of manufacturer rebates on the performance of the manufacturer and the retailer when the manufacturer sells the products in two different markets and found that, while it might seem counterintuitive, higher redemption rates are actually beneficial to channel members. Cao, et al.[17] studied optimal rebate strategies (both trade-in rebates and customer rebates with gift cards or cash coupons) from a business-to-consumer (B2C) platform perspective.

Retailer rebates refer to promotional activities where the retailer provides rebates to consumers. Many researchers have studied this issue. For example, Khouja, et al.[18] studied the promotional behavior of retailers offering gift cards and found that the profitability of retailers using gift cards depends on the repurchase rate of consumers and the redemption rate of the gift cards. Demirag[19] studied the pricing and ordering strategies of retailers with different risk preferences when retailers offer rebates to consumers in order to increase product sales. Yang, et al.[20] constructed a retailer rebates model based on economic order quantity, compared the rebates promotional activities with the everyday low price model, and concluded that the market positioning of retailers plays a vital role in the choice of their promotion strategies. Zhou, et al.[21] studied optimal strategies for an e-commerce provider. The results showed that, in contrast to the traditional supply chain model, the consumer surplus in a centralized structure is lower than that in a decentralized structure.

Much of the research on green product promotion has considered government subsidies to enterprises, with a few studies considering government subsidies on green products to customers. However, there are no extant studies considering rebate strategies in a green supply chain. The rebate strategy, as an effective promotional method, has been widely used in enterprise operations. However, most of the studies on rebate strategies only consider the supply chain model with a single product. In reality, though, competition between products is normal. Therefore, this study considers manufacturer rebates in a green supply chain in a competitive situation, focusing on the following questions: 1) How does the rebate strategy affect the decisions of channel members and the channel performance? 2) Which rebate strategies are preferred by manufacturers and retailers, respectively? 3) What kind of factors affect the strategic preference of channel members.

3 Assumptions and Notations

This study considers a two-echelon green supply chain consisting of one manufacturer and one retailer. The manufacturer produces two products (green products and conventional products) and needs to invest in green technology for producing the green products. The investment cost is $\frac{1}{2}s^2$, where the green level is s. It is assumed that green technology does not affect the marginal cost of green products. Therefore, without loss of generality, the marginal costs of conventional products and green products are assumed to be zero[22]. The manufacturer is the Stackelberg leader, and first decides on wholesale prices w_1 (green products) and w_2 (conventional products), the green level s, and the rebate value r. Then, the retailer determines the retail prices p_1 (green products) and p_2 (conventional products).

Referring to Xing, et al.[23], the demand functions are denoted as follows. Without rebates, the demands for green products and conventional products are $q_1 = \frac{a_1-\theta_1 a_2-p_1+\theta_1 p_2+\lambda s}{1-\theta_1\theta_2}$ and $q_2 = \frac{a_2-\theta_2 a_1-p_2+\theta_2 p_1-\theta_2\lambda s}{1-\theta_1\theta_2}$, respectively. Here, a_1 and a_2 represent the potential market demands for green products and conventional products, respectively, θ_1 is the substitution rate of green products for conventional products, and θ_2 is the substitution rate of conventional products for green products; λ is the green sensitivity factor of consumers.

To simplify the calculation, the potential market demands and the substitution rate of products are assumed to be symmetric ($a_1 = a_2 = a$, $\theta_1 = \theta_2 = \theta$). In the extended section, the models with asymmetric potential market demands and different product substitution rates are discussed.

1) When the manufacturer offers rebates to customers who purchase the green products, the demand for green products and conventional products are $q_1^g = \frac{a(1-\theta)-(p_1-tr)+\theta p_2+\lambda s}{1-\theta^2}$ and $q_2^g = \frac{a(1-\theta)-p_2+\theta(p_1-tr)-\theta\lambda s}{1-\theta^2}$, respectively, where t is the rebate sensitivity factor.

2) When the manufacturer offers rebates to customers who purchase conventional products, the demands for green products and conventional products are $q_1^c = \frac{a(1-\theta)-p_1+\theta(p_2-tr)+\lambda s}{1-\theta^2}$ and $q_2^c = \frac{a(1-\theta)-(p_2-tr)+\theta p_1-\theta\lambda s}{1-\theta^2}$, respectively.

The rebate frequency of manufacturer is β. The redemption rate of customers is k. To avoid trivial cases, it is assumed that $k < t$ and $\lambda \in (0, \sqrt{\frac{(1-\theta^2)[8kt-\beta(k^2+t^2+6kt)]}{2kt(1-\beta)}})$. The superscripts g and c represent rebates for green products and conventional products, respectively. All proofs are given in the Appendix.

4 Model Analysis

4.1 No Rebates Model (Model N)

In this subsection, the model without manufacturer rebates is considered. The manufacturer decides the green level and the wholesale prices first, then the retailer decides the retail prices p_1 and p_2, respectively.

The profit functions for the manufacturer and the retailer are:

$$\pi_m = w_1 q_1 + w_2 q_2 - \frac{1}{2}s^2, \tag{1}$$

$$\pi_r = (p_1 - w_1)q_1 + (p_2 - w_2)q_2. \tag{2}$$

Using backward induction the following are obtained: $w_1^* = \frac{a(\lambda^2\theta+4\theta^2-4)}{2(\lambda^2+4\theta^2-4)}$, $w_2^* = \frac{a}{2}$, $s^* = \frac{-\lambda a(1-\theta)}{\lambda^2+4\theta^2-4}$, $p_1^* = \frac{3a(\lambda^2\theta+4\theta^2-4)}{4(\lambda^2+4\theta^2-4)}$, $p_2^* = \frac{3a}{4}$. Then, the profits of the manufacturer and the retailer are $\pi_m^* = \frac{a^2(\lambda^2+8\theta-8)}{8(\lambda^2+4\theta^2-4)}$ and $\pi_r^* = \frac{a^2[\lambda^4-8(1-\theta^2)(\lambda^2+4\theta-4)]}{16(\lambda^2+4\theta^2-4)^2}$, respectively.

4.2 Rebates for Green Products (Model G)

This subsection considers the model in which the manufacturer offers rebates to customers who purchase green products. The manufacturer is the Stackelberg-leader, and first determines the wholesale prices, green level, and rebate value. Next, the retailer determines the retail prices. The profit functions of channel members are as follows:

$$\pi_m^g = \beta(w_1 - kr)q_1^g + (1-\beta)w_1q_1 + \beta w_2q_2^g + (1-\beta)w_2q_2 - \frac{1}{2}s^2, \tag{3}$$

$$\pi_r^g = (p_1 - w_1)[\beta q_1^g + (1-\beta)q_1] + (p_2 - w_2)[\beta q_2^g + (1-\beta)q_2]. \tag{4}$$

The Hessian Matrix of π_r^g with respect to p_1 and p_2 is

$$H_r^g = \frac{1}{1-\theta^2}\begin{pmatrix} -2 & 2\theta \\ 2\theta & -2 \end{pmatrix}.$$

It is easy to verify that the Hessian matrix is negative definite. Therefore, solving the first order conditional equations, the optimal response functions of the retailer are obtained $p_1^g = \frac{a}{2} + \frac{w_1}{2} + \frac{s\lambda}{2} + \frac{\beta rt}{2}$, $p_2^g = \frac{a}{2} + \frac{w_2}{2}$. Substituting the optimal response functions into π_m^g, the Hessian matrix of π_m^g with respect to s, w_1, w_2, and r are obtained.

$$H_m^g = \frac{1}{2(1-\theta^2)}\begin{pmatrix} 2(\theta^2-1) & -\beta k\lambda & \lambda & -\theta\lambda \\ -\beta k\lambda & 2\beta kt(\beta-2) & -\beta(k+t) & -\beta\theta(k+t) \\ \lambda & -\beta(k+t) & -2 & 2\theta \\ -\theta\lambda & -\beta\theta(k+t) & 2\theta & -2 \end{pmatrix}. \tag{5}$$

Denote $|H_{mi}^g|$ as the i-th order sub-determinant. Therefore,

$$|H_{m1}^g| = -1, \quad |H_{m2}^g| = \beta k\frac{4t(2-\beta)(1-\theta^2) - \beta k\lambda^2}{4(\theta^2-1)^2},$$

