



Numerical modeling of concrete hydraulic fracturing with extended finite element method

REN QingWen^{1*}, DONG YuWen² & YU TianTang¹

¹ Department of Engineering Mechanics, Hohai University, Nanjing 210098, China;

² College of River and Ocean Engineering, Chongqing Jiaotong University, Chongqing 400074, China

* E-mail: renqw@hhu.edu.cn

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The extended finite element method (XFEM) is a new numerical method for modeling discontinuity. Research about numerical modeling for concrete hydraulic fracturing by XFEM is explored. By building the virtual work principle of the fracture problem considering water pressure on the crack surface, the governing equations of XFEM for hydraulic fracture modeling are derived. Implementation of the XFEM for hydraulic fracturing is presented. Finally, the method is verified by two examples and the advantages of the XFEM for hydraulic fracturing analysis are displayed.

extended finite element method (XFEM), hydraulic fracture, concrete, numerical modeling

Introduction Up to now, the research about hydraulic fracturing of the high dam and the large-diameter tunnel under high water pressure is still not mature. It is necessary to research the method of hydraulic fracture analysis.

At present, there are many numerical methods used for hydraulic fracturing analysis, in which finite element method (FEM) is the most widely used method. In essence, FEM is a numerical method of mechanics of continuous media. It is requisite to improve the FEM in order to use FEM to analyze discontinuous problems such as hydraulic fracture. The methods of improving FEM for discontinuous problems can be classified into two types: unfixed-mesh method and fixed-mesh method. Unfixed-mesh method means remeshing after crack growth. The advantage of unfixed-mesh method is that it is unnecessary to develop new FEM software and existing software can satisfy the demand. But for the unfixed mesh method, it is needed to conform crack surfaces with the element boundary and to refine the mesh or use special element near the crack tips. After crack growth,

remeshing is necessary. So unfixed-mesh method is cumbersome and discommodious in application. On the contrary, fixed-mesh method means that the mesh is unchanged and the crack surfaces are presented by modifying approximation or constitutive relation of the elements. For these two methods, fixed-mesh method is more convenient. And the extended finite element method (XFEM) belongs to fixed-mesh method.

XFEM was brought forward by Belytschko, et al. in 1999. XFEM is one of the most effective methods for discontinuous problems [1, 2]. In recent years, XFEM has been widely used in many fields of discontinuous problems, especially in fracture mechanics because XFEM overcomes the difficulties of FEM used for fracture analysis, such as restriction in meshing and remeshing after crack growth [3–5]. To date, most researches about XFEM in fracture mechanics have aimed at cracks with free surfaces and linear elastic problems. By far, researches about cohesive crack growth and contact problems considering surface loading by XFEM were ...



Laser-generated thermoelastic acoustic sources and acoustic waves in anisotropic plate

XU BaiQiang^{*}, WANG Feng, FENG Jun, WANG JiJun, SUN HongXiang & LUO Ying

Faculty of Science, Jiangsu University, Zhenjiang 212013, China

^{*}E-mail: bqxu@ujs.edu.cn

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The effect of anisotropy on the ultrasound wave generation and propagation in the unidirectional fiber-reinforced composite plate has been investigated. A quantitative numerical model for the laser-generated ultrasound in the thermoelastic regime was presented by using a finite element method. All factors, such as spatial and time distributions of the incident laser beam, optical penetration, thermal diffusivity, and source-receiver distance can be taken into account. Numerical results show that the effect on ultrasound waveform of the size of the laser volume source produces strong bipolar longitudinal waves and improves the amplitude and directivity of the longitudinal waves. A fiber-reinforced composite material exhibits isotropic or homogenous behavior for ultrasonic wave propagation perpendicular to the fiber direction. For ultrasonic propagation along the fiber direction, ultrasonic dispersion resulting from the inhomogeneous nature of the material affects the laser ultrasonic waveforms. As the dimensions of the laser pulse are increased in space and time, the displacement waveform becomes broader and its magnitude decreases.

laser ultrasound, numerical simulation, anisotropic material

Efforts to model the process of laser generation and propagation of ultrasound in isotropic and homogeneous materials have been extremely successful [1–5]. For materials that display isotropic and homogeneous behavior on a macroscopic scale, laser ultrasound has been proven to be an extremely robust materials characterization tool [6–10]. For materials that cannot be considered isotropic or homogenous, laser ultrasound can only be applied with limited success without a quantitative model describing ultrasonic behavior of these materials.

The early work by Achenbach [11] on wave propagation in fiber-reinforced composites assumed that the scale on which the composite could be viewed as inhomogeneous was much smaller than the wavelength of

the ultrasound. It follows that for a range of ultrasonic frequencies, composites can be viewed as anisotropic, homogeneous materials. This anisotropy induced by the symmetry of the material inhomogeneities is best illustrated in its simplest form by considering the fibers with a hexagonal symmetry. This type of symmetry has led researchers to model a unidirectional fiber composite as a transversely isotropic solid. Work by Dubois et al. [12] derived results for laser ultrasonic in composites of orthotropic symmetry, but the elastic behavior of the composite was assumed to be the same as for single crystal solids of the same symmetry. Hurley and Spicer [13] have developed analytical solutions for generation in transversely isotropic materials by a laser line source and a laser point source. However, ...



A hybrid-stress solid-shell element for non-linear analysis of piezoelectric structures

YAO LinQuan^{1*} & SZE K Y²

¹ School of Mathematical Science, Soochow University, Suzhou 215006, China;

² Department of Mechanical Engineering, University of Hong Kong, Hong Kong, China

* E-mail: lqyao@suda.edu.cn

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This paper presents eight-node solid-shell elements for geometric non-linear analysis of piezoelectric structures. To subdue shear, trapezoidal and thickness locking, the assumed natural strain method and an ad hoc modified generalized laminate stiffness matrix are employed. With the generalized stresses arising from the modified generalized laminate stiffness matrix assumed to be independent from the ones obtained from the displacement, an extended Hellinger-Reissner functional can be derived. By choosing the assumed generalized stresses similar to the assumed stresses of a previous solid element, a hybrid-stress solid-shell element is formulated. The presented finite shell element is able to model arbitrary curved shell structures. Non-linear numerical examples demonstrate the ability of the proposed model to analyze nonlinear piezoelectric devices.

piezoelectric structures, solid-shell element, hybrid-stress, geometric non-linear

The non-linear characteristics can significantly influence the performance of piezoelectric structures. The non-linear problems of piezoelectric structures comprise of geometrically nonlinear and material nonlinearity. Generally, only the strong electric field subject to piezoelectric material, just the material nonlinearity of piezoelectric structures is considered [1–4]. When the electric field is not too large, geometrically nonlinear analysis is only considered for nonlinear problem. The researcher has already carried out many beneficial researches concerning geometrically non-linearly [5–11]. The nonlinear electric potential distribution in piezoelectric layer was described by introducing internal electric potential node in ref. [5]. A solid shell element for large deformable composite structures with piezoelectric layers and active vibration control was presented in ref. [6]. But, an optimal number of enhancing assumed strain (EAS) param-

eters was used for composite structures in this reference. A modified generalized laminate stiffness matrix is adopted for composite structures in this paper.

In this paper, geometric non-linear is considered only, but piezoelectric non-linearity is not taken into account. An eight-node solid-shell element is derived for geometric non-linear analysis of homogeneous and laminated elastic shells to analyze piezoelectric structures. Following the common practice of resolving shear and trapezoidal lockings in this particular element configuration, ANS is employed to interpolate the natural thickness and transverse shear strains [12–16]. To overcome the thickness locking, an *ad hoc* modified generalized laminate stiffness matrix is adopted. Unlike EAS which can only reproduce accurate thickness stress and strain for homogeneous shells, the matrix also works properly for laminated shells. As a low-cost vehicle to enhance the element inplane response, ...



Three-dimensional elasticity solution of simple-supported rectangular plate on point supports, line supports and elastic foundation

XU YePeng¹ & ZHOU Ding^{2*}

¹ School of Science, Nanjing University of Science and Technology, Nanjing 210094, China;

² College of Civil Engineering, Nanjing University of Technology, Nanjing 210009, China

* E-mail: dingzhou57@yahoo.com

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This paper studies the bending of simple-supported rectangular plate on point supports, line supports and elastic foundation. On the basis of three-dimensional elasticity theory, the exact expressions of the displacement functions, which satisfy the governing differential equations and the simply supported boundary conditions at four edges of the plate, are analytically derived. The reaction forces of the intermediate supports are regarded as the unknown external forces acting on the lower surface of the plate. The unknown coefficients are then determined by the boundary conditions on the upper and lower surfaces of the plate. Comparing the numerical results obtained from the proposed method to those obtained from Kirchhoff plate theory, Mindlin plate theory and those obtained from the commercial finite element software ANSYS, the high accuracy of the present method has been demonstrated.

rectangular plate, point support, line support, elastic foundation, displacement, stress, three-dimensional elasticity solution

Plates are widely used as structural elements in aerospace, mechanical and civil engineering. In practical projects, the intermediate supports, such as elastic foundation, elastic/rigid point-supports and line-supports in one or two directions, may be used to improve the load-bearing capacity of the plates. Therefore, investigating the effect of intermediate supports on the mechanical properties of plates has significance in actual engineering. A great deal of investigations have been carried out on rectangular plates. Zhou [1] presented an analytical solution of transverse vibration of rectangular plates simply supported at two opposite edges with arbitrary number of elastic line supports in one way. Liu and Wang [2] analyzed the bending problem of anisotropic rectangular plates with four free edges on an elastic foundation. Sun et al. [3] worked out the general analytic solution for a

thick plate on biparametric foundation. Zhou and Cheung [4] investigated free vibration of line-supported rectangular plates using a set of static beam functions. Cheung and Zhou [5] studied free vibration of rectangular unsymmetrically laminated composite plates with internal line supports. Zhou and Ji [6] developed free vibration of rectangular plates with internal column supports. Zhou [7] further developed vibration of point-supported rectangular plates with variable thickness using a set of static tapered beam functions. Recently, Xu and Zhou [8] presented the three-dimensional elasticity solution for simply supported rectangular plates with variable thickness.

