SCIENCE CHINA
Information Sciences

• **RESEARCH PAPERS** •

October 2010 Vol. 53 No. 10: 2116–2126
doi: 10.1007/s11432-010-4063-0

Rectangle blocking matrices based unitary multistage Wiener reduced-rank joint detection algorithm for multiple input multiple output systems

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Received May 25, 2009; accepted October 21, 2009; published online August 20, 2010

Abstract Traditional equalization algorithms for multiple input multiple output (MIMO) systems suffer from high complexity and low convergence rate. So an improved adaptive reduced-rank joint detection algorithm of multistage Wiener filter (MSWF) based on rectangle blocking matrices is proposed. The MSWF is implemented by the correlation subtraction algorithm (CSA) structure and is called unitary multistage Wiener filter (UMSWF). The new scheme adopts rectangle submatrix as blocking matrix, which is chosen from the square blocking matrix for UMSWF. The proposed algorithm can reduce the size of the observation data vectors step by step in the forward recursion decomposition of UMSWF. Thus, the computational complexity is reduced and the convergence rate is increased. Theoretical analysis and simulation results show that this improved adaptive reduced-rank joint detection algorithm of UMSWF based on rectangle blocking matrix has better performance such as lower complexity and faster convergence rate. In particular, simulations are conducted in the vertical-Bell Labs layered space-time (V-BLAST) system which adopts BPSK modulation, where 4 and 8 antennas are equipped at the transmitter and receiver, respectively. Compared with traditional equalization algorithm based on UMSWF, our new method can achieve the same BER performance at high SNR with only 0.5 times that of computational complexity.

Keywords multiple input multiple output (MIMO) systems, unitary multistage Wiener filter (UMSWF), joint detection, blocking matrix

Citation Ren P Y, Wang R, Zhang S J. Rectangle blocking matrices based unitary multistage Wiener reduced-rank joint detection algorithm for multiple input multiple output systems. Sci China Inf Sci, 2010, 53: 2116–2126, doi: 10.1007/s11432-010-4063-0

1 Introduction

The equalization technology can effectively eliminate inter-symbol interference (ISI) and improve performance in frequency selective MIMO systems. So it is very important to study an equalization algorithm for frequency selective fading channels. The computational complexity and convergence rate are two of the most important metrics to evaluate the equalization algorithms. Therefore, the design of the equalizer aims at achieving a faster and more stable convergence performance and a better bit error rate (BER) performance with fewer taps [1].

The essence of reduced-rank filtering algorithm is to project the received signal vectors onto a low-dimensional subspace to obtain the optimal filter coefficients in a low-dimensional sub-space. Thereby it

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can reduce the complexity of the receiver and speed up the algorithm convergence rate. The reduced-rank filtering algorithm performs better than full-rank algorithm with fewer samples. The most commonly used strategies for reduced-rank are principal component (PC) algorithm and cross-spectral (CS) measurement method [2]. These two algorithms are both based on eigenvalue decomposition with significant high computational complexity. The performance of PC decreases rapidly with the reduction of the subspace dimension, and CS is better than the PS in the sense of mean square error (MSE). However, the computational complexity of CS is much higher than that of PS. In order to reduce the complexity, the multistage Wiener filter (MSWF) implemented by a nested chain of scalar Wiener filters was presented and analyzed in [3]. When the rank of the filter is much lower than the dimension of the receive signals, this new approach obtains lower-dimensional subspace-based vector by a series of update processes and the performance of MSWF is similar to that of the full rank MMSE. The computational complexity of this method is low and the convergence rate is fast, and thus it can track the time-varying channels quickly.

Based on the analysis of the algorithms for implementing MSWF, an efficient implementation of MSWF named correlation subtraction algorithm (CSA) is proved to be an UMSWF. The rank reduction performance of UMSWF is superior to the original MSWF proposed by Goldstein et al. [4]. In previous work, the MSWF has been applied in an asynchronous code-division multiple-access (CDMA) system to suppress the multiple-access interference [5, 6]. It has been shown that MSWF can offer a significant reduction in subspace dimension compared to other reduced-rank algorithms for a target performance [5]. In time dispersive radio channels, the performance of linear filters degrades dramatically by more frequent nulls in their spectral characteristics. The well-known nonlinear decision feedback equalizer (DFE) is used to overcome this problem in [7]. The authors in [8, 9] studied the adaptive reduced-rank equalization based on joint iterative least squares optimization. It has been shown that the performance of these algorithms is much better than that of traditional algorithms, but the complexity of them is relatively high. Moreover, a computationally efficient preconditioned block conjugate gradient (PBCG) algorithm is discussed in [10], which turns out to outperform the standard BCG algorithm concerning the complexity-performance ratio. The work of [11] applied RUMSWF to adaptive array, and the simulation results showed that the RUMSWF had superior performance. An equalization algorithm based on UMSWF using square matrix as blocking matrix was proposed in [12, 13]. This algorithm can attain near full-rank BER performance with a high computational complexity in the forward recursion decomposition. A joint antenna detection scheme for time-varying channels was proposed for low-complexity MIMO multi-user receiver [14]. This scheme was also used in MIMO uplink, which decreases the computational complexity by an order of magnitude in multi-carrier code division multiple access (CDMA) system using parallel interference cancellation (PIC) after the matched filter. Earlier equalization algorithms based on MSWF have better BER performance; however, the computational complexity is a little higher.