$|H_{m3}^g| = \frac{-\beta A}{4(1-\theta^2)^3}$, and $|H_{m4}^g| = \frac{\beta A}{4(1-\theta^2)^3}$, where $A = (2\beta kt - 2kt)\lambda^2 + (\theta^2-1)(\beta k^2 - 8kt + \beta t^2 + 6\beta kt)$. It is easy to know that $|H_{m1}^g| < 0$, $|H_{m2}^g| > 0$, $|H_{m3}^g| < 0$, and $|H_{m4}^g| > 0$. Therefore, the profit function of the manufacturer π_m^g is jointly concave in s, w_1, w_2, and r. Solving the first-order conditions, the optimal decisions of manufacturer are obtained: $w_1^g = a\frac{(1-\theta^2)[2k(4t-3\beta t-\beta k)+\beta\theta(k^2-t^2)]+2kt\theta\lambda^2(\beta-1)}{2A}$, $w_2^g = \frac{a}{2}$, $s^g = \frac{2akt\lambda(1-\beta)(1-\theta)}{A}$, $r^g = \frac{a(1-\theta^2)(t-k)(1-\theta)}{A}$. Substituting the manufacturer's optimal decisions into the retailer's reaction functions, optimal decisions of retailer are obtained: $p_1^g = a\frac{(1-\theta^2)[4k(6t-\beta k-5\beta t)+\beta\theta(k^2+2kt-3t^2)]+6kt\theta\lambda^2(\beta-1)}{4A}$, $p_2^g = \frac{3a}{4}$. Then, the profit of the manufacturer and the retailer are, respectively:

$$\pi_m^g = a^2\frac{\beta(k^2+t^2)(1-\theta^2) + 2kt[(1-\beta)(\lambda^2-7-8\theta) - 1 + \beta\theta^2]}{-8A},$$

$$\pi_r^g = a^2\frac{\begin{array}{l}\beta^2(k^4+t^4)(\theta^2-1)^2 + k^2t^2(\theta^2-1)[8\lambda^2(\beta-1)(3\beta-4) - \beta^2(13\theta^2-128\theta+51) + \\ 32(\theta-1)(4+\beta\theta-7\beta)] + 4k^2t^2\lambda^4(\beta-1)^2 + 4\beta kt(k^2+t^2)[(\theta^4+1)(3\beta-4) + \\ \lambda^2(\theta^2-1)(\beta-1) + 2\theta(4\theta-3\beta)]\end{array}}{16A^2}.$$

Proposition 1 $\frac{\partial \pi_m^g}{\partial \beta} > 0$, $\frac{\partial \pi_r^g}{\partial \beta} > 0$, $\frac{\partial q_1^g}{\partial \beta} > 0$, $\frac{\partial q_2^g}{\partial \beta} < 0$, $\frac{\partial s^g}{\partial \beta} > 0$, $\frac{\partial r^g}{\partial \beta} > 0$, $\frac{\partial p_1^g}{\partial \beta} > 0$, $\frac{\partial q_1}{\partial \beta} < 0$, $\frac{\partial q_2}{\partial \beta} > 0$, $\frac{\partial w_1^g}{\partial \beta} > 0$.

The profits of channel members, the green level, the rebate value, the wholesale price, the retail price and the market demand for green products in the rebate period are positively correlated with the rebate frequency, while the market demand for green products outside the rebate period is negatively correlated with the rebate frequency. With the increase of rebate frequency, the differentiation of green products and conventional products is more significant, and the manufacturer attracts more customers to buy green products by improving the rebate value and green level. At the same time, the manufacturer raises her wholesale prices to ensure profitability, which leads to an increase in retail prices. However, consumers do not reduce their purchases in the rebate period, because the positive impacts of the increase in the rebate value and the green level dominate the negative impacts of the increase in retail price. Although consumers reduce their purchases of green products outside the rebate period and purchase conventional products instead, the impacts of manufacturer rebates on the aggregate demand for green products are positive and more significant than that for conventional products. So, the profits of channel members increase with the increase of rebate frequency.

Proposition 2 $\frac{\partial \pi_m^g}{\partial k} < 0$, $\frac{\partial \pi_r^g}{\partial k} < 0$, $\frac{\partial q_1^g}{\partial k} < 0$, $\frac{\partial q_2^g}{\partial k} > 0$, $\frac{\partial s^g}{\partial k} < 0$, $\frac{\partial r^g}{\partial k} < 0$, $\frac{\partial p_1^g}{\partial k} < 0$, $\frac{\partial w_1^g}{\partial k} < 0$.

Proposition 2 shows that the higher the redemption rate, the lower the rebate value. Thus, consumers get less surplus when they purchase green products than when they purchase conventional products, and eventually purchase conventional products. This reduces the demand for green products and also reduces the enthusiasm of manufacturers to invest in green technology, resulting in a decline in green levels. Ultimately, it reduces the profits of the manufacturer and the retailer.

Proposition 3 $\frac{\partial \pi_m^g}{\partial t} > 0$, $\frac{\partial \pi_r^g}{\partial t} > 0$, $\frac{\partial q_1^g}{\partial t} > 0$, $\frac{\partial q_2^g}{\partial t} < 0$, $\frac{\partial s^g}{\partial t} > 0$, $\frac{\partial r^g}{\partial t} > 0$, $\frac{\partial p_1^g}{\partial t} > 0$, $\frac{\partial w_1^g}{\partial t} > 0$.

Proposition 3 shows that the rebate sensitivity factor and the redemption rate have opposite effects on channel members. Specifically, with an increase in the rebate sensitivity factor, the manufacturer increases the rebate value and the green level, which is preferred by consumers. This leads to an increase in the sales of green products in the rebate period. Eventually, the profits of channel members are improved.

4.3 Rebates for Conventional Products (Model C)

This subsection considers a supply chain model in which the manufacturer offers rebates to customers who purchase conventional products. The manufacturer first decides the wholesale prices, the green level and the rebate value. Then, the retailer decides the retail prices. The profit functions of the manufacturer and the retailer are, respectively.

$$\pi_m^c = \beta w_1 q_1^c + (1-\beta) w_1 q_1 + \beta (w_2 - kr) q_2^c + (1-\beta) w_2 q_2 - \frac{1}{2} s^2, \tag{6}$$

$$\pi_r^c = \beta (p_1 - w_1) q_1^c + (1-\beta)(p_1 - w_1) q_1 + \beta (p_2 - w_2) q_2^c + (1-\beta)(p_2 - w_2) q_2. \tag{7}$$

Similar to Model G, the optimal decisions can be obtained by backward induction, and are shown as follows:

$$w_1^c = \frac{2a\{(1-\theta^2)[8kt - 4\beta kt - \beta(k+t)^2] + 2(\beta-1)kt\theta\lambda^2\}}{C},$$

$$w_2^c = \frac{a\{k\lambda^2[\beta k - 4t + 3\beta t + \beta\theta^2(t-k)] + 2(\theta^2-1)[\beta k^2(2-\theta) + 2(3\beta-4)kt + \beta\theta t^2]\}}{C},$$
$$s^c = \frac{a\lambda(\theta-1)(\beta k^2 - 8kt + \beta t^2 + \beta\theta t^2 + 6\beta kt + \beta\theta k^2 - 2\beta\theta kt)}{C},$$
$$r^c = \frac{a(1-\theta^2)(k-t)(\lambda^2 - 4 + 4\theta)}{C},$$
$$p_1^c = \frac{3a\{(1-\theta^2)[4kt(2-\beta) - \beta(k+t)^2] + 2kt\theta\lambda^2(\beta-1)\}}{C},$$
$$p_2^c = \frac{a\{(1-\theta^2)[kt(24-20\beta) + \beta k^2(\lambda^2-4) + \beta\theta(k^2 - 3t^2 + 2kt)] + kt\lambda^2[(5\beta-6) + \beta\theta^2]\}}{C},$$

where $C = [\beta(k+t)^2 - 4kt(2-\beta) - \beta\theta^2(k-t)^2]\lambda^2 - 4(1-\theta^2)[\beta(k+t)^2 - 4kt(2-\beta)]$.