In the present study, the exact expressions of the displacement functions, which satisfy the governing differential equations and the simply supported boundary conditions at four edges of the plate, are analytically ...

Equilibrium equations for nonlinear buckling analysis of drill-strings in 3D curved well-bores

TAN MeiLan^{1*} & GAN LiFei²

¹ Faculty of Science, Jiangsu University, Zhenjiang 212013, China;

² 94973 Unit in Hangzhou of Zhejiang Province, Hangzhou 310021, China

* E-mail: tanmeilan@gmail.com

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With the development of drilling technology, the oil/gas well has evolved from its early vertical straight form to the inclined, horizontal, plane curved, or even 3D curved well-bore. Understanding of the buckling behavior of a drill-string in a well-bore is crucial for the success of a drilling operation. Therefore, equilibrium equations for analyzing the buckling behavior of a drill-string in a 3D curved well-bore are required. Based on Love's equilibrium equations for a curved and twisted rod in space, a set of equilibrium equations for the nonlinear buckling analysis of a drill-string in a 3D curved well-bore are derived by introducing a radial constraint of the well-bore. The proposed formulae can account for the well curvature and tortuosity. Thus, it can be used to analyze the buckling behaviors of a drill-string constrained in a well-bore and subjected to axial compression, torsion at its upper end, and gravity simultaneously. It is worth noting that the existing equations in the literature for a drill-string in a straight and plane curved well-bore with a constant curvature are a special case of the proposed model. Thus, the present model can provide a theoretical basis for the nonlinear buckling analysis of a drill-string constrained in a 3D curved well-bore.

nonlinear buckling, tortuosity, constraint, equilibrium equations, drill-strings, 3D curved well-bores

Introduction Buckling of drill-strings has been a serious problem in oil/gas field operations for many years. Some of the associated problems are casing failure, drill-string failure, drill-string lock up, casing wear, increase in drill-string drag and torque, and extended reach drilling. Therefore, the buckling of radial-constrained drill-strings in a well-bore has become a hot topic in the area of petroleum engineering in recent years. Due to the complexity of the problem, conflict results are still reported. Gao et al. [1, 2] included torsion when evaluating the buckling load of a weightless tubular for the case of a constrained Euler sinusoidal buckling. The impact of torsion is taken into account via the dimensionless parameter $\sqrt{2T}/\sqrt{FEI}$, in which T is the torsional load, F the axial compressive load, E Young's modulus of the iso-

tropic material, and I the moment of inertia of the string's cross-section. This parameter showed that when neglecting friction, the influence of torsion on axial compression force was relatively small, suggesting that the model by neglecting the torsion effect [3] was a good approximation. However, experimental results [4] were nearly double the buckling load predicted by Wu and Juvkam-Wold's model [5]. On the other hand, numerical results for helical buckling only agreed well with existing experimental results for compressive loads alone [6]. Based on the principle of energy balance, Wu [7] concluded that the larger the torsional load, the lower the buckling load; torsional loads had a very little effect on the buckling length (pitch); and the tube would not buckle under torsional loads alone. Qiu et al. [8] ...



In-plane elastic stability of fixed parabolic shallow arches

CAI JianGuo, FENG Jian^{*}, CHEN Yao & HUANG LiFeng

School of Civil Engineering, Southeast University, Nanjing 210096, China

^{*}E-mail: fengjian@seu.edu.cn

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The nonlinear behavior of fixed parabolic shallow arches subjected to a vertical uniform load is investigated to evaluate the in-plane buckling load. The virtual work principle method is used to establish the non-linear equilibrium and buckling equations. Analytical solutions for the non-linear in-plane symmetric snap-through and antisymmetric bifurcation buckling loads are obtained. Based on the least square method, an approximation for the symmetric buckling load of fixed parabolic arch is proposed to simplify the solution process. And the relation between modified slenderness and buckling modes are discussed. Comparisons with the results of finite element analysis demonstrate that the solutions are accurate. A cable-arch structure is presented to improve the in-plane stability of parabolic arches. The comparison of buckling loads between cable-arch systems and arches only show that the effect of cables becomes more evident with the increase of arch's modified slenderness.

parabolic arch, elastic stability, shallow, bifurcation, snap-through

Arch is the combination of structure and architecture, and it is well known that arch has been widely used as a structural element. Arches resist general loading by a combination of axial compression and bending actions. Under these actions, when the lateral displacements and twist rotations of an arch are fully restrained, it may buckle in an in-plane symmetric snap-through mode or in an in-plane antisymmetric bifurcation mode as shown in Figure 1.

Owing to their importance in many fields of technology and engineering, the elastic stability of arches has been the subject of a large number of investigations. The solutions have traditionally been obtained by the finite element method and analytical method. The numerical analysis of arches has been extensive in recent years [1–7]. Approximate solution for the classical buckling load for sinusoidal shallow arches under uniformly distributed load was given by Timoshenko and Gere [8]. Then the closed form solutions with high precision were sought

by many researchers. The efforts in the nonlinear differential equilibrium equation have been devoted to two main groups: One is the equilibrium method and the other is energy method. With the first group, the stability of shallow arches on elastic foundation was studied by Simitses [9] and the stability of shallow arches under multiple loads and with elastic support were investigated by Plaut [10, 11].

Schreyer and Masur [12], Kerr and Soifer [13] performed an exact analysis for shallow fixed circular arches subjected to a radial load uniformly distributed around the arch axis and derived analytical solutions with the energy method. Dickie and Broughton [14] obtained the closed form solutions and series solutions of shallow circular arches with different supports. Wicks [15] deduced the buckling equations for shallow arches of arbitrary shapes. Pi et al. [16] and Bradford et al. [17] used the same method as Schreyer and Masur to study the stability of shallow circular arches with arbitrary symmetry section, and pointed out that classical buckling theory overestimated both the ...



On fractional control method for four-wheel-steering vehicle

CHEN Ning^{1,2*}, CHEN Nan¹ & CHEN YanDong²

¹ College of Mechanical Engineering, Southeast University, Nanjing 210096, China;

² College of Mechanical and Electronic Engineering, Nanjing Forestry University, Nanjing 210037, China

* E-mail: chenning@njfu.com.cn

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Four-wheel-steering (4WS) system can enhance vehicle cornering ability by steering the rear wheels in accordance with the front wheels steering and vehicle status. With such steering control system, it becomes possible to improve the lateral stability and handling performance. In this paper, a new control method for 4WS vehicle is proposed, its rear wheels steering angle is in accordance with the angle of front wheels steering and vehicle yaw rate, and the effects of front wheels steering angle velocity are considered by adopting the fractional derivative theory. Some design specifications for control law are also given. The effects of the control method are verified by a kind of numerical scheme presented in this paper. The dynamic characteristics such as the side-slip angle and the yaw angle velocity of the vehicle gravity center are compared among three kinds of vehicles with different control methods. And the kinematics characteristics such as turning radius between 4WS and 2WS are also discussed. Numerical simulation shows that the control method presented can improve the transient response and reduce the turning radius of 4WS vehicle.

fractional derivative, four-wheel-steering, computation simulation, chassis control system

Introduction Four-wheel-steering (4WS) system is one of the three main chassis control systems in vehicle, it can enhance vehicle cornering ability by steering the rear wheels in accordance with the front wheels steering and vehicle status. With such steering control system, it becomes possible to improve the lateral stability and handling performance [1–3].

In the past years, many researchers have proposed a lot of control algorithms and strategies for 4WS vehicle, such as LQR, slide mode variable structure control, fuzzy control, adapting control, H_∞ and μ synthesis robust control and so on. It is not too hard to realize that, the control method of “front and rear wheels’ steering ratio as a function of vehicle speed” [4] has become a prevailing trend in 4WS control. Namely, people try to

keep the side-slip angle β being zero at vehicle’s gravity center. However, zero side-slip angle β approach seems to be infeasible since the driver might have a feeling of discomfort caused by its motion different from that of the conventional vehicle. So keeping the angle to a small β is a more practical control algorithm as compared to zero β . On the other hand, the most dangerous moment for vehicle often occurs when front steering wheel angle is too large, or the vehicle turns too fast or lateral acceleration overshoots. In this research, we propose a new rear wheel steering control method for 4WS vehicle by introducing the fractional derivative theory so that the effects of front wheels steering angle and steering angular velocity are taken into consideration.



