

An extended high-dimensional Melnikov analysis for global and chaotic dynamics of a non-autonomous rectangular buckled thin plate

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By using an extended Melnikov method on multi-degree-of-freedom Hamiltonian systems with perturbations, the global bifurcations and chaotic dynamics are investigated for a parametrically excited, simply supported rectangular buckled thin plate. The formulas of the rectangular buckled thin plate are derived by using the von Karman type equation. The two cases of the buckling for the rectangular thin plate are considered. With the aid of Galerkin's approach, a two-degree-of-freedom non-autonomous nonlinear system is obtained for the non-autonomous rectangular buckled thin plate. The high-dimensional Melnikov method developed by Yagasaki is directly employed to the non-autonomous ordinary differential equation of motion to analyze the global bifurcations and chaotic dynamics of the rectangular buckled thin plate. Numerical method is used to find the chaotic responses of the non-autonomous rectangular buckled thin plate. The results obtained here indicate that the chaotic motions can occur in the parametrically excited, simply supported rectangular buckled thin plate.

extended high-dimensional Melnikov method, rectangular buckled thin plate, non-autonomous nonlinear system, chaotic motions

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1 Introduction

With the use of thin plates in airplanes, shuttle and large space station, many researchers in engineering have paid considerable attention to the nonlinear oscillations, global bifurcations and chaotic motions of the thin plates in the case of large deformation. These obtained results are largely due to the development of the theories of the nonlinear dynamics, bifurcations and chaotic dynamics. In the past two decades, the researchers have done a number of studies on nonlinear oscillations, bifurcations and chaos of the thin plates. Yang and Sethna [1] used the averaging method to

study the local and global bifurcations of a parametrically excited nearly square plate. The results obtained indicated that the heteroclinic loops and the Smale horse exist. Based on research given in ref. [1], Feng and Sethna [2] made use of the global perturbation method developed by Kovacic and Wiggins [3] to study further the bifurcations and chaotic dynamics of a thin plate under parametric excitation and obtained the conditions in which the Shilnikov-type homoclinic orbits and chaos can occur. Chang et al. [4] investigated the bifurcations and chaos of a rectangular thin plate with 1:1 internal resonance. Sassi and Ostiguy [5] studied the effects of initial geometric imperfections on the interaction between forced and parametric nonlinear oscillations for a simply supported rectangular thin plate subjected to the in-plane periodic excitation. Anlas and Elbey-

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li [6] investigated the nonlinear dynamics of a simply supported rectangular plate subjected to transverse harmonic excitation.

Other results on the nonlinear oscillations, bifurcations and chaotic dynamics of flexible structures were also obtained by several researchers. Malhotra and Sri Namachivaya [7] used the averaging method and Melnikov method to investigate the local, global bifurcations and chaos of a two-degree-of-freedom shallow arch subjected to simple harmonic excitation for the case of 1:1 internal resonance. Abe et al. [8] used the method of multiple scales to analyze the two-mode responses of a simply supported rectangular laminated thin plate subjected to harmonic excitation. The global bifurcations and Shilnikov type chaotic dynamics were investigated by Zhang et al. [9] and Zhang [10] for both parametrically and externally excited and parametrically excited simply supported rectangular thin plates. Yeh et al. [11] employed the criteria of the fractal dimension and the maximum Lyapunov exponent to study the bifurcations and chaotic dynamics of a simply supported rectangular thin plate with thermo-mechanical coupling and large deflection. Awrejcewicz et al. [12] used the Bubnov-Galerkin with high-order approximations and finite difference methods to investigate the complex vibrations and bifurcations of a thin plate-strip excited transversally and axially. He [13] studied nonlinear dynamic responses and chaotic motions of a simply supported isotropic laminated thin plate subject to the in-plane and transverse non-uniform thermal field. Recently, Hao et al. [14] studied the nonlinear oscillations, bifurcations and chaos of a functionally graded materials plate. Zhang et al. [15] investigated the periodic and chaotic dynamics of a composite laminated piezoelectric rectangular plate.

The global bifurcations and chaotic dynamics of high-dimensional nonlinear systems have been at the forefront of nonlinear dynamics for the last two decades. Several researchers have paid considerable attention to the development of new methods for studying the global bifurcations and chaotic dynamics of high-dimensional nonlinear systems. However, due to lack of analytical methods to study the global bifurcations and chaotic dynamics for high-dimensional nonlinear systems, it is extremely challenging to develop theories of the global bifurcations and chaotic dynamics for high-dimensional nonlinear systems and to solve systematically engineering problems. Although there are many challenges and difficulties, certain progress has been achieved in this field in the past two decades. Based on the study given by Wiggins [16], Kovacic and Wiggins [3] developed a new global perturbation method which may be used to detect the Shilnikov type homoclinic and heteroclinic orbits for four-dimensional autonomous ordinary differential equations. Yagasaki [17] extended the standard high-dimensional Melnikov method which can be utilized to study the global bifurcations and chaotic dynamics of high-dimensional nonlinear systems with non-

com-pletely integrable unperturbed systems. The ref. [17] improved and developed the methods in refs. [18,19]. Furthermore, the method presented by Yagasaki [17] can deal with the global bifurcations and chaotic dynamics of high-dimensional non-autonomous nonlinear dynamical systems.

The extended subharmonic Melnikov method and modified homoclinic Melnikov method were employed by Yagasaki [20] to analyze the periodic orbits and homoclinic motions in periodically forced weakly coupled oscillators with the perturbations. Yagasaki [21] also utilized a Melnikov-type global perturbation technique to investigate homoclinic and heteroclinic bifurcations and chaotic dynamics of a two-degree-of-freedom Hamiltonian system with saddle-center. Recently, Yagasaki [22] developed a Melnikov-type global perturbation technique to study the homoclinic and heteroclinic orbits to invariant tori in multi-degree-of-freedom Hamiltonian systems with saddle centers.

Besides the aforementioned research on the theories of the global bifurcations and chaotic dynamics of high-dimensional nonlinear systems, other research works on applying the developed theories to engineering problems also deserve mentioning. Feng and Liew [23] analyzed the existence of the Shilnikov type homoclinic orbits of the averaged equation which represents the modal interactions between two modes with zero-to-one internal resonance and influence of the fast mode on the slow mode. Zhang and Li [24] employed the global perturbation approach to investigate the global bifurcations and chaotic dynamics of a two-degree-of-freedom nonlinear vibration absorber. Zhang and Tang [25] studied the global bifurcations and chaotic dynamics of the suspended elastic cable to small tangential vibration of one support which causes simultaneously the parametric excitation of the out-of-plane motion and the parametric and external excitations of the in-plane motion. Malhotra et al. [26] used the energy-phase method to investigate multi-pulse homoclinic orbits and chaotic dynamics for the motion of flexible spinning discs. Recently, Zhang et al. [27] studied the global bifurcations and chaotic dynamics for the nonlinear nonplanar oscillations of a parametrically excited cantilever beam. Yao and Zhang [28] utilized the energy-phase method to analyze the Shilnikov type multi-pulse homoclinic and heteroclinic orbits and chaotic dynamics of the nonlinear nonplanar oscillations of the cantilever beam. Yao and Zhang [29] used the energy-phase method to study the Shilnikov-type multi-pulse orbits and chaotic dynamics of a parametrically and externally excited rectangular thin plate. Zhang et al. [30] studied the multi-pulse global bifurcations and chaos of a nonplanar cantilever beam.

