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Distributed cooperative control of multiple high-speed trains under a moving block system by nonlinear mapping-based feedback

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Abstract Although the high-speed railways in China have been greatly advanced in the past decades with respect to expanding networks and increasing speed, a fixed block system, which separates the trains with several stationary track block sections, is utilized to guarantee the safe operation of multiple trains. A moving block system, which enables the moving authority of a high-speed train to be the real-time positioning point of its preceding one (plus some necessary safe redundant distance, of course), is under development to further make full use of the high-speed railway lines and improve the automation level by automatic train operation for high-speed trains. The aim of this paper is to design a distributed cooperative control for high-speed trains under a moving block system by giving a cooperative model with a back-fence communication topology. A nonlinear mapping-based feedback control method together with a rigorous mathematic proof for the global stability and ultimate bound of the closed-loop control systems is proposed. Comparative results are given to demonstrate the effectiveness and advantages of the proposed method.

Keywords high-speed trains, moving block system, cooperative control

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1 Introduction

A moving block system has been proved to be an effective and efficient signal system for railway systems by its wide practical applications in subway systems, which lays both theoretical and practical foundations for its extension to high-speed railway systems [1–7]. Actually, fixed block and quasi-moving block systems are implemented in high-speed railway lines in China from the viewpoint of operation safety, where the moving authorities are calculated from the target points behind several block sections and the section entry point of preceding trains. In the moving block system, the moving authority of a high-speed train is calculated using the real-time location information of the preceding trains by virtue of advanced positioning, communication, and control technologies, which enables a smaller separation distance among trains and a more efficient utilization of railway lines.

As the foundational core function, the distributed cooperative control plays an essential role in guaranteeing efficient operation of multiple high-speed trains, and some previous theoretical efforts can be found in developing the modelling and control methods for high-speed trains and subway systems. To name a

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few, in [8], the problem of planning energy-saving trajectories for multiple high-speed train movements is studied using online distributed cooperative model predictive control. In [9], a cooperative train control model for energy saving is designed. In [10], a cooperative scheduling model is designed for timetable optimization in subway systems. In [11], a neural adaptive coordination control of multiple trains under a bidirectional communication topology is proposed. In [1], a cooperative control design together with stability analysis for multiple trains' moving signalling systems are proposed. It can be concluded that the studies in [8–10] require the precise information of train dynamics to formulation the optimization problem, and the studies in [1, 11] propose control methods using fixed-gain feedback cooperative control, which is not flexible under different operation situations. Inspired by these observations, this paper proposes a distributed cooperative control for multiple high-speed trains in the mode of moving block using a nonlinear mapping-based feedback method. The features and merits of the proposed method can be briefly summarized as follows: (i) a linear weight model with a back-fence communication topology is proposed for the cooperative control of multiple trains with nonlinear characteristics and uncertain parameters; (ii) the proposed control method requires no information of the experimental parameters of the operational resistances and external disturbances; (iii) the proposed control method adjusts the feedback gain nonlinearly by virtue of a new continuous differentiable nonlinear mapping function, and a Lyapunov function with non-quadratic form is utilized to rigorously prove the closed-loop stability by the Lyapunov stability theorem.

The rest of the paper is organized as follows. Section 2 presents the formulated problem and some necessary preliminaries. The main results, including the detailed design procedures of the proposed control method and the rigorous stability proof, are given in Section 3. Section 4 gives comparative simulation results to demonstrate the effectiveness and advantages of the proposed control. Section 5 concludes this paper.

2 Problem formulation and preliminaries

Without loss of generality, the dynamics of a moving train i along the railway line can be captured by the following second-order differential equation:

$$\begin{cases} \dot{p}_i(t) = v_i(t), \\ \ddot{p}_i(t) = \dfrac{F_i}{m_i} - f_i(v_i) - d_i(X_i), \end{cases} \tag{1}$$

where $p_i(t)$, $v_i(t)$ and $\ddot{p}_i(t)$ are the actual position, speed, and acceleration values of train i, respectively. m_i is the train mass, and F_i is the force implemented on the train. $f_i(v_i) = a_i + b_i \cdot v_i + c_i \cdot v_i^2$ is the Davis equation that models the operational resistance forces, with a, b, and c being unknown positive constants. $d_i(X_i)$ denotes the combinations of external disturbances and modelling uncertainties with $X_i := [v_i, p_i, t]$, meaning that the unknown term $d_i(X)$ varies with respect to the real-time speed, position of train i and time. Define $u_i = F_i/m_i$, the acceleration and deceleration values of train i, and choose u_i as the control signal to be later designed. To design a feasible controller, it is necessary to assume that $d_i(X_i)$ is bounded by some unknown constant d_i^+, that is, $|d_i(X_i)| \leqslant d_i^+$, and it is necessary to point out that this assumption is quite regular from the viewpoints of both theoretical analysis and practical considerations.

The target is to design a distributed cooperative control for multiple trains such that:

(1) The target is to let the headmost train 1 track a prescribed speed trajectory versus distance accurately, and the following trains $i = 2, \ldots, n$ track the minimum separation distance points (MSDPs) calculated behind the preceding trains in moving block mode; the MSDPs are calculated in consideration of the braking distance L_b and proper redundant safe distance L_s, as shown in Figure 1;

(2) All of the closed-loop signals are guaranteed to be bounded for each train, and string stability is also guaranteed for multiple trains in such a manner that $\sup_i \|p_i - p_{i-1} + (L_b + L_s)\|_\infty < \vartheta$, $\forall i = 2, \ldots, n$ and $\sup_1 \|p_1 - p_r\|_\infty < \vartheta$, with p_r and ϑ being the prescribed distance trajectory versus time and a positive small constant, respectively.

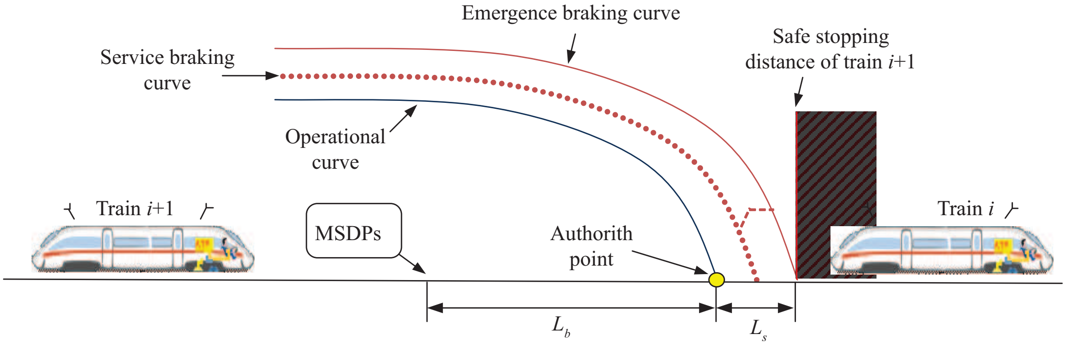

Figure 1 (Color online) The schematic diagram of MSDPs.

