基于 d 维六粒子Cluster态的任意qudit态远程联合制备

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摘要 提出利用 d 维六粒子Cluster态的任意高维量子态确定远程联合制备方案. 首先考虑了基于 d 维六粒子Cluster态的任意双粒子qudit态远程联合制备方案, 然后提出了基于六粒子Cluster态的任意单粒子qudit态双向远程联合制备方案. 在方案中, 两个发送方共享需制备量子态信息, 第一发送方依据已知需制备量子态信息对纠缠粒子执行半正定算子测量(POVM), 第二发送方依据第一发送方测量结果以及已知需制备量子态信息对纠缠粒子选择执行适当的正交投影测量, 接收方依据所有发送方测量结果对纠缠粒子执行相应的局域幺正操作, 就可以在他的粒子上重建需制备量子态. 通信方使用高维Cluster态为量子纠缠信道完成高维量子态远程联合制备, 与其他方案相比, 具有信道容量高的优点.

关键词 量子态远程联合制备, d 维Cluster态, 半正定算子测量, 量子态双向远程联合制备

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1 引言

量子态在量子通信中的应用为量子信息提供了一些经典信息处理中没有对应的信息处理方法^[1-26]. 利用量子纠缠信道的非定域相关性, 量子通信可以完成任意已知^[27-30]或未知量子态的远程传送^[31,32]. 在量子隐形传态中, 发送方未知需制备量子态信息. 量子隐形传态中所消耗的量子纠缠资源和经典通信是固定的, 即每传送1量子比特信息需消耗1个双量子比特最大纠缠态和传送2比特的经典信息^[33-37]. 与量子隐形

传态类似, 在量子态远程制备中通信双方基于预先共享量子纠缠信道远程传送任意已知量子态^[38-40]. 量子态远程制备中发送方已知需制备量子态信息, 接收方未知需制备量子态信息, 发送方协助接收方在接收端量子系统上制备需制备量子态^[41]. 由于发送方已知需制备量子态信息, 因此可以依据已知需制备量子态信息选取适当量子测量, 减少量子态远程制备所需通信资源^[42,43]. Pati^[44]指出通信双方仅需共享一个双粒子最大纠缠态, 传送1比特的经典信息就可完成量子比特Bloch球赤道圆上任意量子态, 即赤道态(Equatorial

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Lv L, Zhou P, He L M, et al. Joint multi-party remote preparation of arbitrary qudit states via a d -level six-qudit Cluster state (in Chinese). Sci Sin-Phys Mech Astron, 2020, 50: 080301, doi: [10.1360/SSPMA-2019-0409](https://doi.org/10.1360/SSPMA-2019-0409)

State)的远程制备. 此外, 量子态远程制备中所消耗的量子纠缠资源和经典通信可相互协调^[45,46]. 由于在量子通信网络, 分布式量子计算等量子信息处理任务中的重要应用, 量子态远程制备引起了研究者的广泛关注. 一方面针对量子态远程制备不同环节开展理论研究, 如高维的量子态远程制备^[47]、低纠缠的量子态远程制备^[48]、优化的量子态远程制备^[49]、连续变量的量子态远程制备^[50]、混合态的远程制备^[51]等. 另一方面, 基于纠缠光子和核磁共振技术完成了不同量子态的远程制备实验^[52-54].

量子态远程制备中发送方完全掌握需制备量子态信息. 为提高远程量子态制备安全, 人们提出量子态远程联合制备方法. 在量子态远程联合制备中多个发送方共享需制备量子态信息, 接收方与所有发送方合作完成量子态制备. 量子态远程联合制备引起研究者广泛关注. 2007年, Xia等人^[55]提出基于 $(N+1)$ 粒子部分纠缠态 $\gamma|00\cdots 0\rangle + \delta|00\cdots 0\rangle$ 的任意单量子比特态 $\alpha|0\rangle + \beta|1\rangle$ 概率多方远程联合制备模型. Nguyen和Kim^[56]提出了基于三粒子类GHZ态任意单量子比特三方概率联合制备方案. 2011年, Xiao等人^[57]提出了确定的2量子比特联合制备方案. 2015年, Choudhury和Dhara^[58]提出基于GHZ态的双粒子赤道态 $\frac{1}{2}(|00\rangle + e^{i\theta_1}|01\rangle + e^{i\theta_2}|10\rangle + e^{i\theta_3}|11\rangle)$ 远程联合制备方案, 量子态制备成功率为25%. Li等人^[59]提出了基于GHZ态的双粒子赤道态确定远程联合制备模型, 在使用相同量子纠缠信道的情况下将双粒子赤道态远程联合制备成功率提高到100%. 2018年, Bich等人^[60]开展了基于高维EPR态的杂化赤道态确定远程联合制备研究.