However, most studies on using the global perturbation method to analyze the complex dynamics of higher-dimensional nonlinear systems in terms of engineering are focused on the autonomous differential equations. For example, in

refs. [9,10] the non-autonomous ordinary differential equation of motion of a rectangular thin plate with two-degree-of-freedom was derived by using the von Karman-type equation and the Galerkin's approach. Then, the method of multiple scales was used to transfer the non-autonomous governing equation of motion to the autonomous averaged equation. Based on the averaged equation, the theory of normal form and the global perturbation method were employed to study the global bifurcations and Shilnikov type chaotic dynamics of the rectangular thin plate. The analytical procedure of using directly an extended Melnikov method to a two-degree-of-freedom non-autonomous nonlinear system is simplified without use of the method of multiple scales or the theory of normal form.

In this paper, we will study the global bifurcations and chaotic dynamics on the non-autonomous system of a parametrically excited, simply supported buckling rectangular thin plate by using directly an extended high-dimensional Melnikov method. The non-autonomous nonlinear system of two-degree-of-freedom for the rectangular thin plate is derived by using the von Karman type equation and the Galerkin's approach. The extended high-dimensional Melnikov method is directly utilized to investigate the global bifurcations and chaotic dynamics of the rectangular buckled thin plate. Thus, the simplification procedures on the equation of motion for the rectangular buckled plate are reduced twice in this paper as compared with the research given in refs. [9,10]. Actually, the method of multiple scales and the theory of normal form, which are respectively used to obtain and simplify the averaged equation of the rectangular thin plate, are not utilized in this paper. Two cases of the buckling for the rectangular thin plate are considered. To our best knowledge, few papers have studied the global bifurcations and chaotic dynamics of rectangular buckled thin plates by using the Melnikov method in the buckling cases. In refs. [9,10], the authors only investigated the chaotic dynamics of the non-buckled rectangular thin plates. The theoretical analysis indicates that the chaotic motions can occur in the parametrically excited simply supported buckling rectangular thin plate. It is also found from the numerical results that there exist chaotic motions of the simply supported rectangular buckled thin plate subjected to the in-plane excitation.

2 Formulation

In this section, we will give a brief outline of the Melnikov method on non-autonomous multi-degree-of-freedom Hamiltonian systems with the perturbations. Consider a Hamilton system with the perturbations:

$$\dot{x} = JDH(x) + \varepsilon g(x, \omega t, \mu), \quad x \in \mathbf{R}^{2n}, \quad (1)$$

where $0 < \varepsilon \ll 1$, $H : \mathbf{R}^{2n} \rightarrow \mathbf{R}$ is the Hamiltonian of the

unperturbed system, functions H and $g : \mathbf{R}^{2n} \times \mathbf{R} \times \mathbf{R} \rightarrow \mathbf{R}^{2n}$ are sufficiently smooth functions, the function $g(x, \theta, \mu)$ is 2π period with respect to θ , $\mu \in \mathbf{R}$ is the parameter of the system and J is a $2n \times 2n$ symplectic matrix:

$$J = \begin{pmatrix} 0 & I_n \\ -I_n & 0 \end{pmatrix} \quad (2)$$

with I_n and $\mathbf{0}$ are $n \times n$ identity and zero matrices, respectively.

We suspend system (1) over the space $\mathbf{R}^{2n} \times S^1$ as follows:

$$\dot{x} = JDH(x) + \varepsilon g(x, \theta, \mu), \quad (3a)$$

$$\dot{\theta} = \omega, \quad (3b)$$

where $S^1 = \mathbf{R} / 2\pi$ is the circle of the length 2π .

When $\varepsilon=0$, system (1) becomes an n -degree-of-freedom Hamiltonian system:

$$\dot{x} = JDH(x). \quad (4)$$

In order to apply the extended high-dimensional Melnikov method to a specific system, the following assumptions [17] are given for the unperturbed system (4).

(1) There exists a saddle-center at $x=x_0$ such that the matrix $A_0 = JD^2H(x_0)$ has n_h pairs of eigenvalues whose real parts have the same magnitude but opposite signs, and n_c pairs of pure imaginary eigenvalues, where $n_h + n_c = n$.

(2) The saddle-center has an $(l-1)$ parameter family of the homoclinic orbits $x^h(t, \alpha)$ $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_{l-1}) \in \mathbf{A}$, where $l \leq n_h$ and $\mathbf{A}$ is an open subset of $\mathbf{R}^{l-1}$, such that the l vectors in $\mathbf{R}^{2n}$ given by

$$\begin{aligned} \frac{\partial x^h}{\partial t}(t, \alpha) &= JDH(x^h(t, \alpha)) \\ \text{and } \frac{\partial x^h}{\partial \alpha_j}(t, \alpha), \quad j &= 1, 2, \dots, l-1 \end{aligned} \quad (5)$$

are linearly independent for $t \in \mathbf{R}$ and $\alpha \in \mathbf{A}$.

Define the following set

$$\gamma_0 = \{(x, \theta) \in \mathbf{R}^{2n} \times S^1 \mid x = x_0, \theta \in S^1\}.$$

It is known that γ_0 is a periodic orbit of the unperturbed system (4) in full $2n+1$ dimensional phase space. It is an invariant manifold which has $n+1$ dimensional stable manifold and unstable manifold $W^s(\gamma_0)$ and $W^u(\gamma_0)$, and the manifolds $W^s(\gamma_0)$ and $W^u(\gamma_0)$ intersect a $l+1$ dimensional homoclinic manifold denoted by Γ

$$\Gamma = \{(x, \theta) \mid x = x^h(t, \alpha), \theta = t + \theta_0, (t, \alpha, \theta_0) \in \mathbf{R}^1 \times \mathbf{R}^{l-1} \times S^1\}.$$

In this paper, the system is not required to be completely integrable. However, the following assumption 3 has to be

satisfied.

(3) The variational equation of the homoclinic orbit $x^h(t, \alpha)$

$$\dot{\xi} = JD^2 H(x^h(t, \alpha))\xi \quad (6)$$

has l linearly independent bounded solutions so that $|\xi(t)|$ exponentially tends to zero as $t \rightarrow \infty$.

We also assume the following condition.

(4) There exists a permutation σ on $2n$ symbols so that $n_h < \sigma(j) \leq n_h + 2n_c$ for $n_h < j \leq n_h + 2n_c$.

For the perturbed system (3), the following assumptions are given.

(5) System (3) has a hyperbolic periodic orbit γ_ε which is $O(\varepsilon)$ -close to and has the same period as periodic orbit γ_0 of the unperturbed system, that is $|\gamma_\varepsilon - \gamma| = O(\varepsilon)$.

(6) The eigenvalues of the perturbed system corresponding to center part of the unperturbed system are all either positive or negative.