To these ends, Assumption 1 and Lemmas 1–3 are introduced.

Assumption 1. The prescribed speed trajectory versus distance is smooth enough such that the prescribed distance trajectory p_r, speed trajectory v_r and acceleration trajectory a_r versus time are all bounded.

Lemma 1 ([12])**.** Consider a continuous function $V(t) \geqslant 0$ defined $\forall t \in \mathbb{R}^+$ and the bounded initial value $V(0)$ and a real-valued function $h(t) \in \mathcal{L}_\infty$. If $\dot{V}(t) \leqslant -h_1 V(t) + h_2 h(t)$ with $h_1 > 0$, h_2 are constants, then it can be concluded that $V(t)$ is bounded.

Lemma 2 ([13])**.** For some given positive constant $\gamma > 0$ and any $\vartheta \in \mathbb{R}$, the inequality $0 \leqslant |\vartheta| - \vartheta \tanh(\frac{\vartheta}{\gamma}) \leqslant \kappa\gamma$ is always true with $\kappa = 0.2785$.

Lemma 3 ([14])**.** A continuous differentiable nonlinear mapping function, presented as

$$\mathfrak{N}(s) = \begin{cases} s, & \text{if } |s| \leqslant \Delta, \\ [\log_a (1 - \ln a \cdot \Delta + \ln a \cdot |s|) + \Delta] \operatorname{sign}(s), & \text{if } |s| > \Delta, \end{cases} \tag{2}$$

where $a > 1$, $\Delta > 0$, holds the following twofold properties: (i) the function $\mathfrak{N}(s)$ is a continuous differentiable strictly monotone increasing function versus its argument s, and its derivative is equal to 1 if $|s| \leqslant \Delta$, and $(1 - \ln a \cdot \Delta + \ln a \cdot |s|)^{-1}$ otherwise; (ii) $\mathfrak{N}_f(s) := \mathfrak{N}_d(s) \cdot s + \mathfrak{N}(s)$ is a monotone increasing function versus s, and $\mathfrak{N}_f(s) \cdot s \geqslant \mathfrak{N}(s) \cdot s$ is true.

3 Main results

To develop a proper distributed cooperative control, the following MSDP error variables can be defined for the trains:

$$\begin{cases} e_1 = p_r - p_1, \\ e_i = (p_{i-1} - p_i) + (L_b + L_s), \end{cases} \tag{3}$$

where L_b and L_s are the braking distance and redundant safe distance as shown in Figure 1, respectively. It is necessary to make an assumption that p_i is the distance of train i without consideration of the train length. To facilitate the design procedure and stability proof, a filtered error $n_i = \dot{e}_i + \alpha_i e_i$ with α_i being some positive constant is defined, which is partially inspired by the robust adaptive control of automatic train operation [15]. The convergences of e_i and n_i are equivalent, as briefly proved as follows.

Proof. The solution of e_i in the first-order differential equation $n_i = \dot{e}_i + \alpha_i e_i$ with 0 initial time can be obtained as follows:

$$e_i(t) = \frac{n_i}{\alpha_i} + \left(e_i(0) - \frac{n_i}{\alpha_i}\right) \exp(-\alpha t), \tag{4}$$

where $e_i(0)$ is the initial value of e_i. It is clear that $(e_i(0) - \frac{n_i}{\alpha_i}) \exp(-\alpha t)$ decays as time goes on, that is, $(e_i(0) - \frac{n_i}{\alpha_i}) \exp(-\alpha t) \approx 0$ after some time moment $t > T$. In this sense, $e_i \approx n_i/\alpha_i$ after some time moment $t > T$. As a result, the convergences of e_i and n_i are thus equivalent. The proof ends.

Remark 1. In this study, the separation distance $L_b + L_s$ among trains is some constant value with the following practical considerations: (i) the braking distance for some specific train is calculated using the maximum constant speed value for safety consideration, and (ii) the hit-hard-wall mode is utilized to separate the distance of neighbouring trains, that is, the moving authority of a train is calculated using the real-time location information and without consideration of the braking distance of the preceding train.

It is known from (3) that the error variables are defined between train i and its preceding one. To realize the cooperation with its following train, the following linear weighting transformation can be utilized to couple the distance between train i and train $i+1$:

$$\begin{cases} N_i = \beta_i n_i - n_{i+1}, & i = 1, \ldots, n-1, \\ N_n = \beta_n n_n, \end{cases} \tag{5}$$

with β_i, $i = 1, \ldots, n$ being positive constants. Similarly, the convergences of n_i and N_i are equivalent, as briefly proved as follows.

Proof. Define $\mathcal{N}_n = [n_1, n_2, \ldots, n_n]^{\mathrm{T}}$ and $\mathcal{N}_N = [N_1, N_2, \ldots, N_n]^{\mathrm{T}}$; the relationship of $\mathcal{N}_n$ and $\mathcal{N}_N$ is thus $\mathcal{N}_N = \mathcal{B}\mathcal{N}_n$ with

$$\mathcal{B} = \begin{pmatrix} \beta_1 & -1 & & & \\ & \beta_2 & -1 & & \\ & & \ddots & \ddots & \\ & & & \ddots & -1 \\ & & & & \beta_n \end{pmatrix}.$$

Because β_i, $i = 1, \ldots, n$ are positive constants, the matrix $\mathcal{B}$ is thus non-singular. This guarantees the above statements. The proof ends.

Differentiating $N_i = \beta_i n_i - n_{i+1}$, $\forall i = 1, \ldots, n-1$ and $N_n = \beta_n n_n$ yields

$$\dot{N}_i = -(\beta_i + 1)\left[u_i - \left(a_i + b_i v_i + c_i v_i^2\right)\right] + \beta_i \ddot{p}_{i-1} + \ddot{p}_{i+1} + \alpha_i \beta_i \dot{e}_i - \alpha_{i+1} \dot{e}_{i+1}, \tag{6}$$

$$\dot{N}_n = -\beta_n\left[u_n - \left(a_n + b_n v_n + c_n v_n^2\right)\right] + \beta_n \ddot{p}_{n-1} + \alpha_n \beta_n \dot{e}_n. \tag{7}$$