In this paper, a new scheme based on UMSWFRE, named RUMSWFRE, is proposed. This new reduced-rank joint detection algorithm modifies the blocking matrix of UMSWF into a rectangular matrix, thus reducing the size of the observation data vectors step by step in the forward recursion decomposition of UMSWF. Moreover, the new rectangular blocking matrix is implemented by CSA to ensure the good reduced-rank performance. Consequently, the computational complexity of the proposed reduced-rank equalization algorithm is lower and the convergence rate is faster than the traditional equalization algorithms.

The rest of the paper is organized as follows: In section 2, the system model for frequency selective MIMO system is introduced. In section 3, the details of proposed adaptive reduced-rank joint detection algorithm based on RUMSWF are described. Section 4 is devoted to the performance analysis of the new approach. In section 5, numerical results are provided, followed by conclusions in section 6.

2 System model

Consider a MIMO system with M antennas at the transmitter and N antennas at the receiver. The signals from M antennas are transmitted over frequency selective fading channels with L propagation paths, which are received by N antennas. The channel impulse response is described by

$$\boldsymbol{H} = \sum_{l=0}^{L-1} \boldsymbol{H}_l \delta(i-l), \tag{1}$$

where the $N \times M$ matrix $\boldsymbol{H}_l$ denotes the fading channel coefficient of the lth path. The $M \times 1$ vector $\boldsymbol{s}(k) = [s_1(k), \ldots, s_M(k)]^{\mathrm{T}}$ is composed of the data symbols transmitted from the M antennas at the kth time instant. We assume that all the $s_i(k)$ of $\boldsymbol{s}(k)$ are independent and identically distributed (i.i.d.). The $N \times 1$ received signals vector is given by

$$\boldsymbol{y}(k) = \sum_{l=0}^{L} \boldsymbol{H}_l \boldsymbol{s}(k-l) + \boldsymbol{n}(k), \tag{2}$$

where the $N \times 1$ vector $\boldsymbol{n}(k) = [n_1(k), \ldots, n_N(k)]^{\mathrm{T}}$ is a complex Gaussian noise vector with zero mean and $E[n(k)n^{\mathrm{H}}(k)] = \sigma^2 I$.

Assuming that the length of the linear equalizer is N_f (i.e. there are N_f taps in the filter), we expand $\boldsymbol{s}(k)$, $\boldsymbol{y}(k)$ and $\boldsymbol{n}(k)$ in time domain at the time instant $k = 0, \ldots, N_f - 1$. We can get

$$\begin{aligned}
\tilde{\boldsymbol{s}}(k) &= \begin{bmatrix} \boldsymbol{s}^{\mathrm{T}}(k) & \cdots & \boldsymbol{s}^{\mathrm{T}}(k-N_f-L) \end{bmatrix}^{\mathrm{T}}, \\
\tilde{\boldsymbol{y}}(k) &= \begin{bmatrix} \boldsymbol{y}^{\mathrm{T}}(k) & \boldsymbol{y}^{\mathrm{T}}(k-1) & \cdots & \boldsymbol{y}^{\mathrm{T}}(k-N_f) \end{bmatrix}^{\mathrm{T}}, \\
\tilde{\boldsymbol{n}}(k) &= \begin{bmatrix} \boldsymbol{n}^{\mathrm{T}}(k) & \cdots & \boldsymbol{n}^{\mathrm{T}}(k-N_f) \end{bmatrix}^{\mathrm{T}}.
\end{aligned}$$

So, the matrix can be expressed as

$$\tilde{\boldsymbol{y}}(k) = \tilde{\boldsymbol{H}}(k)\tilde{\boldsymbol{s}}(k) + \tilde{\boldsymbol{n}}(k), \tag{3}$$

where

$$\tilde{\boldsymbol{H}} = \begin{bmatrix} \boldsymbol{H}_0 & \cdots & \boldsymbol{H}_L & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{H}_0 & \cdots & \boldsymbol{H}_L & \ddots & \vdots \\ \vdots & \ddots & \ddots & & & \\ \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{H}_0 & \cdots & \boldsymbol{H}_L \end{bmatrix}$$

is an L-line diagonal matrix, composed of $N \times M$ multi-path channel matrix $\boldsymbol{H}_l$.

3 Adaptive reduced-rank joint detection algorithm of unitary multistage Wiener filter based on rectangle blocking matrix

The work of [11] shows that the convergence rate of equalizer can be improved and the computational complexity can be reduced by optimizing the blocking matrix of the MSWF. In this paper, motivated by the idea in [9], we apply RUMSWF to adaptive equalization to reduce the complexity of the equalization algorithm and speed up the convergence rate. The new structure adopts a rectangle block matrix as a blocking matrix. It has the advantages of both square blocking matrix and rectangular blocking matrix.

