Open logic based on total-ordered partition model*

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Abstract The problem of uniqueness of reconstruction in open logic is dealt with by introducing the concept of total-ordered partitions, an ordering structure for modeling belief degrees of knowledge, and redefining the reconstruction operation. Based on the resulting definition, a nontrivial condition for the convergency of cognitive processes is given. It is shown that if new knowledge is not always accepted with an extremely skeptical attitude and the changes of belief degrees follow the criterion of minimal changes, the cognitive process will converge. The results provide an approach to unifying two kinds of theories for knowledge base maintenance: belief revision and open logic.

Keywords: open logic, reconstruction, cognitive process, belief revision.

Open logic, developed by Li[1], is a formal system for modeling the growing and updating of knowledge and the dynamics of formal theories. It also serves as a logical foundation for knowledge base maintenance and software evolution.

Reconstruction and cognitive process are the two basic concepts in open logic. The former specifies the process of knowledge base updating. It is very similar to the revision operator in belief revision[2], but there are two essential differences between them. The first one is the closedness of operands and outputs. Reconstruction does not need to operate on a logical closed set, but revision does. Nor is the output of reconstruction necessary to be closed (which means that reconstruction is not a belief base revision operator, cf. ref. [2]). The second one is the uniqueness. Reconstruction is not a function because its output does not need to be unique. In point of closedness, it is a good property that reconstruction can work on a nonclosed knowledge base for most practical knowledge bases are not closed at all (Closedness may be objectionable under some circumstances. See the Tichy counterexample on page 68 of ref. [3]). As far as the uniqueness is concerned, however, the result of a reconstruction operation is generally necessary to be determined and unique because reconstruction is usually used for constructing the new knowledge base from a given knowledge base and pieces of new knowledge. The present paper offers a solution to the problem by introducing the concept of belief degrees of knowledge to open logic and presenting a new definition of reconstruction based on the principle that the result of reconstruction should include as much knowledge with relatively high degree of belief as possible.

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Cognitive process specifies the vertical evolution of a knowledge base. A good strategy for the evolution of a knowledge base could lead the base to evolve or converge to an ideal state. It has been shown[4] that if new knowledge is unconditionally retained in every step of knowledge base reconstructing, the produced sequence in the knowledge base evolution will converge to the set of all true sentences of an ideal model. We call such a strategy "learning without oblivion", which may be efficient but too idealized to be practical. In fact, the oblivion is an important characteristic in the learning of human beings and many artificial intelligent systems. It is proved in the present paper that if new knowledge is not always accepted with an extremely skeptical attitude and changes of belief degrees follow the criterion of minimal changes, the cognitive process will converge.

Throughout this paper, we consider the first-order language $\mathscr{L}$ as the object language. L is the set of all sentences in $\mathscr{L}$. We denote individual sentences by A, B, or C, and denote sets of sentences by Γ, Δ etc. We shall assume that the underlying logic includes the classical first-order logic with the standard interpretation. The notation $\vdash$ means the classical first-order derivability and Cn the corresponding closure operator, i.e.

$$A \in Cn(\Gamma) \text{ if and only if } \Gamma \vdash A.$$

1 Total-ordered partitions

In order to capture the idea of strength of belief, ref. [5] introduced the concept of total-ordered partitions, which was a key notion in the establishment of general belief revisions (see ref. [6]) and the analyses of relationships between belief revision and nonmonotonic reasoning (see ref. [7]). In this section, we use the concept to specify reconstruction and cognitive process.

Definition 1.1. Let Γ be a set of sentences, $\mathscr{P}$ be a partition of Γ, and $<$ be a total ordering relation on $\mathscr{P}$. The triple $\Sigma = (\Gamma, \mathscr{P}, <)$ is called a total-ordered partition(TOP) of Γ, or a total-ordered partition model. If $<$ is a well-ordering on $\mathscr{P}$, Σ is called a well-ordered partition (WOP) of Γ.

Intuitively, every agent builds his knowledge base with his beliefs. Although all sentences in the knowledge base are accepted by the agent, this does not means that they are believed with equal strength. We may suppose that all sentences in the base have been divided into several groups according to their belief degrees. Sentences in the same group are of nearly equal degree. All the groups are arranged in a total ordering.