近年来, 基于Cluster态的量子信息处理研究得到快速发展^[61-67]. 2007年, Vallone等人^[61]通过对光子极化、路径自由度量子态的同时操控, 实现了双光子四量子比特Cluster态制备. Ceccarelli等人^[62]通过对三个自由度同时纠缠的双光子超纠缠态的转化完成了双光子六量子比特Cluster态的制备. 2008年, 田东平等人^[63]开展了基于非最大纠缠Cluster态量子隐形传态研究, 以高维四粒子Cluster态为量子纠缠信道, 实现了任意两粒子qudit态的概率离物传态. 2013年, 孙新梅等人^[64]提出了基于五粒子Cluster态的高效量子态共享方案. 2014年, Liao等人^[65]开展了基于六粒子Cluster态的

任意双量子比特远程联合制备研究, 通过增加经典通信将基于Cluster态的双量子比特联合制备成功率由25%提高到100%. 2017年, 郑晓毅和龙银香^[66]提出了基于Cluster态信道容量可控的量子安全直接通信方案. 2019年, Reimer等人^[67]提出了基于 d 维Cluster态的单向量子信息处理方法.

虽然已有一些基于多粒子Cluster态的远程量子态联合制备模型, 大多数方案都是任意单量子比特态或双量子比特态远程联合制备研究^[56-58], 还没有基于Cluster态的任意高维量子态远程联合制备方案. 本文提出两个基于六粒子 d 维Cluster态的任意高维量子态远程联合制备方案, 一个方案用于实现任意高维双粒子态远程联合制备, 另一个用于实现任意高维单粒子态的双向远程联合制备. 两个发送方共享需制备量子态信息, 第一发送方依据已知信息对纠缠粒子执行相应广义量子测量, 第二发送方依据已知需制备量子态信息和第一发送方测量结果选择执行相应正交投影测量. 接受方与所有发送方合作, 对手中纠缠粒子执行与发送方测量结果相应的局域么正测量完成量子态确定远程联合制备. 基于高维Cluster态的量子态远程联合制备和量子态双向多方联合制备可提高远程量子通信信道容量, 节省量子通信资源.

2 基于 d 维六粒子Cluster态的任意高维双粒子态远程联合制备

与文献[63]类似, 任意双粒子qudit态可表示为

$$|\Phi\rangle = \sum_{j_1, j_2=0}^{d-1} \alpha_{j_1 j_2} e^{i\lambda_{j_1 j_2}} |j_1 j_2\rangle, \quad (1)$$

其中任意实数 $\alpha_{j_1 j_2}, \lambda_{j_1 j_2} (j_1, j_2 = 0, \dots, d-1)$ 满足归一化条件

$$\sum_{j_1, j_2=0}^{d-1} |\alpha_{j_1 j_2}|^2 = 1. \quad (2)$$

第一发送方Alice和第二发送方Bob共享需制备量子态 $|\Phi\rangle$ 信息, 即发送方Alice已知 $\alpha_{j_1 j_2} (j_1, j_2 = 0, \dots, d-1)$, 发送方Bob已知 $\lambda_{j_1 j_2} (j_1, j_2 = 0, \dots, d-1)$. 接收方Charlie与所有发送方合作完成双粒子任意高维量子态 $|\Phi\rangle$ 远程联合制备.

广义X基可表示为^[67]

$$|\xi_r\rangle = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} e^{\frac{2\pi i}{d} r j} |j\rangle, \quad (3)$$

$r = 0, \dots, d-1$ 用于表示 d 个彼此互相正交的量子态 $|\xi_r\rangle$.

为完成任意高维量子态 $|\Phi\rangle$ 远程联合制备, 第一发送方Alice、第二发送方Bob以及接收方Charlie以 d 维六粒子Cluster态为量子纠缠信道^[67]:

$$|\Psi\rangle_{A_1 A_2 A_3 B_1 B_2 B_3} = \frac{1}{d} \sum_{j_1, j_2=0}^{d-1} e^{\frac{2\pi i}{d} j_1 j_2} |j_1 j_1 j_1 j_2 j_2 j_2\rangle, \quad (4)$$

其中Alice拥有粒子 A_1, B_1 , Bob拥有粒子 A_2, B_2 , 接收方Charlie拥有粒子 A_3, B_3 .