For $n_c=0$, assumption (5) immediately follows from assumption (1) and assumptions (4) and (6) will be unnecessary. Generally, it is difficult for one to verify assumption (3). This is equivalent to the conditions which ensure the existence of the normal hyperbolic invariant manifold for the system as compared with the method given in ref. [16]. Then, the following proposition can be obtained

Proposition 1 Suppose that system (1) satisfies the assumptions (1)–(6). The extended high-dimensional Melnikov function is of the form:

$$M_1(\theta, \alpha) = \int_{-\infty}^{+\infty} DH(x^h(t, \alpha)) \cdot g(x^h(t, \alpha), \omega t + \theta) dt, \quad (7a)$$

$$M_{j-1}(\theta, \alpha) = \int_{-\infty}^{+\infty} J \frac{\partial x^h}{\partial \alpha_j}(t, \alpha) \cdot g(x^h(t, \alpha), \omega t + \theta) dt, \quad (7b)$$

$$j = 1, 2, \dots, l-1.$$

Then, there exists a point $(\theta, \alpha) = (\theta_0, \alpha_0)$ so that

$$M(\theta_0, \alpha_0) = 0 \text{ and } \det DM(\theta_0, \alpha_0) \neq 0. \quad (8)$$

Therefore, for $\varepsilon > 0$ sufficiently small, the manifolds $W^s(\gamma_\varepsilon)$ and $W^u(\gamma_\varepsilon)$ intersect transversely near $(x^h(t, \alpha), \omega t + \theta_0)$.

The transverse intersection of the stable and unstable manifolds allows us to determine the existence of transverse homoclinic orbits in the perturbed system. Based on the Smale-Birkhoff theorem, the presence of transverse homoclinic orbits implies that the system has the chaos in the Smale horseshoe sense, namely, there exists chaotic motions in system (1).

3 Equation of motion of a thin plate

We consider a simply supported rectangular thin plate at

four edges, whose edge lengths are a and b and thickness is h , respectively. The thin plate is subjected to in-plane excitation which can be expressed in the form $P = P_0 - P_1 \cos \Omega t$. We establish a Cartesian coordinate system shown in Figure 1 such that the coordinate Oxy is located in the middle surface of the thin plate. It is assumed that the u , v and w represent the displacements of a point in the middle plane of the thin plate in the x , y and z directions, respectively. From van Karman type equation for the thin plate [31], we obtain the governing equation of motion for the rectangular thin plate as follows:

$$D \nabla^4 w + \rho h \frac{\partial^2 w}{\partial t^2} - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 \phi}{\partial y^2} - \frac{\partial^2 w}{\partial y^2} \frac{\partial^2 \phi}{\partial x^2} + 2 \frac{\partial^2 w}{\partial x \partial y} \frac{\partial^2 \phi}{\partial x \partial y} + \mu \frac{\partial w}{\partial t} = 0, \quad (9)$$

$$\nabla^4 \phi = E h \left[\left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right], \quad (10)$$

where ρ is the density of the thin plate, $D = \frac{E h^3}{12(1-\nu^2)}$ is

the bending rigidity, E is the Young's modulus, ν is the Poisson's ratio, ϕ is the stress function, and μ is the damping coefficient.

We assume that the boundary conditions of the simply supported at four edges rectangular thin plate can be written as:

$$\begin{aligned} \text{at } x=0 \text{ and } x=a, \quad w = \frac{\partial^2 w}{\partial x^2} = 0; \\ \text{at } y=0 \text{ and } y=b, \quad w = \frac{\partial^2 w}{\partial y^2} = 0. \end{aligned} \quad (11)$$

The boundary conditions satisfied by the stress function ϕ may be expressed in the following form:

$$u = \int_0^a \left[\frac{1}{E} \left(\frac{\partial^2 \phi}{\partial y^2} - \nu \frac{\partial^2 \phi}{\partial x^2} \right) - \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \right] dx = \delta_x, \quad (12)$$

and

$$h \int_0^b \frac{\partial^2 \phi}{\partial y^2} dy = p \text{ at } x=0 \text{ and } x=a, \quad (13)$$

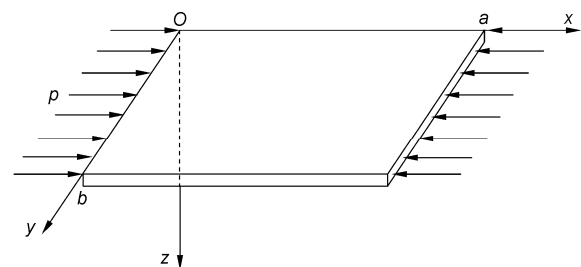


Figure 1 The model of a rectangular thin plate and the coordinate system are given. $P = P_0 - P_1 \cos \Omega t$.

$$v = \int_0^b \left[\frac{1}{E} \left(\frac{\partial^2 \varphi}{\partial x^2} - \nu \frac{\partial^2 \varphi}{\partial y^2} \right) - \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 \right] dx = 0, \quad (14)$$

and

$$\int_0^a \frac{\partial^2 \varphi}{\partial x^2} dx = 0, \quad \text{at } y=0 \text{ and } y=b, \quad (15)$$

where δ_x is the corresponding displacement in the x direction at the boundary.

We mainly consider the nonlinear oscillations of the rectangular thin plate in the first two modes. Thus, we write w in the following form:

$$w(x, y, t) = u_1(t) \sin \frac{\pi x}{a} \sin \frac{3\pi y}{b} + u_2(t) \sin \frac{3\pi x}{a} \sin \frac{\pi y}{b}, \quad (16)$$

where $u_i(t)$ ($i=1, 2$) are the amplitudes of the first two modes, respectively.

Substituting eq. (16) into eq. (10), considering the boundary conditions (12)–(15) and integrating, we obtain the stress function as follows:

$$\begin{aligned} \varphi(x, y, t) = & \varphi_{20}(t) \cos \frac{2\pi x}{a} + \varphi_{02}(t) \cos \frac{2\pi y}{b} \\ & + \varphi_{60}(t) \cos \frac{6\pi x}{a} \\ & + \varphi_{06}(t) \cos \frac{6\pi y}{b} + \varphi_{22}(t) \cos \frac{2\pi x}{a} \cos \frac{2\pi y}{b} \\ & + \varphi_{24}(t) \cos \frac{2\pi x}{a} \cos \frac{4\pi y}{b} \\ & + \varphi_{42}(t) \cos \frac{4\pi x}{a} \cos \frac{2\pi y}{b} \\ & + \varphi_{44}(t) \cos \frac{4\pi x}{a} \cos \frac{4\pi y}{b} - \frac{1}{2} p y^2, \end{aligned} \quad (17)$$

where

$$\begin{aligned} \varphi_{20}(t) = & \frac{9Eh}{32\lambda^2} u_1^2, \quad \varphi_{02}(t) = \frac{9\lambda^2 Eh}{32} u_2^2, \quad \varphi_{60}(t) = \frac{Eh}{288\lambda^2} u_1^2, \\ \varphi_{06}(t) = & \frac{\lambda^2 Eh}{288} u_2^2, \quad \varphi_{22}(t) = -\frac{\lambda^2 Eh}{(\lambda^2 + 1)^2} u_1 u_2, \\ \varphi_{24}(t) = & \frac{25\lambda^2 Eh}{16(\lambda^2 + 4)^2} u_1 u_2, \quad \varphi_{42}(t) = \frac{25\lambda^2 Eh}{16(4\lambda^2 + 1)^2} u_1 u_2, \\ \varphi_{44}(t) = & -\frac{\lambda^2 Eh}{16(\lambda^2 + 1)^2} u_1 u_2, \quad \lambda = \frac{b}{a}. \end{aligned} \quad (18)$$

In order to obtain the dimensionless equation of motion, we introduce the transformations of the variables and parameters:

$$\bar{x}_i = \frac{(ab)^{1/2}}{h^2} u_i \quad (i=1, 2), \quad \bar{p}_0 = \frac{b^2}{\pi^2 D} p_0,$$

$$\begin{aligned} \bar{p}_1 = & \frac{ab^3}{\pi^2 E h^5} p_1, \quad \bar{\Omega} = \frac{ab}{\pi^2} \left(\frac{\rho h}{D} \right)^{1/2} \Omega, \\ \varepsilon = & \frac{12(1-\nu^2)h^2}{ab}, \quad \bar{t} = \frac{\pi^2}{ab} \left(\frac{D}{\rho h} \right)^{1/2} t, \\ \bar{\mu} = & \frac{a^2 b^2}{\pi^2 h^4} \left(\frac{1}{12(1-\nu^2)\rho E} \right)^{1/2} \mu, \end{aligned} \quad (19)$$

where ε is a small parameter.