For any $P \in \mathscr{P}$ and $A \in P$, P is called the rank of A, denoted by $r(A)$. The converse ordering of the rank is called the degree of belief. In other words, sentences in lower rank are considered with higher degree of belief (For technical consideration, we did not arrange the ranks directly according to the degrees of belief. Instead of that, the converse ordering of the ranks is the ordering of belief degrees).

When $\Delta \subseteq \Gamma$, the following notation will be useful:

$$\Delta_P \overset{\text{Def}}{=} \Delta \cap P, \quad \Delta_{<P} \overset{\text{Def}}{=} \bigcup_{Q<P} \Delta_Q, \quad \Delta_{\leqslant P} \overset{\text{Def}}{=} \bigcup_{Q \leqslant P} \Delta_Q.$$

Definition 1.2. Let $\Sigma = (\Gamma, \mathscr{P}, <)$ be a TOP of Γ, A an arbitrary sentence. $\Gamma \Downarrow A$, called the family of maximal consistent subsets of Γ for Σ, is defined as the family of all subsets, $\Delta = \bigcup_{P \in \mathscr{P}} \Delta_P$, of Γ, where for any $P \in \mathscr{P}$

$$\Delta_P \text{ is a maximal subset of } P \text{ such that } \Delta_{\leqslant P} \cup \{\neg A\} \text{ is consistent.}$$

The notation $\Downarrow$ comes from reference [8].

If we write $\Gamma \perp A = \{\Gamma_0 \subseteq \Gamma : \Gamma_0 \nvdash A \wedge \forall \Gamma'(\Gamma_0 \subset \Gamma' \subseteq \Gamma \rightarrow \Gamma' \vdash A)\}$, it is easy to show that $\Gamma \Downarrow A \subseteq \Gamma \perp A$.

2 Belief degrees of sets of sentences and uniqueness of reconstruction

Let Γ be a knowledge base and A a piece of new knowledge. If A is logically independent of Γ, that is, in terms of ref. [1], A is a new law for Γ, then the N-reconstruction of Γ for A is adding A to Γ, or the set $\Gamma \cup \{A\}$. If A is inconsistent with Γ, or $\neg A$ meets a rejection for Γ, then an R-reconstruction of Γ for A is the process of removing some sentences from the old knowledge base so that the result is consistent with A and then adding A to the remainder. If A is a theorem of Γ, then the E-reconstruction of Γ for A is just the old knowledge base Γ. N-reconstruction, R-reconstruction and E-reconstruction are all referred to as reconstruction (see ref. [1] for more details. To unify the three types of reconstruction, the meaning of R-reconstruction was slightly changed.).

We denote the reconstruction of Γ for A by $\mathrm{Rec}(\Gamma, A)$. Therefore, if $\Gamma \vdash A$, then $\mathrm{Rec}(\Gamma, A) = \Gamma$; if A is independent of Γ, $\mathrm{Rec}(\Gamma, A) = \Gamma \cup \{A\}$. If A is inconsistent with Γ, according to Theorem 3.1 in ref. [1], $\mathrm{Rec}(\Gamma, A) = \Delta \cup \{A\}$, where $\Delta \in \Gamma \perp \neg A$. In this case, $\Gamma \perp \neg A$ is not a singleton; therefore R-reconstruction of Γ for A is not unique. This means that reconstruction is not a function (or an operator), which is a serious obstacle to its further application. To make reconstruction a function, a natural idea is choosing an element with the maximal degree of belief among the elements of $\Gamma \perp \neg A$ for the reconstruction of Γ for A. However, the concept of belief degrees only lies in the level of knowledge instead of sets of pieces of knowledge. That is to say we need to lift up an ordering on 2^Γ from belief ordering on Γ in order to specify the belief degree of elements of $\Gamma \perp A$.

To this end, we give the following definition.

