

# Deformation dependence of symmetry energy coefficients of nuclei

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Based on the semi-classical Thomas-Fermi approximation together with the Skyrme energy-density functional, we study the deformation dependence of symmetry energy coefficients of finite nuclei. The symmetry energy coefficients of nuclei with mass number  $A = 40, 100, 150, 208$  are extracted from two-parameter parabola fitting to the calculated energy per particle. We find that the symmetry energy coefficients decrease with the increase of nuclear quadrupole deformations, which is mainly due to the isospin dependence of the difference between the proton and neutron surface diffuseness. Large deformations of nuclei can cause the change of the symmetry energy coefficient by about 0.5 MeV and the influence of nuclear deformations on the symmetry energy coefficients is more evident for light and intermediate nuclei.

**symmetry energy, Skyrme energy-density functional, nuclear deformation, Thomas-Fermi**

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## 1 Introduction

Nuclear symmetry energy has attracted a lot of attention in recent years. In addition to its importance in the study of nuclear physics such as some crucial information on nuclear structure and reactions, the symmetry energy is also closely related to a large number of astrophysical phenomena. For example, the symmetry energy is of great significance for the study of the structure, the evolution mechanism and the mass-radius relationship of neutron stars [1]. Although a great effort has been devoted in recent decades to investigate the symmetry energy [2–14], the density dependence of the symmetry energy, especially at sub-saturation and super-saturation density regions, is far from clear. More investigations on the symmetry energy are still required.

In ref. [15], Liu et al. studied the nuclear symmetry energy at subnormal densities from the measured nuclear masses [16]. The symmetry energy coefficients were ex-

tracted from the masses and some evident oscillations in the mass-dependence of the symmetry energy coefficients were observed. When the shell corrections are taken into account, the oscillations in the extracted symmetry energy coefficients are reduced effectively. Because the shell effects in the measured masses are correlated with nuclear deformations, it is therefore interesting to see how large the nuclear deformations individually affect the symmetry energy coefficients due to the surface-symmetry term and the nuclear surface diffuseness, based on theoretical models and removing the influence of the shell effects.

The density functional theory is widely used in the study of the nuclear ground state which provides us with a useful balance between accuracy and computation cost, allowing large systems with a simple self-consistent manner. In the framework of the effective Skyrme interaction [17] and the extended Thomas-Fermi approximation, the Skyrme energy density functional approach was proposed for the study of nuclear structure and reactions [18–20]. One of the advantages in the semi-classical extended Thomas-Fermi approach

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is that the microscopic shell and pairing corrections can be clearly removed, which is helpful to study the deformation energies without the influence of the shell effects.

In this work, we will study the “macroscopic” energies of nuclei, especially the deformation dependence of the symmetry energy coefficient, based on the extended Thomas-Fermi approximation together with the Skyrme energy-density functional.

## 2 The Skyrme energy-density functional and the extended thomas-fermi approach

The energy of a nucleus at its ground state can be expressed as the integral of nuclear energy density functional:

$$E = \int \mathcal{H} \, dr. \quad (1)$$

The energy density functional  $\mathcal{H}(\mathbf{r})$  contains the Skyrme energy density  $\mathcal{H}_{\text{sky}}(\mathbf{r})$  and the Coulomb energy density  $\mathcal{H}_{\text{Coul}}(\mathbf{r})$ :

$$\mathcal{H}(\mathbf{r}) = \mathcal{H}_{\text{sky}}(\mathbf{r}) + \mathcal{H}_{\text{Coul}}(\mathbf{r}). \quad (2)$$

The Skyrme energy density can be expressed in terms of the local nucleon densities  $\rho_q(\mathbf{r})$ , kinetic energy densities  $\tau_q(\mathbf{r})$  and spin-orbit densities  $\mathbf{J}_q(\mathbf{r})$  ( $q = n$  for neutrons and  $q = p$  for protons):