The profits of the manufacturer and the retailer are, respectively:

$$\pi_m^c = \frac{a^2\beta(1-\theta^2)(k-t)^2 + 2kt(1-\beta)[a^2(\lambda^2-8) + 8\theta]}{-2C}$$
$$\pi_r^c = a^2\frac{\begin{array}{l}(\theta^2-1)\{4\lambda^2(k^2+t^2)(\beta-1) + 32k^2t^2[\lambda^2 + 4\beta^2\theta + (\theta-1)(\beta\theta - 7\beta + 4)] + \\ 8\beta k^2t^2\lambda^2[\theta(1-\beta) - 8 - 4\beta]\} + (\theta^2-1)^2[4\beta kt(k^2+t^2)(3\beta-4) + \beta^2(k^4+t^4)] + \\ 2k^2t^2[2\lambda^4(\beta-1)^2 - \beta^2(\theta-1)(13\theta+51)]\end{array}}{C^2}.$$

Proposition 4 $\frac{\partial \pi_m^c}{\partial \beta} > 0$, $\frac{\partial s^c}{\partial \beta} < 0$, $\frac{\partial r^c}{\partial \beta} > 0$, $\frac{\partial w_1^c}{\partial \beta} < 0$, $\frac{\partial w_2^c}{\partial \beta} > 0$, $\frac{\partial q_1^c}{\partial \beta} < 0$, $\frac{\partial q_2^c}{\partial \beta} > 0$, $\frac{\partial p_1^c}{\partial \beta} < 0$, $\frac{\partial p_2^c}{\partial \beta} > 0$.

Proposition 4 shows that the manufacturer's profit, the rebate value, the demand for conventional products in the rebate period, the wholesale price and the retail price of conventional products are positively related to the rebate frequency. The wholesale price and retail price of green products are negatively correlated with the rebate frequency. Proposition 1 and Proposition 4 show that it is beneficial for the manufacturer to offer rebates, regardless of the type of products. However, unlike with Model G, the rebates for conventional products increase the demand for conventional products and inhibit the demand of green products, so the manufacturer has no incentive to invest in green technology, resulting in a decline in the green level.

Proposition 5 $\frac{\partial \pi_m^c}{\partial k} < 0$, $\frac{\partial p_1^c}{\partial k} > 0$, $\frac{\partial p_2^c}{\partial k} < 0$, $\frac{\partial s^c}{\partial k} > 0$, $\frac{\partial r^c}{\partial k} < 0$, $\frac{\partial w_1^c}{\partial k} > 0$, $\frac{\partial w_2^c}{\partial k} < 0$, $\frac{\partial q_1^c}{\partial k} > 0$, $\frac{\partial q_2^c}{\partial k} < 0$.

Proposition 5 shows that the rebate value, the wholesale price, the retail price and the demand of conventional products in the rebate period decrease with the increase of the redemption rate, while the green level and demand for green products in the rebate period increase with the increase of the redemption rate. This demonstrates that the higher the redemption rate of consumers, the greater the promotional costs for manufacturer, which reduces the rebate value and lead to a decline in the demand for conventional products in the rebate period. To maintain margins, the manufacturer increases the green level and wholesale price of green products, thus improving the market competitiveness of green products. However, with the increase of the redemption rate, the slippage effect is not significant, and the gains for green products cannot compensate for the losses for conventional products, which leads to a decline in the manufacturer's profit.

Proposition 6 $\frac{\partial \pi_m^c}{\partial t} > 0$, $\frac{\partial p_1^c}{\partial t} < 0$, $\frac{\partial p_2^c}{\partial t} > 0$, $\frac{\partial s^c}{\partial t} < 0$, $\frac{\partial r^c}{\partial t} > 0$, $\frac{\partial w_1^c}{\partial t} < 0$, $\frac{\partial w_2^c}{\partial t} > 0$, $\frac{\partial q_1^c}{\partial t} < 0$, $\frac{\partial q_2^c}{\partial t} > 0$, $\frac{\partial q_1}{\partial t} > 0$, $\frac{\partial q_2}{\partial t} < 0$.

Proposition 6 shows that the higher the rebate sensitivity factor of conventional products, the lower the green level of green products, and the less beneficial the sales of green products in the rebate period are. However, the higher the rebate sensitivity factor of conventional products, the more competitive the conventional products are, which ultimately leads to an increase in the manufacturer's profit.

Because the retailer's profit in Model C is complex, the sensitivity analysis is not feasible. It is discussed through numerical analysis. The parameter values are given as follows: $a = 10$, $\theta = 0.4$, $\lambda = 0.6$, The results are shown in Figure 1.

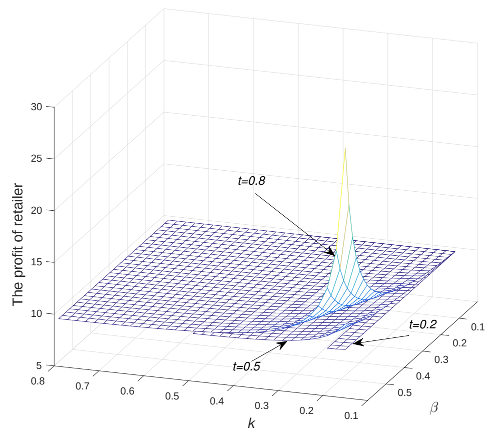

Figure 1 The effects of rebate parameters on the retailer's profit in Model C

Figure 1 shows that the profit of the retailer increases with an increase in the rebate frequency and decreases with an increase in the redemption rate in Model C. These trends become increasingly obvious with the increase in the rebate sensitivity factor. In other words, the higher rebate sensitivity factor aggravates the impacts of rebate frequency and redemption rate.

5 Strategies Analysis

5.1 The Effects of Rebate Strategies

This subsection discusses the effects of rebate strategies on the performances of channel members. Firstly, the effects of rebates for green products are explored, and the results are shown in Table 1.

Table 1 The effects of rebates for green products

$w_1^* - w_1^g$	$w_2^* - w_2^g$	$p_1^* - p_1^g$	$p_2^* - p_2^g$	$s^* - s^g$	$\pi_m^* - \pi_m^g$	$\pi_r^* - \pi_r^g$
$(-)$	(0)	$(-)$	(0)	$(-)$	$(-)$	$(-)$

Table 1 shows that: 1) The wholesale price and retail price of green products in Model N are lower than those in Model G. 2) Rebates for green products have no effects on the optimal decisions for conventional products. 3) Although the wholesale price and the retail price of green products increase in Model G, the rebate strategy increases the demand for green products, ultimately improving the profits of channel members. 4) The green level in Model G is higher than that in Model N. This shows that rebates for green products are not only beneficial to channel members, but also beneficial to the environment.

Secondly, the effects of rebates for conventional products are discussed, and the results are shown in Table 2.

Table 2 The effects of rebates for conventional products

$w_1^* - w_1^c$	$w_2^* - w_2^c$	$p_1^* - p_1^c$	$p_2^* - p_2^c$	$s^* - s^c$	$\pi_m^* - \pi_m^c$
(+)	(−)	(+)	(−)	(+)	(−)

Table 2 shows that, 1) The retail prices and wholesale prices of green products are lower in Model C than those in Model N. 2) The wholesale prices and retail prices of conventional products are higher in Model C than those in Model N. 3) Consistent with Model G, the manufacturer's profit in Model C is higher than that in Model N. 4) As the competitiveness of green products declines, the manufacturer reduces her investment in green levels to reduce the loss of profits, which eventually leads to the decline of green levels.

Because the retailer's profit in Model C is complex, the sign of $\pi_r^* - \pi_r^c$ could not be judged by theoretical analysis. It is discussed by numerical analysis. The parameter values are given as follow, $a = 10$, $\theta = 0.4$, $\lambda = 0.6$, the results are shown in Figure 2.

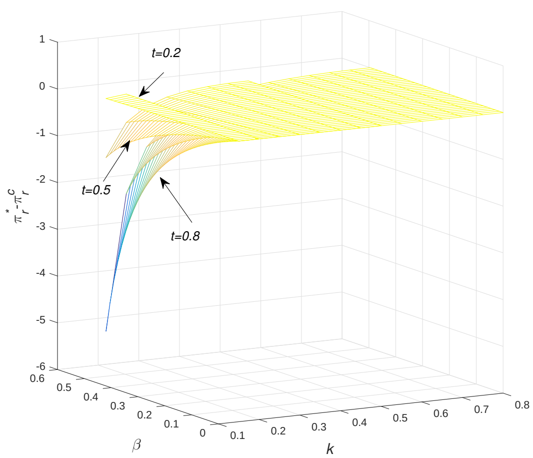

Figure 2 The effects of rebates on retailer's profit in Model C

Figure 2 shows that, 1) The profit for retailers is higher in Model C than that in Model N. 2) The difference in the retailer's profits between Model N and Model C increases with the increase of the rebate frequency and decreases with the increase in the redemption rate, and the increase of the rebate sensitivity factor widens this gap.