安全建立量子纠缠信道后, 第一发送方Alice依据已知需制备量子态信息 $\alpha_{j_1 j_2} (j_1, j_2 = 0, \dots, d-1)$ 对手中纠缠粒子 A_1, B_1 执行相应半正定算子测量和广义X基测量. 第二发送方依据已知需制备量子态信息 $\lambda_{j_1 j_2} (j_1, j_2 = 0, \dots, d-1)$ 以及第一发送方测量结果对纠缠粒子 A_2, B_2 选择执行相应正交投影测量. 接收方依据所有发送方测量对手中纠缠粒子 A_3, B_3 执行相应局域么正操作制备需制备量子态.

与文献^[68]类似, 基于需制备量子态信息 $\alpha_{j_1 j_2} (j_1, j_2 = 0, \dots, d-1)$ 的半正定测量算子可表示为

$$E_{k_1 k_2} = M_{k_1 k_2}^\dagger M_{k_1 k_2}, \quad (5)$$

其中 $k_1, k_2 = 0, \dots, d-1$, 广义测量算子

$$M_{k_1 k_2} = \sum_{j_1, j_2=0}^{d-1} \alpha_{j_1 \oplus k_1, j_2 \oplus k_2} |j_1 j_2\rangle \langle j_1 j_2|, \quad (6)$$

$j_1 \oplus k_1$ 表示 $j_1 + k_1$ 模 d .

半正定算子测量后, 粒子 $A_1, B_1, A_2, B_2, A_3, B_3$ 所组成复合系统量子态可由广义测量算子 $M_{k_1 k_2}$ 确定^[68]. 如果Alice半正定算子测量结果为 $E_{k_1 k_2} (k_1, k_2 = 0, \dots, d-1)$, 则复合系统量子态由 $|\Psi\rangle$ 转化为

$$\begin{aligned} |\Psi_{k_1 k_2}\rangle &= \frac{M_{k_1 k_2} |\Psi\rangle}{\sqrt{\langle \Psi | M_{k_1 k_2}^\dagger M_{k_1 k_2} | \Psi \rangle}} \\ &= \sum_{j_1, j_2=0}^{d-1} \alpha_{j_1 \oplus k_1, j_2 \oplus k_2} e^{\frac{2\pi i}{d} j_1 j_2} |j_1 j_1 j_1 j_2 j_2 j_2\rangle, \end{aligned} \quad (7)$$

半正定算子测量后, Alice对纠缠粒子 A_1, B_1 执行广义X基测量, 复合系统量子态可改写为

$$\begin{aligned} |\Psi_{k_1 k_2}\rangle &= \frac{1}{d} \sum_{j_1, j_2, r_1, r_2=0}^{d-1} |\xi_{r_1}\rangle_{A_1} |\xi_{r_2}\rangle_{B_1} \alpha_{j_1 \oplus k_1, j_2 \oplus k_2} \\ &\times e^{\frac{2\pi i}{d} j_1 j_2} e^{-\frac{2\pi i}{d} j_1 r_1} e^{-\frac{2\pi i}{d} j_2 r_2} |j_1 j_1 j_2 j_2\rangle_{A_2 A_3 B_2 B_3}, \end{aligned} \quad (8)$$

其中 $r_1, r_2 = 0, \dots, d-1$, $|\xi_{r_1}\rangle_{A_1}, |\xi_{r_2}\rangle_{B_1}$ 分别表示纠缠粒子 A_1, B_1 的广义X基测量结果. 如果纠缠粒子 A_1, B_1 的广义X基测量结果为 $|\xi_{r_1}\rangle_{A_1}, |\xi_{r_2}\rangle_{B_1}$, 剩余粒子 A_2, B_2, A_3, B_3 状态塌缩为

$$\begin{aligned} |\Psi_{r_1 r_2}\rangle_{A_2 A_3 B_2 B_3} &= \sum_{j_1, j_2=0}^{d-1} \alpha_{j_1 \oplus k_1, j_2 \oplus k_2} \\ &\times e^{\frac{2\pi i}{d} j_1 j_2} e^{-\frac{2\pi i}{d} j_1 r_1} e^{-\frac{2\pi i}{d} j_2 r_2} |j_1 j_1 j_2 j_2\rangle. \end{aligned} \quad (9)$$