Consider the matrix form of the system model described in section 2. At the kth time instant, the input of the UMSWF is $\tilde{\boldsymbol{y}}(k) = \tilde{\boldsymbol{H}}(k)\tilde{\boldsymbol{s}}(k) + \tilde{\boldsymbol{n}}(k)$. Then after the Dth forward recursion decomposition [3], we can get the desired D dimensional signal from the $N \times 1$ observation signal vector $x_0(k)$. So, the Dth reduced-rank after RUMSWF can be described as

$$\boldsymbol{d}(k) = \begin{bmatrix} d_1(k), & \ldots, & d_D(k) \end{bmatrix} = \boldsymbol{T}_D^{\mathrm{H}} \boldsymbol{x}_0(k), \tag{4}$$

$$\boldsymbol{T}_D = \left[h_1, h_2\boldsymbol{B}_1, \ldots, h_{D-1}\prod_{i=D-2}^{1}\boldsymbol{B}_i, h_D\prod_{i=D-1}^{1}\boldsymbol{B}_i\right] = [t_1, t_2, \ldots, t_D], \tag{5}$$

where $d_i(k)$ is the ith ideal scalar signal. The normalized cross-correlation vector is $h_1, \ldots, h_N$, a unit vector in the direction of the cross-correlation vector between the desired signal and the observation vector. Then $I - h_i h_i^{\mathrm{H}}$, i.e. the $N \times N$ matrix $\boldsymbol{B}_i$, is the blocking matrix which spans the nullspace of the cross-correlation, such that $B_i h_i = 0, i = 1, \ldots, N$. The rank of $\boldsymbol{B}_i$ is $N - i$. The pre-filter matrix is $\boldsymbol{T}_D$. The rank of the $N \times 1$ observation signal vector can be reduced by the pre-filter matrix and we can get the desired D dimensional signal. The pre-filter matrix can be attained by truncating the Dth forward recursion equations of the MSWF.

Let us define the superscript of the matrix or vector $(\cdot)^i$ as the first i rows of the matrix or vector, such as $h_1^{(N-1)}$ denoting the first $N-1$ rows of h_1. The equation $\boldsymbol{B}_1 = I - h_1 h_1^{\mathrm{H}}$ must satisfy equation $\boldsymbol{B}_1^{N-1} h_1 = 0$ while MSWF is implemented by the CSA. In this paper, we replace $\boldsymbol{B}_1$ with $\boldsymbol{B}_1^{N-1}$, i.e., choosing the first $N-1$ rows of $\boldsymbol{B}_1$ as the $(N-1) \times N$ blocking matrix. This alternative retains all the information of $\tilde{y}(k)$, because the rank of $\boldsymbol{B}_1$ is $N-1$. The details of the conclusion are shown in Lemma 1.

Lemma 1. For the specific structure (i.e. the orthogonal projection matrix) given by the $(N-i+1) \times (N-i+1)$ matrix $\boldsymbol{B}_i$, the rank of that new matrix which is composed of the first $N-i$ of $\boldsymbol{B}_i$ rows will not be reduced (proof omitted due to lack of space).

It can be seen from Lemma 1 that we adopt the $(N-i) \times (N-i+1)$ rectangle matrix $\boldsymbol{B}_i^{(N-i)}$ as the blocking matrix, which can retain all the information of $\tilde{y}(k)$. This structure takes advantage of the rectangle blocking matrix to reduce the size of the observation vectors step by step; on the other hand, it does not need a separate solution for blocking matrix by using CSA. By this way, it can also reduce the storage capacity and reduce the computational complexity. For example, the computational complexity of $\boldsymbol{x}_i(k) = \boldsymbol{B}_i \boldsymbol{x}_{i-1}(k)$ is $O(N^2)$, while by adopting CSA we only need the complexity of $O(N)$ to compute $\boldsymbol{x}_i(k) = \boldsymbol{x}_{i-1}(k) - h_i\, d_i(k)$. As a result, the reduction of computational complexity is significant when the number of receiving antennas (i.e., N) is large. To sum up, we give the structure of RUMSWF in Figure 1. In Figure 1, $x_0(k)$ is the $N \times 1$ received signal vector and $d_i(k)$ is the ith ideal scalar signal, while w_i is the "weight" vector for the Wiener filter to estimate the scalar $d_{i-1}(k)$ from the vector $x_{i-1}(k)$, and ε_i is the error signal at each stage.

The adaptive reduced-rank RLS joint detection algorithm based on RUMSWF is similar to the full-rank RLS algorithm. The problem of updating both the normalized cross-correlation vector and the blocking matrix is to be solved. Let us define s_1 and λ as the reference signal and forgetting factor, respectively. The forgetting factor can reduce the importance of the sampled data, which is used for eliminating the impact of the current input vector. So, it actually represents a temporal correlation and it satisfies $0 < \lambda \leqslant 1$. In summary, the reduced-rank adaptive MIMO joint detection algorithm based on RUMSWF can be presented as follows:

Step 1. Update the normalized cross-correlation vector and the blocking matrix. At the kth time instant,

$$\hat{\boldsymbol{R}}(k) = \sum_{i=0}^{k} \lambda^{k-i} \tilde{\boldsymbol{y}}(i) \tilde{\boldsymbol{y}}(i)^{\mathrm{H}} = \tilde{\boldsymbol{y}}(k) \tilde{\boldsymbol{y}}(k)^{\mathrm{H}} + \lambda \hat{\boldsymbol{R}}(k-1), \tag{6}$$

$$\hat{\boldsymbol{r}}(k) = \sum_{i=0}^{k} \lambda^{k-i} \tilde{\boldsymbol{y}}(i) \boldsymbol{s}_m(i)^* = \tilde{\boldsymbol{y}}(k) \boldsymbol{s}_m(k)^* + \lambda \hat{\boldsymbol{r}}(k-1), \tag{7}$$

where $\hat{\boldsymbol{R}}(k)$ denotes the auto-correlation matrix $E[\tilde{\boldsymbol{y}}(k)\tilde{\boldsymbol{y}}(k)^{\mathrm{H}}]$ of the observation signal vector for Classical Wiener solution. At the same time, $\hat{\boldsymbol{r}}(k)$ is the cross-correlation vector $E[\tilde{\boldsymbol{y}}(k) s_m^*(k)]$ of the desired signal $\boldsymbol{s}_m(i)$ and the observation signal vector $\tilde{\boldsymbol{y}}(k)$. Then, we can get

$$h_1(k) = \tilde{y}(k) s_1^*(k) + \lambda h_1(k-1), \tag{8}$$

$$\boldsymbol{B}_1(k) = \boldsymbol{I}_N^{(N-1)} - \boldsymbol{h}_1(k)^{(N-1)} (\boldsymbol{h}_1(k)^{(N-1)})^{\mathrm{H}}. \tag{9}$$

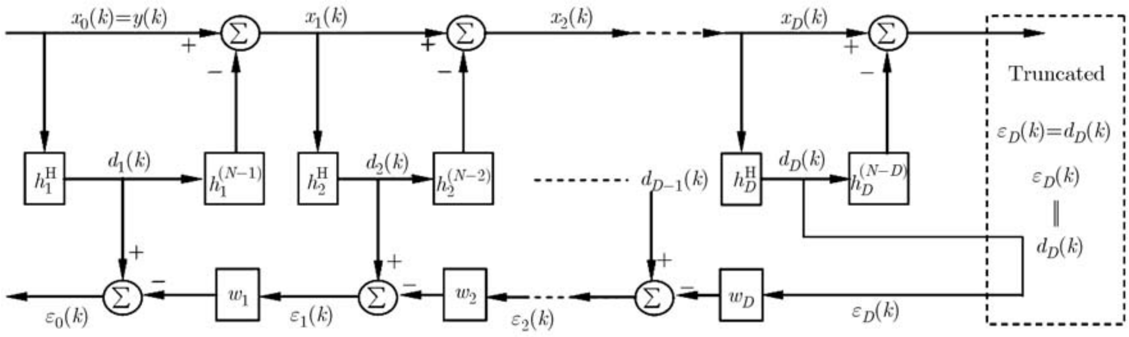

Figure 1 The implement structure of RUMSWF.

Step 2. According to $x_0(k) = \tilde{y}(k)$, get the forward recursion decomposition equation for $i = 1, \dots, D$.

$$d_i(k) = \boldsymbol{h}_i^{\mathrm{H}} \boldsymbol{x}_{i-1}(k), \quad 1 \leqslant i \leqslant D,$$

$$\boldsymbol{x}_i(k) = \boldsymbol{B}_i \boldsymbol{x}_{i-1}(k) = \boldsymbol{x}_{i-1}^{(N-i)}(k) - \boldsymbol{h}_i^{(N-i)}\ d_i(k), \tag{10}$$

$$h_{i+1}(k) = x_i(k) d_1^*(k) + \lambda h_{i+1}(k-1), \tag{11}$$

$$\boldsymbol{B}_{i+1}(k) = \boldsymbol{I}_{N-i}^{(N-i-1)} - (\boldsymbol{h}_{i+1}(k))^{(N-i-1)}((\boldsymbol{h}_{i+1}(k))^{(N-i-1)})^{\mathrm{H}}, \tag{12}$$

where superscript $(\cdot)^*$ denotes conjugate operation, h_i is the $(N-i+1) \times 1$ vector and $\boldsymbol{x}_i(k)$ is the $(N-i) \times 1$ vector. The dimension of $\boldsymbol{x}_i(k)$ is decreased step by step with the increase of the filter rank.

Step 3. Adopt the backward recursion decomposition equation for $i = 1, \dots, D$, which is the same as that of UMSWF [11].

$$d_D(k) = \varepsilon_D(k), \tag{13}$$

$$w_i(k) = \frac{\varepsilon_i(k) d_{i-1}^*(k)}{|\varepsilon_i(k)|^2}, \tag{14}$$

$$\varepsilon_{i-1}(k) = d_{i-1}(k) - w_i^* \varepsilon_i(k). \tag{15}$$

Finally, we can get the output of the equalization

$$\hat{s}_1(k) = w_1^*(k) \varepsilon_1(k). \tag{16}$$

4 Performance analysis

4.1 Complexity

Since most of the operations in MSWF are vector multiplication, in the case of single transmitting antenna, we define the length of the equalizer as N_f and the rank of reduced-rank MSWF as D. The main factors of the complexity in equalization algorithm are the forward and backward recursion decomposition of MSWF and the updating of the reduced-rank RLS. The complexity comparison of the three equalization algorithms based on MSWF is shown in Table 1. The GRS-MSWFRE represents the traditional equalization algorithm based on MSWF and the UMSWFRE is the equalization algorithm in [11] based on UMSWFRE. The RUMSWFRE represents the joint detection algorithm based on rectangle blocking matrix UMSWF proposed in this paper.