5.2 Rebate Strategy Preference

This subsection discusses the strategic preferences of channel members. First, the influence of the redemption rate on the rebate strategy preference of channel members is considered, and the parameter values are given as follows: $\beta = 0.3$, $t = 0.8$, $\lambda = 0.6$, $a = 10$, $\theta = 0.4$. The results are shown in Figure 3. Second, the influence of the rebate sensitivity factor on the rebate strategy preference of channel members is considered, and the value of the parameters are given as follows: $\beta = 0.3$, $k = 0.2$, $\lambda = 0.6$, $a = 10$, $\theta = 0.4$. The results are shown in Figure 4. Third, the influence of the rebate frequency on the rebate strategy preference of channel members is considered, and the value of the parameters are given as follows: $k = 0.3$, $t = 0.5$, $\lambda = 0.6$, $a = 10$, $\theta = 0.4$ and the results are shown in Figure 5.

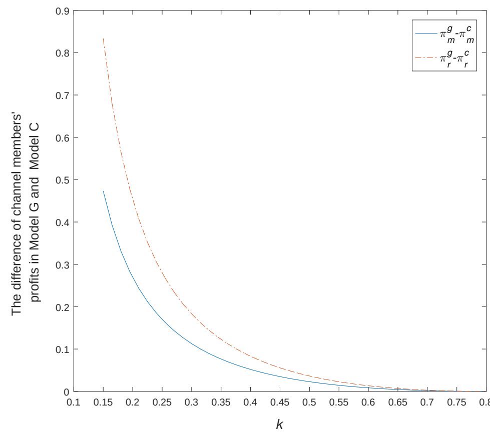

Figure 3 The influence of k on the rebate strategy preference

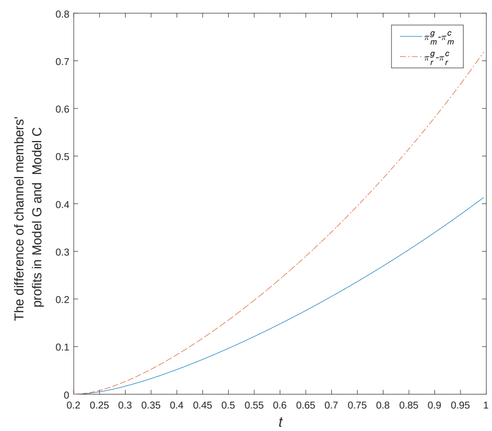

Figure 4 The influence of t on the rebate strategy preference

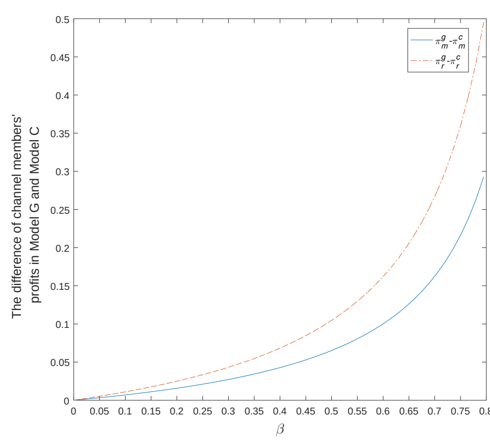

Figure 5 The influence of β on the rebate strategy preference

Figures 3∼5 show that, 1) the redemption rate, the rebate sensitivity factor, and the rebate frequency do not change the strategies preference of channel members. Both members prefer the rebate strategy for green products, which not only helps to increase the profits of channel members, but also helps to improve the green level. 2) The advantages of rebate strategies for green products are more significant with the decrease of the redemption rate and the increases of the rebate sensitivity factor and the rebate frequency. Moreover, the impact of different rebate strategies on the retailer is more significant than that on the manufacturer.

6 Model Extension

Based on the previous analysis, this section expands the model from the following two aspects: 1) The potential market demands are asymmetric, being a_1 and a_2 for green products and conventional products, respectively; 2) The substitution rates are asymmetric, i.e. $\theta_1 \neq \theta_2$.

6.1 Asymmetric Potential Market Demands

The demands for green products and conventional products without rebates are, respectively: $\hat{q}_1 = \frac{a_1-\theta a_2-p_1+\theta p_2+\lambda s}{1-\theta^2}$ and $\hat{q}_2 = \frac{a_2-\theta a_1-p_2+\theta p_1-\theta\lambda s}{1-\theta^2}$. The demands for green products and conventional products with the rebate strategy for green products are, respectively: $\hat{q}_1^g = \frac{a_1-\theta a_2-(p_1-tr)+\theta p_2+\lambda s}{1-\theta^2}$ and $\hat{q}_2^g = \frac{a_2-\theta a_1-p_2+\theta(p_1-tr)-\theta\lambda s}{1-\theta^2}$. The profit functions of the manufacturer and the retailer with the rebates for green products are, respectively:

$$\hat{\pi}_m^g = \beta(w_1-kr)\hat{q}_1^g + (1-\beta)w_1\hat{q}_1 + \beta w_2\hat{q}_2^g + (1-\beta)w_2\hat{q}_2 - \frac{1}{2}s^2, \tag{8}$$

$$\hat{\pi}_r^g = (p_1-w_1)[\beta\hat{q}_1^g + (1-\beta)\hat{q}_1] + (p_2-w_2)[\beta\hat{q}_2^g + (1-\beta)\hat{q}_2]. \tag{9}$$

The demands for green products and conventional products with the rebate strategy for conventional products are, respectively: $\hat{q}_1^c = \frac{a_1-\theta a_2-p_1+\theta(p_2-tr)+\lambda s}{1-\theta^2}$ and $\hat{q}_2^c = \frac{1}{1-\theta^2}[a_2-\theta a_1-(p_2-tr)+\theta p_1-\theta\lambda s]$. The profit functions of the manufacturer and the retailer with the rebates for conventional products are, respectively:

$$\hat{\pi}_m^c = \beta w_1\hat{q}_1^c + (1-\beta)w_1\hat{q}_1 + \beta(w_2-kr)\hat{q}_2^c + (1-\beta)w_2\hat{q}_2 - \frac{1}{2}s^2, \tag{10}$$

$$\hat{\pi}_r^c = \beta(p_1-w_1)\hat{q}_1^c + (1-\beta)(p_1-w_1)\hat{q}_1 + \beta(p_2-w_2)\hat{q}_2^c + (1-\beta)(p_2-w_2)\hat{q}_2. \tag{11}$$

6.2 Asymmetric Product Substitution Rate

The demands for green products and conventional products without rebate are, respectively: $\overline{q}_1 = \frac{a-\theta_1 a-p_1+\theta_1 p_2+\lambda s}{1-\theta_1\theta_2}$ and $\overline{q}_2 = \frac{a-\theta_2 a-p_2+\theta_2 p_1-\theta_2\lambda s}{1-\theta_1\theta_2}$. The demands for green products and conventional products with the rebate strategy for green products are, respectively: $\overline{q}_1^g = \frac{a-\theta_1 a-(p_1-tr)+\theta_1 p_2+\lambda s}{1-\theta_1\theta_2}$ and $\overline{q}_2^g = \frac{a-\theta_2 a-p_2+\theta_2(p_1-tr)-\theta_2\lambda s}{1-\theta_1\theta_2}$. The profit functions of the manufacturer and the retailer with the rebates for green product are, respectively:

$$\overline{\pi}_m^g = \beta(w_1-kr)\overline{q}_1^g + (1-\beta)w_1\overline{q}_1 + \beta w_2\overline{q}_2^g + (1-\beta)w_2\overline{q}_2 - \frac{1}{2}s^2, \tag{12}$$

$$\overline{\pi}_r^g = (p_1-w_1)[\beta\overline{q}_1^g + (1-\beta)\overline{q}_1] + (p_2-w_2)[\beta\overline{q}_2^g + (1-\beta)\overline{q}_2]. \tag{13}$$