For simplicity, we drop overbar in the following analysis. By means of the Galerkin's method, substituting eqs. (16) and (17) into eq. (9) and integrating, we obtain the governing equation of motion of the rectangular thin plate for the dimensionless as follows:

$$\ddot{x}_1 + \varepsilon \mu \dot{x}_1 - g_1 x_1 + 2\varepsilon x_1 f_1 \cos \Omega t + \alpha_1 x_1^3 + \alpha_2 x_1 x_2^2 = 0, \quad (20a)$$

$$\ddot{x}_2 + \varepsilon \mu \dot{x}_2 - g_2 x_2 + 2\varepsilon x_2 f_2 \cos \Omega t + \beta_1 x_2^3 + \beta_2 x_1^2 x_2 = 0, \quad (20b)$$

where

$$\begin{aligned} \alpha_1 = & \frac{12(1-\nu^2)h^2}{ab} \frac{\lambda^4 + 81}{16\lambda^2}, \quad \beta_1 = \frac{3(1-\nu^2)h^2}{4ab} \left(81\lambda^2 + \frac{1}{\lambda^2} \right), \\ \alpha_2 = & \beta_2 \\ = & \frac{12(1-\nu^2)h^2}{ab} \left[\frac{17\lambda^2}{(1+\lambda^2)^2} + \frac{625\lambda^2}{16(4+\lambda^2)^2} + \frac{625\lambda^2}{16(1+4\lambda^2)^2} \right] \\ g_k = & \left(h_k p_0 - (\omega_k^*)^2 \right) \quad \text{and} \\ h_k = & \begin{cases} 1, & k=1, \\ 9, & k=2, \end{cases} \quad p_1^* = (\omega_1^*)^2 = \frac{(9+\lambda^2)^2}{\lambda^2}, \\ p_2^* = & (\omega_2^*)^2 = \frac{(9\lambda^2+1)^2}{\lambda^2}, \quad f_k = \frac{1}{2} h_k p_1, \quad k=1, 2, \end{aligned} \quad (21)$$

where g_k ($k=1, 2$) are two linear stiffness coefficients of the thin plate, p_k^* ($k=1, 2$) are the critical forces corresponding to two buckled modes at which the thin plate loses the stability, ω_k^* ($k=1, 2$) are the natural frequencies of the two buckled modes, and f_k ($k=1, 2$) are the amplitudes of parametric excitation.

It is found from the aforementioned analysis that the buckling load is $p_{0c} = p_k^*$. In this paper, we restrict our attention to the case in which the applied static load is larger than the buckling load, namely, $p_0 > p_{0c}/h_k$. In the following analysis, the two cases of the buckling of the rectangular thin plate will be considered.

4 Global perturbation analysis

Introducing the coordinate transformations $x_1=y_1$, $\dot{x}_1 = y_2$,

$x_2=y_3$, $\dot{x}_2=y_4$ on eq. (20), we obtain the following equivalent form of system (20):

$$\dot{y}_1 = y_2, \quad (22a)$$

$$\dot{y}_2 = g_1 y_1 - \alpha_1 y_1^3 - \alpha_2 y_1 y_3^2 - \varepsilon \mu y_2 - 2\varepsilon y_1 f_1 \cos \theta, \quad (22b)$$

$$\dot{y}_3 = y_4, \quad (22c)$$

$$\dot{y}_4 = g_2 y_3 - \beta_1 y_3^3 - \alpha_2 y_1^2 y_3 - \varepsilon \mu y_4 - 2\varepsilon y_3 f_2 \cos \theta, \quad (22d)$$

$$\dot{\theta} = \Omega. \quad (22e)$$

Let $\varepsilon=0$ in eq. (22), the unperturbed system is of the form:

$$\dot{y}_1 = y_2, \quad (23a)$$

$$\dot{y}_2 = g_1 y_1 - \alpha_1 y_1^3 - \alpha_2 y_1 y_3^2, \quad (23b)$$

$$\dot{y}_3 = y_4, \quad (23c)$$

$$\dot{y}_4 = g_2 y_3 - \beta_1 y_3^3 - \alpha_2 y_1^2 y_3. \quad (23d)$$

The Hamiltonian function of system (23) is obtained as:

$$H = \frac{1}{2} y_2^2 + \frac{1}{2} y_4^2 - \frac{1}{2} g_1 y_1^2 - \frac{1}{2} g_2 y_3^2 + \frac{1}{4} \alpha_1 y_1^4 + \frac{1}{4} \beta_1 y_3^4 + \frac{1}{2} y_1^2 y_3^2. \quad (24)$$

It is found that the point $y = (0, 0, 0, 0)^T$ is an equilibrium of system (23) at which there exists $H(0, 0, 0, 0) = 0$. Two cases of the buckled for the rectangular thin plate will be considered in the following analysis.

We first consider the case of $g_1, g_2 > 0$. It is known that the buckling of both modes for the rectangular thin plate simultaneously occurs in this case.

It is observed that the singular point $O(0, 0, 0, 0)$ is a saddle point of system (23) whose eigenvalues are $\lambda = \pm\sqrt{g_1}$ and $\lambda = \pm\sqrt{g_2}$, respectively. Therefore, it is observed that assumption 1 holds.

The homoclinic orbits are calculated as follows:

$$y^h(t) = \left(\pm \sqrt{\frac{2g_1}{\alpha_1}} \operatorname{sech} \sqrt{g_1} t, \pm \sqrt{\frac{2}{\alpha_1}} g_1 \operatorname{sech} \sqrt{g_1} t \tanh \sqrt{g_1} t, 0, 0 \right), \quad (25)$$

and

$$y^h(t) = \left(0, 0, \pm \sqrt{\frac{2g_2}{\beta_1}} \operatorname{sech} \sqrt{g_2} t, \pm \sqrt{\frac{2}{\beta_1}} g_2 \operatorname{sech} \sqrt{g_2} t \tanh \sqrt{g_2} t \right). \quad (26)$$

Therefore, it is known that assumption 2 holds when $l=1$.