The backwardness of recursive synthesis is identical in three algorithms, so the difference in complexity attributes to the forward recursion. The improved joint detection algorithm based on RUMSWF takes advantages of both rectangle blocking matrix and CSA. In this new algorithm, we do not need to make a separate solution for blocking matrix and the size of the observation vector $\boldsymbol{x}_i(k)$ is decreased step by step with the forward recursion. Hence, it can effectively reduce the computational complexity. For ex-

Table 1 The complexities of three equalization algorithms (the times of the complex multiplication)

Algorithm	Forward recursion: The complexity for $x_i(k) = B_i x_{i-1}(k)$	Forward recursion: The complexity for $d_i(k) = h_i^H x_{i-1}(k)$	The complexity for reduced-rank RLS updating
GRS-MSWFRE	$N_f[N^2 + (N-1)^2 + \cdots + (N-D+1)^2]$	$N_f[N + (N-1) + \cdots + (N-D)]$	$O(MDN_fN^2)$
UMSWFRE	N_fND	$N_fN(D+1)$	$O(MD[N_f+1]N)$
RUMSWFRE	$N_f[(N-1) + \cdots + (N-D)]$	$N_f[N + (N-1) + \cdots + (N-D)]$	$O(MD[(N_f+1)N - N_fD(D+1)])$

Table 2 The complexities of algorithms with different transceiver antennas (the times of the complex multiplication)

System V-BLAST $N_f = 3, L = 3$ BPSK modulation	GRS-MSWFRE		UMSWFRE		RUMSWFRE	
	Forward recursion	Reduced-rank RLS updating	Forward recursion	Reduced-rank RLS updating	Forward recursion	Reduced-rank RLS updating
$M = 4, N = 16, D = 4$	2748	12288 $O(10^4)$	432	1024 $O(10^3)$	372	784 $O(10^2)$
$M = 4, N = 16, D = 8$	4200	24576 $O(10^4)$	816	2048 $O(10^3)$	600	1184 $O(10^3)$
$M = 4, N = 16, D = 10$	4578	30720 $O(10^4)$	1008	2560 $O(10^3)$	678	1240 $O(10^3)$
$M = 4, N = 16, D = 16$	4896	49152 $O(10^4)$	1584	4096 $O(10^3)$	768	1320 $O(10^3)$

ample, the computational complexity of $\boldsymbol{x}_i(k) = \boldsymbol{B}_i\boldsymbol{x}_{i-1}(k)$ is $O(N^2)$, while it is reduced to only $O(N)$ adopting CSA to compute $\boldsymbol{x}_i(k) = \boldsymbol{x}_{i-1}(k) - h_i\, d_i(k)$.

As shown in Table 1, the times of multiplications of the improved adaptive equalization algorithm based on RUMSWF are reduced by $MDN_fD(D+1)$ compared to that of the algorithm based on UMSWF for each stage. Consequently, the reduction for overall updating equalization is $MN_f[1^2 \times 2 + 2^2 \times 3 + \cdots (D-1)^2 \times D]$. It reduces almost $\frac{N_f(D+1)}{MN(N_f+1)}$ computational complexity of the UMSWFRE [11]. It can be seen that there is a significant computational complexity reduction in the improved equalization algorithm.

Table 2 lists the complexities of different algorithms with different reduced-rank dimensions. The complexity of the improved adaptive joint detection algorithm based on RUMSWF increases with the stage of the filter. In the same case, the updating of improved equalization algorithm is an order of magnitude lower than that of GRS-MSWF algorithm, about half that of UMSWF algorithm. The efficiency of the improved equalization algorithm is higher than those of the other two algorithms.

4.2 BER

Let us take $s_1(k)$ as the reference signal, $w_1(k)$ as the linear weighted vector, and $y(k)$ as the received signal. The output $\hat{s}_1(k)$ is the estimation of $s_1(k)$. The signal to interference plus noise ratio (SINR) of the linear equalizer output is defined as (see [5])

$$SINR = \sigma_s^2(|\boldsymbol{w}_1^{\mathrm{H}}\boldsymbol{h}_1|^2/\boldsymbol{w}_1^{\mathrm{H}}\boldsymbol{R}_I\boldsymbol{w}_1), \tag{17}$$

where σ_s^2 denotes the power of $s_1(k)$ and h_1 is the cross-correlation of the transmitted and received signals. The covariance matrix of the interference plus noise is $\boldsymbol{R}_I = \sum_{i=2}^{M}\boldsymbol{h}_i\boldsymbol{h}_i^{\mathrm{H}} + \sigma^2\boldsymbol{I}$. Then, we can define the BER as

$$\begin{aligned} p_e(1) &= \frac{1}{2^{M-1}} \sum_{[s_2 \quad \cdots \quad s_M] \in \{1,-1\}^{M-1}} Q\left(\sigma_s\boldsymbol{w}_1^{\mathrm{H}}\boldsymbol{h}_1 - \sum_{m=2}^{M}\sigma_s\boldsymbol{w}_1^{\mathrm{H}}\boldsymbol{h}_1 s_m/\sigma\sqrt{\sigma_s\boldsymbol{w}_1^{\mathrm{H}}\boldsymbol{w}_1}\right) \\ &\approx Q(\sqrt{SINR}), \end{aligned} \tag{18}$$