$$\begin{aligned} \mathcal{H}_{\text{sky}}(\mathbf{r}) = & \frac{\hbar^2}{2m} \tau + \frac{1}{2} t_0 \left[ \left( 1 + \frac{1}{2} x_0 \right) \rho^2 - \left( x_0 + \frac{1}{2} \right) (\rho_n^2 + \rho_p^2) \right] \\ & + \frac{1}{12} t_3 \rho^\alpha \left[ \left( 1 + \frac{1}{2} x_3 \right) \rho^2 - \left( x_3 + \frac{1}{2} \right) (\rho_n^2 + \rho_p^2) \right] \\ & + \frac{1}{4} \left[ t_1 \left( 1 + \frac{1}{2} x_1 \right) + t_2 \left( 1 + \frac{1}{2} x_2 \right) \right] \tau \rho \\ & + \frac{1}{4} \left[ t_2 \left( x_2 + \frac{1}{2} \right) - t_1 \left( x_1 + \frac{1}{2} \right) \right] (\tau_n \rho_n + \tau_p \rho_p) \\ & + \frac{1}{16} \left[ 3t_1 \left( 1 + \frac{1}{2} x_1 \right) - t_2 \left( 1 + \frac{1}{2} x_2 \right) \right] (\nabla \rho)^2 \\ & - \frac{1}{16} \left[ 3t_1 \left( x_1 + \frac{1}{2} \right) + t_2 \left( x_2 + \frac{1}{2} \right) \right] [(\nabla \rho_n)^2 + (\nabla \rho_p)^2] \\ & + \frac{1}{2} W_0 [\mathbf{J} \cdot \nabla \rho + \mathbf{J}_n \cdot \nabla \rho_n + \mathbf{J}_p \cdot \nabla \rho_p], \end{aligned} \quad (3)$$

where  $t_0, t_1, t_2, t_3, x_0, x_1, x_2, x_3, \alpha$  are the Skyrme interaction parameters.  $W_0$  denotes the strength of the spin-orbit interaction. In the calculations, we adopt the Skyrme parameter set SkM\* [21] and SLy4 [22]. The kinetic energy density  $\tau = \tau_p + \tau_n$  and the spin-orbit density  $\mathbf{J} = \mathbf{J}_p + \mathbf{J}_n$  are described by using the extended Thomas-Fermi approach up to the second order in  $\hbar$  (ETF2) [18–20].

The kinetic energy density is then expressed as:

$$\begin{aligned} \tau_q[\rho_q(\mathbf{r})] = & \frac{3}{5} (3\pi^2)^{2/3} \rho_q^{5/3} + \frac{1}{36} \frac{(\nabla \rho_q)^2}{\rho_q} \\ & + \frac{1}{3} \Delta \rho_q + \frac{1}{6} \frac{\nabla \rho_q \nabla f_q + \rho_q \Delta f_q}{f_q} \end{aligned}$$

$$- \frac{1}{12} \rho_q \left( \frac{\nabla f_q}{f_q} \right)^2 + \frac{1}{2} \rho_q \left[ \frac{2m}{\hbar^2} \frac{W_0}{2} \frac{\nabla(\rho + \rho_q)}{f_q} \right]^2 \quad (4)$$

with the effective-mass form factor

$$\begin{aligned} f_q(\mathbf{r}) = & 1 + \frac{2m}{\hbar^2} \left\{ \frac{1}{4} \left[ t_1 \left( 1 + \frac{1}{2} x_1 \right) + t_2 \left( 1 + \frac{1}{2} x_2 \right) \right] \rho(\mathbf{r}) \right. \\ & \left. + \frac{1}{4} \left[ t_2 \left( x_2 + \frac{1}{2} \right) - t_1 \left( x_1 + \frac{1}{2} \right) \right] \rho_q(\mathbf{r}) \right\} \end{aligned} \quad (5)$$

and the spin-orbit density is expressed as:

$$\mathbf{J}_q = -\frac{1}{2} W_0 \left( \frac{2m}{\hbar^2} \right) \frac{1}{f_q} \rho_q \nabla \rho_q. \quad (6)$$

The Coulomb energy density is a functional of proton density. Here we consider the contributions of the direct term and exchange term:

$$\mathcal{H}_{\text{Coul}}(\mathbf{r}) = \frac{e^2}{2} \rho_p(\mathbf{r}) \int \frac{\rho_p(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r}' - \frac{3e^2}{4} \left( \frac{3}{\pi} \right)^{1/3} [\rho_p(\mathbf{r})]^{4/3}. \quad (7)$$

From eqs. (1)–(7), one can see that the total energy of a nucleus can be expressed as a functional of nuclear density and its gradients.