The demands for green products and conventional products under the rebate strategy for conventional products are, respectively: $\overline{q}_1^c = \frac{a-\theta_1 a-p_1+\theta_1(p_2-tr)+\lambda s}{1-\theta_1\theta_2}$ and $\overline{q}_2^c = \frac{1}{1-\theta_1\theta_2}[a-\theta_2 a-$

$(p_2 - tr) + \theta_2 p_1 - \theta_2 \lambda s]$. The profit functions of the manufacturer and the retailer with the rebates for conventional products are, respectively:

$$\overline{\pi}_m^c = \beta w_1 \overline{q}_1^g + (1-\beta) w_1 \overline{q}_1 + \beta(w_2 - kr)\overline{q}_2^g + (1-\beta) w_2 \overline{q}_2 - \frac{1}{2}s^2, \tag{14}$$

$$\overline{\pi}_r^c = (p_1 - w_1)[\beta \overline{q}_1^g + (1-\beta)\overline{q}_1] + (p_2 - w_2)[\beta \overline{q}_2^g + (1-\beta)\overline{q}_2]. \tag{15}$$

6.3 Numerical Analysis

In this subsection, the extended models are discussed by numerical analysis. First, the influence of asymmetric potential market demands on the rebate strategy preference of channel members is considered. The parameter values are given as follows: $\theta = 0.4$, $k = 0.3$, $t = 0.5$, $\beta = 0.7$, $\lambda = 0.6$, $a_2 = 10$, with the results shown in Figure 6.

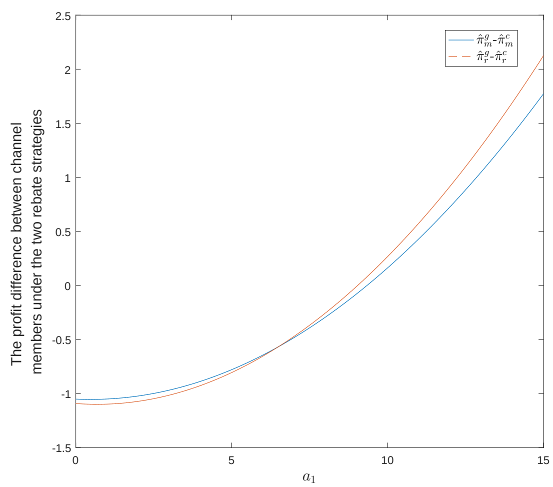

Figure 6 The influence of a_1 on the rebate strategy preference

Figure 6 shows that, 1) with the increasing of potential market demand for green products, the profit difference of channel members under different rebate strategies first decreases and then increases. 2) When the demand for green products is small, it is more favorable for channel members to choose the rebate strategy for conventional products. However, when the potential market of green products is high, channel members prefer the rebate strategy for green products. 3) The influence of a_1 on the profit difference for retailer is more significant than that on the profit difference for manufacturer.

Second, the influence of asymmetric product substitution rates on the rebate strategies preference of channel members is discussed. The parameter values are given as follows $k = 0.3$, $t = 0.5$, $\beta = 0.7$, $\lambda = 0.6$, $a = 10$, $\theta_2 = 0.4$. The results are shown in Figure 7.

Figure 7 shows that, as θ_1 increases, the profit difference between channel members with different rebate strategies first decreases and then increases. When θ_1 is small, the rebates for green products decrease the actual price of green products, thereby attracting the consumers in the potential market of green products, ultimately leading to an increase in channel members' profits. When θ_1 is large, green products have a high substitution value for conventional products, and then the rebates for green products are attractive for customers both from the conventional products market and green products market, which increase the market demand for green products, ultimately increasing the profits of channel members. In summary, when θ_1 is small or large, the advantage of rebates for green products is more significant, but asymmetric product substitution rates do not change the channel members' rebate strategy preferences.

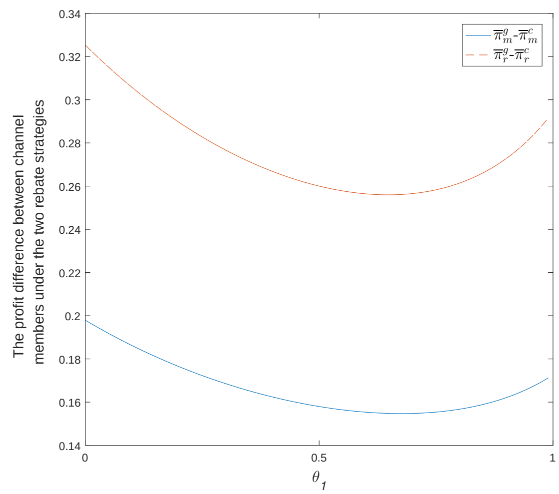

Figure 7 The influence of θ_1 on the rebate strategy preference

7 Conclusion

This study constructs a green supply chain model composed of one manufacturer and one retailer, and analyzes two rebate strategies for manufacturers simultaneously producing green products and conventional products. The differences between the rebate strategies are compared, with the following results.

1) In the green supply chain, the rebate strategies help increase the profits of channel members. Specifically, the more significant the slippage effects are (the smaller the k or the larger the t), the more beneficial the rebate strategies are, and the effects of the rebate strategies are more significant for the retailer than they are for the manufacturer. In addition, the rebate strategy for green products is more beneficial to channel members and to the environment than the rebate strategy for conventional products.

2) In Model G, the higher the rebate sensitivity factor is, the higher the green level is. This means that the channel members can improve the rebate sensitivity factor through promotions and achieve a win-win situation in terms of protecting the environment and enhancing their profits. However, in Model C, the increase in the rebate sensitivity factor reduces the green level. In this case, the channel members should be cautious about promoting rebate strategies.

3) When the potential market demands of products are asymmetric, it is more beneficial for channel members to choose products with higher potential market demand to employ the rebate strategy. This shows that the potential market demand is an important factor affecting the choice of the rebate strategy for channel members.

Although this study provides important conclusions, there are still many areas that call for further study. 1) This study only considers manufacturer rebate promotions and does not consider the rebate strategy of retailers. 2) It is assumed that consumers are homogeneous, and the heterogeneity of consumers is not considered. 3) This study mainly assumes a deterministic market demand, and does not consider the uncertainty of market demand caused by consumers' unfamiliarity with green products.

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Appendix

Proof of Proposition 1

$$\frac{\partial \pi_m^g}{\partial \beta}=\frac{a^2kt(k-t)^2(1-\theta)^3(\theta+1)}{A^2}>0,$$
$$\frac{\partial \pi_r^g}{\partial \beta}=\frac{8a^2k^2t^2(k-t)^2(1-\beta)(\theta-1)^4(\theta+1)^2}{A^3}>0,$$
$$\frac{\partial q_1^g}{\partial \beta}=\frac{at(k+t)(k-t)^2(\theta-1)^2(\theta+1)}{A^2}>0,$$
$$\frac{\partial q_2^g}{\partial \beta}=-\frac{a\theta t(k+t)(k-t)^2(\theta-1)^2(\theta+1)}{A^2}<0,$$
$$\frac{\partial s^g}{\partial \beta}=\frac{2akt\lambda(k-t)^2(\theta-1)^2(\theta+1)}{A^2}>0,$$
$$\frac{\partial r^g}{\partial \beta}=\frac{a(\theta^2-1)(k-t)(\theta-1)[2kt\lambda^2-(1-\theta^2)(k^2+t^2+6kt)]}{A^2}.$$

Because $0<\lambda<\sqrt{\frac{(1-\theta^2)[8kt-\beta(k^2+t^2+6kt)]}{2kt(1-\beta)}}$, then $0<2kt\lambda^2<\frac{1}{1-\beta}(1-\theta^2)[8kt-\beta(k^2+t^2+6kt)]$. So, $2kt\lambda^2-(1-\theta^2)(k^2+t^2+6kt)<-\frac{(1-\theta^2)(k-t)^2}{1-\beta}<0$, it means $\frac{\partial r^g}{\partial \beta}>0$.

$$\frac{\partial p_1^g}{\partial \beta}=\frac{2akt(k-t)(\theta-1)^2(\theta+1)[k\lambda^2-(1-\theta^2)(k+3t)]}{A^2}.$$

Because $0<\lambda<\sqrt{\frac{(1-\theta^2)[8kt-\beta(k^2+t^2+6kt)]}{2kt(1-\beta)}}$, then $0<k\lambda^2<\frac{(1-\theta^2)[8kt-\beta(k^2+t^2+6kt)]}{2t(1-\beta)}$. So, $k\lambda^2-(1-\theta^2)(k+3t)<\frac{(1-\theta^2)(k-t)[6t-\beta(k+5t)]}{2t(1-\beta)}<0$, it means $\frac{\partial p_1^g}{\partial \beta}>0$.