For eq. (25), the variational equations of the homoclinic orbits are obtained as:

$$\dot{\xi}_1 = \xi_2, \quad (27a)$$

$$\dot{\xi}_2 = \left(g_1 - 6g_1 \operatorname{sech}^2 \sqrt{g_1} t \right) \xi_1, \quad (27b)$$

$$\dot{\xi}_3 = \xi_4, \quad (27c)$$

$$\dot{\xi}_4 = \left(g_2 - \alpha_2 \frac{2g_1}{\alpha_1} \operatorname{sech}^2 \sqrt{g_1} t \right) \xi_3. \quad (27d)$$

For eq. (26), the variational equations of the homoclinic orbits are obtained as:

$$\dot{\xi}_1 = \xi_2, \quad (28a)$$

$$\dot{\xi}_2 = \left(g_1 - \alpha_2 \frac{2g_2}{\alpha_1} \operatorname{sech}^2 \sqrt{g_2} t \right) \xi_1, \quad (28b)$$

$$\dot{\xi}_3 = \xi_4, \quad (28c)$$

$$\dot{\xi}_4 = \left(g_2 - 6g_2 \operatorname{sech}^2 \sqrt{g_2} t \right) \xi_3. \quad (28d)$$

Introduce the following transformations on eq. (28):

$$\tau = \sqrt{g_i} t, \quad \xi_1 = \eta_1, \quad \xi_2 = \eta_2 \sqrt{g_i}, \quad \xi_3 = \eta_3, \quad \xi_4 = \eta_4 \sqrt{g_i},$$

where $i=1$ in eq. (27) and $i=2$ in eq. (28).

Then, each of eqs. (27) and (28) can be reduced to two independent two-dimensional systems which are of the form

$$\dot{\zeta}_1 = \zeta_2, \quad (29a)$$

$$\dot{\zeta}_2 = (\lambda - \kappa \operatorname{sech}^2 t) \zeta_1. \quad (29b)$$

Thus, eq. (29) has the same form as that in ref. [17]. In eqs. (27a) and (27b), we have $\lambda=g_1$ and $\kappa=6g_1$. In eqs. (27c) and (27d), there are $\lambda=g_2$ and $\kappa=2g_1\alpha_2/\alpha_1$. It is known from ref. [17] that these solutions which satisfy assumption (3) are possible if and only if $\lambda=1$ and $\kappa=2$, or $\lambda=1, 4$ and $\kappa=6$. Therefore, we let $g_2=1$ and $2g_1\alpha_2/\alpha_1=2, 6$, or $g_2=4$ and $2g_1\alpha_2/\alpha_1=6$. In eqs. (28a) and (28b), we have $\lambda=g_1$ and $\kappa=2g_2\alpha_2/\alpha_1$. In eqs. (28c) and (28d), there are $\lambda=g_2$ and $\kappa=6g_2$. Thus, we let $g_1=1$ and $2g_2\alpha_2/\alpha_1=2, 6$, or $g_1=4$ and $2g_2\alpha_2/\alpha_1=6$. Assumption 3 holds based on ref. [17] when $g_1=1, g_2=1$ and $\alpha_1=\alpha_2$ or $\alpha_2=3\alpha_1$ and $l=1$. Assumption (5) immediately follows from assumption (1). Assumptions (4) and (6) do not have any meaning here.

The extended high-dimensional Melnikov function of the homoclinic orbit (25) is obtained as follows:

$$\begin{aligned}
M(\theta) &= \int_{-\infty}^{+\infty} y_2 [-\mu y_2 - 2y_1 f_1 \cos(\Omega t + \theta)] dt \\
&= -\frac{2g_1^{3/2} \mu}{\alpha_1} \int_{-\infty}^{+\infty} \tanh^2 \sqrt{g_1} t d(\tanh^2 \sqrt{g_1} t) \\
&\quad + \frac{8g_1}{\alpha_1} f_1 \sin \theta \int_0^{+\infty} \sin \Omega t \frac{\text{sh} \sqrt{g_1} t}{\text{ch}^3 \sqrt{g_1} t} d(\sqrt{g_1} t) \\
&= \pm \frac{2\Omega^2 \pi}{\alpha_1} f_1 \sin \theta \csc h \left(\frac{\Omega \pi}{2\sqrt{g_1}} \right) - \frac{4\mu g_1^{3/2}}{3\alpha_1}. \quad (30)
\end{aligned}$$

In addition, the extended high-dimensional Melnikov function of the homoclinic orbit (26) is obtained as follows:

$$\begin{aligned}
M(\theta) &= \int_{-\infty}^{+\infty} y_4 [-\mu y_4 - 2y_3 f_2 \cos(\Omega t + \theta)] dt \\
&= \pm \frac{2\Omega^2 \pi}{\beta_1} f_2 \sin \theta \csc h \left(\frac{\Omega \pi}{2\sqrt{g_2}} \right) - \frac{4\mu g_2^{3/2}}{3\beta_1}. \quad (31)
\end{aligned}$$

The control parameters may be chosen as f_1 and f_2 . From Proposition 1, it can be seen that if there are the conditions

$$f_1 > \frac{2\mu g_1^{3/2}}{3\Omega^2 \pi} \text{sh} \left(\frac{\Omega \pi}{2\sqrt{g_1}} \right) \text{ and } f_2 > \frac{2\mu g_2^{3/2}}{3\Omega^2 \pi} \text{sh} \left(\frac{\Omega \pi}{2\sqrt{g_2}} \right), \quad (32)$$

the manifolds $W^s(\gamma_\varepsilon)$ and $W^u(\gamma_\varepsilon)$ intersect transversely.

In the second case, it is found that there exists the condition $g_1 g_2 < 0$. Let $g_1 > 0$ and $g_2 < 0$. It is known that the buckling of the first-order mode and the non-buckling of the second-order mode for the rectangular thin plate simultaneously occur in this case.

If the square root of $-g_2$ is equal to Ω , then, the resonance occurs so that the method given in ref. [17] can not be applied. Therefore, the square root of $-g_2$ is not near Ω . It is found that the singular point $O(0, 0, 0, 0)$ is a saddle-center of system (23) whose eigenvalues are $\lambda = \pm\sqrt{g_1}$ and $\lambda = \pm\sqrt{-g_2}i$, respectively. Thus, assumption 1 holds. The homoclinic orbits of y_1 and y_2 are the same as those of eq. (25) in the first case, namely

$$y^h(t) = \left(\pm \sqrt{\frac{2g_1}{\alpha_1}} \text{sech} \sqrt{g_1} t, \pm \sqrt{\frac{2}{\alpha_1}} g_1 \text{sech} \sqrt{g_1} t \tanh \sqrt{g_1} t, 0, 0 \right). \quad (33)$$

Therefore, assumption (2) holds when $l=1$. For the first case, assumption (3) holds. Next, we will verify that assumptions (4)–(6) hold. The two-dimensional subspace $\{\xi \in R^4 \mid \xi_1 = \xi_2 = 0\}$ is invariant under the flow of system (27) and consists of the eigenvalues $\pm i\sqrt{-g_2}$. Therefore, it is found that assumption (4) holds.

In the following analysis, we verify that assumptions (5) and (6) hold. It is known that $A_1 = D_x g(0, \theta)$ is a constant matrix. We obtain the following relations:

$$A_0 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ g_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & g_2 & 0 \end{pmatrix}, \quad \varepsilon A_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ -2\varepsilon f_1 \cos \theta & -\varepsilon \mu & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -2\varepsilon f_2 \cos \theta & 0 & -\varepsilon \mu \end{pmatrix}, \quad (34)$$

$$A_0 + \varepsilon A_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -2\varepsilon f_1 \cos \theta & -\varepsilon \mu & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -2\varepsilon f_2 \cos \theta & g_2 & -\varepsilon \mu \end{pmatrix}. \quad (35)$$

Eigenvalues of the function $A_\varepsilon = A_0 + \varepsilon A_1$ corresponding to y_3 and y_4 are $\lambda_{3,4} = -\varepsilon \mu \pm i\sqrt{-4g_2 - \varepsilon^2 \mu^2}$, respectively.