为实现任意高维量子态远程联合制备, Bob依据已知需制备量子态信息以及Alice测量结果选取相应正交投影测量基. 与需制备量子态信息 $\lambda_{j_1 j_2} (j_1, j_2 = 0, \dots, d-1)$ 以及Alice半正定算子测量结果 k_1, k_2 相应的双粒子正交投影测量基可表示为

$$\begin{aligned} |\varphi_{s_1 s_2}^{(k_1 k_2)}\rangle &= \frac{1}{d} \sum_{j_1, j_2=0}^{d-1} e^{-\frac{2\pi i}{d} j_1 \oplus k_1, j_2 \oplus k_2} \\ &\times e^{\frac{2\pi i}{d^2} (j_1 d + j_2)(s_1 d + s_2)} |j_1 j_2\rangle_{A_2 B_2}, \end{aligned} \quad (10)$$

其中 $s_1, s_2 = 0, \dots, d-1$. $|\varphi_{s_1 s_2}^{(k_1 k_2)}\rangle$ 表示第 $k_1 k_2$ 正交基中的第 $s_1 s_2$ 正交态. 粒子 A_2, B_2, A_3, B_3 状态可改写为

$$\begin{aligned} |\Psi_{r_1 r_2}\rangle_{A_2 B_2 A_3 B_3} &= \frac{1}{d} \sum_{j_1, j_2, s_1, s_2=0}^{d-1} |\varphi_{s_1 s_2}^{(k_1 k_2)}\rangle_{A_2 B_2} \\ &\times \alpha_{j_1 \oplus k_1, j_2 \oplus k_2} e^{\frac{2\pi i}{d} j_1 \oplus k_1, j_2 \oplus k_2} \\ &\times e^{\frac{2\pi i}{d} j_1 j_2} e^{-\frac{2\pi i}{d} j_1 r_1} e^{-\frac{2\pi i}{d} j_2 r_2} \\ &\times e^{\frac{2\pi i}{d^2} (j_1 d + j_2)(s_1 d + s_2)} |j_1 j_2\rangle_{A_3 B_3}. \end{aligned} \quad (11)$$

若Bob正交投影测量结果为 $|\varphi_{s_1 s_2}^{(k_1 k_2)}\rangle$, 则剩余粒子 A_3, B_3 状态塌缩为

$$\begin{aligned} |\Psi_{s_1 s_2}\rangle_{A_3 B_3} &= \sum_{j_1, j_2=0}^{d-1} \alpha_{j_1 \oplus k_1, j_2 \oplus k_2} e^{i j_1 \oplus k_1 j_2 \oplus k_2} \\ &\times e^{\frac{2\pi i}{d} j_1 j_2} e^{-\frac{2\pi i}{d} j_1 r_1} e^{-\frac{2\pi i}{d} j_2 r_2} \\ &\times e^{\frac{2\pi i}{d^2} (j_1 d + j_2)(s_1 d + s_2)} |j_1 j_2\rangle_{A_3 B_3}. \end{aligned} \quad (12)$$

接收方Charlie依据Alice, Bob测量结果 $k_1, k_2, r_1, r_2, s_1, s_2$ 对手中粒子 A_3, B_3 执行相应局域么正操作

$$\begin{aligned} U_T &= \sum_{j_1, j_2=0}^{d-1} e^{-\frac{2\pi i}{d} j_1 j_2} e^{\frac{2\pi i}{d} j_1 r_1} e^{\frac{2\pi i}{d} j_2 r_2} e^{\frac{2\pi i}{d^2} (j_1 d + j_2)(s_1 d + s_2)} \\ &\times |j_1 \oplus k_1, j_2 \oplus k_2\rangle \langle j_1 j_2|, \end{aligned} \quad (13)$$