where $Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\frac{t^2}{2}} dt$ is the Gaussian cumulative distribution function. It is obvious that the higher the output SINR, the better the BER performance. When we fix the transmitted signal power at $\sigma_s^2 = 1$, the BER of the output signal is $p_e(1) \approx Q\left(\sqrt{\frac{\boldsymbol{w}_1^{\mathrm{H}} \boldsymbol{h}_1 \boldsymbol{h}_1^{\mathrm{H}} \boldsymbol{w}_1}{\boldsymbol{w}_1^{\mathrm{H}} \boldsymbol{R}_I \boldsymbol{w}_1}}\right)$. We can get the Dth output SINR of the RUMSWF from the derivation for UMSWF [11],

$$SINR^{(D)} = \boldsymbol{h}_1^{\mathrm{H}} \boldsymbol{T}_D (\boldsymbol{T}_D^{\mathrm{H}} \boldsymbol{R}_I \boldsymbol{T}_D)^{-1} \boldsymbol{T}_D^{\mathrm{H}} \boldsymbol{h}_1. \tag{19}$$

Finally, we can obtain the BER of the reduced-rank equalization based on RUMSWF for MIMO

$$p_e(1) \approx Q(\sqrt{SINR^{(D)}}). \tag{20}$$

4.3 Stage of the RUMSWF

When the stage of the MSWF is different, i.e., the rank D is different, the BER of the reduced-rank joint detection is different, proving that the Dth SINR of MSWF satisfies the following formula with the stage for $D \geqslant 0$:

$$SINR^{(D+1)} = \frac{P}{\sigma^2 + \alpha \frac{P}{1+SINR^{(D)}}}, \tag{21}$$

where P is the received power of each user. If there is only one user, it has the received signal power. The ratio of the number of the users to the received antenna is $\alpha = K/N$ and the noise power is σ^2. Formula (19) converges to the full-rank MMSE BER [5] with the increase of D. Especially, if $\sigma^2 = 0$, as $SINR^{(0)} = 0$ and $SINR^{(1)} = P/(\sigma^2 + \alpha P)$, we can get

$$SINR^{(D)} = \sum_{n=1}^{D} \alpha^{-n} = \begin{cases} \dfrac{\alpha^{-D} - 1}{1 - \alpha}, & \alpha < 1, \\ D, & \alpha = 1, \\ \dfrac{1 - \alpha^{-D+1}}{1 - \alpha}, & \alpha > 1. \end{cases} \tag{22}$$

In this paper, we just discuss the case $\alpha = (1/N) < 1$. So, the output SINR of MSWF converges to the full rank BER. At the same time, we know that MSWF is equivalent to projecting the received signal onto a low dimensional subspace. The dimension of the subspace is the stage of the MSWF. Consequently, the performance of RUMSWF depends largely on the approximation to the signal subspace. Generally speaking, in order to get the full rank SINR, the required stage of the MSWF is gradually increased. But when the stage is increased to a certain level, the BER is close to the full rank BER and the performance improvement will become less obvious. The reason is that when we project the received signal onto a low dimensional subspace, the noise vector included in received signal is also projected onto the same subspace. We expect that the low dimensional subspace can suppress the noise outside the projection subspace as much as possible. There are two effects of the subspace projection: on the one hand, the higher the subspace projection dimension, i.e., the more the stage of MSWF, the more the signal components of subspace and the better the approximation we can get, but the subspace eliminates more noise subspace components. When the dimension of the projection subspace equals the signal subspace dimension, the two spaces become identical, but no noise is eliminated. On the other hand, the lower dimension of the projection subspace can eliminate more noise components. When the projection subspace is Null Space, it can completely eliminate the noises, but it has no ability to approximate the signal subspace. Thus the noise component plays a major role in the system performance for low SNR cases. In these cases, although the projection subspace cannot well approximate the signal subspace, the performance gain brought by eliminating the noise component is greater than the loss caused by bad approximation of signal subspace. So, the overall performance is better. Similarly, the noise component is very small in high SNR cases; the performance gain brought by eliminating the noise component is far from enough to offset the loss of that caused by less accurate approximation of signal subspace. Therefore, the projection subspace dimension must be increased to achieve the desired performance in these cases.

5 Simulations results

We consider a V-BLAT system with $M = 4$ antennas at the transmitter and $N = 8$ antennas at the receiver. The signal from each transmitting antenna is modulated by BPSK and has the same power. The channel is a frequency selective Rayleigh fading channel with propagation paths $L = 3$. Moreover, the length of the equalizer is $N_f = 7$. The power in each path is exponentially distributed and the corresponding spectrum follows Jake's model. The multipath fading coefficient of each antenna is independent and follows complex Gaussian distribution with unit variance. There is no channel coding in this system. The noise is complex Gaussian white noise with zero mean and the power changes with the signal-to-noise ratio (SNR). In our simulations, there are 1000 bits in each frame [11]. Monto Carlo simulation is used for simulations.