The distributed cooperative control is proposed as follows:

$$\begin{cases} u_i = \dfrac{k_i[\mathfrak{N}_f(N_i) + \frac{N_i}{M_i}]}{\beta_i + 1} + \dfrac{\mathcal{F}_i}{\beta_i + 1} + \hat{a}_i + \hat{b}_i \dot{p}_i + \hat{c}_i \dot{p}_i^2 + \tanh\left[\lambda_i \mathfrak{N}_f(N_i)\right] \hat{d}_i^+, \quad i = 1, 2, \ldots, n-1, \\ u_n = \dfrac{k_n[\mathfrak{N}_f(N_n) + \frac{N_n}{M_n}]}{\beta_n} + \dfrac{\mathcal{F}_n}{\beta_n} + \hat{a}_n + \hat{b}_n \dot{p}_n + \hat{c}_n \dot{p}_n^2 + \tanh\left[\lambda_n \mathfrak{N}_f(N_n)\right] \hat{d}_n^+, \end{cases} \tag{8}$$

where k_i, $i = 1, 2, \ldots, n$ are positive design parameters, M_i are positive constants satisfying $M_i \gg k_i$, $\hat{a}_i$, $\hat{b}_i$, $\hat{c}_i$ and $\hat{d}_i^+$ are the estimated values of the unknown parameters a_i, b_i, c_i and d_i^+ of train i, respectively, $\mathcal{F}_i = \beta_i \ddot{p}_{i-1} + \ddot{p}_{i+1} + \alpha_i \beta_i \dot{e}_i - \alpha_{i+1} \dot{e}_{i+1}$ with $p_0 = p_r$, and $\mathcal{F}_n = \beta_n \ddot{p}_{n-1} + \alpha_n \beta_n \dot{e}_n$. The adaptation laws are designed as follows:

$$\begin{cases} \dot{\hat{a}}_i = \varepsilon_{ai}\left[(\beta_i + 1)\mathfrak{N}_f(N_i) - \sigma_{i1}\hat{a}_i\right], \\ \dot{\hat{b}}_i = \varepsilon_{bi}\left[(\beta_i + 1)\mathfrak{N}_f(N_i)\dot{p}_i - \sigma_{i2}\hat{b}_i\right], \\ \dot{\hat{c}}_i = \varepsilon_{ci}\left[(\beta_i + 1)\mathfrak{N}_f(N_i)\dot{p}_i^2 - \sigma_{i3}\hat{c}_i\right], \\ \dot{\hat{d}}_i^+ = \varepsilon_{di}\left[(\beta_i + 1)\tanh\left(\lambda_i \mathfrak{N}_f(N_i)\right) - \sigma_{i4}\hat{d}_i^+\right], \\ \dot{\hat{a}}_n = \varepsilon_{an}\left[\beta_n \mathfrak{N}_f(N_n) - \sigma_{n1}\hat{a}_n\right], \\ \dot{\hat{b}}_n = \varepsilon_{bn}\left[\beta_n \mathfrak{N}_f(N_n)\dot{p}_n - \sigma_{n2}\hat{b}_n\right], \\ \dot{\hat{c}}_n = \varepsilon_{cn}\left[\beta_n \mathfrak{N}_f(N_n)\dot{p}_n^2 - \sigma_{n3}\hat{c}_n\right], \\ \dot{\hat{d}}_n^+ = \varepsilon_{dn}\left[\beta_n \tanh\left(\lambda_n \mathfrak{N}_f(N_n)\right) - \sigma_{n4}\hat{d}_n^+\right], \end{cases} \tag{9}$$

where ε_{ai}, ε_{bi}, ε_{ci} and ε_{di}, $i = 1, 2, \ldots, n$ are the adaptation rate coefficients and σ_{i1}, σ_{i2}, σ_{i3} and σ_{i4}, $i = 1, 2, \ldots, n$ are small positive constants.

Remark 2. It is noted that each controller in (8) contains six parts. The terms $\frac{k_i[\mathfrak{N}_f(N_i)+\frac{N_i}{M_i}]}{\beta_i+1}$ for $i = 1, \ldots, n-1$ and $\frac{k_n[\mathfrak{N}_f(N_n)+\frac{N_n}{M_n}]}{\beta_n}$ are the negative feedback components, which take charge of driving the tracking errors to converge to values as small as possible. The terms $\frac{\mathcal{F}_i}{\beta_i+1}$ for $i = 1, \ldots, n-1$ and $\frac{\mathcal{F}_n}{\beta_n}$ are available information for controller design. The terms $\hat{a}_i + \hat{b}_i\dot{p}_i + \hat{c}_i\dot{p}_i^2$ for $i = 1, \ldots, n$ are the on-line estimations of resistances $a_i + b_i\dot{p}_i + c_i\dot{p}_i^2$. Finally, the terms $\tanh[\lambda_i\mathfrak{N}_f(N_i)]\hat{d}_i^+$ for $i = 1, \ldots, n$ are components to guarantee the robustness of the proposed control law against the disturbances. It is necessary to point out that an extra feedback term $\frac{N_i}{M_i}$ is added in the negative feedback components, and the design principle of the introduction of this term is explained in Remark 3.

The above design procedures can be summarized as Theorem 1.

Theorem 1. Consider a group of high-speed trains operating in moving block mode with the dynamics of train i modelled as (1). If the distributed cooperative control (8) and the corresponding adaptation laws (9) are implemented, the twofold targets stated in Section 2 can be achieved.

Proof. Incorporating the open-loop dynamics (6), (7) and the proposed control law (8), the closed-loop dynamics can be obtained as follows:

$$\begin{cases} \dot{N}_i = -k_i\left[\mathfrak{N}_f(N_i) + \dfrac{N_i}{M_i}\right] - (\beta_i+1)\left[\tilde{a}_i + \tilde{b}_i\dot{p}_i + \tilde{c}_i\dot{p}_i^2 + \tanh\left(\lambda_i\mathfrak{N}_f(N_i)\right)\hat{d}_i^+ - d_i(X)\right], \\ \dot{N}_n = -k_n\left[\mathfrak{N}_f(N_n) + \dfrac{N_n}{M_n}\right] - \beta_n\left[\tilde{a}_n + \tilde{b}_n\dot{p}_n + \tilde{c}_n\dot{p}_n^2 + \tanh\left(\lambda_n\mathfrak{N}_f(N_n)\right)\hat{d}_n^+ - d_n(X)\right], \end{cases} \tag{10}$$

where $\tilde{a}_i = \hat{a}_i - a_i$, $\tilde{b}_i = \hat{b}_i - b_i$, and $\tilde{c}_i = \hat{c}_i - c_i$ are the estimated error variables. Similarly, $\tilde{d}_i^+ := \hat{d}_i^+ - d_i^+$ is also the estimated error variable in the following analysis.