Micromechanical analysis of interaction energy for SMA reinforced composite

ZHU YuPing^{1,2*}, DUI GuanSuo¹ & DUO Liu³

¹ Institute of Mechanics, Beijing Jiaotong University, Beijing 100044, China;

² Department of Engineering Mechanics, Jiangsu University, Zhenjiang 212013, China;

³ State Key Lab of Crystal Materials, Shandong University, Jinan 250100, China

* E-mail: zhuyuping@126.com

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The energy of the interaction between the matrix and the inclusions in shape memory alloy (SMA) reinforced composite is one of the most important and complicated parts in thermodynamic constitutive theory. In this paper, the interaction energy is derived based on the classical theory of micromechanics and the thermodynamic theory. The SMA composite is treated as three phases, namely the austenitic phase, the martensite phase and the matrix phase. The interaction among the three phases is analyzed in a way close to the fact. The present expression is used to calculate the interaction energy of a typical SMA composite with attentions paid to understand of the effects of the matrix material, the fiber geometry, and the fiber/matrix volume ratio. It is shown that the method developed in this paper is credible compared with the references. Some useful conclusions are obtained.

shape memory alloy, interaction energy, SMA composite, micromechanics, thermodynamics

The shape memory alloys (SMAs), extensively used as sensors and large strain actuators, have recently attracted considerable attention due to their unique properties such as pseudoelasticity and shape memory effect. When SMAs are integrated with composite materials, the resulting structure also provides distinctive properties such as variable Young's modulus and damping capacity with the ability to generate large internal forces, to allow active control of their static and dynamic behaviors [1]. As a result, the concept of "intelligent SMA composite" has been proposed in the field of composites.

However, in order to design SMA devices more efficiently, a physics-based constitutive model is required to accurately simulate the mechanical or thermodynamic behavior of SMAs. In the literature, SMA macroscopic modeling usually involves free energy functions proposed and derived based on phenomenological thermo-

mechanics through micromechanical analysis. In general, the free energy functions include contributions from elastic energy, chemical free energy and surface energy. The interaction energy is an important part of the elastic energy. It represents the elastic energy associated to phase's accommodation. It also represents the effects associated with inclusion, eigenstrain and matrix. Although it could be small, it is a key factor to understand the transformation mechanism. Consequently, appropriate formulation of the interaction energy becomes extremely important and usually requires in-depth knowledge of microstructure and its evolution.

There have been several approaches to construct the interaction energy for SMA. Boyd and Lagoudas [2, 3] assumed an approximate expression of the interaction energy according to potential factors. The coefficients in the expression were determined by phase transformation ...



Nonlinear vibrations of nano-beams accounting for nonlocal effect using a multiple scale method

YANG XiaoDong^{1*} & C. W. LIM²

¹ Department of Engineering Mechanics, Shenyang Institute of Aeronautical Engineering, Shenyang 110136, China;

² Department of Building and Construction, City University of Hong Kong, Hong Kong 999077, China

* E-mail: jxdyang@163.com

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The nonlinear free transverse vibrations of a nano-beam on simple supports are investigated based on nonlocal elasticity theory. The governing equation is proposed by considering geometric nonlinearity due to finite stretching of the beam. The method of multiple scales is applied to the governing equation to evaluate the nonlinear natural frequencies. Numerical examples are presented to demonstrate the analytical results and highlight the contributions of the nonlinear term and nonlocal effect.

nonlocal elasticity theory, nanobeam, natural frequency, method of multiple scales

Introduction Nanotechnology covers a broad range of topics in the field of applied sciences. The small-size scale and nano-scale surface effect associated with nanotechnology become significant and consequently the classical continuum theory can not predict the behavior of the nano-scale structures. In the 1970s and later, Eringen [1, 2] proposed the nonlocal elasticity theory to model materials with nonlocal stress not only dependent on the classical local stress at a particular point but also dependent on a spatial integral which represents weighted averages of the contributions of local stress of all points in the domain. The application of nonlocal elasticity to beam theory in micro- and nano- structures, especially the version proposed by Peddieson et al. [3], has received much attention in the nanotechnology community because of the simple constitutive equation accounting for the size scale param-

eter. Subsequently, deformation of nano-beams based on Euler-Bernoulli, Timoshenko or shell model was investigated to predict the contribution of nonlocal effects [3–8]. Recently, Lim and Wang [9] used a higher-order strain gradient nonlocal stress model to derive the bending solutions of nanobeams subject to various boundary conditions.

The afore-mentioned investigations deal mainly with linear phenomena accounting for the nonlocal effect. In most practical vibrations of beam structures, the nonlinearity can not be neglected because of the existence of finite stretching of beam. In this paper, the nonlinear dynamics of the nonlocal Euler-Bernoulli beam is investigated by the method of multiple scales. Numerical method is used to evaluate the nonlinear natural frequencies and the contribution of the nonlocal effect is discussed.



Dynamic substructure model for multiple impact responses of micro/nano piezoelectric precision drive system

SHEN YuNian & YIN XiaoChun^{*}

Department of Mechanics and Engineering Science, Nanjing University of Science and Technology, Nanjing 210094, China

^{*}E-mail: yinxiaochun2000@yahoo.com.cn

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A dynamic substructure technique which considers the electromechanical coupling effect of the PZT and the inertial effect of flexible components is presented to study the multiple impact dynamic behavior of micro/nano piezoelectric impact drive systems. It can investigate the step-like motion of object body and the multiple impacts behaviors reasonably by the comparison of the experimental data and the numerical solution of the spring-mass model. It is expected to have higher accuracy in the numerical simulations of the motion and the responses, especially to high frequency pulse voltage excitations. The present dynamic substructure technique has firstly studied reasonably the propagations of piezoelectric-induced transient waves and impact-induced transient waves. It is helpful to the failure analysis and the design of piezoelectric stack and flexible components. The present dynamic substructure technique can be applied to the transient dynamics optimization design and the precision control of the micro/nano piezoelectric impact drive systems.

piezoelectric micro actuator, multiple impacts, dynamic substructure technique, electromechanical coupling, transient wave propagation, precision positioning

Introduction With the rapid development of the ultraprecision machining technology and the micro/nanotechnology, the measurement, location and controlling technology in micrometer and nanometer displacement become the urgent requirements of ultraprecision machining, micro mechanical assembling, micromanipulation robot, MEMS fabrication, optical adjustment, microscopic medical, bioengineering and et al. [1–4]. The positioning accuracy of micro/nano piezoelectric precision drive systems can reach 1–10 nm. It has high control accuracy, high speed response, higher output power and low power dissipation. It can work in the hostile environments, such as low temperature, vacuum and super-clean environments. Hence, the investigation

of its positioning process and mechanical behaviors is a hot spot recently.

The whole working process of a micro/nano piezoelectric precision drive system is a typical transient response process. Its piezoelectric elements, such as a PZT stack, can response quickly to a sudden step pulse voltage and a high frequency intermittent voltage. The repeated contact-impact or/and the repeated friction between the impact drive mechanism (IDM) and the object body are common driving manners of the object body to cumulate micro-displacement and power. The repeated impact [5], the piezoelectric driving [6–8] and the repeated friction will excite the transient waves travelling throughout the connecting flexible components, ...



A macro-meso constitutive law for concrete having imperfect interface and nonlinear matrix

ZHANG Qing^{1*} & XIA XiaoZhou^{1,2}

¹ Department of Engineering Mechanics, Hohai University, Nanjing 210098, China;

² Institute of Construct, Guangdong University of Technology, Guangzhou 510006, China

* E-mail: lxzhangqing@hhu.edu.cn

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The overall behavior of concrete depends on its meso structures such as aggregate shape, interface status, and mortar matrix property. The two key meso structure characters of concrete, bond status of interface and nonlinear property of matrix, are considered in focus. The variational structure principle is adopted to establish the macro-meso constitutive law of concrete. Specially, a linear reference composite material is selected to make its effective behavior approach the nonlinear overall behavior of concrete. And the overall property of linear reference composite can be estimated by classical estimation method such as self-consistent estimates method and Mori-Tanaka method. This variational structure method involves an optimum problem ultimately. Finally, the macro-meso constitutive law of concrete is established by optimizing the shear modulus of matrix of the linear reference composite. By analyzing the constitutive relation of concrete established, we find that the brittleness of concrete stems from the imperfect interface and the shear dilation property of concrete comes from the micro holes contained in concrete.

imperfect interface, matrix nonlinear property, variational structure principle, macro-meso constitutive law, concrete

Introduction Concrete-like material is indicative of the different characteristics from metal-like material in macroscopic such as compression hardening, shear dilation and strain softening. These different mechanical characteristics in macroscopic must come from their different inner structures. In mesoscopic, concrete can be taken as an inhomogeneous multi-phase composite consisting of mortar matrix, aggregate, the bond between matrix and aggregate, microcrack and microhole. The mechanical properties of the concrete take on non-homogeneous characteristic in distribution, and there are many strong or weak discontinuous surfaces in the concrete inner structure. To comprehensively discover the intrinsic mechanism of the macro mechanical behavior of concrete-like material, the meso structure characteristic must be focused on and meso inclusion

theory should be applied.