In the semi-classical ETF2 approach, the ground state deformations of deformed nuclei can not be accurately described due to the neglecting of the microscopic shell and pairing effects. However, based on the restricted density variational method together with the form of Woods-Saxon distribution:

$$\rho_q(\mathbf{r}) = \frac{\rho_0^{(q)}}{1 + \exp \left[ \frac{r - \mathcal{R}_q(\theta)}{a_q} \right]}, \quad (8)$$

for describing the density of an axially deformed nucleus, one can calculate the “macroscopic” part of the binding energy  $\tilde{E}$  of a nucleus as a function of quadrupole deformation  $\beta_2$ .  $\rho_0^{(q)}$  and  $a_q$  denote the central density and the surface diffuseness of nuclei, respectively.  $\mathcal{R}_q$  defines the distance from the origin of the coordinate system to the point on the nuclear surface

$$\mathcal{R}_q(\theta) = R_q [1 + \beta_2 Y_{20}(\theta)]. \quad (9)$$

The central density  $\rho_0^{(q)}$  is determined from the conservation of particle number. At a given nuclear quadrupole deformation  $\beta_2$ , one can obtain the minimal energy  $\tilde{E}(\beta_2)$  of the nuclear system since the total energy is expressed as a function of density and its gradients under the ETF2 approximation, by using the optimization algorithm [20] and varying the four variables  $R_p, a_p, R_n, a_n$  for the description of the proton and neutron density distributions.

To test the ETF2 approach, we study the root-mean-square (rms) charge radii and deformation energies of nuclei with this approach. In Table 1 we list the calculated rms charge radii of some spherical nuclei with the ETF2 approach by adopting the parameters set SkM\*. We find that the deviations from the experimental data are generally smaller than 0.03 fm for these nuclei, which is comparable with the rms

error (0.028 fm) of the Hartree-Fock-Bogoliubov (HFB27) model [23]. In addition, the deformation energies, defined as the difference in energy of a nucleus between its spherical and equilibrium shapes, for isotopes of Uranium are also investigated with the ETF2 approach. To describe the ground state deformations of nuclei in this semi-classical model, the Strutinsky shell corrections from the universal parameterization [24] of deformed Woods-Saxon potential are added to the macroscopic energy  $\tilde{E}$ . In Figure 1, we show the calculated deformation energies for the isotopes of Uranium. The solid curve and squares denote the results of HFB27 and WS\* [25] model, respectively. One sees that the results of ETF2 approach (solid circles) are comparable with those from the HFB27 and WS\* mass models. These tests indicate that the semi-classical ETF2 approach together with the Skyrme energy-density functional is appropriate for the description of nuclear deformation energies.

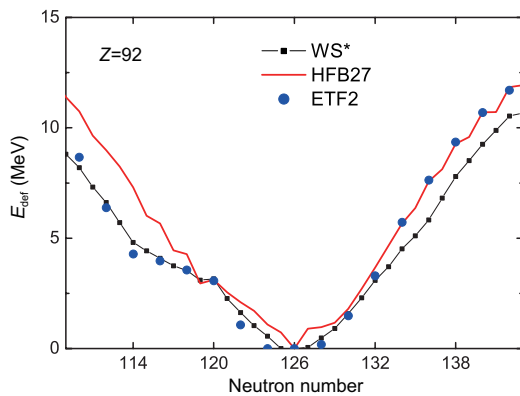
### 3 Symmetry energy coefficients of finite nuclei

In this section, we first investigate the symmetry energy coefficient  $a_{\text{sym}}$  in the liquid drop mass formula at spherical shapes as a function of nuclear mass number, based on the semi-classical ETF2 approach. Then, the macroscopic energy of a deformed nuclear system is studied with the ETF2 approach and the deformation dependence of  $a_{\text{sym}}$  is also investigated.