$$\frac{\partial q_1}{\partial \beta}=\frac{2akt(k-t)(\theta-1)(k\theta^2-3t-k+3\theta^2t+t\lambda^2)}{A^2}<0,$$
$$\frac{\partial q_2}{\partial \beta}=-\frac{2ak\theta t(k-t)(\theta-1)(k\theta^2-3t-k+3\theta^2t+t\lambda^2)}{A^2}>0,$$
$$\frac{\partial w_1^g}{\partial \beta}=\frac{2akt(k-t)(\theta-1)^2(\theta+1)(2k\theta^2-2t-2k+k\lambda^2+2\theta^2t)}{A^2}>0.$$

Proof of Proposition 2

$$\frac{\partial \pi_m^g}{\partial k}=\frac{a^2\beta t(k^2-t^2)(\beta-1)(\theta-1)^3(\theta+1)}{A^2}<0,$$
$$\frac{\partial \pi_r^g}{\partial k}=\frac{8a^2\beta kt^2(k^2-t^2)(\beta-1)^2(\theta-1)^4(\theta+1)^2}{A^3}<0,$$
$$\frac{\partial q_1^g}{\partial k}=\frac{at(\beta-1)(\theta-1)\{2t^2(1-\beta)\lambda^2-(1-\theta^2)[8t^2-\beta(5t^2+k^2+2kt)]\}}{A^2}.$$

Because $0<\lambda<\sqrt{\frac{(1-\theta^2)[8kt-\beta(k^2+t^2+6kt)]}{2kt(1-\beta)}}$, then $2t^2(1-\beta)\lambda^2<\frac{t}{k}(1-\theta^2)[8kt-\beta(k^2+t^2+6kt)]$. So $2t^2(1-\beta)\lambda^2-(1-\theta^2)[8t^2-\beta(5t^2+k^2+2kt)]<\frac{\beta(1-\theta^2)(k^2-t^2)(k+t)}{k}<0$, it means $\frac{\partial q_1^g}{\partial k}<0$.

$$\frac{\partial q_2^g}{\partial k} = -\theta \frac{\partial q_1^g}{\partial k} > 0.$$
$$\frac{\partial s^g}{\partial k} = -\frac{2a\beta t\lambda(k^2-t^2)(\beta-1)(\theta-1)^2(\theta+1)}{A^2} < 0.$$
$$\frac{\partial r^g}{\partial k} = \frac{a(\theta-1)^2(\theta+1)\{2t^2(1-\beta)\lambda^2-(1-\theta^2)[8t^2-\beta(7t^2-k^2+2kt)]\}}{A^2}.$$

Because $0<\lambda<\sqrt{\frac{(1-\theta^2)[8kt-\beta(k^2+t^2+6kt)]}{2kt(1-\beta)}}$, then $2t^2(1-\beta)\lambda^2 < \frac{t}{k}(1-\theta^2)[8kt-\beta(k^2+t^2+6kt)]$. So $2t^2(1-\beta)\lambda^2-(1-\theta^2)[8t^2-\beta(7t^2-k^2+2kt)] < \frac{(1-\theta^2)\beta(k^2-t^2)(t-k)}{k} < 0$, it means $\frac{\partial r^g}{\partial k} < 0$.

$$\frac{\partial p_1^g}{\partial k} = \frac{a\beta t(\theta-1)^2(\theta+1)\{(1-\beta)2k^2\lambda^2-(\theta^2-1)[\beta(k^2+5t^2+2kt)-2k^2-6t^2]\}}{A^2}.$$

Because $0<\lambda<\sqrt{\frac{(1-\theta^2)[8kt-\beta(k^2+t^2+6kt)]}{2kt(1-\beta)}}$, then $2k^2(1-\beta)\lambda^2 < \frac{k}{t}(1-\theta^2)[8kt-\beta(k^2+t^2+6kt)]$. So $(1-\beta)2k^2\lambda^2-(\theta^2-1)[\beta(k^2+5t^2+2kt)-2k^2-6t^2] < \frac{1}{t}(1-\theta^2)(k^2-t^2)[6t-\beta(k+5t)] < 0$, it means $\frac{\partial p_1^g}{\partial k} < 0$. $\frac{\partial w_1^g}{\partial k} = \frac{\partial p_1^g}{\partial k} < 0$.

Proof of Proposition 3

$$\frac{\partial \pi_m^g}{\partial t} = -\frac{a^2\beta k(k^2-t^2)(\beta-1)(\theta-1)^3(\theta+1)}{A^2} > 0,$$
$$\frac{\partial \pi_r^g}{\partial t} = -\frac{8a^2\beta k^2 t(k^2-t^2)(\beta-1)^2(\beta-1)^4(\theta+1)^2}{A^3} > 0,$$
$$\frac{\partial q_1^g}{\partial t} = -\frac{ak(\beta-1)(\theta-1)\{2t^2(1-\beta)\lambda^2-(1-\theta^2)[8t^2-\beta(5t^2+k^2+2kt)]\}}{A^2}.$$

Because $0<\lambda<\sqrt{\frac{(1-\theta^2)[8kt-\beta(k^2+t^2+6kt)]}{2kt(1-\beta)}}$, then $2t^2(1-\beta)\lambda^2 < \frac{t}{k}(1-\theta^2)[8kt-\beta(k^2+t^2+6kt)]$. So $2t^2(1-\beta)\lambda^2-(1-\theta^2)[8t^2-\beta(5t^2+k^2+2kt)] < \frac{\beta(1-\theta^2)(k^2-t^2)(k+t)}{k} < 0$, it means $\frac{\partial q_1^g}{\partial t} > 0$. $\frac{\partial q_2^g}{\partial t} = -\frac{\partial q_1^g}{\partial t} < 0$.

$$\frac{\partial r^g}{\partial t} = -\frac{a(\theta-1)^2(\theta+1)\{2k^2(1-\beta)\lambda^2-(1-\theta^2)[8t^2-\beta(7t^2-k^2+2kt)]\}}{A^2}.$$

Because $0<\lambda<\sqrt{\frac{(1-\theta^2)[8kt-\beta(k^2+t^2+6kt)]}{2kt(1-\beta)}}$, then $2k^2(1-\beta)\lambda^2 < \frac{k}{t}(1-\theta^2)[8kt-\beta(k^2+t^2+6kt)]$. So $2k^2(1-\beta)\lambda^2-(1-\theta^2)[8t^2-\beta(7t^2-k^2+2kt)] < \frac{(1-\theta^2)\beta(k^2-t^2)(t-k)}{t} < 0$, it means $\frac{\partial r^g}{\partial k} < 0$.

$$\frac{\partial s^g}{\partial t} = \frac{2a\beta k\lambda(k^2-t^2)(\beta-1)(\theta-1)^2(\theta+1)}{A^2} > 0.$$
$$\frac{\partial p_1^g}{\partial t} = -\frac{a\beta k(\theta-1)^2(\theta+1)\{(1-\beta)2k^2\lambda^2-(\theta^2-1)[\beta(k^2+5t^2+2kt)-2k^2-6t^2]\}}{A^2}.$$

Because $0<\lambda<\sqrt{\frac{(1-\theta^2)[8kt-\beta(k^2+t^2+6kt)]}{2kt(1-\beta)}}$, then $2k^2(1-\beta)\lambda^2 < \frac{k}{t}(1-\theta^2)[8kt-\beta(k^2+t^2+6kt)]$. So $(1-\beta)2k^2\lambda^2-(\theta^2-1)[\beta(k^2+5t^2+2kt)-2k^2-6t^2] < \frac{1}{t}(1-\theta^2)(k^2-t^2)[6t-\beta(k+5t)] < 0$, it means $\frac{\partial p_1^g}{\partial t} > 0$. $\frac{\partial w_1^g}{\partial t} = \frac{\partial p_1^g}{\partial t} > 0$.