In previous research, the interface between matrix and aggregate of concrete is assumed perfect and little consideration has been given to the nonlinear property of matrix. In this paper, the bond status of interface and nonlinear property of matrix are considered. The variational structure principle proposed by Castaneda [1, 2] is adopted to estimate the nonlinear mechanical property. Specially, by selecting linear “comparison composite” material with the distribution and the shape of aggregate being the same as the concrete, we make the effective behavior of “comparison composite” approach the nonlinear overall behavior of concrete, and the overall property of linear reference composite can be estimated by classical estimation method. Finally, the macro-meso constitutive law of concrete is established, which ...



Experimental study on the mechanical properties of rocks at high temperature

ZHANG LianYing^{1,2*}, MAO XianBiao^{1,3} & LU AiHong^{1,3}

¹ School of Sciences, China University of Mining and Technology, Xuzhou 221116, China;

² School of Math and Physical Science, Xuzhou Institute of Technology, Xuzhou 221008, China

³ State Key Laboratory of Geomechanics and Deep Underground Engineering, China University of Mining and Technology, Xuzhou 221008, China

* E-mail: zhanglianying@126.com

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The mechanical properties of marble, limestone, and sandstone as well as the stress-strain curve, the varying characteristics of the peak strength, the peak strain and elastic modulus were studied by using the MTS810 Rock Mechanics Servo-controlled Testing System under the action of temperatures ranging from room temperature to 800°C. Results show that (1) the peak strength and elastic modulus of marble fluctuate at the temperature from normal to 400°C; and they decrease gradually over 400°C. (2) With the rise of the temperature, the peak strength and elastic modulus of limestone show downward trend from normal temperature to 200°C; have little change from 200°C to 600°C; and decrease sharply over 600°C. (3) The peak strength of sandstone shows a downward trend while a little change for elastic modulus at normal temperature to 200°C; and from 200°C to 600°C, the peak strength of sandstone increases while a little change for elastic modulus; the peak strength and elastic modulus decrease rapidly at the temperature over 600°C. (4) The peak strain of limestone shows little change at normal temperature to 600°C, however, the peak strain increases rapidly over 600°C; and for marble and sandstone, the peak strain decreases with the rise of the temperature from normal temperature to 200°C, the peak strain increases rapidly over 200°C. The result can provide valuable references for the rock engineering design at high temperature.

rock mechanics, high temperature effect, mechanical properties, experimental study

It is a new challenge for rock mechanics to deal with rock engineering problems at high temperature. Most rock mass in nuclear waste bury, geothermic exploitation and underground development in big city may undergo high temperature, so rock strength and deformation characteristics under high temperature need to be studied in detail. Related mechanical parameters are the basic foundations for excavation of rock underground engineering, support design and stability analysis of the surrounding rock.

The effect of temperature on the physical and me-

chanical properties of rocks has been an important study topic in rock mechanics. Heuze and Lau et al. [1–3] tested a number of rock's deformation modulus, Poisson's ratio, the tensile strength, compressive strength, cohesion and internal friction angle, viscosity, and other parameters and discussed the dependence of lateral pressure and high temperature on thermal expansions as well as the creep characteristics of the rock. Du et al. [4–8] studied the mechanical properties of granite, marble, and sandstone which had been cooled down after experiencing high temperature, and analyzed the variations of stress-strain curve, ...



Measurement of creep of optical fiber by a low coherent white light double interferometer system

XU ZhiHong^{1*} & Farhad ANSARI²

¹ School of Science, Nanjing University of Science and Technology, Nanjing 210014, China;

² Department of Civil and Material Engineering, University of Illinois at Chicago, IL 60607, USA

* E-mail: xuzh@mail.njust.edu.cn

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The creep properties of optical fiber used in fiber optical sensors were studied in this paper. A low coherent white light double interferometer system was designed and calibrated and the creep deformations of optical fibers under static and cyclic loadings were measured with this device. The research results showed that polymer coated optical fibers crept at the beginning when they were under static or cyclic load. As the number of the cyclic loading or the static loading times increased the creep tended to stop. Thus to ensure that the optical fiber keeps pre-stress for long time in pressure transducer, it is recommended that the optical fiber should be tensioned cyclically before being fixed into the sensor device.

fiber optic sensor, optical fiber, creep, interferometer, low coherent

Introduction Pressure is a fundamental reservoir-engineering parameter in down-hole in oil well and its permanent monitoring is utilized to guarantee the production process to go on safely [1]. The use of fiber optic technology, particularly for down-hole applications, is driven by the inherent advantages of fiber optics over conventional sensor technology such as immunity to electromagnetic interference, lightweight, small size, stability and durability in harsh environment [1–3]. In 2005, Xu and Ansari [2] indicated that reliability in fiber optic sensor was one of the most important factors that should be considered in the sensor system design. In the pressure fiber optic sensor, the optical fiber with Bragg grating is often used in mechanical transducer as sensing element and the sensing direction is the longitude of the fiber. The optical fiber is very thin and it can only take tensile stress axially so in pressure sensor device the optic fiber is intrinsically under a permanent tension. For the permanent down-hole monitoring system, the fiber

optic sensors have no chance to re-calibrate once they are put to use, so to keep the pre-tensioned stress in the fiber for long time is the key problem for the reliability of the sensing system. Optical fiber is a kind of composite material, typically consisting of a silica-based core and cladding surrounded by one or two layers of polymeric material. Silica is a brittle material and the polymer coating can be considered as a visco-elastic one. When they are combined, the appearing properties of this composite material will be visco-elastic [4]. For the pre-stressed optical fiber, the creep property is the most important to be considered to avoid the stress relaxation [5–8]. In this paper, the creep properties of the optical fiber were studied experimentally, the optical fibers were under cyclic loading and static loading respectively and the creeps were measured with a low coherence double reflected interferometer. The experiment methodologies, instruments and experiment results are outlined in the following sections.



Effects of volumetric allocation on heave response of semisubmersible in deep sea

ZHANG HuiQin & LI JiaChun*

Institute of Mechanics, Chinese Academy of Sciences, Beijing 100190, China

* E-mail: jcli05@imech.ac.cn

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The configuration of semisubmersibles consisting of pontoons and columns and their corresponding heave motion response in incident progressive waves are examined. The purpose of the present study is to provide a theoretical approach to estimating the effects of volumetric allocation on natural period and response amplitude operator (RAO) in heave motion. We conclude that the amplitude of heave motion response can be considerably suppressed by appropriately adjusting volumetric allocation so that the natural heave period keeps away from the range of wave energy. The theoretical formulae are found in good agreement with the corresponding computational results by WAMIT.

semisubmersible, heave, natural period, RAO, WAMIT

Introduction With ever-growing needs for oil and gas resources, the ocean engineering has been paid much attention to by world engineering community. In the South China Sea known as the second Persia Bay, there reserves a large amount of oil and natural gas. China has started to stride forward from coastal regions with depth under 300 m to offshore regions as deep as 3000 m. Ongoing worldwide trends in deepwater oil and gas production call for in-depth research in the interaction of fluid and structures as a basis of platform design. Major types of platforms such as tension leg platforms (TLP), semisubmersibles and spar platforms have been successfully used in deepwater oil and gas exploitation during the last two decades [1].

Floating structures may experience resonant motions, which should be avoided as much as possible under installation, operation and survival conditions. In particular, the heave motion response of a floating structure should be kept adequately low to guarantee the safety of risers and umbilical pipes as most important

components in the equipment of oil production. Therefore, we should make every effort to minimize vertical motion of floating structures.

The natural period in heave motion can be effectively enhanced simply by adding structural mass, which, however, seems to be infeasible due to a number of restrictions in the design. Haslum and Faltinsen [2] proposed that heave response to wave frequency is reduced by increasing system damping. As an example, Tao [3] considered the suppression of heave resonant response by altering hull shapes with larger damping. A new large floating platform was designed by Srinivasan et al. [4, 5]. They tried to control its heave motion by applying the concept of both hydrodynamic added mass and separated-flow damping intelligently. Do and Pan [6] developed a nonlinear controller for an active heave compensation system, which may reduce heave motion of a vessel connected to the riser. Chen et al. [7] demonstrated that the location or draft of heave plates exerted a significant effect on the heave motion of a semisubmersible. ...



Numerical simulations of self-propelled swimming of 3D bionic fish school

WU ChuiJie^{1*} & WANG Liang²

¹ State Key Laboratory of Structural Analysis for Industrial Equipment, Dalian University of Technology, Dalian 116024, China;

² Research Center for Fluid Dynamics, People's Liberation Army University of Science and Technology, Nanjing 211101, China

* E-mail: cjwudut@dlut.edu.cn

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Numerical simulations of self-propelled swimming of a three dimensional bionic fish and fish school in a viscous fluid are carried out. This is done with the assistance of a parallel software package produced for 3D moving boundary problems. This computational fluid dynamics package combines the adaptive multi-grid finite volume method, the immersed boundary method and VOF (volume of fluid) method. By using the package results of the self-propelled swimming of a 3D bionic fish and fish school in a viscous fluid are obtained. With comparison to the existing experimental measurements of living fishes, the predicted structure of vortical wakes is in good agreement with the measurements.

adaptive multi-grid, immersed boundary method, self-propelled swimming, fish school, 3D bionic fish

Introduction The mechanics of fish swimming is an attractive research topic in fluid mechanics. However, since fishes are uncontrollable, it is difficult to make them follow up a designer's command. Instead of experiments, numerical simulations of bionic fishes are applied in this study.