Definition 2.1. Let $\Sigma = (\Gamma, \mathscr{P}, <)$ be a TOP of Γ. Define a relation on 2^Γ as follows: for any $\Delta', \Delta'' \in 2^\Gamma$,

$$\Delta' \prec \Delta'' \quad \text{if and only if} \quad \exists P \in \mathscr{P}(\Delta''_P \subset \Delta'_P \wedge \forall Q < P(\Delta'_Q = \Delta''_Q)).$$

Intuitively, $\Delta' \prec \Delta''$, saying "the belief degree of Δ' is higher than that of Δ''", means that Δ' includes more pieces of knowledge with relatively high belief degrees than Δ'' does.

Proposition 2.1. *$\prec$ is a strict partial ordering on 2^Γ.*

Proof. Omitted.

For any sentence A, let $\min(\Gamma \perp A)$ be the set of all minimal elements of $\Gamma \perp A$ under the ordering $\prec$, that is,

$$\min(\Gamma \perp A) = \{\Delta \in \Gamma \perp A : \forall \Delta' \in \Gamma \perp A(\Delta' \nprec \Delta)\}.$$

Proposition 2.2. *Let $\Sigma = (\Gamma, \mathscr{P}, <)$ be a TOP of Γ. For any sentence A,*

$$\min(\Gamma \perp A) = \Gamma \Downarrow A.$$

Proof. Omitted.

This means that $\Gamma \Downarrow A$ just consists of all the elements of $\Gamma \perp A$ with the highest degree of belief. For an arbitrary total-ordered partition of Γ, however, the elements of $\min(\Gamma \perp A)$ are not necessary to be unique. In fact, the size of $\min(\Gamma \perp A)$ is dependent on the partition of Γ: if the elements of $\mathscr{P}$ are all singletons, $\min(\Gamma \perp A)$ is a singleton; but if $\mathscr{P} = \{\Gamma\}$, then $\min(\Gamma \perp$

$A) = \Gamma \perp A$.

To show how to decrease the size of $\min(\Gamma \perp A)$, let us see an example.

Example 2.1 (Cited from ref. [9]). Let

A = "All European swans are white.";

B = "The bird caught in the trap is a swan.";

C = "The bird caught in the trap comes from Sweden.";

D = "Sweden is part of Europe.";

E = "The bird caught in the trap is white.".

Let $\Gamma = \{A, B, C, D, A \wedge B \wedge C \wedge D \rightarrow E\}$. Then $\Gamma \perp E = \{\{A \wedge B \wedge C \wedge D \rightarrow E, A, B, C\}, \{A \wedge B \wedge C \wedge D \rightarrow E, A, B, D\}, \{A \wedge B \wedge C \wedge D \rightarrow E, A, C, D\}, \{A \wedge B \wedge C \wedge D \rightarrow E, B, C, D\}, \{A, B, C, D\}\}$.

If the partition Σ_1 of Γ is the following:

$$P_0 = \{A \wedge B \wedge C \wedge D \rightarrow E\}, P_1 = \{D, B\}, P_2 = \{C, A\},$$

then $\min(\Gamma \perp E) = \{\{A \wedge B \wedge C \wedge D \rightarrow E, B, C, D\}, \{A \wedge B \wedge C \wedge D \rightarrow E, A, B, D\}\}$.

If Σ_1 is refined to the following partition Σ_2:

$$P_0 = \{A \wedge B \wedge C \wedge D \rightarrow E\}, P_1 = \{D, B\}, P_2 = \{C\}, P_3 = \{A\},$$

then $\min(\Gamma \perp E) = \{\{A \wedge B \wedge C \wedge D \rightarrow E, B, C, D\}\}$.

This example demonstrates that there exist two ways to deal with the problem of reconstruction uniqueness:

(i) If $\min(\Gamma \perp A)$ is not a singleton for a given total-ordered partition Σ of Γ, we could gradually refine Σ so that the number of elements of $\min(\Gamma \perp A)$ can be decreased to one. Then we take the unique element of $\min(\Gamma \perp A)$ as the reconstruction of Γ for A.

(ii) If there is no information available for the refinement of a given partition, then we take $\bigcap \min(\Gamma \perp A)$ as the reconstruction of Γ for A, just like the conservative strategy used in belief revision.