Proof of Proposition 4

$$\frac{\partial \pi_m^c}{\partial \beta} = -\frac{a^2kt(\theta^2-1)(k-t)^2(\lambda^2+4\theta-4)^2}{C^2} > 0,$$
$$\frac{\partial s^c}{\partial \beta} = -\frac{8ak\theta t\lambda(\theta^2-1)(k-t)^2(\lambda^2+4\theta-4)}{C^2} < 0,$$

$$\frac{\partial r^c}{\partial \beta} = \frac{a}{C^2}(\theta^2-1)(k-t)(\lambda^2+4\theta-4)\{[(1-\theta^2)(k+t)^2+4kt(1+\theta^2)]\lambda^2 +4(\theta^2-1)(k^2+6kt+t^2)\}<0,$$

$$\frac{\partial w_1^c}{\partial \beta} = -\frac{4ak\theta t\lambda^2(\theta^2-1)(k-t)^2(\lambda^2+4\theta-4)}{C^2}<0,$$

$$\frac{\partial w_2^c}{\partial \beta} = \frac{4akt(\theta^2-1)(k^2-t^2)(\lambda^2+4\theta-4)(4\theta^2+\lambda^2-4)}{C^2}>0,$$

$$\frac{\partial q_1^c}{\partial \beta} = -\frac{a\theta t(\theta^2-1)(k-t)^2(\lambda^2+4\theta-4)(4k+4t+k\lambda^2-t\lambda^2)}{C^2}<0,$$

$$\frac{\partial q_2^c}{\partial \beta} = -\frac{at(\theta^2-1)(\lambda^2-4)(k+t)(k-t)^2(\lambda^2+4\theta-4)}{C^2}>0,$$

$$\frac{\partial p_1^c}{\partial \beta} = -\frac{6ak\theta t\lambda^2(\theta^2-1)(k-t)^2(\lambda^2+4\theta-4)}{C^2}<0,$$

$$\frac{\partial p_2^c}{\partial \beta} = \frac{2akt(\theta^2-1)(k^2+2kt-3t^2)[16(\theta-1)^2(\theta+1)+\lambda^2(4\theta^2+4\theta+\lambda^2-8)]}{C^2}>0.$$

Proof of Proposition 5

$$\frac{\partial \pi_m^c}{\partial k} = \frac{a^2\beta t(\theta^2-1)(k^2-t^2)(\beta-1)(\lambda^2+4\theta-4)^2}{C^2}<0,$$

$$\frac{\partial p_1^c}{\partial k} = \frac{6a\beta\theta t\lambda^2(\theta^2-1)(k^2-t^2)(\beta-1)(\lambda^2+4\theta-4)}{C^2}>0,$$

$$\frac{\partial p_2^c}{\partial k} = \frac{a\beta t}{C^2}(1-\theta^2)(\lambda^2+4\theta-4)\{[\lambda^2-4(1-\theta^2)][\beta(k^2+5t^2+2kt)-2k^2-6t^2]-\beta\theta^2(k-t)^2\lambda^2\}.$$

Because

$$\lambda < \sqrt{\frac{(1-\theta^2)[\beta(k^2+t^2+6kt)-8kt]}{2kt(\beta-1)}} < \sqrt{\frac{4(1-\theta^2)[\beta(k+t)^2-4kt(2-\beta)]}{\beta(k+t)^2-4kt(2-\beta)-\beta\theta^2(k-t)^2}} < \sqrt{\frac{4(1-\theta^2)[\beta(k^2+5t^2+2kt)-2k^2-6t^2]}{\beta(k^2+5t^2+2kt)-2k^2-6t^2-\beta\theta^2(k-t)^2}},$$

so $[\lambda^2-4(1-\theta^2)][\beta(k^2+5t^2+2kt)-2k^2-6t^2]-\beta\theta^2(k-t)^2\lambda^2>0$, it means $\frac{\partial p_2^c}{\partial k}<0$.

$$\frac{\partial s^c}{\partial k} = \frac{8a\beta\theta t\lambda(\theta^2-1)(k^2-t^2)(\theta-1)(\lambda^2+4\theta-4)}{C^2}>0,$$

$$\frac{\partial r^c}{\partial k} = \frac{a(1-\theta^2)(\lambda^2+4\theta-4)\{\beta\theta^2(k-t)^2\lambda^2+[\lambda^2-4(1-\theta^2)][\beta(7t^2-k^2+2kt)-8t^2]\}}{C^2}.$$

Because

$$\lambda < \sqrt{\frac{(1-\theta^2)[\beta(k^2+t^2+6kt)-8kt]}{2kt(\beta-1)}} < \sqrt{\frac{4(1-\theta^2)[\beta(k+t)^2-4kt(2-\beta)]}{\beta(k+t)^2-4kt(2-\beta)-\beta\theta^2(k-t)^2}} < \sqrt{\frac{4(1-\theta^2)[\beta(7t^2-k^2+2kt)-8t^2]}{\beta\theta^2(k-t)^2+\beta(7t^2-k^2+2kt)-8t^2}},$$

so $[\beta\theta^2(k-t)^2+\beta(7t^2-k^2+2kt)-8t^2]\lambda^2+4(\theta^2-1)[\beta(7t^2-k^2+2kt)-8t^2]>0$, it means $\frac{\partial r^c}{\partial k}<0$.

$$\frac{\partial w_1^c}{\partial k} = \frac{4a\beta\theta t\lambda^2(\theta^2-1)(k^2-t^2)(\beta-1)(\lambda^2+4\theta-4)}{C^2}>0,$$

$$\frac{\partial w_2^c}{\partial k} = \frac{a\beta t}{-C^2}(\theta^2-1)(\lambda^2+4\theta-4)\{[\lambda^2-4(1-\theta^2)][\beta(3k^2+3t^2+2kt)-4(k^2+t^2)]+\beta\theta^2(k-t)^2\lambda^2\}.$$

Because

$$\lambda < \sqrt{\frac{(1-\theta^2)[\beta(k^2+t^2+6kt)-8kt]}{2kt(\beta-1)}} < \sqrt{\frac{4(1-\theta^2)[\beta(k+t)^2-4kt(2-\beta)]}{\beta(k+t)^2-4kt(2-\beta)-\beta\theta^2(k-t)^2}} < \sqrt{\frac{4(1-\theta^2)[\beta(3k^2+3t^2+2kt)-4k^2-4t^2]}{\beta(3k^2+3t^2+2kt)-4k^2-4t^2+\beta\theta^2(k-t)^2}},$$

so $[\beta(3k^2+3t^2+2kt)-4k^2-4t^2+\beta\theta^2(k-t)^2]\lambda^2-4(1-\theta^2)[\beta(3k^2+3t^2+2kt)-4k^2-4t^2]>0$, it means $\frac{\partial w_2^c}{\partial k}<0$.

$$\frac{\partial q_1^c}{\partial k} = \frac{a\theta t}{C^2}(\beta-1)(\lambda^2+4\theta-4)\{[\beta(7t^2-k^2+2kt)-8t^2+\beta\theta^2(k-t)^2]\lambda^2 -4(1-\theta^2)[\beta(k^2+5t^2+2kt)-8t^2]\}.$$

Because

$$\lambda < \sqrt{\frac{(1-\theta^2)[\beta(k^2+t^2+6kt)-8kt]}{2kt(\beta-1)}} < \sqrt{\frac{4(1-\theta^2)[\beta(k+t)^2-4kt(2-\beta)]}{\beta(k+t)^2-4kt(2-\beta)-\beta\theta^2(k-t)^2}} < \sqrt{\frac{4(1-\theta^2)[\beta(k^2+5t^2+2kt)-8t^2]}{\beta(7t^2-k^2+2kt)-8t^2+\beta\theta^2(k-t)^2}},$$

so $[\beta(7t^2-k^2+2kt)-8t^2+\beta\theta^2(k-t)^2]\lambda^2-4(1-\theta^2)[\beta(k^2+5t^2+2kt)-8t^2]>0$, it means $\frac{\partial q_1^c}{\partial k}>0$.

$$\frac{\partial q_2^c}{\partial k} = -\frac{at}{C^2}(\beta-1)(\lambda^2+4\theta-4)\{[\beta(k^2+5t^2+2kt)-8t^2-\beta\theta^2(k^2+2kt-3t^2)]\lambda^2 -4(1-\theta^2)[\beta(k^2+5t^2+2kt)-8t^2]\}.$$

Because

$$\lambda < \sqrt{\frac{(1-\theta^2)[\beta(k^2+t^2+6kt)-8kt]}{2kt(\beta-1)}} < \sqrt{\frac{4(1-\theta^2)[\beta(k+t)^2-4kt(2-\beta)]}{\beta(k+t)^2-4kt(2-\beta)-\beta\theta^2(k-t)^2}} < \sqrt{\frac{4(1-\theta^2)[\beta(k^2+5t^2+2kt)-8t^2]}{\beta(k^2+5t^2+2kt)-8t^2-\beta\theta^2(k^2+2kt-3t^2)}},$$

so $[\beta(k^2+5t^2+2kt)-8t^2-\beta\theta^2(k^2+2kt-3t^2)]\lambda^2-4(1-\theta^2)[\beta(k^2+5t^2+2kt)-8t^2]>0$, it means $\frac{\partial q_2^c}{\partial k}<0$.