A parallel software package for 2D and 3D moving

boundary problems is first produced, which combines the adaptive multi-grid finite volume method, the immersed boundary and VOF (volume of fluid) methods. Before applying the code to self-propelled swimming of a bionic fish in a tank, two standard flow cases are used to verify this complex code. The two flows are flows around a 2D circular cylinder and around a 3D sphere.



Large-Eddy Simulations of turbulent flows with lattice Boltzmann dynamics and dynamical system sub-grid models

GUAN Hui¹ & WU ChuiJie^{2*}

¹ Research Center for Fluid Dynamics, PLA University of Science and Technology, Nanjing 211101, China;

² School of Aeronautics and Astronautics, Dalian University of Technology, Dalian 116024, China

* E-mail: cjwudut@dlut.edu.cn

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In this paper, two sub-grid scale (SGS) models are introduced into the Lattice Boltzmann Method (LBM), i.e., the dynamics SGS model and the dynamical system SGS model, and applied to numerically solving three-dimensional high Re turbulent cavity flows. Results are compared with those obtained from the Smagorinsky model and direct numerical simulation for the same cases. It is shown that the method with LBM dynamics SGS model has advantages of fast computation speed, suitable to simulate high Re turbulent flows. In addition, it can capture detailed fine structures of turbulent flow fields. The method with LBM dynamical system SGS model does not contain any adjustable parameters, and can be used in simulations of various complicated turbulent flows to obtain correct information of sub-grid flow field, such as the backscatter of energy transportation between large and small scales. A new average method of eliminating the inherent unphysical oscillation of LBM is also given in the paper.

Large-Eddy Simulation, Lattice Boltzmann Method, dynamics sub-grid scale model, dynamical system sub-grid scale model, parallel computation

Introduction The LES method is a powerful tool in numerical simulations of turbulence. The basic idea of LES is based on the following assumptions: the small scale structures of sub-grid flow field is not sensitive to the large scale structures of flow field nor to the influence of boundary conditions. Therefore, small scale structures are more general, and easier to model.

LES consists of three basic steps. First, a filtering method is applied to decompose the turbulent instantaneous flow field into large and small scale parts. Then computational fluid dynamics (CFD) method is used to numerically solve the filtered LES equations and the

continuity equation. At the same time, the terms in LES equations, which involve the influence of small scale back to the large scale field, are modeled with a proper sub-grid scale (SGS) model. The main difficulty of LES is the correct understanding of the essence of turbulence. Hence the SGS models used are unable to meet requirements of various complex turbulent flows.

Two SGS models, i.e., the dynamics SGS model and the dynamical system SGS model with LBM, are described in detail below. Following that, computational results obtained from the two methods for three-dimensional cavity flows are given.



Fractal derivative multi-scale model of fluid particle transverse accelerations in fully developed turbulence

SUN HongGuang & CHEN Wen*

Department of Engineering Mechanics, Hohai University, Nanjing 210098, China

* E-mail: chenwen@hhu.edu.cn

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The Tsallis distribution and the stretched exponential distribution were successfully used to fit the experimental data of turbulence particle acceleration published in Nature (2001), which manifested a clear departure from the normal distribution. These studies, however, fall short of a clear physical mechanism behind the statistical phenomenological description. In this study, we propose a multi-scale diffusion model which considers both normal diffusion in molecular-scale and anomalous diffusion in vortex-scale, and the latter is described by a novel fractal derivative modeling approach. This multi-scale model gives rise to a new probability density function which fits experimental data very well.

fully developed homogeneous turbulence, probability density function (PDF), anomalous diffusion, power-stretched Gaussian distribution (PSGD), fractal derivative

Introduction Turbulence phenomena extensively occur in diverse nature and engineering fields [1]. Although for several centuries enormous effort has been made in the study of turbulence, there is still no satisfactory dynamics description of such a flow field [2, 3] until now. On the other hand, subjected to the experimental technique, the measurement of turbulence particle movement has also not been satisfactory for many decades. Recently, with the help of the advanced technology, La Porta et al. [4] made an accurate measurement of acceleration transverse distribution of fluid particle in fully developed pipe turbulence. More precise experimental data were achieved via the further improved technology by Mordant et al. [5]. These experimental results clearly show that the probability density function (PDF) of a Lagrangian acceleration component of turbulence is highly non-Gaussian. In their 2001 Nature paper, La Porta et al. [4] proposed an empirical stretched exponential distribution function to fit the experimental data.

However, a physical explanation of their statistical model was notably missing.

In recent years, the Tsallis distribution, based on the well-known Tsallis non-extensive entropy [6], has gained a lot of attention in many research fields. Beck et al. [7–10] exploited a Tsallis distribution to refit the experimental data given in refs. [4, 5] and tried to interpret the model via the Tsallis entropy theory. But his fitting did not agree well with the data [5]. To remedy this drawback, Arimitsu et al. [11] proposed a new PDF, which fits the data better, via the multi-fractal analysis. The drawback of Arimitsu's model is that it has a far more complex mathematical expression.

In this study, we try to develop a novel turbulence diffusion model via the new fractal derivative, recently introduced in ref. [12]. The basic idea of the new model is to depict the complex turbulence diffusion by the trans-scale coupling of molecular-scale normal diffusion and vortex-scale anomalous diffusion. ...



Absolute scaling law for temperature data in Rayleigh-Benard convection

FU Qiang

Research Center of Fluid Mechanics, Institute of Science, People's Liberation Army University of Science and Technology, Nanjing 211101, China

E-mail: njfqhp@sohu.com

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In addition to the hierarchical-structure (H-S) model, this paper further explores the most intensive intermittent structure of Rayleigh-Bénard convection at the high Ra numbers proportional to temperature. With respect to the discovery and by means of the scale, both of Bolgiano, there are two regions of the structure holding the absolute scaling law given by Ching's paper. Through theoretic analysis of data, this paper indicates that the regions act as two local intensive intermittent structures, by which the statistical absolute scaling performance of region is induced, rather than the statistical result of the entire time series in belief since 1941. In terms of statistical theory, the local structure in fluid, therefore, is the essence governing the absolute scaling performance of region, especially in high intensity.

Rayleigh-Benard convection, Bolgiano scale, the most intensive structure, absolute scaling law, local structure

Introduction In 1941, Kolmogorov suggested a universal law of turbulence [1] that the p th-order velocity structure functions ζ_p has a simple and universal scaling law in fully developed turbulence with respect to the inertial range l (hereafter K41). Moreover, he concluded that ζ_p becomes into a linear function of p when it is assumed that the energy dissipation ε_l over a body in size l of the turbulence is constant. Then came the famous law of $l^{2/3}$, or the $k^{-5/3}$ law in terms of energy spectrum. However, Landau [2] remarked that K41 did not reckon the intermittency of real turbulence. Thus, Kolmogorov inferred the anomalous scaling law [3] (K62), after introducing a modified but similar assumption. Since then, a chain of achievements of applying statistics to turbulence, initiated by Komlogorov and then by many other scientists, have been referred to as the absolute scaling law.

In 1993, Benzi et al. [4] showed that the scaling properties of velocity increments could be extended to the relative scaling law, referred to as the extended self-similarity (hereafter ESS). The ESS significantly fitted experimental measurements in terms of the scaling exponent of for-

mula [4] for various turbulences in a wide range of Reynolds number (Re), unlike K41 that requires fully developed turbulences of quite high Re numbers. Having suggested several concerned hypotheses and introduced the most intensive state in turbulence, She and Leveque [5] proposed a hierarchical structure (H-S) model, which was referred to as the SL scaling law, in order to describe the strongest fluctuation in turbulence signal. The analysis of experimental data has shown that the SL scaling law is in excellent agreement with the ESS in relative scaling exponent [6].

In 2000, Emily Ching [7] found that there was not an absolute scaling law of K41 but two ESS scaling ranges in the light of Rayleigh-Bénard convection at high Ra numbers, between which is a separating scale known as Bolgiano scale. In 2007, Fu [8] further studied the most intensive structure proposed first in the hierarchical-structure (H-S) model. The results show that there exists an absolute scaling law for this structure, and the statistical absolute scaling behavior is only generated by the local strongest intermittence structure.



Compensation for time-delayed feedback bang-bang control of quasi-integrable Hamiltonian systems

LIU ZhongHua¹ & ZHU WeiQiu^{2*}

¹Department of Civil Engineering, Xiamen University, Xiamen 361005, China;

²Department of Mechanics, State Key Laboratory of Fluid Power Transmission and Control, Zhejiang University, Hangzhou 310027, China

* E-mail: wqzhu@zju.edu.cn

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The stochastic averaging method for quasi-integrable Hamiltonian systems with time-delayed feedback bang-bang control is first introduced. Then, two time delay compensation methods, namely the method of changing control force amplitude (CFA) and the method of changing control delay time (CDT), are proposed. The conditions applicable to each compensation method are discussed. Finally, an example is worked out in detail to illustrate the application and effectiveness of the proposed methods and the two compensation methods in combination.

time-delayed feedback control, compensation method, bang-bang control, quasi-integrable Hamiltonian system

Introduction Time delay is unavoidable in feedback control due to the time used to measure and estimate system state, calculate and execute control forces, etc. Time-delayed control may not only deteriorate the control performance but also cause instability of the controlled system [1–3]. Thus, the time delay control draws much attention of the control community, and different compensation methods have been proposed for time-delayed control [4–6]. On the other hand, however, purposeful-injected time delay may improve both the system's stability and performance [7].