Choose the following global Lyapunov function for multiple trains:

$$V = \sum_{i=1}^{n}\left[\mathfrak{N}(N_i)N_i + \frac{\tilde{a}_i^2}{2\varepsilon_{ai}} + \frac{\tilde{b}_i^2}{2\varepsilon_{bi}} + \frac{\tilde{c}_i^2}{2\varepsilon_{ci}} + \frac{\tilde{d}_i^{+2}}{2\varepsilon_{di}}\right]. \tag{11}$$

The derivative of (11) along (10) can be calculated as

$$\begin{aligned} \dot{V} =& \sum_{i=1}^{n}\left[\mathfrak{N}_f(N_i)\dot{N}_i + \frac{\tilde{a}_i\dot{\hat{a}}_i}{\varepsilon_{ai}} + \frac{\tilde{b}_i\dot{\hat{a}}_i}{\varepsilon_{bi}} + \frac{\tilde{c}_i\dot{\hat{a}}_i}{\varepsilon_{ci}} + \frac{\tilde{d}_i^+\dot{\hat{d}}_i^+}{\varepsilon_{di}}\right] \\ =& \sum_{i=1}^{n}\left[-k_i\mathfrak{N}_f^2(N_i) - \frac{k_i\mathfrak{N}_f(N_i)N_i}{M_i} + \frac{\tilde{a}_i\dot{\hat{a}}_i}{\varepsilon_{ai}} + \frac{\tilde{b}_i\dot{\hat{a}}_i}{\varepsilon_{bi}} + \frac{\tilde{c}_i\dot{\hat{a}}_i}{\varepsilon_{ci}} + \frac{\tilde{d}_i^+\dot{\hat{d}}_i^+}{\varepsilon_{di}}\right] \\ &- \sum_{i=1}^{n-1}(\beta_i+1)\,\mathfrak{N}_f(N_i)\left[\tilde{a}_i + \tilde{b}_i\dot{p}_i + \tilde{c}_i\dot{p}_i^2 + \tanh\left(\lambda_i\mathfrak{N}_f(N_i)\right)\hat{d}_i^+ - d_i(X)\right] \\ &- \beta_n\mathfrak{N}_f(N_n)\left[\tilde{a}_n + \tilde{b}_n\dot{p}_n + \tilde{c}_n\dot{p}_n^2 + \tanh\left(\lambda_n\mathfrak{N}_f(N_n)\right)\hat{d}_n^+ - d_n(X)\right]. \end{aligned} \tag{12}$$

Incorporating the adaptation laws (9) into (12) yields

$$\begin{aligned} \dot{V} =& \sum_{i=1}^{n}\left[-k_i\mathfrak{N}_f^2(N_i) - \frac{k_i\mathfrak{N}_f(N_i)N_i}{M_i} - \sigma_{i1}\tilde{a}_i\hat{a}_i - \sigma_{i2}\tilde{b}_i\hat{b}_i - \sigma_{i3}\tilde{c}_i\hat{c}_i - \sigma_{i4}\tilde{d}_i^+\hat{d}_i^+\right] \\ &- \sum_{i=1}^{n-1}(\beta_i+1)\,\mathfrak{N}_f(N_i)\left[\tanh\left(\lambda_i\mathfrak{N}_f(N_i)\right)\hat{d}_i^+ - d_i(X) - \tanh\left(\lambda_i\mathfrak{N}_f(N_i)\right)\tilde{d}_i^+\right] \\ &- \beta_n\mathfrak{N}_f(N_n)\left[\tanh\left(\lambda_n\mathfrak{N}_f(N_n)\right)\hat{d}_n^+ - d_n(X) - \tanh\left(\lambda_n\mathfrak{N}_f(N_n)\right)\tilde{d}_n^+\right]. \end{aligned} \tag{13}$$

It is worth noticing that the inequalities $-\sigma_{i1}\tilde{a}_i\hat{a}_i = -\sigma_{i1}\tilde{a}_i(\tilde{a}_i + a_i) \leqslant -\frac{\sigma_{i1}\tilde{a}_i^2}{2} + \frac{\sigma_{i1}a_i^2}{2}$, $-\sigma_{i2}\tilde{b}_i\hat{b}_i \leqslant -\frac{\sigma_{i2}\tilde{b}_i^2}{2} + \frac{\sigma_{i2}b_i^2}{2}$, $-\sigma_{i3}\tilde{c}_i\hat{c}_i \leqslant -\frac{\sigma_{i3}\tilde{c}_i^2}{2} + \frac{\sigma_{i3}c_i^2}{2}$, and $-\sigma_{i4}\tilde{d}_i^+\hat{d}_i^+ \leqslant -\frac{\sigma_{i4}\tilde{d}_i^{+2}}{2} + \frac{\sigma_{i4}d_i^{+2}}{2}$ are always true in view of Young's inequality [16]. Invoking Lemma 2, one has

$$\begin{aligned}
&-\sum_{i=1}^{n-1}(\beta_i+1)\mathfrak{N}_f(N_i)\left[\tanh\left(\lambda_i\mathfrak{N}_f(N_i)\right)\hat{d}_i^+ - d_i(X) - \tanh\left(\lambda_i\mathfrak{N}_f(N_i)\right)\tilde{d}_i^+\right] \\
&\quad= \sum_{i=1}^{n-1}(\beta_i+1)\mathfrak{N}_f(N_i)\left[d_i(X) - \tanh\left(\lambda_i\mathfrak{N}_f(N_i)\right)d_i^+\right] \\
&\quad\leqslant \sum_{i=1}^{n-1}(\beta_i+1)\left[d_i^+\left|\mathfrak{N}_f(N_i)\right| - \mathfrak{N}_f(N_i)\tanh\left(\lambda_i\mathfrak{N}_f(N_i)\right)d_i^+\right] \\
&\quad\leqslant \sum_{i=1}^{n-1}\frac{0.2785\,(\beta_i+1)\,d_i^+}{\lambda_i},
\end{aligned} \tag{14}$$

$$\begin{aligned}
&-\beta_n\mathfrak{N}_f(N_n)\left[\tanh\left(\lambda_n\mathfrak{N}_f(N_n)\right)\hat{d}_n^+ - d_n(X) - \tanh\left(\lambda_n\mathfrak{N}_f(N_n)\right)\tilde{d}_n^+\right] \\
&\quad= \beta_n\mathfrak{N}_f(N_n)\left[d_n(X) - \tanh\left(\lambda_n\mathfrak{N}_f(N_n)\right)d_n^+\right] \leqslant \frac{0.2785\beta_n d_n^+}{\lambda_n}.
\end{aligned} \tag{15}$$

Then, Eq. (13) becomes

$$\begin{aligned}
\dot{V} \leqslant& \sum_{i=1}^{n}\left[-k_i\mathfrak{N}_f^2(N_i) - \frac{k_i\mathfrak{N}_f(N_i)N_i}{M_i} - \frac{\sigma_{i1}\tilde{a}_i^2}{2} - \frac{\sigma_{i2}\tilde{b}_i^2}{2} - \frac{\sigma_{i3}\tilde{c}_i^2}{2} - \frac{\sigma_{i4}\tilde{d}_i^{+2}}{2}\right] \\
&+ \sum_{i=1}^{n}\left(\frac{\sigma_{i1}a_i^2}{2} + \frac{\sigma_{i2}b_i^2}{2} + \frac{\sigma_{i3}c_i^2}{2} + \frac{\sigma_{i4}d_i^{+2}}{2}\right) + \sum_{i=1}^{n-1}\frac{0.2785\,(\beta_i+1)\,d_i^+}{\lambda_i} + \frac{0.2785\beta_n d_n^+}{\lambda_n}.
\end{aligned} \tag{16}$$