The above two methods could coordinate with each other so that the high believable information is lost as less as possible while no arbitrary decision is made in the absence of any additional information.

With this idea, we redefine the reconstruction as follows:

Definition 2.2. Let Σ be a TOP of Γ, A an arbitrary sentence. We define the reconstruction $\mathrm{Rec}(\Gamma, A)$ of Γ for A based on Σ as follows:

$$\mathrm{Rec}(\Gamma, A) = (\bigcap (\Gamma \Downarrow \neg A)) \cup (\{A\} \setminus Cn(\bigcap (\Gamma \Downarrow \neg A))).$$

The following observations show the underlying motivation for the definition:

For any set Γ and a sentence A,

(i) If $\Gamma \vdash A$, $\mathrm{Rec}(\Gamma, A) = \Gamma$, which means that $\mathrm{Rec}(\Gamma, A)$ is the E-reconstruction of Γ for A under the original definition of reconstruction.

(ii) If $\Gamma \nvdash A$ and $\Gamma \nvdash \neg A$, $\mathrm{Rec}(\Gamma, A) = \Gamma \cup \{A\}$, so $\mathrm{Rec}(\Gamma, A)$ is the N-reconstruction of Γ for A.

(iii) If $\Gamma \vdash \neg A$, $\mathrm{Rec}(\Gamma, A) = (\bigcap \Gamma \Downarrow \neg A) \cup \{A\}$. In this case, $\mathrm{Rec}(\Gamma, A)$ is similar but generally not equal to the R-reconstruction of Γ for A. It is easy to see that they coincide when $\Gamma \Downarrow \neg A$ is a singleton.

From this point of view, $\mathrm{Rec}(\Gamma, A)$ is a kind of generalization of the original definition of reconstruction.

Proposition 2.3.

(i) $A \in Cn(\mathrm{Rec}(\Gamma, A))$.

(ii) $\mathrm{Rec}(\Gamma, A) \subseteq \Gamma \cup \{A\}$.

(iii) *For any set* Γ, *if* $A \dashv\vdash B$, $\mathrm{Rec}(\Gamma, A) \dashv\vdash \mathrm{Rec}(\Gamma, B)$.

Proof. Straightforward from the definition of reconstruction.

The item (i) shows that the new knowledge is at least a piece of implicit knowledge in new knowledge base, so it is always accepted (if the new knowledge is refused, no reconstruction operation is needed); item (ii) shows that the new knowledge is the only thing being added; the last item says that newly defined reconstruction is independent of the syntax of new knowledge (but not necessary to be independent on the old knowledge base, which is an important difference between belief revision and open logic).

Example 2.2. Let $\Gamma, \Sigma_1, \Sigma_2$ be defined as in Example 3.1. From the definition of reconstruction, we have:

(i) The reconstruction $\mathrm{Rec}(\Gamma, \neg E)$ based on Σ_1 is $\{A \wedge B \wedge C \wedge D \rightarrow E, B, D, \neg E\}$.

(ii) The reconstruction $\mathrm{Rec}(\Gamma, \neg E)$ based on Σ_2 is $\{A \wedge B \wedge C \wedge D \rightarrow E, B, C, D, \neg E\}$.

This means that a finer partition would save more information.

3 Cognitive process and its convergency

Cognitive process is an important concept which differentiates open logic from other theories of knowledge base maintenance. Let $\{\Gamma_i\}_{i=0}^{\infty}$ and $\{A_i\}_{i=1}^{\infty}$ be a sequence of sets of sentences and a sequence of sentences, respectively. For any n, let Γ_n be the reconstruction of Γ_{n-1} for A_n. Then $\{\Gamma_i\}_{i=0}^{\infty}$ is called a cognitive process for $\{A_i\}_{i=1}^{\infty}$. In ref. [4], the following Limit Theorem is showed:

Let M be a model of the language $\mathscr{L}$ and τ_M be the set of all true sentences in the model M. For any numerating $\{A_i\}_{i=1}^{\infty}$ of τ_M and a set Γ_0, there exists a cognitive process $\{\Gamma_n\}$ such that $\lim_{n \to \infty} Cn(\Gamma_n) = \tau_M$.