In the present paper, the stochastic averaging method for quasi-integrable Hamiltonian systems with time-delayed feedback bang-bang control [8] is first presented. Then two compensation methods, namely the method of changing control force amplitude (CFA) and the method of changing control delay time (CDT), are proposed. The conditions under which each method can be appropriately applied are discussed. Finally, an example is worked out in detail to illustrate the application and effectiveness of the proposed methods and the two compensation methods in combination.



A perturbation-incremental scheme for studying Hopf bifurcation in delayed differential systems

XU Jian^{1*} & CHUNG Kwok Wai²

¹ School of Aerospace Engineering and Applied Mechanics, Tongji University, Shanghai 200092, China;

² Department of Mathematics, City University of Hong Kong Kowloon, Hong Kong, China

* E-mail: xujian@mail.tongji.edu.cn

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A new method, called perturbation-incremental scheme (PIS), is presented to investigate the periodic solution derived from Hopf bifurcation due to time delay in a system of first-order delayed differential equations. The method is summarized as three steps, namely linear analysis at critical value, perturbation and increment for continuation. The PIS can bypass and avoid the tedious calculation of the center manifold reduction (CMR) and normal form. Meanwhile, the PIS not only inherits the advantages of the method of multiple scales (MMS) but also overcomes the disadvantages of the incremental harmonic balance (IHB) method. Three delayed systems are used as illustrative examples to demonstrate the validity of the present method. The periodic solution derived from the delay-induced Hopf bifurcation is obtained in a closed form by the PIS procedure. The validity of the results is shown by their consistency with the numerical simulation. Furthermore, an approximate solution can be calculated in any required accuracy.

delayed differential equation, perturbation-incremental scheme, Hopf bifurcation, synchronization, center manifold

Introduction Dynamics of systems with time delay is of interest since time delay is ubiquitous in nature, science and engineering. A general mathematical model can be written as

$$\dot{\mathbf{Z}}(t) = \mathbf{C}\mathbf{Z}(t) + \mathbf{D}\mathbf{Z}(t - \tau) + \varepsilon \mathbf{F}(\mathbf{Z}(t), \mathbf{Z}(t - \tau)), \quad (1)$$

where $\mathbf{Z}(t) \in \mathbf{R}^n$, \mathbf{C} and \mathbf{D} are $n \times n$ real constant matrixes, $\mathbf{F}(\cdot)$ is a nonlinear function of its variables with $\mathbf{F}(0, 0) = 0$, ε is a parameter representing the couple degree between nonlinearities, τ is the time delay, and n is a positive integer. Eq. (1) may model many real systems, such as neural [1, 2], ecological [3], biological [4], mechanical [5–8], controlling [9], secure communication via chaotic synchronization [10, 11] and other natural systems subject to finite propagation speeds

of signals, finite reaction times and finite processing times [12]. It has been shown that the time delay in various systems has not only quantitative but also qualitative effects on dynamics even for small time delays [13, 14].

As a result, various qualitative and quantitative theories for delayed differential equations (DDEs) are developed and extended in recent years. Eq. (1) has been used as a mathematical model for the investigation of stability of systems with time delay. The delay-induced Hopf bifurcation may be the most simple but basic bifurcation on stability analysis.

In the qualitative treatment of Hopf bifurcation, many authors [1, 2, 15–17] suggested a center manifold reduction (CMR) then a normal form procedure to classify stability of the periodic solution derived from the Hopf bifurcation. The main steps of the analysis are ...



Bifurcation and chaos of the bladed overhang rotor system with squeeze film dampers

CAO DengQing^{*}, WANG LiGang, CHEN YuShu & HUANG WenHu

School of Astronautics, Harbin Institute of Technology, Harbin 150001, China

* E-mail: dqcao@hit.edu.cn

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To study the nonlinear dynamic behavior of the bladed overhang rotor system with squeeze film damper (SFD), a blade-overhang rotor-SFD model is formulated using the lumped mass method and the Lagrange approach. The cavitated short bearing model is employed to describe the nonlinear oil force of the SFD. To reduce the scale of the nonlinear coupling system, a set of orthogonal transformations is employed to decouple the one nodal diameter equations of blades, which are coupled with the dynamical equations of the rotor, with other equations of blades. In this way, the original system with $16+4n$ ($n \geq 3$) degrees of freedom (DoF) is reduced to a system with 24 DoF only. Then the parametric excitation terms in the blade-overhang rotor-SFD model are simplified in terms of periodic transformations. The coupling equations are numerically solved and the solutions are used to analyze the dynamic behavior of the system in terms of the bifurcation diagram, whirl orbit, Poincaré map and spectrum plot. A variety of motion types are found such as multi-periodic, quasi-periodic, and chaotic motions. Moreover, the typical nonlinear dynamic evolutions including the periodic-doubling bifurcation and reverse bifurcation are noted. It is noticed that there exist apparent differences in the dynamic behavior between the blade-overhang rotor-SFD models without and with considering the effect of blades.

bladed overhang rotor system, squeeze film damper, bifurcation, chaos

Introduction In aeroengines, the bladed overhang rotor assembly with SFD is widely used, and its vibration may affect the safety and stability of the machinery. The interaction between blades and the rotor may cause excessive vibration in a resonant condition, which may result in the failure of the machine. An SFD is actually a special type of journal bearing which is used to improve the stability of the rotating system. However, in some operating conditions, the nonlinear phenomena, such as nonsynchronous motion and chaotic motion, are observed in the rotor system with SFD, which means SFD may cause the instability of the system.

At present, it is a common practice to analyze the dynamic behavior in separate parts: the shaft bladed-disk

model and the rotor-bearing model [1, 2]. The shaft bladed-disk model is often used to analyze the interaction between shaft and blade, but neglects the inherent nonlinearity of oil film force at the journal bearing. And the rotor-bearing model takes the nonlinearity of oil film force into account, but ignores the effect of the vibration of blades on the rotor. Even if the approach simplifies the rotor dynamic analysis, it may lead the results to deviate from the facts. Therefore, an effective modelling strategy that addresses very well the nonlinear dynamic behavior of the bladed rotor-SFD system is crucial in estimating system performance and guiding the reliability verification process.

The discussions of Crawley et al. [1] and Chun and ...



Dynamics of vehicle-pavement coupled system based on a revised flexible roller contact tire model

YANG ShaoPu^{*}, LI ShaoHua & LU YongJie

School of Mechanical Engineering, Shijiazhuang Railway Institute, Shijiazhuang 050043, China

^{*}E-mail: yangsp@sjzri.edu.cn

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A revised flexible roller contact tire model (RFRC tire model) is proposed, which considers not only the geometric and flexible filtering effect, but also tire damping and pavement displacement. A vehicle-pavement coupled system is modeled as a two DOF oscillator moving along a simply supported beam on a linear viscoelastic foundation. By using the Galerkin's and Direct Integral method, dynamical responses of the vehicle-pavement coupled system are obtained based on the RFRC tire model and the traditional single point contact tire model (SPC tire model). The simulation results are compared with test data and the validity of the proposed RFRC tire model is verified. Differences between the two models are also investigated. It is found that the dynamical behaviors for both models agree with each other quite well when road surface roughness is a long harmonic wave. On the other hand, they are different under short harmonic wave or impulse road excitation. Thus the RFRC tire model should be used to compute the tire force and investigate dynamical responses of vehicle and pavement.

tire model, dynamics, flexible roller contact, vehicle-pavement coupled system, viscoelastic foundation

Introduction The contact characteristics between tire and road, which is one of the key issues to affect the vehicle dynamics and dynamical pavement forces, have received more and more attention. Many tire models were proposed during the past decades. The simplest one is the point contact tire model, which describes the tire's vertical force and is most widely used [1, 2]. However, the contact characteristics between tire and road are rather complicated. Tire has a great geometric size and elastic distortion. Enveloping effect on road should not be neglected when vehicle is running on a rough ground. In addition, the contact print length between tire and road varies in time and the tire may jump away from the ground. Thus when the road surface roughness is complex, the point contact tire model may pose rather large uncertainties. Besides the point contact model, a lot of enveloping models for tires were also presented, including fixed print model, rigid roller contact

model and flexible roller contact model. Guo presented a flexible roller contact tire model, and carried out the simulations of a vehicle vibration system based on rigid and flexible roller contact tire model [3–5]. The simulation results showed a good agreement with experimental data. Huang et al. [6] gave a new method for measuring effective road profiles to study tire displacement enveloping characteristics. Guan and Dong [7] presented a vehicle system with enveloping tire model to study the performances of an active suspension system. Nevertheless, these enveloping models didn't consider the effects of tire damping and pavement vibration on tire force.

Many researchers showed that an important reason causing road damage is the dynamical vehicle loads. Thus, it is necessary to model the contact characteristics between the tire and the road in order to investigate the interactions between vehicle and pavement thoroughly. ...