From Lemma 3, it is known that $\frac{k_i\mathfrak{N}_f(N_i)N_i}{M_i} \geqslant \frac{k_i\mathfrak{N}(N_i)N_i}{M_i}$. $k_i\mathfrak{N}_f^2(N_i) \geqslant 0$ is also true. Eq. (16) becomes

$$\begin{aligned}
\dot{V} \leqslant& \sum_{i=1}^{n}\left[-\frac{k_i\mathfrak{N}(N_i)N_i}{M_i} - \frac{\sigma_{i1}\tilde{a}_i^2}{2} - \frac{\sigma_{i2}\tilde{b}_i^2}{2} - \frac{\sigma_{i3}\tilde{c}_i^2}{2} - \frac{\sigma_{i4}\tilde{d}_i^{+2}}{2}\right] + \mathfrak{D} \\
=& -\sum_{i=1}^{n}\left[\frac{k_i}{M_i}\mathfrak{N}(N_i)N_i + \sigma_{i1}\varepsilon_{ai}\frac{\tilde{a}_i^2}{2\varepsilon_{ai}} + \sigma_{i2}\varepsilon_{bi}\frac{\tilde{b}_i^2}{2\varepsilon_{bi}} + \sigma_{i3}\varepsilon_{ci}\frac{\tilde{c}_i^2}{2\varepsilon_{ci}} + \sigma_{i4}\varepsilon_{di}\frac{\tilde{d}_i^{+2}}{2\varepsilon_{di}}\right] + \mathfrak{D} \\
\leqslant& -\min_{i=1,\ldots,n}\left\{\frac{k_i}{M_i}, \sigma_{i1}\varepsilon_{ai}, \sigma_{i2}\varepsilon_{bi}, \sigma_{i3}\varepsilon_{ci}, \sigma_{i4}\varepsilon_{di}\right\}V + \mathfrak{D},
\end{aligned} \tag{17}$$

where $\mathfrak{D} := \sum_{i=1}^{n}\left(\frac{\sigma_{i1}a_i^2}{2} + \frac{\sigma_{i2}b_i^2}{2} + \frac{\sigma_{i3}c_i^2}{2} + \frac{\sigma_{i4}d_i^{+2}}{2}\right) + \sum_{i=1}^{n-1}\frac{0.2785(\beta_i+1)d_i^+}{\lambda_i} + \frac{0.2785\beta_n d_n^+}{\lambda_n}$. Invoking Lemma 1, one knows that $V(t)$ is bounded, and it is easy to find that the components of $V(t)$, including N_i, $\tilde{a}_i$, $\tilde{b}_i$, $\tilde{c}_i$, and $\tilde{d}_i^+$, are all bounded. From (17), it can be obtained that

$$V(t) \leqslant V(0)\exp(-\mathfrak{M}t) + \frac{\mathfrak{D}}{\min_{i=1,\ldots,n}\left\{\frac{k_i}{M_i}, \sigma_{i1}\varepsilon_{ai}, \sigma_{i2}\varepsilon_{bi}, \sigma_{i3}\varepsilon_{ci}, \sigma_{i4}\varepsilon_{di}\right\}}.$$

After some time moment T, one has $\lim_{t>T\to\infty}V(0)\exp(-\mathfrak{M}t) \approx 0$, where the variable $\mathfrak{M} := \min_{i=1,\ldots,n}\left\{\frac{k_i}{M_i}, \sigma_{i1}\varepsilon_{ai}, \sigma_{i2}\varepsilon_{bi}, \sigma_{i3}\varepsilon_{ci}, \sigma_{i4}\varepsilon_{di}\right\}$, that is, $V(t) \leqslant \frac{\mathfrak{D}}{\mathfrak{M}}$ ultimately. According to the property of the nonlinear mapping function $\mathfrak{N}(\cdot)$ given in Lemma 3, one knows that $\mathfrak{N}(N_i)N_i = N_i^2$ if $\left|N_i\right| \leqslant \Delta$, or else, $\mathfrak{N}(N_i)N_i = [\log_a(1 - \ln a\Delta t + \ln a|N_i|) + \Delta]|N_i| \geqslant \Delta|N_i|$. From the definition of V, one knows that $\mathfrak{N}(N_i)N_i \leqslant V$ is always true, and as a result,

$$|N_i| \leqslant \sqrt{\frac{\mathfrak{D}}{\min_{i=1,\ldots,n}\left\{\frac{k_i}{M_i}, \sigma_{i1}\varepsilon_{ai}, \sigma_{i2}\varepsilon_{bi}, \sigma_{i3}\varepsilon_{ci}, \sigma_{i4}\varepsilon_{di}\right\}}} := \mathfrak{B}_1$$

is thus true if $|N_i| \leqslant \Delta$, or else, if $|N_i| > \Delta$ holds,

$$|N_i| \leqslant \frac{\mathfrak{D}}{\min_{i=1,\ldots,n}\left\{\frac{k_i}{M_i}, \sigma_{i1}\varepsilon_{ai}, \sigma_{i2}\varepsilon_{bi}, \sigma_{i3}\varepsilon_{ci}, \sigma_{i4}\varepsilon_{di}\right\} \cdot \Delta} := \mathfrak{B}_2.$$

In conclusion, the ultimate bound of the error variable N_i is obtained as $|N_i| \leqslant \max\{\mathfrak{B}_2, \min\{\mathfrak{B}_1, \Delta\}\}$. This bound value can be adjusted to be sufficiently small by choosing proper control parameters, that is, the string stability of multiple trains is achieved. The proof is completed.

Remark 3. If the control in (8) is modified as follows:

$$\begin{cases} u_i = \dfrac{k_i \mathfrak{N}_f(N_i)}{\beta_i + 1} + \dfrac{\mathcal{F}_i}{\beta_i + 1} + \hat{a}_i + \hat{b}_i \dot{p}_i + \hat{c}_i \dot{p}_i^2 + \tanh\left[\lambda_i \mathfrak{N}_f(N_i)\right] \hat{d}_i^+, \quad i = 1, 2, \ldots, n-1, \\ u_n = \dfrac{k_n \mathfrak{N}_f(N_n)}{\beta_n} + \dfrac{\mathcal{F}_n}{\beta_n} + \hat{a}_n + \hat{b}_n \dot{p}_n + \hat{c}_n \dot{p}_n^2 + \tanh\left[\lambda_n \mathfrak{N}_f(N_n)\right] \hat{d}_n^+, \end{cases} \tag{18}$$

Eq. (17) will become as follows after some similar derivations:

$$\dot{V} \leqslant \sum_{i=1}^{n} \left[-k_i \mathfrak{N}_f^2(N_i) - \frac{\sigma_{i1}\tilde{a}_i^2}{2} - \frac{\sigma_{i2}\tilde{b}_i^2}{2} - \frac{\sigma_{i3}\tilde{c}_i^2}{2} - \frac{\sigma_{i4}\tilde{d}_i^{+2}}{2} \right] + \mathfrak{D}. \tag{19}$$

By choosing the proper design parameters k_i, σ_{i1}, σ_{i2}, σ_{i3} and σ_{i4} with consideration of the initial values of $N_i(0)$, one can also find the proper parameter selection principle to guarantee the semi-definitive property of the Lyapunov function V. By introducing $k_i \frac{N_i}{M_i}$ as the numerator in the first term with $M_i \gg k_i$ in the proposed distributed cooperative control (8), one obtains the nonlinearity (17) that facilitates the acquisitions of the ultimate bound of the error variables N_i, $\tilde{a}_i$, $\tilde{b}_i$, $\tilde{c}_i$, and $\tilde{d}_i^+$.