This means that if model M is the real reflection of the object world (or every true sentence in M is "truth"), there exists a cognitive process (or learning process) through which an agent can reach all the truth eventually no matter what his initial knowledge state is. However, not all cognitive processes can converge to the truth. Li[4] gave an idealized condition for convergence of cognitive processes under which learning is out of oblivion. In this section, we will present a more natural condition based on our new definition of reconstruction.

In the process of knowledge growing and updating of an agent, knowledge changes not only in numbers but also in the degrees of belief. In general, an epistemic agent would have an estimation of belief degrees for his knowledge. Such estimation in each stage will influence the reconstruction in the next stage. Although belief degrees may be the agent's subjective evaluation of knowledge, it would be a rational assumption that the changes of belief degrees should be minimal in the case of the absence of subjective information from the epistemic agent. In other words, the ordering on the original knowledge base should be preserved as much as possible. We call such a principle the criterion of minimal changes of belief degrees, which is also embodied in the work of Boutilier[10] and Williams[11].

Definition 3.1. Let $\Sigma = (\Gamma, \mathscr{P}, <)$ be a WOP of Γ, η be the order-type of $\mathscr{P}$. For any sentence A and an ordinal α, define a well-ordered partition, $\Sigma_A^\Gamma(\alpha) = (\mathrm{Rec}(\Gamma, A), \mathscr{P}_A^\Gamma, <_A^\Gamma)$, or $\mathrm{Rec}(\Gamma, A)$ as follows:

For any $\beta < \max\{\alpha + 1, \eta\}$, let

$$P_\beta^\Gamma = \begin{cases} P_\beta \cap \mathrm{Rec}(\Gamma, A), & \text{if } \beta \neq \alpha, \\ (P_\beta \cup \{A\}) \cap \mathrm{Rec}(\Gamma, A), & \text{otherwise.} \end{cases}$$

Let $\mathscr{P}_A^\Gamma = \{P_\beta^\Gamma : \beta < \max\{\alpha + 1, \eta\}\}$.

For any $P_\beta^\Gamma, P_\gamma^\Gamma \in \mathscr{P}_A^\Gamma$, define

$$P_\beta^\Gamma <_A^\Gamma P_\gamma^\Gamma \quad \text{if and only if} \quad \beta < \gamma.$$

$\Sigma_A^\Gamma(\alpha)$ is called the minimal change of belief degrees for Σ with respect to A and α, which means that the new knowledge A is accepted in the belief degree α.

Definition 3.2. Let Γ_0 be a set of sentences with a WOP Σ_0. Let $\{A_i\}_{i=1}^n$ be a sequence of sentences, $\{\alpha_i\}_{i=1}^n$ a sequence of ordinal numbers. Define recursively a sequence of sets $\{\Gamma_i\}_{i=1}^n$ and a sequence of well-ordered partitions $\{\Sigma\}_{i=1}^n$ as follows:

(i) $\Gamma_i = \mathrm{Rec}(\Gamma_{i-1}, A_i)$, where the reconstruction operation is based on the well-ordered partition Σ_{i-1} of Γ_{i-1}.

(ii) Σ_i is the minimal change of belief degree for Σ_{i-1} with respect to A_i and α_i.

$\{\Gamma_i\}_{i=0}^n$ is called the cognitive process with respect to $\{A_i\}_{i=1}^n$ and $\{\alpha_i\}_{i=1}^n$ starting from Γ_0.

Theorem 3.1. *Let M be a model of $\mathscr{L}$. $\tau_M = \{A \in L : M \models A\}$. Γ_0 is a consistent set of sentences with a WOP Σ_0. For any sequence of ordinals $\{\alpha_i\}_{i=1}^\infty$ and an enumeration $\{A_i\}_{i=1}^\infty$ of τ_M, if $\{\Gamma_i\}_{i=1}^\infty$ is a cognitive process with respect to $\{A_i\}_{i=1}^\infty$ and $\{\alpha_i\}_{i=1}^\infty$ starting from Γ_0, then*

$$\underline{\lim_{n\to\infty}} Cn(\Gamma_n) \subseteq \tau_M \subseteq \overline{\lim_{n\to\infty}} Cn(\Gamma_n). \tag{1}$$