Periodic and chaotic dynamics of composite laminated piezoelectric rectangular plate with one-to-two internal resonance

ZHANG Wei^{*}, YAO ZhiGang & YAO MingHui

College of Mechanical Engineering, Beijing University of Technology, Beijing 100124, China

^{*}E-mail: sandyzhang0@yahoo.com

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The bifurcations and chaotic dynamics of a simply supported symmetric cross-ply composite laminated piezoelectric rectangular plate are studied for the first time, which are simultaneously forced by the transverse, in-plane excitations and the excitation loaded by piezoelectric layers. Based on the Reddy's third-order shear deformation plate theory, the nonlinear governing equations of motion for the composite laminated piezoelectric rectangular plate are derived by using the Hamilton's principle. The Galerkin's approach is used to discretize partial differential governing equations to a two-degree-of-freedom nonlinear system under combined the parametric and external excitations. The method of multiple scales is employed to obtain the four-dimensional averaged equation. Numerical method is utilized to find the periodic and chaotic responses of the composite laminated piezoelectric rectangular plate. The numerical results indicate the existence of the periodic and chaotic responses in the averaged equation. The influence of the transverse, in-plane and piezoelectric excitations on the bifurcations and chaotic behaviors of the composite laminated piezoelectric rectangular plate is investigated numerically.

composite laminated piezoelectric rectangular plate, third-order shear deformation, parametric excitation, bifurcation, chaos

Introduction Nowadays piezoelectric materials, which include piezoelectric lead-zirconate-titanate (PZT) and piezoelectric polyvinylidene fluoride (PVDF), are new type functional materials in engineering fields. Piezoelectric materials can be used as the actuators and sensors in engineering structures. Therefore, composite laminated piezoelectric plates have been widely applied to aircraft, large space station and shuttle in the two past decades [1, 2]. With the increased applications of composite laminated piezoelectric plates in engineering fields, for example morphing structures or morphing wings, composite laminated plates with piezoelectric materials can undergo large oscillating deformation, which leads

to nonlinear oscillations of plates. Research on the nonlinear dynamics, bifurcations, and chaos of composite laminated piezoelectric plate will play a significant role in engineering applications. However, up to now, only a few studies on the bifurcations and chaotic dynamics of composite laminated piezoelectric plate have been conducted.

Several researchers have focused their attention on investigating the nonlinear dynamic responses of composite laminated plates. Srinivasamurthy and Chia [3] presented a nonlinear shear-deformable theory for dynamic behaviors of generally laminated circular plates with movable and immovable in-plane boundary conditions. ...



On two transverse nonlinear models of axially moving beams

DING Hu¹ & CHEN LiQun^{1,2*}

¹ Shanghai Institute of Applied Mathematics and Mechanics, Shanghai University, Shanghai 200072, China;

² Department of Mechanics, Shanghai University, Shanghai 200444, China

* E-mail: lqchen@staff.shu.edu.cn

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Nonlinear models of transverse vibration of axially moving beams are computationally investigated. A partial-differential equation is derived from the governing equation of coupled planar motion by omitting its longitudinal terms. The model can be reduced to an integro-partial-differential equation by averaging the beam disturbed tension. Numerical schemes are respectively presented for the governing equations of coupled planar and the two governing equations of transverse motion via the finite difference method and differential quadrature method under the fixed boundary and the simple support boundary. A steel beam and a copper beam are treated as examples to demonstrate the deviations of the solutions to the two transverse equations from the solution to the coupled equation. The numerical results indicate that the differences increase with the amplitude of vibration and the axial speed. Both models yield almost the same precision results for small amplitude vibration and the integro-partial-differential equation gives better results for large amplitude vibration.

axially moving beam, nonlinearity, transverse vibration, finite difference method, differential quadrature method

Introduction Axially moving beams are extensively investigated because they can model many engineering devices such as power transmission band, belt saws, aerial cable tramways, and elevator cables. If the large amplitude motion is concerned, the geometric nonlinearity should be taken into consideration, and transverse and longitudinal motions of the beam are usually coupled. Thurman and Mote [1] first developed the full governing equations of planar motion of an axially moving beam. Under the quasi-static stretch assumption, Wickert [2] developed a nonlinear model for transverse motion of axially moving beams. The approach is also referred to as Kirchhoff's approach [3]. The model is a nonlinear integro-partial-differential equation, which has been used by Suweken and Van Horssen [4], Pellicano and Zirilli [5], Ravindra and Zhu [6], Chakraborty and Mallik [7], Chakraborty et al. [8], Parker and Lin [9], Pellicano et al. [10],

Pellicano and Vestroni [11], Chen and Yang [12], Chen and Zhao [13], and Chen and Yang [14]. Nevertheless, there is another nonlinear model for transverse motion derived from the coupled model by setting all longitudinal term zero and omitting higher order nonlinear terms. The model is a nonlinear partial-differential equation, which has been used by Öz et al. [15], Feng and Hu [16], Marynowski and Kapitaniak [17], Zhang et al. [18], Marynowski [19], Chen and Yang [12], Sze et al. [20], Chen and Yang [14], Chen et al. [21], Marynowski and Kapitaniak [22], Zhang and Song [23], and Ghayesh and Khadem [24].

Very limited attentions have been paid to the difference between the two models, namely, the integro-differential equation and the partial-differential equation. Approximate analytical results based on the two models were compared for parametric vibration [12] and free ...



Costate estimation for dynamic systems of the second order

WEN Hao, JIN DongPing & HU HaiYan*

Institute of Vibration Engineering Research, MOE Key Lab of Structure Mechanics and Control for Aircraft, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China

* E-mail: hhyae@nuaa.edu.cn

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The dynamics of a mechanical system in the Lagrange space yields a set of differential equations of the second order and involves much less variables and constraints than that described in the state space. This paper presents a so-called Legendre pseudo-spectral (PS) approach for directly estimating the costates of the Bolza problem of optimal control of a set of dynamic equations of the second order. Under a set of closure conditions, it is proved that the Karush-Kuhn-Tucker (KKT) multipliers satisfy the same conditions as those determined by collocating the costate equations of the second order. Hence, the KKT multipliers can be used to estimate the costates of the Bolza problem via a simple linear mapping. The proposed approach can be used to check the optimality of the direct solution for a trajectory optimization problem involving the dynamic equations of the second order and to remove any conversion of the dynamic system from the second order to the first order. The new approach is demonstrated via two classical benchmark problems.

costate estimation, second order, pseudo-spectral method

Introduction It is a fundamental, but open problem in control theory to synthesize the nonlinear optimal control for a mechanical system. One of motivations for studying such a tough problem comes from the aerospace engineering, where the stringent requirements of any mission always demand optimality. With the advent of digital computers, a number of computational approaches have been available to solve the problem of nonlinear optimal control efficiently and reliably [1]. Among them, a class of approaches, often referred to as indirect approaches, entails forming the optimality conditions and then solving the resulting equations with associated boundary conditions. However, the indirect approaches suffer from establishing a set of necessary conditions for the optimization via Pontryagin's maximum principle. In addition, it is extremely difficult to obtain the initial estimates for the costates and the switching structure for the problems with path inequali-

ties if an indirect approach is put into use [1, 2]. In contrast, the so-called direct approaches discretize the optimal control problem first and then solve the resulting nonlinear programming problem (NLP). Hence, the direct approaches are preferable for solving complex problems since they are free of the above-mentioned shortcomings and usually enjoy a much larger region for convergence [1]. However, the direct approaches do not tie the resulting solutions to the maximum principle. That is, it is uncertain whether the direct solution to the NLP is truly the optimal solution to the original continuous control problem. The costate profiles have to be estimated and then used for error analysis, mesh refinement and solution verification.

A significant progress has been made recently on the costate estimation for validating or refining the solutions determined by using some special direct approaches, such as the direct shooting method [3], and PS methods [4–8]. ...



Decomposition of almost Poisson structure of non-self-adjoint dynamical systems

GUO YongXin^{1*}, LIU Chang², LIU ShiXing^{1,2} & CHANG Peng¹

¹ College of Physics, Liaoning University, Shenyang 110036, China;

² Department of Applied Mechanics, Beijing Institute of Technology, Beijing 100081, China

* E-mail: yxguo@lnu.edu.cn

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Non-self-adjoint dynamical systems, e.g., nonholonomic systems, can admit an almost Poisson structure, which is formulated by a kind of Poisson bracket satisfying the usual properties except for the Jacobi identity. A general theory of the almost Poisson structure is investigated based on a decomposition of the bracket into a sum of a Poisson one and an almost Poisson one. The corresponding relation between Poisson structure and symplectic structure is proved, making use of Jacobiizer and symplecticizer. Based on analysis of pseudo-symplectic structure of constraint submanifold of Chaplygin's nonholonomic systems, an almost Poisson bracket for the systems is constructed and decomposed into a sum of a canonical Poisson one and an almost Poisson one. Similarly, an almost Poisson structure, which can be decomposed into a sum of canonical one and an almost "Lie-Poisson" one, is also constructed on an affine space with torsion whose autoparallels are utilized to describe the free motion of some non-self-adjoint systems. The decomposition of the almost Poisson bracket directly leads to a decomposition of a dynamical vector field into a sum of usual Hamiltonian vector field and an almost Hamiltonian one, which is useful to simplifying the integration of vector fields.

almost-Poisson structure, non-self-adjointness, nonholonomic systems, symplectic form, Jacobi identity, torsion

Introduction In the framework of inverse problem of dynamics, dynamical systems can be classified into self-adjoint and non-self-adjoint ones. From the viewpoint of calculus of variation, a system of ordinary variational forms is termed self-adjoint when it coincides with its adjoint system for all admissible variations. A set of ordinary differential equations is called self-adjoint if the corresponding variational forms are self-adjoint. Otherwise, it is called non-self-adjoint [1]. In the modern setting of differential geometry, the self-adjointness of the dynamical systems can also be equivalently defined by the conditions satisfied by symmetries of equations of motion. A dynamical system is called self-adjoint if its dynamical symmetries coincide with its adjoint symme-

tries [2, 3]. Evidently, conservative systems are self-adjoint. The converse is not true. For the Newtonian systems in the fundamental form or kinematic form, the conditions of self-adjointness of the systems are Helmholtz's conditions, which lead to a direct Lagrangian or Hamiltonian representation of the systems. Geometrically, self-adjointness of Lagrangian or Hamiltonian systems can be proved to be accordant with symplecticity of phase space. So such self-adjoint systems can admit Poisson structure and are easy to integrate.