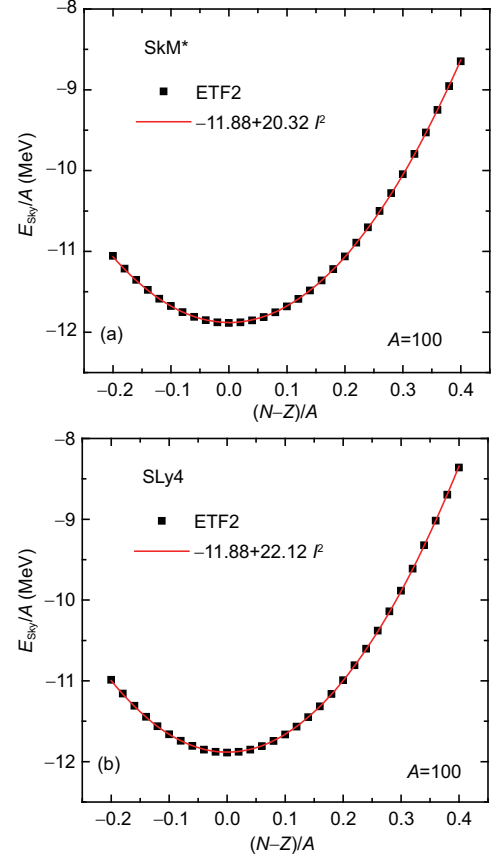
Figure 2 shows the calculated Skyrme energy per particle  $E_{\text{sky}}/A = \frac{1}{A} \int \mathcal{H}_{\text{sky}} d\mathbf{r}$  as a function of isospin asymmetry for nuclei with  $A = 100$ . Here, the Coulomb interaction which

**Table 1** Nuclear root-mean-square charge radii of some nuclei (in fm) from the Skyrme energy density functional plus the ETF2 approach by using the parameters set SkM\*. The experimental data are taken from ref. [26]

Nucleus	$^{48}\text{Ca}$	$^{64}\text{Ni}$	$^{90}\text{Zr}$	$^{116}\text{Sn}$	$^{144}\text{Sm}$	$^{208}\text{Pb}$
SkM*	3.51	3.84	4.27	4.61	4.94	5.53
Experiment	3.48	3.86	4.27	4.63	4.95	5.50



**Figure 1** (Color online) Deformation energies of some isotopes of Uranium. The solid curve, squares and circles denote the results of HFB27, WS\* [25], and ETF2 approach, respectively.



**Figure 2** (Color online) Skyrme energy per particle as a function of isospin asymmetry  $I = (N - Z)/A$  for nuclei with  $A = 100$ . (a) The results with the Skyrme force SkM\*. (b) The results with SLy4. The Coulomb interaction is removed in the calculations.

can influence the density distributions of nuclei has been ignored in order to obtain the information of symmetry energy as clean as possible. In Figure 2, we set  $\beta_2 = 0$  in the calculations. The solid squares denote the calculated results with the Skyrme energy density functional approach. According to the Bethe-Weizsäcker mass formula, the energy per particle of a nucleus subtracting the contribution of the Coulomb energy can be expressed as a function of mass number  $A$  and isospin asymmetry  $I = (N - Z)/A$ :

$$(E - E_c)/A = a_v + a_s A^{-1/3} + a_{\text{sym}} I^2, \quad (10)$$

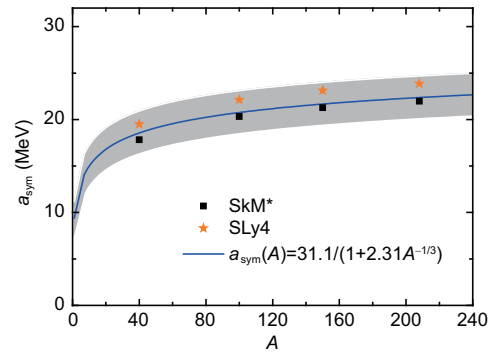
where  $a_v$ ,  $a_s$  denote the coefficients of the volume and the surface term, respectively. The parabolic law of the binding energies of nuclei is due to the isospin symmetry in nuclear physics. In the realistic nuclear system, the dependence of the asymmetry term is more complicated than the  $I^2$  term. To give a qualitative view of the effects of nuclear deformations, we adopt the quadratic term for the description of the isospin asymmetry term. The coefficient  $a_{\text{sym}}$  of the quadratic term for the isospin asymmetry is the symmetry energy coefficient. The solid curves in Figure 2 denote the results from two-parameter parabola fitting to the