The corresponding high-dimensional Melnikov function is the same as eq. (30)

$$\begin{aligned}
M(\theta) &= \int_{-\infty}^{+\infty} y_2 [-\mu y_2 - 2y_1 f_1 \cos(\Omega t + \theta)] dt \\
&= \pm \frac{2\Omega^2 \pi}{\alpha_1} f_1 \sin \theta \csc h \left(\frac{\Omega \pi}{2\sqrt{g_1}} \right) - \frac{4\mu g_1^{3/2}}{3\alpha_1}. \quad (36)
\end{aligned}$$

Choose f_1 as the control parameter. From Proposition 1, it can be seen that if the follow condition holds:

$$f_1 > \frac{2\mu g_1^{3/2}}{3\Omega^2 \pi} \text{sh} \left(\frac{\Omega \pi}{2\sqrt{g_1}} \right), \quad (37)$$

and $-\sqrt{g_2} \neq \Omega$, then, the manifolds $W^s(\gamma_\varepsilon)$ and $W^u(\gamma_\varepsilon)$ of system (22) with $g_2 < 0$ intersect transversely based on Proposition 1.

5 Numerical results

To verify the aforementioned theoretical analysis, we use the fourth-order Runge-Kutta algorithm to explore the existence of the chaotic motions for the parametrically excited simply supported rectangular buckled thin plate. Eq. (22) is used to do numerical simulations.

Figure 2 demonstrates the existence of the chaotic motion of the non-autonomous rectangular buckled thin plate

with two buckled modes when the parameters respectively are $g_1 = 3.6$, $g_2 = 4.5$, $\mu = 0.56$, $\alpha_1 = 6.36$, $\alpha_2 = 8.48$, $\beta_1 = 6.56$, $f_1 = 92$, $f_2 = 86$, $\Omega = 1$. The initial conditions are given as $y_1^0 = 0.9501$, $y_2^0 = 0.2311$, $y_3^0 = 0.6068$, $y_4^0 = 0.4860$. Figure 3 is another type of chaotic motion of the non-autonomous rectangular buckled thin plate with two buckled modes when the parameters are $g_1 = 3.6$, $g_2 = 4.2$, $\mu = 0.36$, $\alpha_1 = 9.36$, $\alpha_2 = 11.48$, $\beta_1 = 8.56$, $f_1 = 89$, $f_2 = 86$, $\Omega = 1$. The initial conditions are the same as those in Figure 2.

Figure 4 indicates that the chaotic motion of the non-autonomous rectangular buckled thin plate with the first-order buckled mode and the second-order non-buckled mode

occurs, which is the same as that in case 2. The non-resonance condition is considered, that is, the square root of $-g_2$ is not near Ω . The parameters of eq. (22) are chosen as $g_1 = 3.6$, $g_2 = -3.2$, $\mu = 0.36$, $\alpha_1 = 9.36$, $\alpha_2 = 6.48$, $\beta_1 = 8.56$, $f_1 = 89$, $f_2 = 96$, $\Omega = 1$. The initial conditions are given $y_1^0 = 0.9501$, $y_2^0 = 0.2311$, $y_3^0 = 0.6068$, $y_4^0 = 0.4860$. Figure 5 illustrates the existence of the chaotic motion of the non-autonomous rectangular buckled thin plate with the first-order buckled mode and the second-order non-buckled mode when the parameters are chosen as $g_1 = 3.6$, $g_2 = -3.2$, $\mu = 0.36$, $\alpha_1 = 9.36$, $\alpha_2 = 10.48$, $\beta_1 = 8.56$, $f_1 = 79$, $f_2 = 76$, $\Omega = 1$. The initial conditions are the same as those in Figure 4.

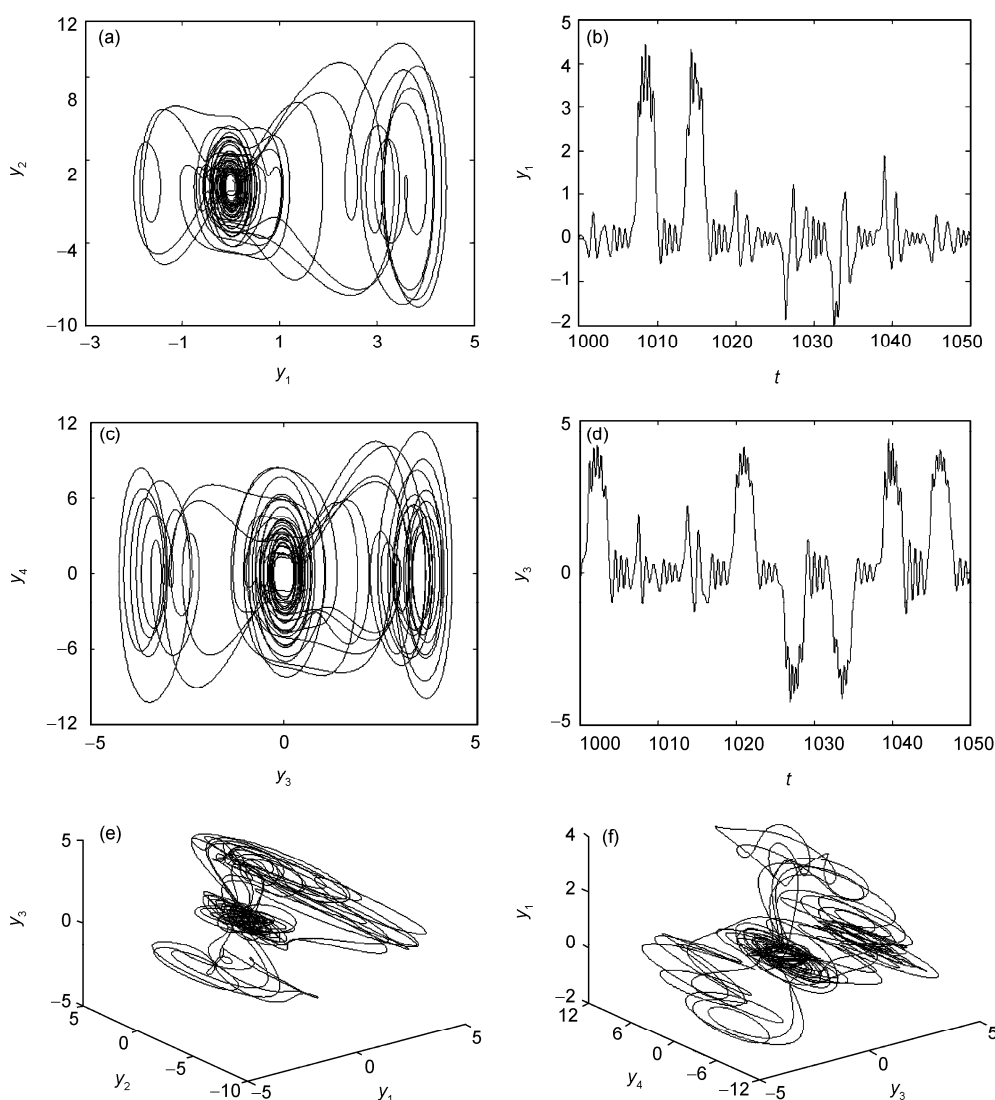


Figure 2 Chaotic motion of the rectangular buckled thin plate is obtained in case 1. (a) and (c) represent the phase portraits on the planes (y_1, y_2) and (y_3, y_4) , respectively; (b) and (d) respectively depict the waveforms on the planes (t, y_1) and (t, y_3) ; (e) and (f) represent three-dimensional phase portraits in spaces (y_1, y_2, y_3) and (y_3, y_4, y_1) , respectively.