可将粒子 A_3, B_3 状态 $|\Psi\rangle_{A_3 B_3}$ 转化为需制备量子态

$$|\Phi\rangle_{A_3 B_3} = U_T |\Psi_{s_1 s_2}\rangle_{A_3 B_3}. \quad (14)$$

与任意单量子比特态确定远程制备类似, 为完成任意双粒子qudit态确定远程联合制备, 第一发送方Alice将粒子 A_1, B_1 半正定算子测量结果 $k_1, k_2 (k_1, k_2 = 0, \dots, d-1)$ 、广义X基测量结果 $r_1, r_2 (r_1, r_2 = 0, \dots, d-1)$ 传送给接收方Charlie; 将粒子 A_1, B_1 半正定算子测量结果 k_1, k_2 传送给第二发送方Bob. 第二发送方Bob需要将粒子 A_2, B_2 正交投影测量结果 $s_1, s_2 (s_1, s_2 = 0, \dots, d-1)$ 传送给接收方Charlie. 为实现任意双粒子qudit态确定远程联合制备, 通信方需要消耗一个 d 维六粒子Cluster态, 传送 $8\log_2 d$ 比特经典信息.

3 基于 d 维六粒子Cluster态的任意高维单粒子态双向远程联合制备

与基于 d 维六粒子Cluster态的任意双粒子qudit态远程联合制备类似, 任意单粒子qudit态可表示为^[63]

$$|\Phi_1\rangle = \sum_{j_1=0}^{d-1} \alpha_{j_1} e^{i j_1} |j_1\rangle, \quad |\Phi_2\rangle = \sum_{j_2=0}^{d-1} \beta_{j_2} e^{i j_2} |j_2\rangle, \quad (15)$$

其中任意实数 $\alpha_{j_1}, \lambda_{j_1} (j_1 = 0, \dots, d-1), \beta_{j_2}, \delta_{j_2} (j_2 = 0, \dots, d-1)$ 满足归一化条件

$$\sum_{j_1=0}^{d-1} |\alpha_{j_1}|^2 = 1, \quad \sum_{j_2=0}^{d-1} |\beta_{j_2}|^2 = 1. \quad (16)$$

为实现量子态 $|\Phi_1\rangle, |\Phi_2\rangle$ 的同时双向远程联合制备, 通信方Alice与Bob共享量子态 $|\Phi_1\rangle$ 信息, 即Alice已知

量子态 $|\Phi_1\rangle$ 振幅信息 $\alpha_{j_1} (j_1 = 0, \dots, d-1)$, Bob已知量子态 $|\Phi_1\rangle$ 相位信息 $\lambda_{j_1} (j_1 = 0, \dots, d-1)$. 同时通信方Charlie与Bob共享量子态 $|\Phi_2\rangle$ 信息, 即Charlie已知量子态 $|\Phi_2\rangle$ 振幅信息 $\beta_{j_2} (j_2 = 0, \dots, d-1)$, Bob已知量子态 $|\Phi_2\rangle$ 相位信息 $\delta_{j_2} (j_2 = 0, \dots, d-1)$. Alice与Bob合作协助Charlie远程制备量子态 $|\Phi_1\rangle$, 同时Charlie与Bob合作远程协助Alice制备量子态 $|\Phi_2\rangle$.

与任意高维双粒子态远程联合制备类似, 通信方共享 d 维六粒子Cluster态 $|\Psi\rangle_{A_1 A_2 A_3 B_1 B_2 B_3}$ ^[67]. 其中Alice拥有粒子 A_1, B_1 , Bob拥有粒子 A_2, B_2 , Charlie拥有粒子 A_3, B_3 . 为完成任意高维单粒子态的双向远程联合制备, Alice和Charlie依据已知需制备量子态信息对粒子 A_1, B_3 执行广义量子测量. Bob依据Alice和Charlie测量结果以及已知需制备量子态信息对粒子 A_2, B_2 选择执行相应正交投影测量. Charlie和Alice依据测量结果选择执行相应局域么正操作, 即可在粒子 A_3, B_1 上制备需制备量子态.

与Alice已知需制备量子态 $|\Phi_1\rangle$ 信息 $\alpha_{j_1} (j_1 = 0, \dots, d-1)$ 类似, Charlie已知需制备量子态 $|\Phi_2\rangle$ 信息 $\beta_{j_2} (j_2 = 0, \dots, d-1)$ 相应的半正定算子测量可表示为^[68]

$$E_{k_1} = M_{k_1}^\dagger M_{k_1}, \quad O_{k_2} = N_{k_2}^\dagger N_{k_2}, \quad (17)$$