squares. The curvature of the solid curve represents the corresponding symmetry energy coefficient  $a_{\text{sym}}$  of finite nuclei with mass number  $A$ . The extracted symmetry energy coefficients for nuclei with  $A = 40, 100, 150, 208$  are shown in Figure 3. With these four isobaric-chains, we can test the model for the description of the mass-dependence of  $a_{\text{sym}}$ . The squares and stars denote the results with the parameter set SkM\* and SLy4, respectively. Here, the nuclear deformations have not yet been taken into account. The values of  $a_{\text{sym}}(A)$  obtained in this approach are close to the results of ref. [15] in which the symmetry energy coefficients are extracted from the measured masses together with the formula  $a_{\text{sym}}(A) = S_0(1 + \kappa A^{-1/3})^{-1}$  [27] for describing the mass-dependence. Here,  $S_0$  is the volume symmetry energy coefficient of the nuclei (i.e. the nuclear symmetry energy at normal density) and  $\kappa$  is the ratio of the surface-symmetry coefficient to the volume-symmetry coefficient. The shades represent the uncertainty of the extracted symmetry energy coefficients [15]. From Figure 3, one can see that the results from the formula  $a_{\text{sym}}(A) = 31.1(1 + 2.31A^{-1/3})^{-1}$  are larger than those from SkM\* and smaller than those from SLy4, because the corresponding values of  $S_0$  are 30.0 and 32.0 MeV for SkM\* and SLy4, respectively [28], and  $S_0 = 31.1$  MeV for the solid curve. Here, we would like to state that the Coulomb energy term (see eq. (7)) is not involved in the calculation of the density distributions and the corresponding macroscopic energy  $\tilde{E}$  of a nucleus. Considering the isospin dependence of the surface diffuseness obtained in the ETF2 approach, the Coulomb energy from the ETF2 approach could not be simply written as  $a_c Z^2/A^{1/3}$  which is assumed for extracting the symmetry energy coefficient  $a_{\text{sym}}$  in ref. [15].

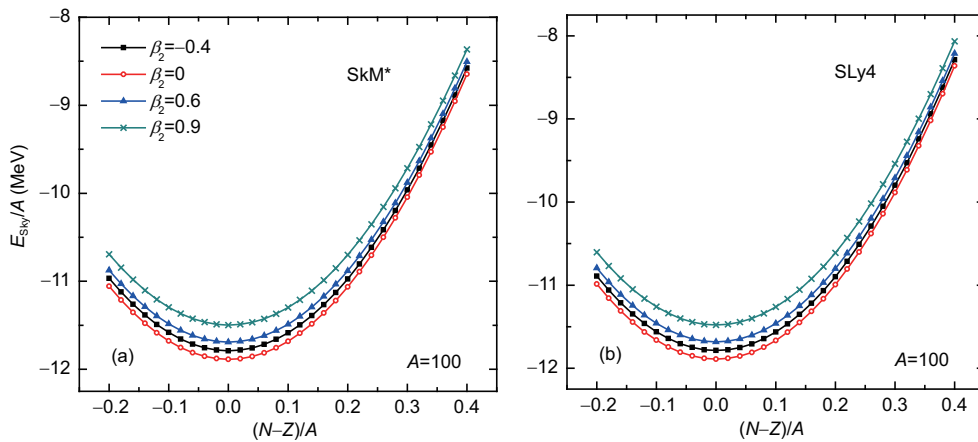
Now we come to the cases for deformed nuclei. With the same approach, the macroscopic energy  $\tilde{E}$  of a nucleus with a certain quadrupole deformation  $\beta_2$  can be calculated. Figure 4 shows the values of  $E_{\text{Sky}}/A$  as a function of isospin asymmetry for a series of nuclei with  $A = 100$  under given nuclear quadrupole deformations. One sees that  $E_{\text{Sky}}/A$  changes with nuclear deformations, which could be due to the influence of

surface-symmetry term and nuclear surface diffuseness. Similar to the spherical cases, the symmetry energy coefficients  $a_{\text{sym}}$  in the liquid drop formula can also be extracted for the deformed nuclei. We show in Figure 5 the symmetry energy coefficients of nuclei as a function of quadrupole deformation with SkM\* and SLy4. The squares, circles, triangles and crosses denote the results for the isobaric nuclei with  $A = 40, 100, 150, 208$ , respectively. One sees that symmetry energy coefficients decrease with the increase of  $|\beta_2|$ . The large ground state deformations for some nuclei can cause the change of the symmetry energy coefficients by about 0.5 MeV. Therefore, the influence of nuclear deformations should be further considered if one wants to determine the symmetry energy coefficients more precisely from the masses.

