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Analysis of crack propagation in concrete structures with structural information entropy

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The propagation of cracks in concrete structures causes energy dissipation and release, and also causes energy redistribution in the structures. Entropy can characterize the energy redistribution. To investigate the relation between the propagation of cracks and the entropy in concrete structures, cracked concrete structures are treated as dissipative structures. Structural information entropy is defined for concrete structures. A compact tension test is conducted. Meanwhile, numerical simulations are also carried out. Both the test and numerical simulation results show that the structural information entropy in the structures can characterize the propagation of cracks in concrete structures.

crack, entropy, concrete sun

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1 Introduction

The propagation of cracks in concrete structures is caused by various potential defects, external loads, or continuous harsh environmental exposure. Actually, it is the process of initiation, propagation, penetration from a micro crack to a macro crack [1, 2]. In the evolution process of crack propagation, it always exchanges material and energy with the outside world. The failure of a structure is the combined result of energy dissipation and release. Energy dissipation generates concrete structure damage, which results in performance degradation and strength loss, and energy release is the internal cause of the fracture of the structure. Energy dissipation and release will inevitably lead to energy redistribution in the structure, and the energy redistribution can be described by entropy. Entropy is a core concept of the dissipative structure theory, which describes the order degree of a complex system or energy distribution [3]. In recent years, with the interdisciplinary development, entropy is applied to not only some natural sciences such as physics, chemistry, and geology, but also many other social sciences [4]. According to the three formation conditions of a dissipative structure, Qin [5] studied the nonlinear characteristics of rock deformation, dimensionality reduction, and entropy reduction during rock fracture from both macro and micro aspects. Based on the study of the negative entropy of a dissipative structural system and its implementation process, Wu [6] pointed out that the existence and development of a dissipative structural system was essential to the process of achieving negative entropy. Wang [7] believed that negative entropy flux was a necessary condition for the formation of a non-equilibrium dissipative structure. The system in the state of negative entropy flux showed that the total entropy of the system continued to decrease and that the energy utilization efficiency of the system became better. Chen et al. [8] mathematically proved that information entropy described by structure strain energy was a convex function defined on a convex set. Moreover, they studied

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the principle of maximum entropy for a statistically determinate truss. Xu et al. [9], who applied nonlinear theories such as entropy and catastrophe theories in the problem of nonlinear stability of rocks, revealed the instability process and the failure mechanism of rock mass and established a practical analysis method of rock mass damage and instability criterion. Du [10] introduced entropy into the field of engineering structural safety factor calculation. To date, application of entropy in analyzing the propagation and evolution process of cracks in concrete structures has not been reported.

In this paper, basic concepts of entropy are first introduced, and then structural information entropy of concrete structures is defined. Finally, tests and numerical simulation are performed to study the entropy variation during the evolution process of cracks in concrete structures.

2 Basic concepts of entropy

Entropy is defined by Clausius, which expresses the irreversibility of heat transfer. For a closed system of thermal isolation, the entropy change dS of a micro process meets the Clausius' inequality

$$\mathrm{d}S \geqslant \frac{\mathrm{d}Q}{T}. \tag{1}$$

The above inequality defines the relation among the macro variables of the system—temperature T, heat Q, and entropy S. Equality holds for reversible processes while inequality does for irreversible processes. T is the temperature after the system absorbs heat from the outside world. For an isolated system, there is no heat exchange between the system and outside world. That is to say, dQ=0. Therefore, inequality (1) becomes

$$\mathrm{d}S \geqslant 0. \tag{2}$$

It shows that the occurrence of any irreversible process in an isolated system will lead to the entropy increase. Namely, for an irreversible process, it always proceeds in the direction of entropy increase. Eventually the system reaches the equilibrium state and the entropy arrives at its maximum. That is the principle of entropy increase [11].

It is known that heat transfer is a type of energy conversion. Energy is composed of very small energy quanta. In the process of conversion, the distribution of the energy quanta will become loose and disorderly. This is the physical meaning of Clausius' thermodynamic entropy [12]. The introduction of the concept of entropy renders that one-way characteristic of irreversible process can be quantified. The irreversible process always proceeds in a single direction from the non-homogeneous state to the homogeneous one, which makes entropy increase. The uniformity degree state of the macro state of a thermodynamic system reflects the clutter degree of molecular motion at the micro level, which reflects the possible molecular movement patterns. The more uniform the macro state of the system is, the more chaotic its micro molecular thermal motion is. The chaotic degree of the thermal motion represents the disorder degree of the system state. Namely, the lower the disorder degree of a non-uniform macro system is, the smaller the corresponding entropy value is. On the contrary, the higher the disorder degree of a non-uniform macro system, the bigger the corresponding entropy value is. Therefore, the entropy of the system is relevant to its disorder degree. This relationship was first established by Boltzmann, which is known as the Boltzmann's Theory [11]

$$S = k_{\mathrm{B}} \ln \omega, \tag{3}$$

where ω is the thermodynamic probability of the macro state of the system, and k_{B} is the Boltzmann constant.

The Boltzmann theory reveals the essence of entropy. For an isolated system, all reversible processes make the system change from the state with a small probability to the state with a large probability. That is to say, all the irreversible processes will make the disorder degree increase. Equilibrium is the most disorderly state and also the state with the maximum probability.