其中 $k_1, k_2 = 0, \dots, d-1$. 广义测量算子

$$M_{k_1} = \sum_{j_1=0}^{d-1} \alpha_{j_1 \oplus k_1} |j_1\rangle \langle j_1|, \quad N_{k_2} = \sum_{j_2=0}^{d-1} \beta_{j_2 \oplus k_2} |j_2\rangle \langle j_2|. \quad (18)$$

与任意高维双粒子态远程联合制备类似, 半正定算子测量后粒子 $A_1, B_1, A_2, B_2, A_3, B_3$ 所组成复合系统量子态可由广义测量算子 M_{k_1}, N_{k_2} 确定^[68]. 若Alice, Charlie对粒子 A_1, B_3 半正定算子测量结果分别为 E_{k_1}, O_{k_2} , 则复合系统状态由 $|\Psi\rangle$ 演化为

$$\begin{aligned} |\Psi_{k_1 k_2}\rangle &= \frac{M_{k_1} N_{k_2} |\Psi\rangle}{\sqrt{\langle \Psi | M_{k_1}^\dagger N_{k_2}^\dagger M_{k_1} N_{k_2} | \Psi \rangle}} \\ &= \sum_{j_1, j_2=0}^{d-1} \alpha_{j_1 \oplus k_1} \beta_{j_2 \oplus k_2} e^{\frac{2\pi i}{d} j_1 j_2} |j_1 j_1 j_1 j_2 j_2 j_2\rangle. \end{aligned} \quad (19)$$

复合系统量子态演化为 $|\psi_{k_1 k_2}\rangle$ 后, Alice和Charlie对手中粒子 A_1, B_3 执行广义X基测量. 复合系统状态可改写为

$$|\psi_{k_1 k_2}\rangle = \frac{1}{d} \sum_{j_1, j_2=0}^{d-1} |\zeta_{r_1}\rangle_{A_1} |\zeta_{r_2}\rangle_{B_3} \alpha_{j_1 \oplus k_1} \beta_{j_2 \oplus k_2} \times e^{\frac{2\pi i}{d} j_1 j_2} e^{-\frac{2\pi i}{d} j_1 r_1} e^{-\frac{2\pi i}{d} j_2 r_2} |j_1 j_2\rangle_{A_2 A_3 B_1 B_2}, \quad (20)$$

其中 $r_1, r_2 = 0, \dots, d-1$, $|\zeta_{r_1}\rangle_{A_1}, |\zeta_{r_2}\rangle_{B_3}$ 表示粒子 A_1, B_3 的广义X基测量结果. 若粒子 A_1, B_3 的广义X基测量结果分别为 $|\zeta_{r_1}\rangle_{A_1}, |\zeta_{r_2}\rangle_{B_3}$, 则剩余粒子 A_2, A_3, B_1, B_2 状态塌缩为

$$|\psi_{r_1 r_2}\rangle_{A_2 A_3 B_1 B_2} = \sum_{j_1, j_2=0}^{d-1} \alpha_{j_1 \oplus k_1} \beta_{j_2 \oplus k_2} e^{\frac{2\pi i}{d} j_1 j_2} \times e^{-\frac{2\pi i}{d} j_1 r_1} e^{-\frac{2\pi i}{d} j_2 r_2} |j_1 j_2\rangle. \quad (21)$$

为完成量子态双向远程联合制备, Bob依据已知需制备量子态 $|\Phi_1\rangle, |\Phi_2\rangle$ 信息对手中纠缠粒子 A_2, B_2 选择执行相应正交投影测量. 与Bob已知需制备量子态 $|\Phi_1\rangle, |\Phi_2\rangle$ 信息 $\lambda_{j_1} (j_1 = 0, \dots, d-1)$, $\delta_{j_2} (j_2 = 0, \dots, d-1)$ 相应的正交投影测量可表示为

$$|\varphi_{s_1 s_2}^{(k_1 k_2)}\rangle = \frac{1}{d} \sum_{j_1, j_2=0}^{d-1} e^{-\frac{2\pi i}{d} j_1 j_2} e^{-i\lambda_{j_1} \oplus k_1} e^{-i\delta_{j_2} \oplus k_2} \times e^{\frac{2\pi i}{d} j_1 s_1} e^{\frac{2\pi i}{d} j_2 s_2} |j_1 j_2\rangle_{A_2 B_2}, \quad (22)$$