Poisson structure for self-adjoint dynamical systems is formulated by Poisson brackets on the set of functions on manifold with the properties of anticommutativity, bilinearity, Leibniz's rule and Jacobi identity [4]. ...



Bursting of Morris-Lecar neuronal model with current-feedback control

DUAN LiXia¹, LU QiShao^{2*} & CHENG DaiZhan³

¹ College of Science, North China University of Technology, Beijing 100144, China;

² School of Science, Beijing University of Aeronautics and Astronautics, Beijing 100191, China;

³ Institute of Systems Science, Chinese Academy of Sciences, Beijing 100190, China

* E-mail: qishaolu@hotmail.com

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The Morris-Lecar (ML) neuronal model with current-feedback control is considered as a typical fast-slow dynamical system to study the combined influences of the reversal potential V_{Ca} of Ca^{2+} and the feedback current I on the generation and transition of different bursting oscillations. Two-parameter bifurcation analysis of the fast subsystem is performed in the parameter (I, V_{Ca}) -plane at first. Three important codimension-2 bifurcation points and some codimension-1 bifurcation curves are obtained which enable one to determine the parameter regions for different types of bursting. Next, we further divide the control parameter (V_0, V_{Ca}) -plane into five different bursting regions, namely, the “fold/fold” bursting region R1, the “fold/Hopf” bursting region R2, the “fold/homoclinic” bursting region R3, the “subHopf/homoclinic” bursting region R4 and the “subHopf/subHopf” bursting region R5, as well as a silence region R6. Codimension-1 and -2 bifurcations are responsible for explanation of transition mechanisms between different types of bursting. The results are instructive for further understanding the dynamical behavior and mechanisms of complex firing activities and information processing in biological nervous systems.

neuron, bursting, bifurcation, fast-slow dynamics analysis, codimension

Introduction Nerve cells (neurons) are responsible for receiving and transmitting signals to and from the Central Nervous System. Neural information is encoded and transmitted as spikes in membrane electrical potential, called action potentials. One of the major challenges in neuroscience is to understand the basic physiological mechanisms underlying the complex spatiotemporal patterns of spiking activity observed during the normal functioning of the brain, and to determine the origins of pathological dynamical states such as epilepsy, Parkinson's disease, Alzheimer's disease, and schizophrenia. A second major challenge is to understand how the patterns of spiking activity provide a substrate for the encoding and transmission of information.

A common variety of firing in neurons and other ex-

citable cells is bursting oscillation. Bursting is a relatively slow rhythmic alternation between an active phase of rapid spiking and a quiescent phase without spiking. It occurs in many nerve and endocrine cells, including thalamic neurons, hypothalamic neurons, pyramidal neurons in the neocortex, respiratory neurons in pre-Bötzinger complex, pituitary cells, and pancreatic β -cells. There are several possible physiological roles of bursting, such as bursting can overcome synaptic transmission failure, facilitate transmitter release, and evoke long-term potentiation and hence affect synaptic plasticity much greater, or differently than single spikes [1]. In addition, bursting have more informational content than single spikes when analyzed as unitary events [2] and have higher signal-to-noise ratio than single spikes [3]. It has been proposed to provide ...



p -moment stability of stochastic impulsive differential equations and its application in impulsive control

XU Wei¹, NIU YuJun^{1*}, RONG HaiWu² & SUN ZhongKui¹

¹ Department of Applied Math, Northwestern Polytechnical University, Xi'an 710072, China;

² Department of mathematics, Fushan University, Fushan 528000, China

* E-mail: nyjyrf@yeah.net

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The exponential p -moment stability of stochastic impulsive differential equations is addressed. A new theorem to ensure the p -moment stability is established for the trivial solution of the stochastic impulsive differential system. As an application of the theorem proposed, the problem of controlling chaos of Lorenz system which is excited by parameter white-noise excitation is considered using impulsive control method. Finally, numerical simulation results are given to verify the feasibility of our approach.

p -moment stability, impulsive, white-noise, stochastic differential equations

Introduction Impulsive phenomenon exists widely in many evolution process in which states are changed abruptly at certain moments of time, involving such fields as medicine, biology, economics, aviation, mechanics, electronics, telecommunications, etc. For the abrupt change, there may emerge jumps in the evolution, which lead to the non-smooth effects of the system. In recent years, significant progress has been made in the study of impulsive differential equations [1–16]: Lakshmi-kantham et al. [1] brought forward a lot of basic methods firstly, which were used in the study of impulsive differential equations. And an important theory, the stability of an impulsive control scheme is equivalent to the stability of trivial solution of an impulsive differential equation, was proved in ref. [1] too. Yang et al. [2–5] studied the problem of impulsive control and impulsive synchronization, they used the comparison method of impulsive differential equations to judge whether the system under consideration is stable or not. But in all these works, most of them put their attention on the system without stochastic factor. As we know, stochastic phenomenon exists in

almost everywhere during the development of our world [10], and a lot of dynamical systems have variable structures subject to stochastic abrupt changes, which may result from abrupt phenomena such as stochastic failures and repairs of components, changes in the interconnections of subsystems, sudden environment changes, and so on. So it is necessary to study the impulsive system with stochastic factor. Yang et al. [6] studied the p -moment stability of stochastic impulsive differential equations with delays, and established a stability criteria of the system. Wu et al. [7] investigated the p -moment stability of stochastic differential equations with jumps, and a theory of the p -moment stability was constructed. In this paper, we aim at gaining an insight into the problem of the p -moment stability of trivial solution in stochastic impulsive differential equations and attempt to study its application in chaos control. As an example, we use the theory to judge whether or not the impulsive control system of Lorenz system with white-noise excitation is stable, which can be used to judge whether this system can be controlled or not with the impulsive control method.



Improved generalized cell mapping for global analysis of dynamical systems

ZOU HaiLin & XU JianXue*

Institute of Nonlinear Dynamics, School of Aerospace, Xi'an Jiaotong University, Xi'an 710049, China

* E-mail: jxxu@mail.xjtu.edu.cn

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Three main parts of generalized cell mapping are improved for global analysis. A simple method, which is not based on the theory of digraphs, is presented to locate complete self-cycling sets that correspond to attractors and unstable invariant sets involving saddle, unstable periodic orbit and chaotic saddle. Refinement for complete self-cycling sets is developed to locate attractors and unstable invariant sets with high degree of accuracy, which can start with a coarse cell structure. A nonuniformly interior-and-boundary sampling technique is used to make the refinement robust. For homeomorphic dissipative dynamical systems, a controlled boundary sampling technique is presented to make generalized cell mapping method with refinement extremely accurate to obtain invariant sets. Recursive laws of group absorption probability and expected absorption time are introduced into generalized cell mapping, and then an optimal order for quantitative analysis of transient cells is established, which leads to the minimal computational work. The improved method is applied to four examples to show its effectiveness in global analysis of dynamical systems.

global analysis, generalized cell mapping, invariant set, chaotic saddle, crisis

Introduction To gain an overall understanding of a nonlinear dynamical system, we usually need to make a global analysis for this system [1]. One important aim of global analysis is to locate the invariant sets usually involving attractors, saddle points, unstable periodic orbits, and chaotic saddles. In some cases, we may need to know the global structure of invariant sets. Conley index is an efficient and rigorous theory to study the existence and structure of invariant sets [2]. For the description of Conley index, see ref. [3]. Basing on Conley index, topological techniques for invariant sets in dynamical systems were presented [4]. An algorithmic approach to chain recurrence was presented in ref. [5]. Recently, the structure of homoclinic orbits of the Smale horseshoe map was studied in ref. [6], a constructive method for verification of hyperbolicity and structural stability of discrete dynamical systems was presented [7] and sym-

bolic dynamics for homoclinic tangles and attractors were studied in refs. [8, 9], respectively.

Invariant sets are important to dynamical systems. Attractors represent the long-term stable motions. Unstable invariant sets involving saddle points, unstable periodic orbits, and chaotic saddles play important roles in bifurcations. Chaotic saddles can cause observable consequences such as chaotic transients, fractal basin boundaries and chaotic scattering [10]. So we mainly focus on how to find the accurate location of invariant sets rather than study the specific structure of an invariant set in this paper. Basins of attraction and basin boundaries are significant in engineering. All the states that will finally settle into attractors are called transient states. One may wish to know how long the system takes to go from a transient state to its attractor. We also study basins of attraction, basin boundaries, and transient states.