相对论点耦合模型结合复动量表象方法研究 ^{74}Ca 奇特结构

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摘要 本文介绍了奇特原子核的研究意义、研究方法及研究进展. 给出了近期在相对论点耦合模型下结合复动量表象(CMR)方法和Bardeen-Cooper-Schrieffer (BCS)方法, 发展的RMFPC-CMR-BCS方法的理论框架. 本文重点是用此方法研究 ^{74}Ca 的奇特结构. 由于那些靠近连续谱阈值的单粒子态对奇特结构的形成有着重要作用, 本文详细研究了对晕现象较为重要的单粒子态的波函数和密度分布, 在用BCS方法考虑对关联作用后, 低轨道角动量 $2d_{3/2}$ 态因其是分布在最靠近连续谱阈值的共振态, 最后一个核子将有很大概率弥散到 $2d_{3/2}$ 共振态, 从而使原子核半径增大, 形成奇特晕现象. 文中对这部分进行了详细研究, 也展示出共振态对奇特结构形成的重要作用.

关键词 相对论点耦合模型, 复动量表象方法, 奇特核结构

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1 引言

自从采用放射性粒子束发现第一个晕核 ^{11}Li ^[1]后, 对那些远离 β 稳定线的奇特原子核的研究已经成为理论和实验原子核物理的研究热点^[2-8]. 而这些奇特核最重要的特征是弱束缚性, 尤其是那些靠近中子和质子滴线的原子核. 它们的费米面非常接近连续谱阈值, 当考虑对关联相互作用后, 价核子容易散射连续谱形

成奇特结构^[9,10], 其中费米面附近的共振态在各种奇特结构形成过程中有着非常关键的作用^[11-17]. 因此, 探寻合适的研究共振态的方法对研究奇特核性质有着十分重要的意义.

近年来, 多种理论方法已经被广泛应用于研究单粒子共振态, 如R-矩阵方法^[18,19]、K-矩阵方法^[20]、S-矩阵方法^[21]、Jost Function方法^[22,23]和格林函数方法^[24,25]. 后来, 人们将一些处理束缚态的方法运用到

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处理共振态问题, 发展了很多种类束缚态方法, 在处理共振态时, 因其计算更加简便而被广泛发展和应用^[26-31].

复动量表象(CMR)方法是一种有效处理共振态的类束缚态方法^[32-36], 它通过在复空间中直接求解粒子的运动方程, 可以同时获得束缚态和共振态. 由于相对论平均场(RMF)理论在描述原子核性质时已经取得很大的成功, 多种类束缚态方法已发展到RMF理论下研究单粒子共振态^[37-43]. 不久前, 复动量表象方法在相对论平均场框架下得到进一步发展, 形成了RMF-CMR方法^[44], 它可以很好地描述原子核中的奇特结构及性质^[45-52]. 在RMF-CMR方法中, 人们可以直观地得到束缚态分布在动量空间的虚轴上, 共振态分布在第四象限, 通过进一步运用Bardeen-Cooper-Schrieffer(BCS)方法考虑对关联效应, 发现那些靠近连续谱阈值附近的共振态在奇特现象形成过程中发挥着重要作用^[53,54]. 文献[55]在相对论点耦合模型下结合复动量表象方法和BCS方法发展了RMFPC-CMR-BCS方法, 不仅研究了单粒子共振态, 而且很好地处理对关联效应, 以Ce偶偶核为例, 验证了这个方法研究原子核基态性质的优越性. 因此, 继续探讨这个方法研究奇特原子核的有效性和适用性将是个有意义的工作.

由于关于Ca原子核性质还存在着很多未解之谜, 对其奇特结构的研究一直是个热门话题. 近年来, 人们对丰中子Ca同位素性质的实验测量已经取得了很大的进展^[56-65]. 同时, 发展了很多研究Ca同位素奇特结构的理论方法^[66-69]. 考虑到RMFPC-CMR-BCS方法在研究单粒子共振态和奇特结构性质方面的优越性, 本文将运用该方法进一步研究Ca原子核的奇特结构.

本文将分为以下几个部分: 第2部分介绍RMFPC-CMR-BCS方法的理论框架; 第3部分详细阐述⁷⁴Ca奇特结构的研究结果及讨论; 第4部分是结论与展望.

2 理论框架

相对论点耦合模型出发点是通过引入有效拉氏量密度

$$\mathcal{L} = \mathcal{L}^{\text{free}} + \mathcal{L}^{4f} + \mathcal{L}^{\text{der}} + \mathcal{L}^{\text{cm}} + \mathcal{L}^{\text{hot}}, \quad (1)$$

其中, $\mathcal{L}^{\text{free}}$ 和 $\mathcal{L}^{4f}$ 分别表示自由核子运动的部分和四费米子点耦合部分, $\mathcal{L}^{\text{der}}$ 是通过引入密度以及流的微商

项来模拟核力的有限程效应, 采用QED标准的拉氏量密度形式 $\mathcal{L}^{\text{cm}}$ 表示质子之间的库仑相互作用, $\mathcal{L}^{\text{hot}}$ 表示拉氏密度中的非线性高阶耦合项. 它们具体的表达式可以参考文献[70].