其中 $s_1, s_2, k_1, k_2 = 0, \dots, d-1$. $|\varphi_{s_1 s_2}^{(k_1 k_2)}\rangle$ 表示第 $k_1 k_2$ 正交基中第 $s_1 s_2$ 正交态. 粒子 A_2, A_3, B_1, B_2 所组成的复合系统量子态可改写为

$$|\psi_{r_1 r_2}\rangle_{A_2 A_3 B_1 B_2} = \frac{1}{d} \sum_{j_1, j_2, s_1, s_2=0}^{d-1} |\varphi_{s_1 s_2}^{(k_1 k_2)}\rangle_{A_2 B_2} \alpha_{j_1 \oplus k_1} \times e^{i\lambda_{j_1} \oplus k_1} \beta_{j_2 \oplus k_2} e^{i\delta_{j_2} \oplus k_2} e^{-\frac{2\pi i}{d} j_1 r_1} \times e^{-\frac{2\pi i}{d} j_2 r_2} e^{-\frac{2\pi i}{d} j_1 s_1} e^{-\frac{2\pi i}{d} j_2 s_2} |j_1 j_2\rangle_{A_3 B_1}. \quad (23)$$

若粒子 A_2, B_2 正交投影测量结果为 $|\varphi_{s_1 s_2}^{(k_1 k_2)}\rangle$, 剩余粒子 A_3, B_1 状态塌缩为

$$|\psi_{s_1 s_2}\rangle_{A_3 B_1} = \sum_{j_1, j_2=0}^{d-1} \alpha_{j_1 \oplus k_1} e^{i\lambda_{j_1} \oplus k_1} \beta_{j_2 \oplus k_2} e^{i\delta_{j_2} \oplus k_2} \times e^{-\frac{2\pi i}{d} j_1 r_1} e^{-\frac{2\pi i}{d} j_2 r_2} e^{-\frac{2\pi i}{d} j_1 s_1} e^{-\frac{2\pi i}{d} j_2 s_2} |j_1 j_2\rangle. \quad (24)$$

Charlie与Alice分别对粒子 A_3, B_1 执行与测量结果 $k_1, k_2, r_1, r_2, s_1, s_2$ 相应的局域么正演化

$$U_{A_3} = \sum_{j_1=0}^{d-1} e^{\frac{2\pi i}{d} j_1 r_1} e^{\frac{2\pi i}{d} j_1 s_1} |j_1 \oplus k_1\rangle \langle j_1|, \quad (25)$$

$$U_{B_1} = \sum_{j_2=0}^{d-1} e^{\frac{2\pi i}{d} j_2 r_2} e^{\frac{2\pi i}{d} j_2 s_2} |j_2 \oplus k_2\rangle \langle j_2|,$$

可将粒子 A_3, B_1 状态转化为需制备量子态

$$U_{A_3} \otimes U_{B_1} |\psi_{s_1 s_2}\rangle_{A_3 B_1} = |\Phi_1\rangle_{A_3} \otimes |\Phi_2\rangle_{B_1}. \quad (26)$$

与双粒子qudit态远程联合制备类似, 为完成任意单粒子qudit态的双向远程联合制备, 发送方Alice将粒子 A_1 的半正定算子测量结果 $k_1 (k_1 = 0, \dots, d-1)$, 广义X基测量结果 $r_1 (r_1 = 0, \dots, d-1)$ 发送给Charlie; 将粒子 A_1 的半正定算子测量结果 k_1 发送给Bob. Bob将粒子正交投影测量结果 $s_1 (s_1 = 0, \dots, d-1)$ 发送给Charlie. 同时, Charlie将粒子 B_3 的半正定算子测量结果 $k_2 (k_2 = 0, \dots, d-1)$, 广义X基测量结果 $r_2 (r_2 = 0, \dots, d-1)$ 发送给Alice; 将粒子 B_3 的半正定算子测量结果 k_2 发送给Bob. Bob将粒子正交投影测量结果 $s_2 (s_2 = 0, \dots, d-1)$ 发送给Alice. 为实现任意单粒子qudit态的双向远程联合制备, 通信方需消耗一个 d 维六粒子Cluster态, 传送 $8\log_2^d$ 比特经典信息.

4 结论

与基于六粒子Cluster态的任意双量子比特态远程联合制备类似^[65], 在基于 d 维六粒子Cluster态任意双粒子qudit远程联合制备中, 发送方Alice和Bob可以同时依据已知需制备量子态信息对手中纠缠粒子执行相应量子测量, 并将测量结果发送给接收方Charlie. 这种方法以减少量子态远程联合制备成功率为代价来缩短纠缠粒子需存储时间.