4 Comparative simulation results

This section presents comparative simulation results to demonstrate the effectiveness and advantages of the proposed control law in Theorem 1. To provide an unprejudiced comparison, the following linear-gain-based control (labelled as LGC for short) can be developed using the similar forgoing design procedures, trivially:

$$\begin{cases} u_i = \dfrac{k_i N_i}{\beta_i + 1} + \dfrac{\mathcal{F}_i}{\beta_i + 1} + \hat{a}_i + \hat{b}_i \dot{p}_i + \hat{c}_i \dot{p}_i^2 + \tanh\left(\lambda_i N_i\right) \hat{d}_i^+, \quad i = 1, 2, \ldots, n-1, \\ u_n = \dfrac{k_n N_n}{\beta_n} + \dfrac{\mathcal{F}_n}{\beta_n} + \hat{a}_n + \hat{b}_n \dot{p}_n + \hat{c}_n \dot{p}_n^2 + \tanh\left(\lambda_n N_n\right) \hat{d}_n^+, \\ \dot{\hat{a}}_i = \varepsilon_{ai}\left[(\beta_i + 1)N_i - \sigma_{i1}\hat{a}_i\right], \\ \dot{\hat{b}}_i = \varepsilon_{bi}\left[(\beta_i + 1)N_i \dot{p}_i - \sigma_{i2}\hat{b}_i\right], \\ \dot{\hat{c}}_i = \varepsilon_{ci}\left[(\beta_i + 1)N_i \dot{p}_i^2 - \sigma_{i3}\hat{c}_i\right], \\ \dot{\hat{d}}_i^+ = \varepsilon_{di}\left[(\beta_i + 1)\tanh\left(\lambda_i N_i\right) - \sigma_{i4}\hat{d}_i^+\right], \\ \dot{\hat{a}}_n = \varepsilon_{an}\left[\beta_n N_n - \sigma_{n1}\hat{a}_n\right], \\ \dot{\hat{b}}_n = \varepsilon_{bn}\left[\beta_n N_n \dot{p}_n - \sigma_{n2}\hat{b}_n\right], \\ \dot{\hat{c}}_n = \varepsilon_{cn}\left[\beta_n N_n \dot{p}_n^2 - \sigma_{n3}\hat{c}_n\right], \\ \dot{\hat{d}}_n^+ = \varepsilon_{dn}\left[\beta_n \tanh\left(\lambda_n N_n\right) - \sigma_{n4}\hat{d}_n^+\right], \end{cases} \tag{20}$$

where the variables and parameters are defined the same as in the main results in Section 3. The prescribed trajectory for the headmost train is given in Figure 2(a).

The initial speed values for all trains are set as 0, and the relative initial distance values are set as $(i-1) \times 6500$ m for train i, $i = 1, 2, 3, 4, 5$. The coefficients of the Davis equation are set as $a_1 = 0.85$,

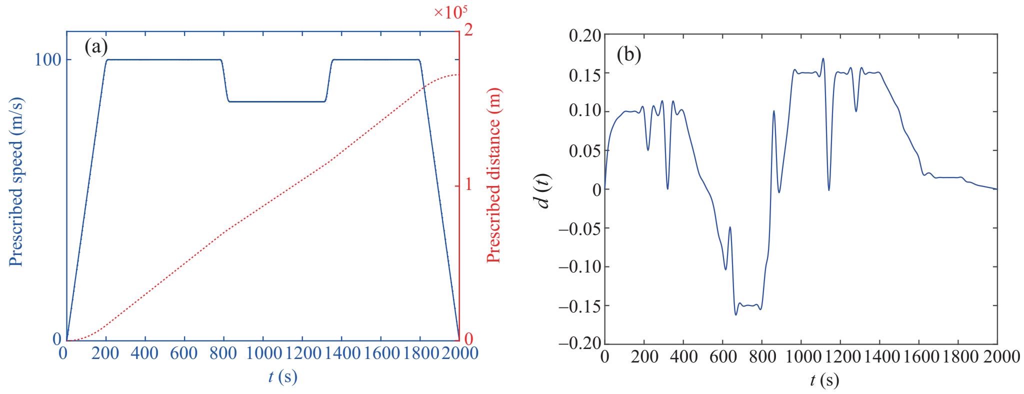

Figure 2 (Color online) Prescribed trajectories and disturbance used in the simulations. (a) Prescribed speed v_r and distance p_r; (b) disturbance $d(t)$.