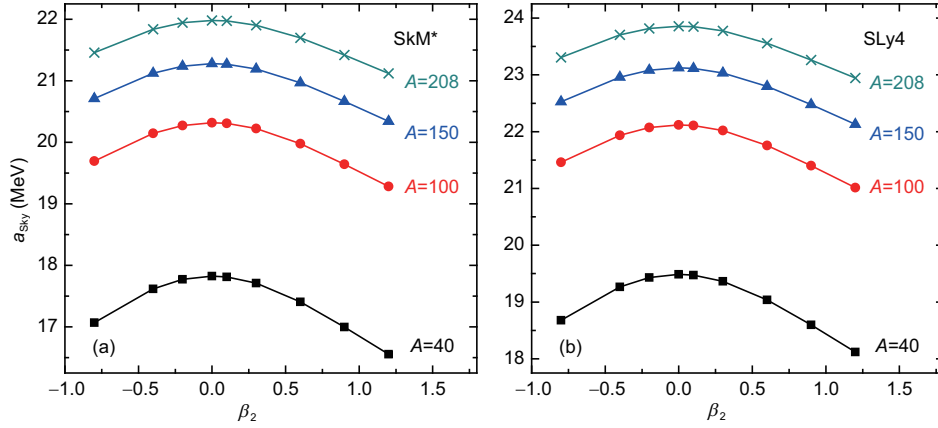
Here, the curvature  $\kappa_\beta = \frac{\partial^2 a_{\text{sym}}}{\partial \beta_2^2}|_{\beta_2=0}$  of the symmetry energy coefficients in Figure 5 is simultaneously studied. Figure 6 shows the curvature of the symmetry energy coefficient as a function of mass number. The squares and stars denote the results of SkM\* and SLy4, respectively. From Figure 6,



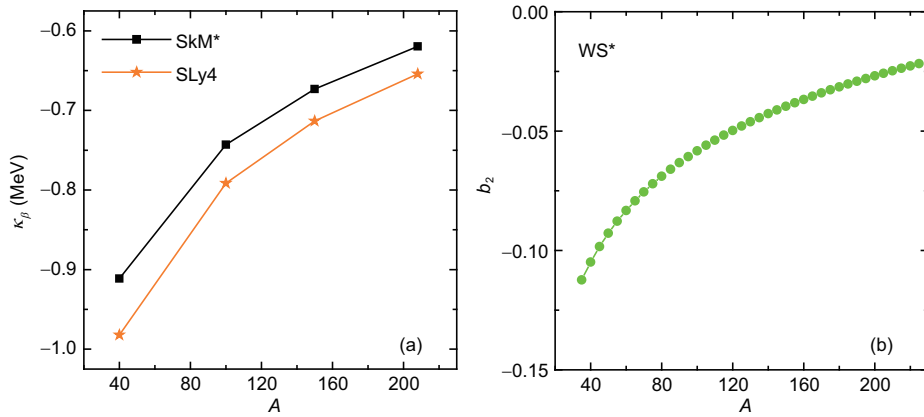
**Figure 3** (Color online) Symmetry energy coefficients of nuclei as a function of mass number. The solid curve denotes the results of  $a_{\text{sym}}(A) = S_0(1 + \kappa A^{-1/3})^{-1}$  with  $S_0 = 31.1$  MeV and  $\kappa = 2.31$  which is taken from ref. [15]. The squares and stars denote the results with SkM\* and SLy4, respectively.



**Figure 4** (Color online) The same as Figure 2, but with nuclear deformations  $\beta_2 = -0.4, 0, 0.6, 0.9$ .



**Figure 5** (Color online) Symmetry energy coefficients of nuclei as a function of quadrupole deformation.



**Figure 6** (Color online) (a) Curvature  $\kappa_\beta$  of the symmetry energy coefficients as a function of mass number. (b) Curvature of the parabola  $b_2$  in the WS mass formula [25] for describing the deformation energies of nuclei.