在文献[58]中, 发送方可依据已知需制备双量子比特态 $|\Phi\rangle = \frac{1}{2}(|00\rangle + e^{i\varphi_1}|01\rangle + e^{i\varphi_2}|10\rangle + e^{i\varphi_3}|11\rangle)$ 信息,

对纠缠粒子执行相应正交投影测量来避免使用与需制备量子态信息相应的半正定算子测量对纠缠信道转化的需要. 但当需制备量子态是任意双粒子 qudit 态

$|\Phi\rangle = \sum_{j_1, j_2=0}^{d-1} \alpha_{j_1 j_2} e^{i\varphi_{j_1 j_2}} |j_1 j_2\rangle$ 时, 文献[58]中方法不可行.

与文献[69]不同, 对于发送方所有测量结果, 接收方都可以选取相应局域么正操作完成量子态重建, 实现任意双粒子 qudit 态的确定远程联合制备.

在量子态远程联合制备实际应用中, 受环境噪声

影响最大纠缠信道 $\sum_{j_1, j_2=0}^{d-1} e^{\frac{2\pi i}{d} j_1 j_2} |j_1 j_1 j_1 j_2 j_2 j_2\rangle$ 往往演化成部分纠缠信道

$$|\Psi\rangle_{A_1 A_2 A_3 B_1 B_2 B_3} = \sum_{j_1, j_2=0}^{d-1} e^{\frac{2\pi i}{d} j_1 j_2} \gamma_{j_1 j_2} |j_1 j_1 j_1 j_2 j_2 j_2\rangle \left(\sum_{j_1, j_2=0}^{d-1} \gamma_{j_1 j_2}^2 = 1 \right).$$

与文献[53]类似, 通信方可通过引入附加粒子与执行联合么正演化来消除部分纠缠信道对量子态远程联合制备影响. 在基于部分纠缠信道 $|\Psi\rangle$ 的任意 qudit 态联

合制备中, Alice 引入初态为 $|0\rangle_a$ 的附加量子比特 a , 依据需制备量子态信息以及纠缠信道信息对手中粒子 A_1, B_1, a 执行联合么正演化并对附加量子比特执行测量基 $\{|0\rangle, |1\rangle\}$ 下的正交投影测量来将部分纠缠信道转化为目标信道 $|\Psi_{k_1 k_2}\rangle$.

本文以 d 维六粒子 Cluster 态为纠缠信道, 提出了 2 个高维量子态远程联合制备方案, 一个方案是基于 d 维六粒子 Cluster 态的任意双粒子 qudit 远程联合制备方案, 另一个方案是基于 d 维六粒子 Cluster 态的任意单粒子 qudit 双向远程联合制备方案. 所有发送方共享需制备量子态信息, 第一发送方依据已知需制备量子态信息对手中纠缠粒子执行广义量子测量, 第二发送方依据第一发送方测量结果以及已知需制备量子态信息选择执行适当正交投影测量, 接收方与所有发送方合作, 执行与发送方测量结果相应的局域么正操作, 就可以在他的纠缠粒子上制备需制备量子态. 在远程量子通信中, 基于高维 Cluster 态的远程量子态联合制备方案可用于提高远程量子信息处理网络信道容量.

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Joint multi-party remote preparation of arbitrary qudit states via a d -level six-qudit Cluster state

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We propose two protocols for joint, multi-party remote preparation of qudit states by using a d -level six-qudit Cluster state as the quantum channel. The first protocol is used for the joint remote preparation of an arbitrary two-qudit state via the d -level six-qudit Cluster state. The second is used for bi-directional joint remote preparation of an arbitrary single-qudit state via a d -level six-qudit Cluster state. All senders share information about the prepared state. The first sender applies Positive Operator-Valued Measurement (POVM) on her entangled particles in accordance with her information about the prepared state, while the second sender performs projective measurements in accordance with the information about the prepared state and the first sender's measurement results. The receiver can prepare the original state by performing a corresponding unitary operation according to the measurement results from all the senders. These two protocols have the advantage of being able to provide a high channel capacity by using a d -level Cluster state as the quantum channel.

joint remote state preparation, d -level Cluster state, positive operator-valued measurement, bidirectional joint remote state preparation

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