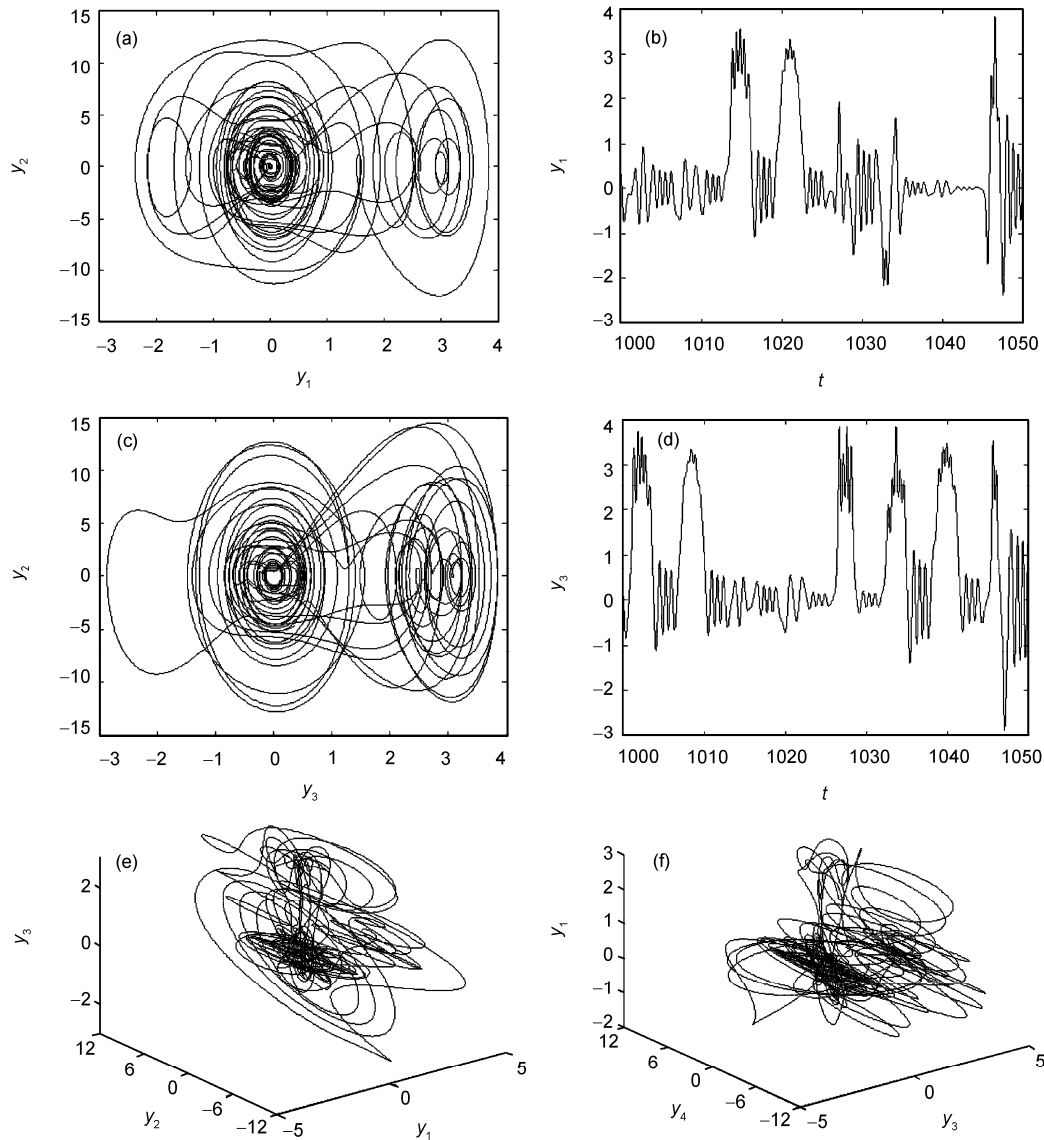


Figure 3 Another type of chaotic motion of the rectangular buckled thin plate is obtained in case 1. (a)–(f) are the same as Figure 2.

Based on the numerical results, it is observed from Figures 5(e) and 5(f) that the shape of the phase portraits in three-dimensional spaces (y_1, y_2, y_3) and (y_3, y_4, y_1) indeed indicates the existence of the multi-pulse chaotic motion for the non-autonomous rectangular buckled thin plate.

6 Conclusions

It is known that the global perturbation method developed by Kovacic and Wiggins in ref. [3] can be only used to deal with the global bifurcations and single-pulse chaotic dynamics of high-dimensional autonomous nonlinear dynamical systems. In order to directly investigate the global bifurcations and chaotic dynamics of high-dimensional non-

autonomous nonlinear dynamical systems, Yagasaki [17] presented a method to deal with high-dimensional non-autonomous nonlinear dynamical systems. In this paper, the global bifurcations and chaotic dynamics of a parametrically excited, simply supported rectangular buckled thin plate are investigated by using an extended Melnikov method of multi-degree-of-freedom Hamiltonian systems with perturbations developed by Yagasaki. The global perturbation analysis is directly based on the non-autonomous nonlinear system of the rectangular buckled thin plate. It is known from the aforementioned analysis that assumption 6 for the perturbed system is further simplified in order to determine conveniently the eigenvalues of the perturbed system corresponding to the center part of the unperturbed system. The Melnikov function is calculated to give the conditions under