5.1 BER performance

We consider the BER performance versus the SNR with optimized parameters (forgetting factor λ=0.998 [12]) for all schemes. SNR [11, 12] is the output SNR, i.e., $SNR = \frac{1}{N}\sum_{i=1}^{N} SNR_i$. SNR_i denotes the power ratio between the received signal and the noise at the ith received antenna. Figure 2 illustrates the BER difference between the improved equalization algorithm based on RUMSWF and that based on UMSWF. The results also indicate the BER performance of the two algorithms with a different rank (the stage D). Our work shows that the required stages for both MSWF are gradually increased in order to obtain full-rank MMSE with SNR from low to high. From the cases of $D = 4, 8, 16$, we can see that the BER performance of UMSWF slightly outperforms RUMSWF. The reason is that we modified the blocking matrix; MSWF is equivalent to projecting the received signal onto a low dimensional subspace. So, the performance depends largely on the approximation of the signal subspace. However, the equalization algorithm based on RUMSWF chooses the first $N-i$ rows from the $(N-i+1)\times(N-i+1)$ square matrix as the blocking matrix. If these $N-i$ rows are linear independent vectors, in other words, if they can form an orthogonal signal space, the $N-i$ rows can exactly express all the information of $\tilde{y}(k)$. If they are not linear independent vectors, they cannot express $\tilde{y}(k)$ exactly. Thus, there exists the BER difference between the two algorithms. However, the difference is very small, which can be neglected especially in the high SINR cases.

In Figure 3, we consider a frequency selective OFDM V-BLAST system with $M = 4$ and $N = 8$. The system parameters are set based on IEEE 802.11a, where the channel bandwidth is B= 20 MHz and the sampling period is 50 ns. Moreover, the subcarrier number is $N_c = 512$ and the guard interval is GI =800 ns. The carrier frequency is 5 GHz and the OFDM symbol period is 3.2 μs (excluding interval) [15]. The channel coefficient adopts indoor channel model [16],

$$h_{ij}(\tau_k, t) = \frac{1}{\sqrt{M}}\sum_{k=1}^{L}\rho[k]\times\sum_{m=1}^{M}\mathrm{e}^{\mathrm{j}(2\pi t f_{i,j,k,m}+\theta_{i,j,k,m})}\delta(\tau-\tau_k), \tag{23}$$

where $f_{i,j,k,m} = f_{d\max}\sin(2\pi u_{i,j,k,m})$ stands for the discrete Doppler frequency shift and $\theta_{i,j,k,m} = 2\pi u_{i,j,k,m}$ is the Doppler phase. The number of oscillator is $M = 60$ and τ_k is the delay of kth path.

A comparison of BER performances between the proposed algorithm and traditional algorithms for OFDM V-BLAST system is shown in Figure 3. The results show that the proposed algorithm has better equalization and detection performance in OFDM V-BLAST system. At the same time, the performance loss of our proposed algorithm is only 1.8 dB between UMSWFRE for a BER of 10^{-3} with rank equal to 8 and the performance gap between them is getting smaller and smaller as SNR increases. This implies that the proposed scheme can infinitely approximate to UMSWFRE at higher SNR.

5.2 Convergence rate

We evaluate and compare the convergence rate of the two reduced-rank equalizers with that of the traditional full-rank RLS algorithm. The adaptive MIMO linear equalizer employs SNR =10 dB and forgetting factor $\lambda = 0.998$, which is used for eliminating the impact of the current input vector. Other

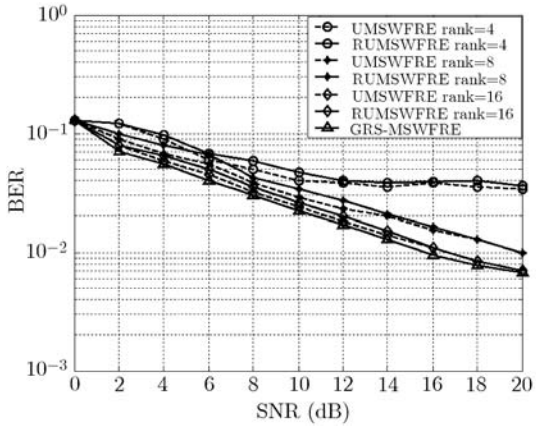

Figure 2 BER performance against SNR of the reduced-rank equalizer based on MSWF.

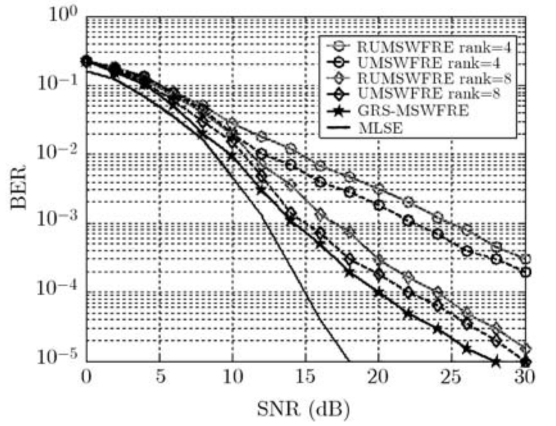

Figure 3 BER performance against SNR of the reduced-rank equalizer for OFDM V-BLAST system.