Specially, if $\Gamma_0 \setminus \tau_M$ is finite, then

$$\lim_{n\to\infty} Cn(\Gamma_n) = \tau_M. \tag{2}$$

Proof. (a) Suppose that $A \in \tau_M$. For there are infinite sentences in τ_M being equivalent to A, let these sentences form a subsequence $\{A_{k_j}\}_{j=1}^\infty$ of $\{A_i\}_{i=1}^\infty$. By the construction of cognitive process, for any $j \geqslant 1$, we have $A_{k_j} \in Cn(\Gamma_{k_j})$, or $A \in Cn(\Gamma_{k_j})$, which means $A \in \overline{\lim_{n\to\infty}} Cn(\Gamma_n)$. Thus $\tau_M \subseteq \overline{\lim_{n\to\infty}} Cn(\Gamma_n)$.

(b) Suppose that $A \in \underline{\lim_{n\to\infty}} Cn(\Gamma_n)$. Then there is a number n_0 such that $A \in Cn(\Gamma_n)$ for any $n \geqslant n_0$. If $A \notin \tau_M$, then $\neg A \in \tau_M$. By the proof of (a), there is a subsequence $\{A_{k_j}\}_{j=1}^\infty$ of $\{A_i\}_{i=1}^\infty$ such that $A_{k_j} \vdash\dashv \neg A$ and $A_{k_j} \in Cn(\Gamma_{k_j})$ for any $j \geqslant 1$. Thus there exists $k_{j_0} \geqslant n_0$ such that $\neg A \in Cn(\Gamma_{k_{j0}})$. But $A \in Cn(\Gamma_{k_{j0}})$, which contradicts the consistency of $\Gamma_{k_{j0}}$. Thus $A \in \tau_M$, that is, $\underline{\lim_{n\to\infty}} Cn(\Gamma_n) \subseteq \tau_M$.

(c) If $\Gamma_0 \setminus \tau_M$ is finite. Let $\Gamma_0 \setminus \tau_M = \{B_0, \cdots, B_m\}$. Then $\{\neg B_0, \cdots, \neg B_m\} \subseteq \tau_M$. There exists n_0 such that $A_{n_0} \vdash\dashv \neg B_0 \wedge \cdots \wedge \neg B_m$. By Proposition 2.3 and the definition of cognitive process, $A_{n_0} \in Cn(\Gamma_{n_0})$. Because $\Gamma_{n_0} \subseteq \Gamma_0 \cup \tau_M$ and Γ_{n_0} is consistent, we have $\Gamma_{n_0} \subseteq \tau_M$. Then, by the definitions of cognitive process and reconstruction, we conclude that $\{\Gamma_n\}_{n=n_0}^\infty$ is a monotonically increasing sequence, which means $\{Cn(\Gamma_n)\}_{n=0}^{\infty}$ converges. Therefore, eq. (1) implies (2).

This theorem shows that any cognitive process starting from a finite set will converge no matter how new knowledge is accepted. This result, however, cannot be generalized to the case that the starting point is an infinite set. In fact, if new knowledge is always accepted with an extremely skeptical attitude, the cognitive process may diverge when new knowledge is inconsistent with old knowledge.

In our opinion, a rational cognitive process should have the following properties:

(i) The new knowledge should not always be accepted with the extremely skeptical attitude and any new knowledge should have an opportunity to be accepted with relatively high degrees of belief.

(ii) When a piece of knowledge is learned several times, its relative belief degrees in the cognitive process should not decrease.

Under this consideration, we give the following definition of rational cognitive processes.