基于拉氏密度, 通过时间反演对称性的变分原理, 核子满足的Dirac方程可表示为

$$[\boldsymbol{\alpha} \cdot \mathbf{p} + \beta(M + S) + V]\psi = \varepsilon\psi, \quad (2)$$

其中, M 是核子质量, $\boldsymbol{\alpha}$ 和 β 是狄拉克矩阵元. S 和 V 分别是标量势和矢量势.

在动量表象中, Dirac方程可以表示为

$$\int d\mathbf{k}' \langle \mathbf{k} | H | \mathbf{k}' \rangle \psi(\mathbf{k}') = \varepsilon \psi(\mathbf{k}), \quad (3)$$

其中, $|\mathbf{k}'\rangle$ 是动量 $\mathbf{p}$ 或波矢为 $\mathbf{k}=\mathbf{p}/\hbar$ 的自由粒子波函数. $\psi(\mathbf{k})$ 是动量波函数. Dirac哈密顿量可以写成 $H = \boldsymbol{\alpha} \cdot \mathbf{p} + \beta(M + S) + V$.

对于球对称系统来说, 波函数 $\psi(\mathbf{k})$ 可以通过变量分离法分成径向和角向部分:

$$\psi(\mathbf{k}) = \begin{pmatrix} f(k)\phi_{lm_j}(\Omega_k) \\ g(k)\phi_{\bar{l}m_j}(\Omega_k) \end{pmatrix}. \quad (4)$$

将方程(4)代入方程(3), 可以得出球对称系统下的径向Dirac方程为

$$Mf(k) - kg(k) + \int k'^2 dk' V^+(k, k') f(k') = \varepsilon f(k), \quad (5a)$$

$$-kf(k) - Mg(k) + \int k'^2 dk' V^-(k, k') g(k') = \varepsilon g(k), \quad (5b)$$

其中,

$$V^+(k, k') = \frac{2}{\pi} \int r^2 dr [V(r) + S(r)] j_l(k'r) j_l(kr), \quad (6a)$$

$$V^-(k, k') = \frac{2}{\pi} \int r^2 dr [V(r) - S(r)] j_{\bar{l}}(k'r) j_{\bar{l}}(kr). \quad (6b)$$

类似于文献[55], 用BCS方法处理对关联相互作用, 可得到具有共振态的能隙方程和粒子数方程为

$$\sum_b \frac{\Omega_b}{\sqrt{(\varepsilon_b - \lambda)^2 + \Delta^2}} + \sum_r \Omega_r \frac{g_r(\varepsilon)}{\sqrt{(\varepsilon - \lambda)^2 + \Delta^2}} = \frac{2}{G}, \quad (7)$$

$$\sum_b \Omega_b \left[1 - \frac{\varepsilon_b - \lambda}{\sqrt{(\varepsilon_b - \lambda)^2 + \Delta^2}} \right] + \sum_r \Omega_r \int g_r(\varepsilon) \left[1 - \frac{\varepsilon - \lambda}{\sqrt{(\varepsilon - \lambda)^2 + \Delta^2}} \right] = N, \quad (8)$$

方程中的能级密度可以为

$$g_r(\varepsilon) = \frac{1}{\pi} \frac{\Gamma/2}{(\varepsilon - \varepsilon_r)^2 + \Gamma^2/4}, \quad (9)$$

其中, $\Delta(G)$ 是对立强度, N 是粒子数, $2\Omega_b = 2j_\sigma + 1$ 和 $2\Omega_r = 2j_\sigma + 1$ 分别表示束缚态和共振态的简并度, j_σ 为总角动量量子数.

通过将方程(9)代入对能隙方程(7)和粒子数方程(8), 共振态宽度的影响可以很自然地包含在内, 细节可参考文献[71]. 并且, 还可以得出束缚态和共振态的占据概率. 当共振态能谱包含在对能隙方程中, 不仅可以自洽处理粒子间的配对相互作用, 还有助于研究共振态对原子核晕现象的形成机制.