Proof of Proposition 6

$$\frac{\partial \pi_m^c}{\partial t} = -\frac{a^2\beta k(\theta^2-1)(k^2-t^2)(\beta-1)(\lambda^2+4\theta-4)^2}{C^2} > 0,$$
$$\frac{\partial p_1^c}{\partial t} = -\frac{6a\beta\theta k\lambda^2(\theta^2-1)(k^2-t^2)(\beta-1)(\lambda^2+4\theta-4)}{C^2} < 0,$$

$$\frac{\partial p_2^c}{\partial t} = \frac{a\beta k}{C^2}(\theta^2-1)(\lambda^2+4\theta-4)\{[\beta(k^2+5t^2+2kt)-2k^2-6t^2-\beta\theta^2(k-t)^2]\lambda^2 \\ -4(1-\theta^2)[\beta(k^2+5t^2+2kt)-2k^2-6t^2]\}.$$

Because

$$\lambda < \sqrt{\frac{(1-\theta^2)[\beta(k^2+t^2+6kt)-8kt]}{2kt(\beta-1)}} < \sqrt{\frac{4(1-\theta^2)[\beta(k+t)^2-4kt(2-\beta)]}{\beta(k+t)^2-4kt(2-\beta)-\beta\theta^2(k-t)^2}} \\ < \sqrt{\frac{4(1-\theta^2)[\beta(k^2+5t^2+2kt)-2k^2-6t^2]}{\beta(k^2+5t^2+2kt)-2k^2-6t^2-\beta\theta^2(k-t)^2}},$$

so $[\beta(k^2+5t^2+2kt)-2k^2-6t^2-\beta\theta^2(k-t)^2]\lambda^2-4(1-\theta^2)[\beta(k^2+5t^2+2kt)-2k^2-6t^2] > 0$, it means $\frac{\partial p_2^c}{\partial t} > 0$.

$$\frac{\partial s^c}{\partial t} = -\frac{8a\beta\theta k\lambda(\theta^2-1)(k^2-t^2)(\beta-1)(\lambda^2+4\theta-4)}{C^2} < 0,$$
$$\frac{\partial r^c}{\partial t} = \frac{a(\theta^2-1)(\lambda^2+4\theta-4)\{\beta\theta^2(k-t)^2\lambda^2+[\lambda^2+4(\theta^2-1)][\beta(7k^2-t^2+2kt)-8k^2]\}}{C^2}.$$

Because

$$\lambda < \sqrt{\frac{(1-\theta^2)[\beta(k^2+t^2+6kt)-8kt]}{2kt(\beta-1)}} < \sqrt{\frac{4(1-\theta^2)[\beta(k+t)^2-4kt(2-\beta)]}{\beta(k+t)^2-4kt(2-\beta)-\beta\theta^2(k-t)^2}} \\ < \sqrt{\frac{4(1-\theta^2)[\beta(7k^2-t^2+2kt)-8k^2]}{\beta(7k^2-t^2+2kt)-8k^2+\beta\theta^2(k-t)^2}},$$

so $\beta\theta^2(k-t)^2\lambda^2+[\lambda^2+4(\theta^2-1)][\beta(7k^2-t^2+2kt)-8k^2] > 0$, it means $\frac{\partial r^c}{\partial t} > 0$.

$$\frac{\partial w_1^c}{\partial t} = -\frac{4a\beta\theta k\lambda^2(\theta^2-1)(k^2-t^2)(\beta-1)(\lambda^2+4\theta-4)}{C^2} < 0,$$
$$\frac{\partial w_2^c}{\partial t} = \frac{a\beta k}{C^2}(\theta^2-1)(\lambda^2+4\theta-4)\{[\lambda^2+4(\theta^2-1)][\beta(3k^2+3t^2+2kt)-4(k^2+t^2)] \\ +\beta\theta^2(k-t)^2\lambda^2\}.$$

Because

$$\lambda < \sqrt{\frac{(1-\theta^2)[\beta(k^2+t^2+6kt)-8kt]}{2kt(\beta-1)}} < \sqrt{\frac{4(1-\theta^2)[\beta(k+t)^2-4kt(2-\beta)]}{\beta(k+t)^2-4kt(2-\beta)-\beta\theta^2(k-t)^2}} \\ < \sqrt{\frac{4(1-\theta^2)[\beta(3k^2+3t^2+2kt)-4k^2-4t^2]}{\beta(3k^2+3t^2+2kt)-4k^2-4t^2+\beta\theta^2(k-t)^2}},$$

so $[\beta(3k^2+3t^2+2kt)-4k^2-4t^2+\beta\theta^2(k-t)^2]\lambda^2+4(\theta^2-1)[\beta(3k^2+3t^2+2kt)-4k^2-4t^2] > 0$, it means $\frac{\partial w_2^c}{\partial t} > 0$.

$$\frac{\partial q_1^c}{\partial t} = -\frac{ak\theta}{C^2}(\beta-1)(\lambda^2+4\theta-4)\{[\beta(7t^2-k^2+2kt)-8t^2+\beta\theta^2(k-t)^2]\lambda^2 \\ -4(1-\theta^2)[\beta(k^2+5t^2+2kt)-8t^2]\}.$$

Because

$$\lambda < \sqrt{\frac{(1-\theta^2)[\beta(k^2+t^2+6kt)-8kt]}{2kt(\beta-1)}} < \sqrt{\frac{4(1-\theta^2)[\beta(k+t)^2-4kt(2-\beta)]}{\beta(k+t)^2-4kt(2-\beta)-\beta\theta^2(k-t)^2}}$$
$$< \sqrt{\frac{4(1-\theta^2)[\beta(k^2+5t^2+2kt)-8t^2]}{\beta(7t^2-k^2+2kt)-8t^2+\beta\theta^2(k-t)^2}},$$

so $[\beta(7t^2-k^2+2kt)-8t^2+\beta\theta^2(k-t)^2]\lambda^2-4(1-\theta^2)[\beta(k^2+5t^2+2kt)-8t^2]>0$, it means $\frac{\partial q_1^c}{\partial t}<0$.

$$\frac{\partial q_2^c}{\partial t} = \frac{ak}{C^2}(\beta-1)(\lambda^2+4\theta-4)\{[\beta(k^2+5t^2+2kt)-8t^2-\beta\theta^2(k^2+2kt-3t^2)]\lambda^2$$
$$-4(1-\theta^2)[\beta(k^2+5t^2+2kt)-8t^2]\}.$$

Because

$$\lambda < \sqrt{\frac{(1-\theta^2)[\beta(k^2+t^2+6kt)-8kt]}{2kt(\beta-1)}} < \sqrt{\frac{4(1-\theta^2)[\beta(k+t)^2-4kt(2-\beta)]}{\beta(k+t)^2-4kt(2-\beta)-\beta\theta^2(k-t)^2}}$$
$$< \sqrt{\frac{4(1-\theta^2)[\beta(k^2+5t^2+2kt)-8t^2]}{\beta(k^2+5t^2+2kt)-8t^2-\beta\theta^2(k^2+2kt-3t^2)}},$$

so $[\beta(k^2+5t^2+2kt)-8t^2-\beta\theta^2(k^2+2kt-3t^2)]\lambda^2-4(1-\theta^2)[\beta(k^2+5t^2+2kt)-8t^2]>0$, it means $\frac{\partial q_2^c}{\partial t}>0$.

Proof of the results in Table 1 and Table 2.

Because the optimal decisions and profits of channel members in the model without manufacturer rebates are the special outcomes in the models with manufacturer rebates while $\beta=0$. So, according to Proposition 1 and Proposition 4, we can easily verify the results in Table 1 and Table 2.