Definition 3.3. Suppose the definitions of M, τ_M and Γ_0 are as in Theorem 3.1. $\{A_i\}_{i=1}$ numerates the set τ_M and $\{\alpha_i\}_{i=1}^{\infty}$ is a sequence of natural numbers. If $\{\Gamma_i\}_{i=1}^{\infty}$ is a cognitive process with respect to $\{A_i\}_{i=1}^{\infty}$ and $\{\alpha_i\}_{i=1}^{\infty}$ starting from Γ_0, let $\{\Sigma_i\}_{i=0}^{\infty}$ be a WOP of $\{\Gamma_i\}_{i=0}^{\infty}$. If the following conditions hold:

(i) for any $A \in \tau_M$, there exists a number n_0 such that $A_{n_0} \dashv\vdash A$ and $\{B \in \Gamma_{n_0} : r^{\Sigma_{n_0}}(B) \leqslant r^{\Sigma_{n_0}}(A_{n_0})\}$ is a finite set ($r^{\Sigma}(A)$ denotes the rank of A under the partition Σ),

(ii) for any $i, j \geqslant 1$, if $i < j$ and $A_i \dashv\vdash A_j$,

$$\forall B \in \Gamma_i \cap \Gamma_j (r^{\Sigma_j}(B) \leqslant r^{\Sigma_j}(A_j) \rightarrow r^{\Sigma_i}(B) \leqslant r^{\Sigma_i}(A_i)),$$

then $\{\Gamma_i\}_{i=1}^{\infty}$ is called a rational cognitive process with respect to $\{A_i\}_{i=1}^{\infty}$ and $\{\alpha_i\}_{i=1}^{\infty}$ starting from Γ_0.

Condition (i) means any new knowledge has an opportunity to be accepted with a relatively high belief degree. Condition (ii) shows that when a piece of knowledge is learned more than once, the relative belief degree in which it is accepted should not be lower than those at last times.

Theorem 3.2. *Any rational cognitive process converges and*

$$\lim_{n \to \infty} Cn(\Gamma_n) = \tau_M.$$

Proof. We split the proof into the following three steps:

(a) We prove that for any $A \in \tau_M$, there is a natural number N such that $A_N \dashv\vdash A$ and $\{B \in \Gamma_N : r^{\Sigma_N}(B) \leqslant r^{\Sigma_N}(A_N)\} \subseteq \tau_M$.

In fact, condition (i) implies that there exists n_0 such that $A_{n_0} \dashv\vdash A$, and $\Delta = \{B \in \Gamma_{n_0} : r^{\Sigma_{n_0}}(B) \leqslant r^{\Sigma_{n_0}}(A_{n_0})\}$ is finite. If $\Delta \setminus \tau_M$ is empty, or $\Delta \subseteq \tau_M$, the proof is ready if we let $N = n_0$. For the case that $\Delta \setminus \tau_M$ is nonempty, let $\Delta \setminus \tau_M = \{C_1, \cdots, C_m\}$. Then $\neg C_1 \wedge \cdots \wedge \neg C_m \in \tau_M$. Thus there is a number $n_1 \geqslant n_0$ such that $A_{n_1} \dashv\vdash \neg C_1 \wedge \cdots \wedge \neg C_m$. According to the construction of cognitive process, $A_{n_1} \in Cn(\Gamma_{n_1})$. Since Γ_{n_1} is consistent, $(\Delta \setminus \tau_M) \cap \Gamma_{n_1} = \varnothing$. On the other hand, again by the construction of cognitive process, the sequence $\{\Gamma_n \setminus \tau_M\}_{n=0}^{\infty}$ decreases monotonically. Therefore,

$$\forall n \geqslant n_1((\Delta \setminus \tau_M) \cap \Gamma_n = \varnothing). \tag{3}$$

Since there are infinite sentences in τ_M which are logically equivalent to A, there exists a number $N \geqslant n_1$ such that $A_N \dashv\vdash A$. Let $\Delta' = \{B \in \Gamma_N : r^{\Gamma_N}(B) \leqslant r^{\Gamma_N}(A_N)\}$. With condition (ii) of the rational cognitive process, we obtain

$$\Delta' \setminus \tau_M \subseteq \Delta \setminus \tau_M .$$

By the expression (3), we have $(\Delta \setminus \tau_M) \cap \Gamma_N = \varnothing$. Thus $(\Delta' \setminus \tau_M) \cap \Gamma_N = \varnothing$. Note that $\Delta' \subseteq \Gamma_N$, so $\Delta' \setminus \tau_M = \varnothing$, that is, $\Delta' \subseteq \tau_M$.