3 结果与分析

本文运用RMFPC-CMR-BCS方法计算研究了 ^{74}Ca 的单粒子能级结构, 并分析其晕结构的形成机制. 在相对论点耦合理论中采用PC-PK1相互作用获得自洽的势场, 然后在动量空间下求解自洽势场中的Dirac方程, 可得出相应的共振参数. 类似于文献[55], 本文选用 $\Delta = 12/\sqrt{A}$ 作为对立强度参数. 由于共振态参数与动量空间中的积分路径无关[37], 所以选取较大的积分路径以获得所有相关共振态. 动量空间的积分路径选取为 $k=0 \text{ fm}^{-1}$, $k=0.26-i0.13 \text{ fm}^{-1}$, $k=1.0 \text{ fm}^{-1}$ 和 $k_{\max}=2.0 \text{ fm}^{-1}$, 计算得到的单粒子能谱展示在图1中. 可以清晰看到红色实心菱形表示的共振态分布在第四象限, 它们是 $2d_{3/2}$, $1g_{7/2}$ 和 $1h_{11/2}$, 并且靠近实轴 k 轴, 虚部较小对应窄共振态. 由于共振态的寿命和宽度成反比, 这意味着共振态 $2d_{3/2}$, $1g_{7/2}$ 和 $1h_{11/2}$ 有长寿命. 而对于轨道角动量较低的 $2d_{3/2}$ 共振态, 在考虑对关联的作用后, 将更加接近连续谱阈值, 对原子核晕现象的形成有更加重要的作用[62].

为了进一步分析 ^{74}Ca 的晕结构, 图2展示了能量在 -10 – 10 MeV 区域内的束缚态和共振态, 这里的共振态能量和宽度是通过 $E_r + iE_i = E_r - i\Gamma/2 = \sqrt{k^2 + M^2} - M$ 得到的. 相应共振态的能量和宽度数值展示在表1中. 当不考虑对关联相互作用时, ^{74}Ca 的费米面将分布在最后一个束缚态 $2d_{5/2}$ 上. 然而, 当采用BCS方法处理原子核中的对关联作用时, 此时费米面为 -0.51 MeV , 文中计算得到那些费米面附近共振态的占据概率, 展示

在图2中. 可以清晰地看到, 最接近费米面附近的共振态 $2d_{3/2}$ 的占据概率最大, 这表现出它有很大可能弥散到连续谱, 从而使得原子核半径增大形成晕现象.

图1 (网络版彩图)展示在复动量空间中, 采用RMFPC-CMR-BCS方法计算 ^{74}Ca 的单粒子能谱, 其中计算过程中采用的是PC-PK1有效相互作用[70]. 束缚态、共振态和连续谱分别用蓝色空心方块、红色实心菱形和黑色空心圆表示. 橄榄绿实线表示动量空间的积分路径

Figure 1 (Color online) Single-neutron spectra for ^{74}Ca in the RMFPC-CMR-BCS calculations with the effective interaction PC-PK1 [70] in the complex k plane. The bound states, the resonant states, and the continuum are denoted by the blue open squares, red filled diamonds, and black open circles, respectively. The olive solid line denotes the contour of integration in the complex momentum plane.

图2 与图1一样的单粒子能谱的能量和占据概率. 灰色矩形的高度表示着共振态相应的宽度

Figure 2 The same single-neutron spectra as Figure 1, but for their energies and occupation probabilities. Heights of the gray rectangles represent the widths of the corresponding resonances.

为了更好地研究 ^{74}Ca 的晕现象, 图3展示了原子核及单粒子态的密度分布. 在图3(a)中看出考虑对关联作用前后质子密度趋势相似, 但考虑对关联后, 黑色实线表示的中子密度将更加弥散, 这是导致原子核半径弥散的主要原因. 图3(b)进一步计算了费米面附近束缚态和共振态的中子密度, 可以看到随着半径 r 的增加, 共振态的密度衰减得很缓慢, 而 $3s_{1/2}$ 和 $2d_{5/2}$ 两条束缚态密度在较大半径(如 $r \geq 10$ fm)处衰减较为缓慢, 这表明弱束缚能级对晕现象的形成有不可忽略的作用.

图4展示了 ^{74}Ca 中单粒子态密度在中子密度中的占比分布, 可以看到随着半径 r 的增加, 尤其是 $r \geq 10$ fm处, 弱束缚态 $3s_{1/2}$ 和 $2d_{5/2}$ 的密度占比开始下降, 而共振态的占比则不断增大. 由于离心位垒会随轨道角动量

表 1 费米面附近共振态的能量和宽度

Table 1 Energy and width of resonant states near the Fermi surface

共振态	能量 (MeV)	宽度 (MeV)
$2d_{3/2}$	0.166	0.00642
$1g_{7/2}$	2.991	-0.117
$1h_{11/2}$	7.179	-0.545

图 3 (网络版彩图) ^{74}Ca 的密度分布. (a) 黑色实线和红色实线分别表示考虑对关联时中子密度和质子密度. 黑色虚线和红色虚线分别表示不考虑对关联时中子密度和质子密度. (b) 单粒子态 $3s_{1/2}$, $2d_{5/2}$, $2d_{3/2}$, $1g_{7/2}$ 和 $1h_{11/2}$ 的密度

Figure 3 (Color online) Density distributions in ^{74}Ca . (a) The neutron and proton densities marked by the black solid line and red solid line with the pairing correlation. The black dashed line and red dashed line respectively represent neutron and proton densities without the pairing correlation. (b) The densities of the single-particle levels $3s_{1/2}$, $2d_{5/2}$, $2d_{3/2}$, $1g_{7/2}$, and $1h_{11/2}$.