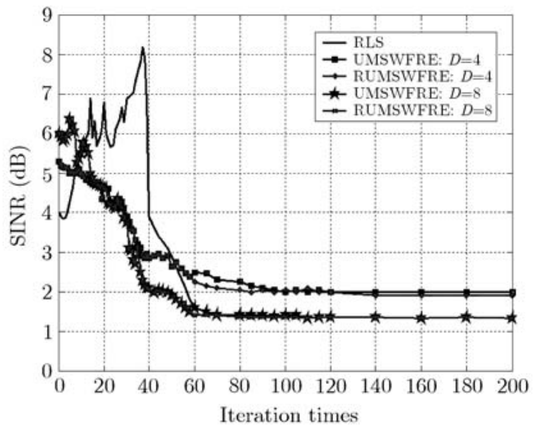

Figure 4 Convergence performance against rank (D).

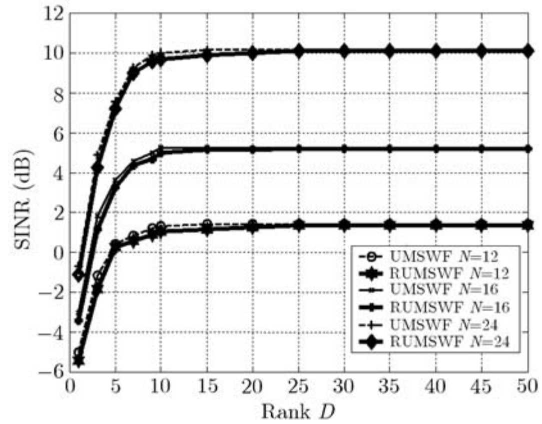

Figure 5 SINR performance against rank (D), with the fixed M=8 and different received antennas N.

parameters are the same as in subsection 5.1. The MSE of these two algorithms versus the iteration times is shown in Figure 4. Figure 4 shows that the proposed scheme converges slightly faster than the algorithm in [11]. Because the improved algorithm needs much lower computational complexity than the algorithm in [11], the convergence rate is faster in obtaining the same BER. It also indicates that these two algorithms converge much faster than the traditional full-rank RLS. Moreover, the smaller the D, the faster the convergence rate.

Convergence rate is defined as the required time for the optimal iteration solution, which can be taken as the speed of the search algorithm. Conventional RLS adaptive algorithm searches for the optimal solution in the entire signal subspace, and the convergence rate is proportional to the sub-space dimension. However, the reduced-rank algorithm is different. The subspace projection greatly reduces the scope of the search and increases the algorithm convergence rate. The lower the projection sub-space dimension, the faster the convergence rate. The reduced-rank algorithm can only find the local optimal solution in the reduce-dimension subspace while the conventional RLS algorithm can find the overall optimal solution. So, the SINR of the reduced-rank algorithm is slightly lower than that of the RLS, as shown in Figure 4. Finally, the results also show that the reduced-rank RLS cannot achieve the full-rank MMSE of RLS when the stage D is too small.

5.3 Rank of the filter stage

We consider the SINR performance versus the rank D. The maximum output SINR for MWSF is derived and analyzed in [10]. It is proved that the maximum output SINR is very close to minimum MSE. MSWF can maximize the output SINR. Figure 5 indicates the output SINR of two UMSWF with 8 transmit antennas and various numbers of receive antennas. The other parameters are the same as those of the simulations for BER.

The results show that the necessary number D of states does not change when employing more receive antennas. The UMSWF algorithm can achieve full-rank SINR with $D = 10$ while the RUMSWF achieves the same SINR as $D = 15$. The reason is that the received signals have a lot of redundancy with more receive antennas, UMSWF finds a low-dimensional subspace by a series of orthogonal decompositions. The subspace contains a majority of signal energy and the solutions in subspace can well approximate the full-rank solution. Considering the property of MSWF, the $M \times 1$ MIMO transmitted signal can get full-rank MSE after M-stage decompositions and the performance will not be further improved with the stage increased. Thus, the performance of reduced-rank algorithm is not affected by the number of the receive antennas. This is also proved by simulations in [5], which shows that the necessary number of states of MSWF to achieve full-rank SINR in CDMA systems does not change as the number of users and/or the length of the spreading sequences increase.

6 Conclusions

In this paper, an adaptive reduced-rank joint detection algorithm based on RUMSWF was proposed. We apply it to frequency selective MIMO systems. The new scheme modifies the blocking matrix of UMSWF, adopting rectangle matrix as blocking matrix to reduce the size of the observation data vectors step by step in the forward recursion decomposition. And it is chosen from the square blocking matrix for UMSWF, which takes advantage of square blocking matrix without a separate solution for the blocking matrix. We implemented RUMSWF into adaptive joint detection to reduce the computational complexity of the algorithm and improve the convergence rate. The RUMSWF algorithm can effectively reduce the times of the complex multiplication in the forward recursion decomposition and speed up convergence rate, especially for more transceiver antennas and reduced-rank stages. Theoretical analysis and simulation results showed that this improved adaptive reduced-rank joint detection algorithm outperforms the algorithm based on UMSWF [11] in complexity and convergence rate.

Acknowledgements

This work was supported by the Key Laboratory of Universal Wireless Communication in Beijing University of Posts and Telecommunications, Ministry of Education (Grant No. 2007101), the National High-Tech Research & Development Program of China (Grant No. 2009AA011801), and the National Natural Science Foundation of China (Grant No. 60832007).

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