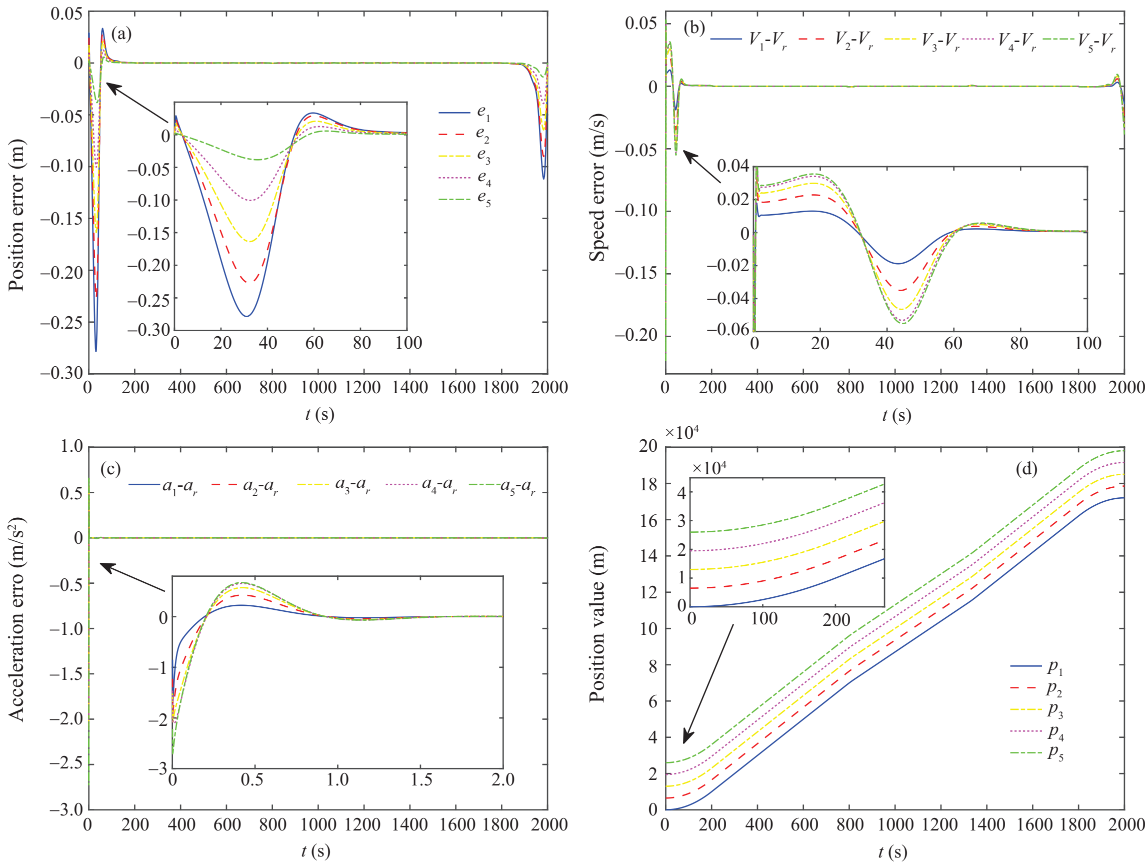

Figure 3 (Color online) Simulation results using Theorem 1. (a) Position errors; (b) speed errors; (c) acceleration errors; (d) position values versus time.

$b_1 = 0.004$, $c_1 = 0.00016$, $a_2 = 0.8$, $b_2 = 0.002$, $c_2 = 0.0002$, $a_3 = 0.9$, $b_3 = 0.003$, $c_3 = 0.00018$, $a_4 = 0.7$, $b_4 = 0.0025$, $c_4 = 0.0004$, $a_5 = 0.75$, $b_5 = 0.0035$, and $c_5 = 0.00025$. An unknown external disturbance term $d(t)$ is given in Figure 2(b) for simulation requirements. To better simulate the unknown terms, $d_i(X_i)$ are set as $d_i(X_i) = a_i \sin(0.1t) + b_i \sin(0.2t)v_i + c_i \sin(0.01t)v_i^2 + \mathcal{P}_i d(t)$ with $\mathcal{P}_1 = 1$, $\mathcal{P}_2 = 1.1$, $\mathcal{P}_3 = 1.2$, $\mathcal{P}_4 = 0.9$, and $\mathcal{P}_5 = 1.15$, respectively. The control parameters are optionally chosen as follows: $k_i = 35$, $m_i = k_i + 5$, $\beta_i = 1$, $\lambda_i = 5$, $\Delta = 1$, $a = 10$, $\varepsilon_{ai} = \varepsilon_{bi} = \varepsilon_{ci} = \varepsilon_{di} = 10^{-5}$, and $\sigma_{i1} = \sigma_{i2} = \sigma_{i3} = 0.01$. The initial estimated values of $\hat{a}_i$, $\hat{b}_i$, $\hat{c}_i$, and $\hat{d}_i^+$ are set as 0, 0.8, 0.001, and 0.0001, respectively.

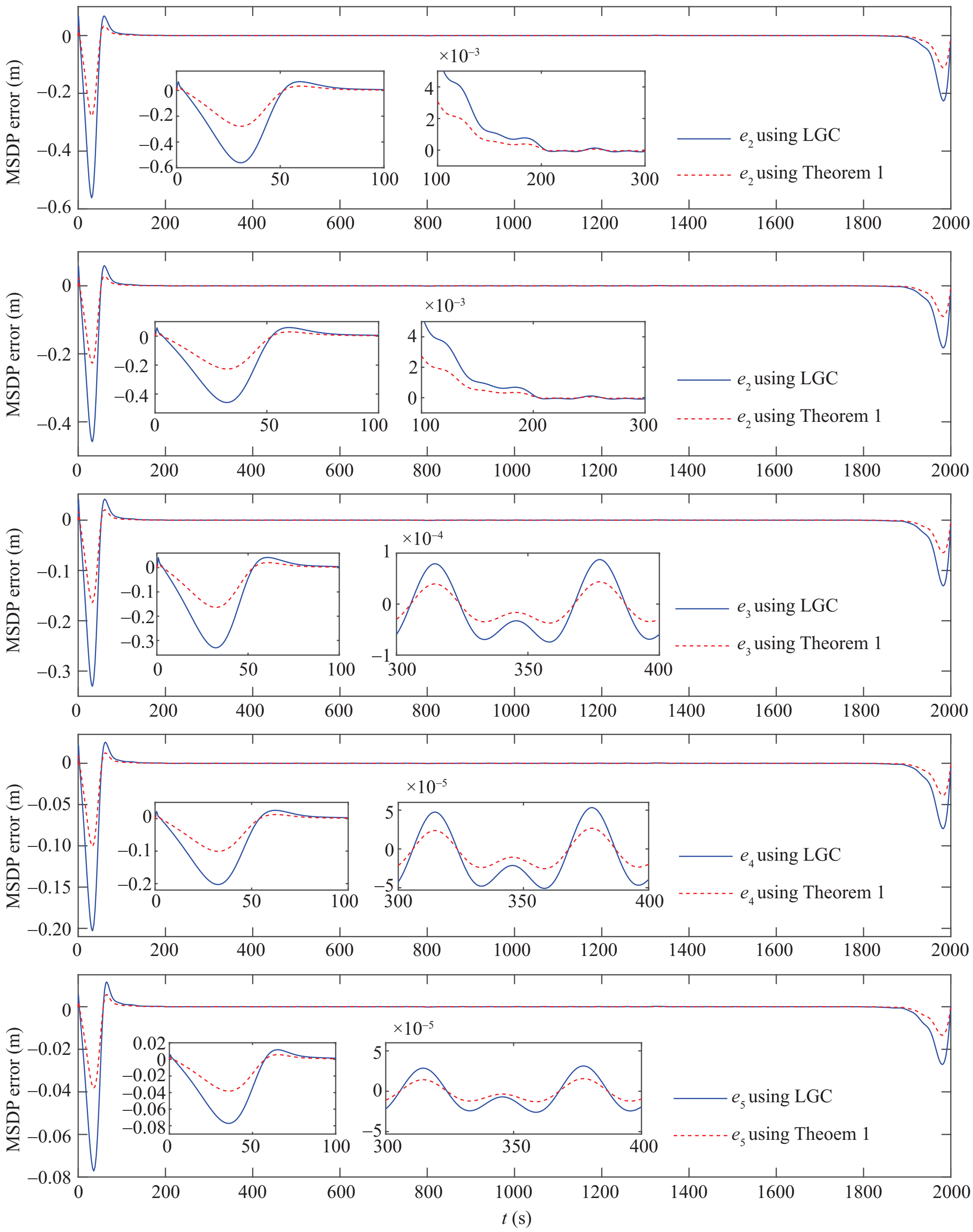

Figure 4 (Color online) Comparative results.