(b) Assume that $A \in \tau_M$. By (a), there exists a natural number N such that $A_N \dashv\vdash A$ and $\Delta = \{B \in \Gamma_N : r^{\Sigma_N}(B) \leqslant r^{\Sigma_N}(A_N)\} \subseteq \tau_M$. Hence $\Delta \cup \{A_n\}$ is consistent for any $n \geqslant N$. According to the construction of cognitive process, $\Delta \subseteq \Gamma_n$. Specially, $A_N \in \Gamma_n$, or $A \in Cn(\Gamma_n)$. Thus $A \in \underline{\lim}_{n\to\infty} Cn(\Gamma_n)$. So we have $\tau_M \subseteq \underline{\lim}_{n\to\infty} Cn(\Gamma_n)$.

(c) Suppose that $A \in \overline{\lim}_{n\to\infty} Cn(\Gamma_n)$. If $A \notin \tau_M$, $\neg A \in \tau_M$. By (b), $\neg A \in \underline{\lim}_{n\to\infty} Cn(\Gamma_n)$. Thus there is a number n_0 such that $A \wedge \neg A \in Cn(\Gamma_{n_0})$, which contradicts the consistency of Γ_{n_0}. Therefore, $\overline{\lim}_{n\to\infty} Cn(\Gamma_n) \subseteq \tau_M$.

Note that the criterion of minimal changes of belief degree and the rationality of cognitive processes are not the necessary conditions for the convergency of cognitive processes. It is not difficult to entail some more loose conditions of the convergency from the proof of the theorem.

4 Conclusion

As mentioned above, the reconstruction operation is very similar to the belief revision operator. However, they are incomparable under the original definition of reconstruction because the original reconstruction operation is not a function. Under its new definition, when the old knowledge base Γ and its total-ordered partition Σ are both given, the reconstruction Rec(Γ, A) is a function with respect to Γ and new knowledge A, so they become comparable. In fact, we have the following results:

Let Γ be a set of sentences and Σ a total-ordered partition of Γ. We have

(i) Belief revision of Γ by A: $\Gamma * A = \bigcap(\Gamma \Downarrow \neg A) + A$, where Γ is logically closed and Σ is a nicely-ordered partition of Γ (Σ is a nicely-ordered partition of Γ if Σ is a total-ordered partition of Γ and satisfies: If $A_1, \cdots, A_n \vdash B$, $\sup\{r(A_1), \cdots, r(A_n)\} \leqslant r(B)$. cf. ref. [5]).

(ii) Belief base revision of Γ by A: $\Gamma \hat{\otimes} A = \bigcap_{\Delta \in \Gamma \Downarrow \neg A} Cn(\Delta) + A$ (see reference [8]).

(iii) Reconstruction of Γ for A: Rec(Γ, A) $= (\bigcap(\Gamma \Downarrow \neg A)) \cup (\{A\} \setminus Cn(\bigcap(\Gamma \Downarrow \neg A)))$.

This shows that the key difference among reconstruction, belief revision and belief base revision is: the belief revision demands both old and new knowledge bases be closed; the belief base revision demands the revised knowledge base be closed, but the reconstruction needs no closedness. In our opinion, the closedness of knowledge bases could benefit the theoretical research, but is over demanding for practical applications.

We conclude that belief revision and open logic specify the knowledge base maintenance from two different levels. The former emphasizes the theoretical analysis and the latter lays more attention on the implementation. One could be used by the other. For instance, the concept of cognitive processes can be used to model the iterated belief revisions (see ref. [12]), and the reconstruction based on the total-ordered partition could be applied to the automatic revision of program specifications. (As a subproject of the NSFC, we are developing a system of automatic revision of program specifications based on open logic and belief revision. Such a system realizes the automatic maintenance of algebraic specifications according to user given importance evaluation of axioms and modules in the specifications, and automatically produces new specifications when users add new functional modules. This system is now in the testing stage. For the underlying idea please see ref. [13].) We believe that this is significant for enriching the research on knowledge base maintenance and reinforcing the applications of the relative theories to the software engineering.

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