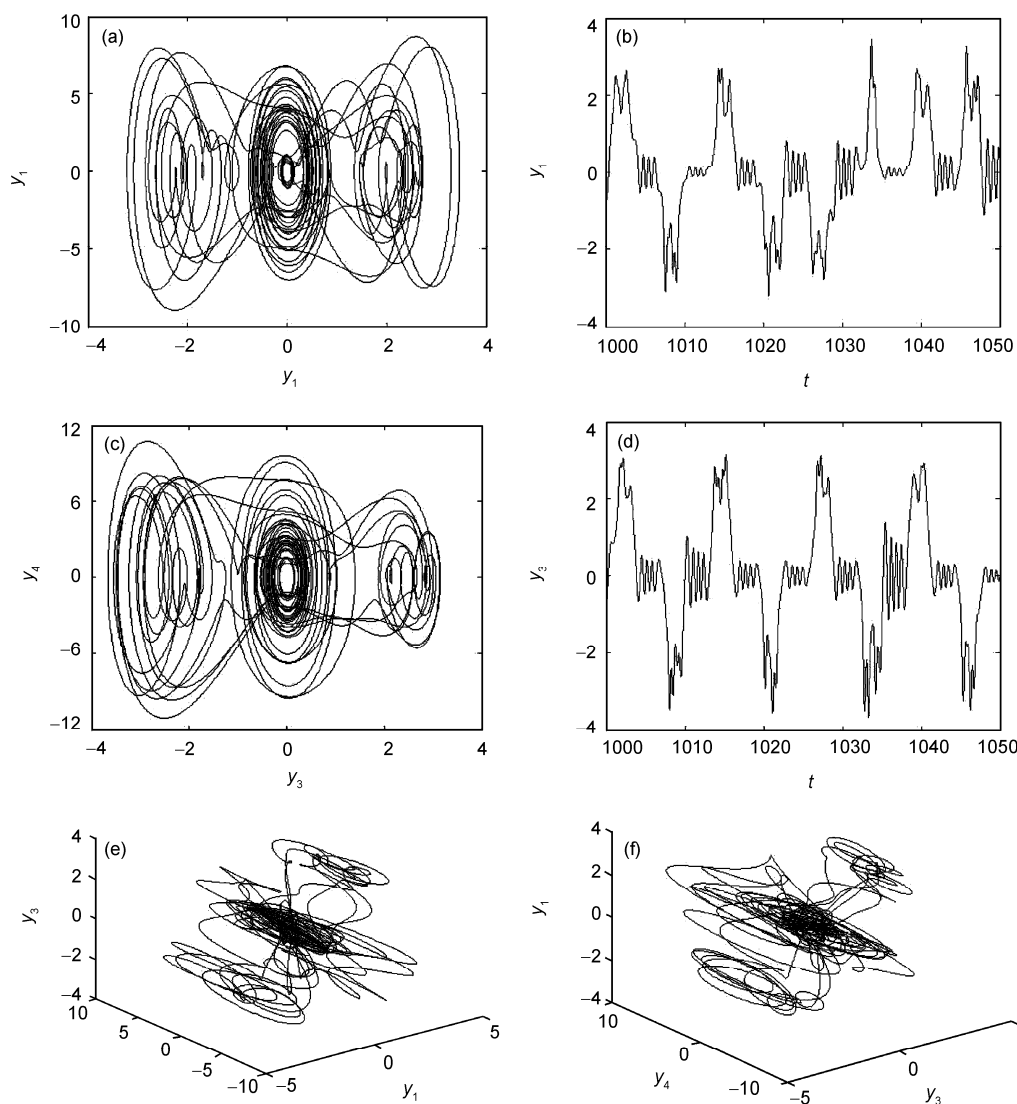


Figure 4 Chaotic motion of the rectangular buckled thin plate is obtained in case 2. (a)–(f) are the same as Figure 2.

which the chaotic motions of the non-autonomous rectangular buckled thin plate occur. The chaotic motions of the non-autonomous rectangular buckled thin plate are also found by numerical simulations. Directly using the extended high-dimensional Melnikov method to analyze the global bifurcations and chaotic dynamics of the non-autonomous nonlinear dynamical systems are advantageous in avoiding transforming the non-autonomous nonlinear systems to the autonomous averaged equation.

The two buckled cases are considered for the rectangular thin plate, which can not be analyzed by using the method in refs. [9,10]. In the first case, the buckling of both modes simultaneously occurs for the rectangular thin plate subjected to the in-plane excitation. In the second case, the buckling of the first-order mode and the non-buckling of the second-order mode simultaneously occur for the rectangular thin plate subjected to the in-plane excitation. The numeri-

cal results also illustrate that chaotic motions exist for the simply supported rectangular buckled thin plate subjected to the in-plane excitation in the two cases of the buckling. It is seen from the numerical results that the shape of three-dimensional phase portraits in the spaces (y_1, y_2, y_3) and (y_3, y_4, y_1) indeed shows the existence of chaotic motion for the rectangular buckled thin plate subjected to the in-plane excitation.

It is difficult to keep good agreement between the theoretical results and numerical simulations. Up to now, we did not find reports in the open literature which were consistent between the theoretical results and numerical simulations. The Melnikov method is a qualitative method for analyzing the chaotic motions of nonlinear systems. In this paper, the theoretical analysis qualitatively shows that the chaotic motions can occur in the parametrically excited, simply supported rectangular buckled thin plate for two cases of the

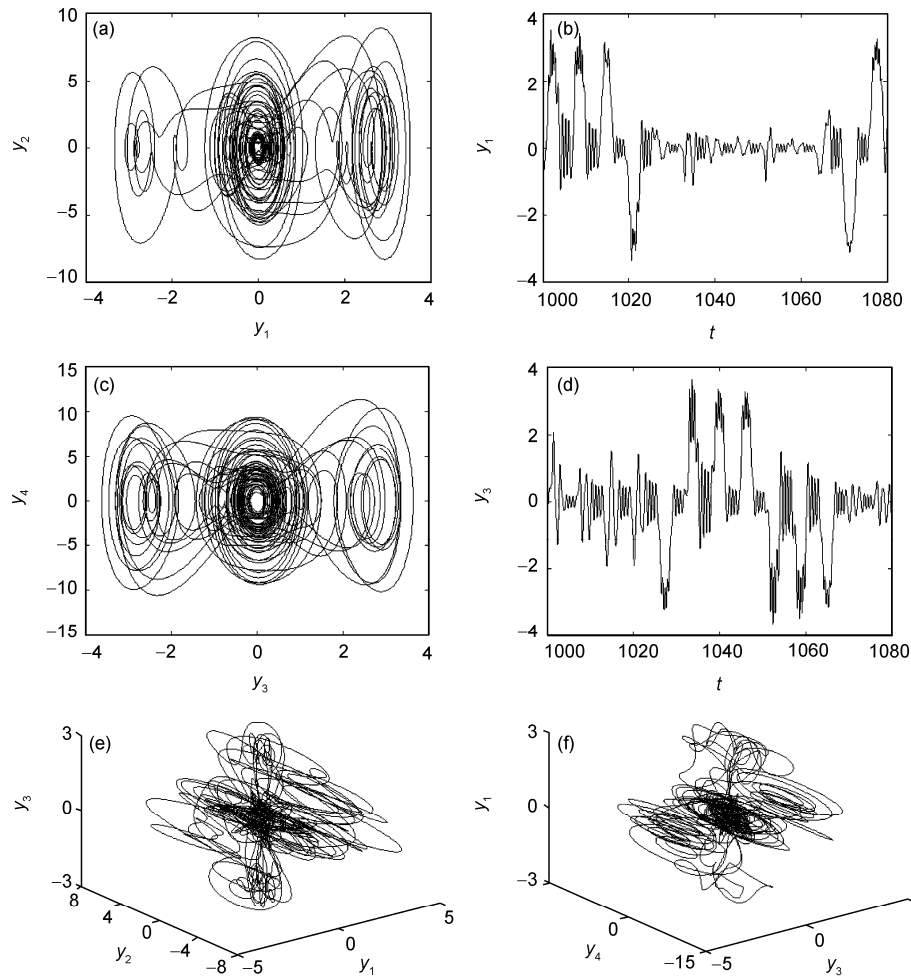


Figure 5 Another type of chaotic motion of the rectangular buckled thin plate is obtained in case 2. (a)–(f) are the same as Figure 2.

buckling and numerical simulations quantitatively demonstrate that the chaotic motions exist in the system.

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