Remark 4. The settings of the initial values of $\hat{a}_i$, $\hat{b}_i$, $\hat{c}_i$ and $\hat{d}_i^+$ are used in the control signals' calculations directly. In this sense, one cannot set these values at will because large settings of these variables may result in an input saturation problem. Actually, the initial values of $\hat{a}_i$, $\hat{b}_i$, $\hat{c}_i$ and $\hat{d}_i^+$ can be set as some rough estimations or zeros for simplicity in practical applications.

The simulation results obtained from Theorem 1 are shown in Figure 3. It is observed that the relative distance errors among 5 trains are guaranteed to be small with a descending error amplitude from train 1 to train 5, which is the achievement of the string stability stated above. The speed and acceleration tracking performance are shown in Figures 3(b) and (c), and it is clear that the tracking performance is satisfactory; in particular, the performance is satisfactory under the situation that the reference speed trajectory v_r and acceleration trajectory a_r are unknown to trains 2 through 5 in moving block mode. Figure 3(d) shows the plots of the distances of the 5 trains versus operating time. It can be concluded that the proposed control method in Theorem 1 guarantees satisfactory performance. The comparative results between the LGC and Theorem 1 are shown in Figure 4. It is observed that the Theorem 1 guarantees better tracking performance. The effectiveness and advantages of the proposed distributed cooperative control for multiple high-speed trains are thus well demonstrated.

5 Conclusion

In this paper, a distributed cooperative control method for multiple high-speed trains is developed under moving block mode. The target is to maintain the minimum distance tracking between a train and its preceding one, which is calculated from real-time moving authority information in practice. Different from the practical operation mode, this paper has proposed a virtual marshalling modelling method to guarantee the string stability of multiple trains, which is achieved by coupling the information of a train with its neighbouring trains, that is, the preceding and following trains. The proposed distributed cooperative control method requires no information of the system parameters and adjusts the control parameters adaptively online. The closed-loop signals are guaranteed to be bounded by a rigorous mathematical proof using Lyapunov stability. Comparative simulation results are presented to demonstrate the effectiveness and advantages of the proposed method.

In future work, the hit-soft-wall mode will be considered to separate the distance of neighbouring trains with the braking distance calculated using the real-time speed of a train, which complicates the open-loop dynamics by considering the braking distance of a train and its preceding one. Besides, we will validate the effectiveness of the proposed method in practical testing systems.

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Profile of Bin NING

Dr. Ning is a professor and President of Beijing Jiaotong University (BJTU). He received the B.S., M.S., and Ph.D. degrees in transportation engineering from Northern Jiaotong University (now Beijing Jiaotong University), China, in 1982, 1987, and 2005, respectively. He was a visiting research fellow in Brunel University, from 1991 to 1992, and a visiting research fellow in University of California, Berkeley, from 2002 to 2003.

For more than 30 years, Dr. Ning has been conducting research and teaching in train operation control systems for high speed railways, urban rail transit systems and main line railways. As an expert in control systems engineering (rail transit train control), his main research interests include train operation control systems, intelligent transport systems, fault-tolerant design and fault-diagnosis, and system safety and reliability for digital systems. He has published over 100 refereed technical papers, and been authorized more than 10 patents. He and his team have successfully developed communication-based train control (CBTC) system for urban rail system, universal cab signaling, and automatic train protection (ATP) systems for Chinese railways, which have been successfully applied in industry.

Dr. Ning is a member of the Chinese Academy of Engineering, the Royal Swedish Academy of Engineering Science, and the International EURASIAN Academy of Sciences; and a Fellow of the Institute of Electrical and Electronics Engineers (IEEE), the Association of International Railway Signaling Engineers (IRSE), and the Institute of Engineering and Technology (IET). He is also a Fellow of China Railway Society, the Executive Director of China Automation Association, and the Deputy Director of China Traffic System Engineering Society. He also served as an Associate Editor of *IEEE Transactions on Intelligent Transportation Systems* (2010–2015) and *Acta Automatica Sinica* (2011–2012).

Identification of key problems in train control systems and designing the CBTC system in urban rail transit

Dr. Ning and his team proposed the theory and design methods for the CBTC system. He applied the tolerance techniques, artificia1 intelligence and modern mobile communication technologies to solve several key technological problems in urban rail traffic control systems, for safety protection, optimal automatic operation, 90-second minimum safety headway, stable operations with passenger comfort, and accurate train stopping (Ning et al. 2015, Ning et al. 2011a, Ning et al. 2011b, Zhu et al. 2017, Xun et al. 2013). He invented a combined vehicle-wayside technology with wireless free wave, leaking waveguide, and leaking cable formats, which achieves multimedia no-gap switching transmission. His invention has been successfully applied to urban rail traffic systems in Beijing and some other cities in China. This made China the fourth country in the world that has such a core technology with successful applications. It has a1so greatly contributed to the technologica1 development in the world as well as intelligent control of rail traffic systems.

Identification of internal mechanism among various track circuits and designing cab signaling systems and devices in railways

According to the requirements of high reliability and safety for Chinese high-speed trains, heavy-loading railways, and mainline railway trains, Dr. Ning designed a computational algorithm which can process the digital signals of 11 different types of track circuits, and thereby solved a series of technical problems such as corresponding speed of universal cab signaling as main signals, anti-unbalancing current in rails caused by track current, as well as recording and monitoring (Ning et al. 2006, Ning 2000, Ning et al. 2005, Dong et al. 2010). He a1so designed train-borne signaling devices that can handle all signaling formats in Chinese Railway Network and satisfy simultaneously high-speed trains, heavy-loading trains, and the ordinary trains. The devices occupied more than 85% China's railway cab signaling market, and made indispensable contributions to safety and efficiency of railway operation in China.

Foundations for the fully automatic operation train control system for urban rail transit

To fulfill the requirements of fully automatic operation (FAO) train control system, Dr. Ning and his team analyzed the typical characteristics and major demands, designed key technologies, studied synthesis methods, and formulated technical specifications, which helps the establishment of an overall solution for the FAO design and synthesis (Wang et al. 2014, Gao et al. 2014, Yang et al. 2014). These theoretical results support the research, development, and applications of the first operational urban rail line, that is, Beijing Yanfang Line, in China.

Dr. Ning has established two national research platforms, the State Key Laboratory of Traffic Control and Safety and the National Engineering Research Center of Train Operation Control, which have been played a very important role in the areas of train operation control in China. His academic recognitions include the National Science and Technology Progress Award (4 times), the National Teaching Award (2 times), the Grand Prize of Zhan-Tianyou Railway Science and Technology (2014), and the Ho Leung Ho Lee Foundation Award (2016).

Selected publications

- Ning B, Xun J, Gao S, et al. An integrated control model for headway regulation and energy saving in urban rail transit. IEEE Trans Intell Transport Syst, 2015, 16: 1469–1478
- Ning B, Tang T, Dong H R, et al. An introduction to parallel control and management for high-speed railway systems. IEEE Trans Intell Transport Syst, 2011, 12: 1473–1483 (2011a)
- Ning B, Dong H R, Wen D, et al. ACP-based control and management of urban rail transportation systems. IEEE Intell Syst, 2011, 26: 84–88 (2011b)
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