量子数的增加而增大, 共振态 $1g_{7/2}$ 和 $1h_{11/2}$ 对中子密度弥散的贡献相对较小, 而共振态 $2d_{3/2}$ 的密度占比越来越大, 这是形成 ^{74}Ca 晕现象最重要的原因, 与图2有一样的结论. 同时, 文中还计算得到 ^{74}Ca 的中子半径为 5.12 fm, 原子核总半径为 4.78 fm, 当关掉对关联后, ^{74}Ca 的中子半径为 4.78 fm, 原子核总半径为 4.52 fm. 这表明当引入BCS方法考虑对关联效应后, 因中子有一定概率占据费米面附近共振态, 从而形成晕现象的结果.

为进一步解释 $2d_{3/2}$ 共振态密度随着半径 r 增加而缓慢减小的现象, 图5展示了Dirac旋量的上下分量的实部和虚部波函数. 可看到随着半径 r 的增加, 波函数

图 4 (网络版彩图)单粒子态密度占中子密度之比

Figure 4 (Color online) Ratio of the neutron density of the single-particle levels to the total neutron density.

图 5 (网络版彩图)共振态 $2d_{3/2}$ 的波函数. 黑色实线和红色虚线分别表示上分量 $f(r)$ 的实部和虚部. 蓝色点虚线和玫红色短虚线分别表示下分量 $g(r)$ 的实部和虚部

Figure 5 (Color online) Wave functions of the resonant state $2d_{3/2}$. The black solid and red dashed lines represent the real and imaginary parts of the upper component $f(r)$. The blue dash-dotted and magenta short-dashed lines represent the real and imaginary parts of the lower component $g(r)$.

的下分量 $g(r)$ 快速衰减为零, 对 neutron 密度的贡献主要来源于波函数的上分量 $f(r)$, 而且在半径较大区域, 波函数上分量的实部还处于较为弥散的状态, 这是引起 neutron 密度弥散的主要原因.

4 结论与展望

本文采用RMFPC-CMR-BCS方法计算研究了 ^{74}Ca 的束缚态和共振态单粒子能级分布, 并在考虑对关联效应后分析了共振态的占据情况以及晕结构形成机制. 文中展示了动量空间的单粒子能级分布, 给出了研究区域内的共振态占据概率, 并比较考虑对关联前后 neutron 密度分布情况, 通过对比展示了共振态密度分布

对 neutron 密度弥散的影响, 直观展示出半径较大区域共振态所占比例. 最后用波函数的上下分量展示出, 共振态密度弥散主要来源于波函数上分量的实部在半径很大区域有较大的弥散分布. 这些结论均说明本文所采用的研究共振态的方法可以很好地计算出共振态参数和单粒子能级分布, 并展示了共振态对原子核晕现象形成的重要作用.

接近连续谱阈值的共振态对奇特现象的形成有着重要作用, 随着 neutron 数增加, 势场不断增大, 那些接近 neutron 滴线核有着特殊的性质, 研究 Ca 的同位素链的性质将为 neutron 滴线核的研究提供一定基础, 而本文对 ^{74}Ca 的研究也将为下一工作提供坚实的理论基础. 相关工作正在整合分析中.

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Exploration of the exotic structure of ^{74}Ca using the relativistic point-coupling model combined with complex momentum representation

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In this paper, the significance, methods, and progress of research on exotic nuclei are introduced. The theoretical framework of the RMFPC-CMR-BCS method developed by combining the complex momentum representation (CMR) and the Bardeen-Cooper-Schrieffer (BCS) methods according to the relativistic point-coupling model is given. This paper focuses on the investigations of the exotic structure of ^{74}Ca using the developed method. Because single-particle states near the continuous spectrum threshold are crucial to the formation of an exotic structure, the wave function and the density distribution of a single-particle state that are essential for the halo phenomenon are studied in detail. As the $2d_{3/2}$ state with low orbital angular momentum is closest to the continuum threshold, the last nucleon is likely to spread to the $2d_{3/2}$ state after the BCS method is used to deal with the pairing correlation, which increases the radius of the nucleus. Details about the $2d_{3/2}$ state can be found, and it shows the relevance of resonance in the formation of exotic structures.

relativistic point-coupling model, complex momentum representation, exotic structure

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