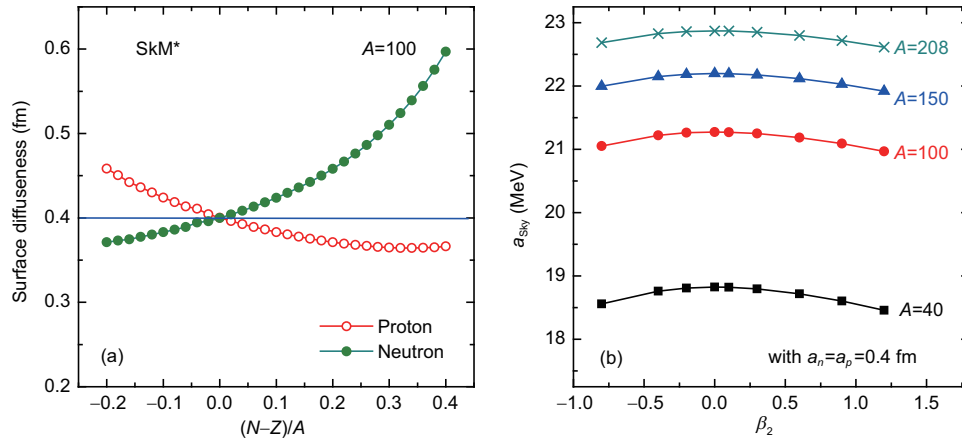
one finds that the absolute value of  $\kappa_\beta$  decreases with the mass number, which is similar to the trend of the coefficient  $b_2$  (see Figure 6(b)) of the nuclear deformation energy in the Weizsäcker-Skyrme mass formula [25, 29]. The influence of the quadrupole deformations on the symmetry energy coefficients gradually decreases with the increase of nuclear size.  $b_2$  in Figure 6(b) denotes the (quadrupole) deformation energy coefficient which has a relationship with nuclear surface of finite nuclei and is therefore related to the surface-symmetry energy of nuclei.

To understand the deformation dependence of the symmetry energy coefficient, we further study the influence of nuclear surface diffuseness on  $a_{\text{sy}}$ . In Figure 7(a), we show the calculated surface diffuseness of neutrons and protons for the isobaric chain  $A = 100$  at spherical cases. Here, the Skyrme force SkM\* is adopted. The open and solid circles denote the results for protons and neutrons, respectively. One sees that the isospin dependence of the surface diffuseness is evident. For neutron-rich nuclei, the symmetry potential will “push” the extra-neutrons to the very low density region [30]. If setting the surface diffuseness  $a_n = a_p = 0.4$  fm in the calculations, we find that the values of  $a_{\text{sy}}$  slightly increase and the deformation dependence of the symmetry energy coefficient

$a_{\text{sy}}$  becomes weak (see Figure 7(b)). It indicates that the isospin dependence of the difference between the proton and neutron diffuseness plays a key role for the deformation dependence of the symmetry energy coefficient of finite nuclei.

## 4 Summary

In summary, by using the Skyrme energy density functional plus the extended Thomas-Fermi (ETF2) approximation, the symmetry energy coefficients of nuclei have been studied with the restricted density variational method. Considering nuclear quadrupole deformations in the calculations, we find that the calculated symmetry energy coefficient depends on not only the mass number  $A$  but also the nuclear deformations. The large deformations of nuclei can cause the change of symmetry energy coefficient by about 0.5 MeV. In the traditional liquid drop model, it is usually thought that only the surface energy and Coulomb energy terms are dependent on nuclear deformations. From the microscopic Skyrme energy density functional plus the ETF2 calculations, it is found that the isospin dependence of the difference between the proton and neutron diffuseness plays a key role for the deformation



**Figure 7** (Color online) (a) Nuclear surface diffuseness for nuclei with  $A = 100$ . (b) The same as Figure 5, but setting the surface diffuseness  $a_p = a_n = 0.4$  fm in the calculations.

dependence of the symmetry energy coefficient of finite nuclei. These investigations are not only important for the study of symmetry energy, but also for the improvement of nuclear mass models.

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