The above-mentioned theory is based on the entropy defined by Clausius, which can be called the thermodynamic entropy. Shannon, when he created the information theory, used entropy as a measurement of the average amount of information [13]. Generally speaking, the entropy defined by Shannon is called the information entropy. The information entropy and thermodynamic entropy are intrinsically linked while they are different in some extent in the aspect of study objects. Shannon took the amount of information as a central concept of information theory. He used the statistical characteristics of Markov process to characterize the properties of information sources. According to Shannon's formulation, suppose one process has n independent results (or states): x_1, x_2, $\cdots$, x_n. The probability of each state is $p(x_1)$, $p(x_2)$, $\cdots$, $p(x_n)$, and

$$\sum_{i=1}^{n} p(x_i) = 1. \tag{4}$$

Then, the information entropy is defined as

$$H(x) = -\sum_{i=1}^{n} p_i \ln p_i, \tag{5}$$

where p_i is the probability that signal x_i appears in the information sources. $H(x)$ is the entropy to characterize the uncertainty of the amount of information, which is a measurement of the system state.

For the continuous random variable, the information entropy is defined by the following formula:

$$H(x) = -\int_R f(x)\ln f(x)\mathrm{d}x, \tag{6}$$

where $f(x)$ is the distribution density function of the continuous random variable x.

Both the physical entropy and the information entropy are statistical expressions of the disorder degree of a system [14]. Although they are applied to different disciplines, their main characteristics are the same:

(1) The mathematical expressions are the same, which are expressed by the functional of a probability distribution function. Both, in essence, represent the system randomness and disorder.

(2) Both comply with the entropy evolution equation with a similar mathematical form and physical meaning.

(3) Both meet the principles of entropy increase, maximum entropy, and Jaynes' maximum entropy. Since the information entropy is not like the physical entropy in which the mechanical background of study objects has to be considered, it can be applied to many other disciplines such as economics, sociology, and ecology. Besides, it is easy to calculate, so it is widely used to estimate the system disorder degree in practical applications.

3 Definition of structural information entropy of concrete structures

A concrete structure is discretized into n elements by finite element method. Under certain boundary conditions and external loads, the strain energy possessed by the ith element is given by

$$q_i = \frac{1}{2}\int_{v_i} \sigma_{ij}\varepsilon_{ij}\mathrm{d}v_i, \tag{7}$$

where σ_{ij} and ε_{ij}, respectively, are the stress and strain tensors of the element. v_i is the volume of the ith element. Superpose the strain energy of all the elements, then the total strain energy Q of the system is obtained

$$Q = \sum_{i=1}^{n} q_i. \tag{8}$$

Set

$$\lambda_i = \frac{q_i}{Q} \quad (i = 1,\ 2,\ \cdots,\ n). \tag{9}$$

And then

$$\sum_{i=1}^{n}\lambda_i = 1, \quad \lambda_i \geqslant 0 \quad (i = 1,\ 2,\ \cdots,\ n). \tag{10}$$

Clearly, the newly introduced physical quantity λ_i (i=1, 2, $\cdots$, n) is complete and non-negative in mathematics. Its mechanical meaning is the share of the strain energy of the ith element in the total strain energy. To comprehensively reflect the strain energy distribution of the structure [8, 9], an information entropy function of the structure is defined as

$$H = -\sum_{i=1}^{n}\lambda_i \ln \lambda_i. \tag{11}$$

Xu et al. [9] have proved that the structural information entropy H of the structure is a convex function with a single peak value, and only if the strain energy of each element is equal, i.e. $\lambda_i = 1/n$ (i=1, 2, $\cdots$, n), the entropy H of the structure reaches the maximum. The dissipation level of energy is called the degree of energy dissipation. The structural information entropy of concrete structures can be used as a measurement of the dissipation degree of system energy. The higher the concentration of system energy is, the more uneven it is and the smaller the system entropy is. On the contrary, the lower the concentration of system energy is, the larger its entropy is. When the system reaches the state of uniform energy distribution, the entropy of the system reaches the maximum.

4 Entropy variation during the crack propagation process

To study the relationship between the crack propagation evolution and entropy variation, a large concrete specimen is used to conduct a compact tension experiment. Meanwhile, numerical simulation is carried out.

4.1 Test program and calculation model

In the experiment, a compact tension notched specimen is used, which is shown in Figure 1, l=h=2.5 m, h_1=0.2 h, f=l/4, a_0=0.4 h, t=0.2 m. The mix proportion by weight of the concrete is: cement: sand: gravel: water = 1:1.87:1.36:0.57. Ordinary Portland cement is employed.

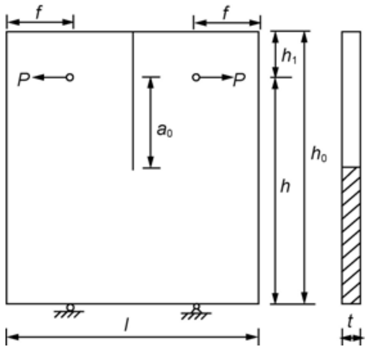

Figure 1 Compact tension specimen.

Limestone gravels with the maximum diameter of 20 mm are used as coarse aggregates, and river sands with the maximum diameter of 5 mm are utilized as fine aggregates. The specimen is cured with sealed plastic to avoid loss of water. After 180 days of curing, the specimen is loaded with a hydraulic jack. The displacements at the loading point are measured by the MTS Company's clip-on extensometer, and together with loads collected by the MTS Company's data collection.

The finite element model is shown in Figure 2. In the model, 8-node solid element is adopted. Nonlinear spring element is used to simulate the cohesive force of the crack. Material parameters of the specimen are shown in Table 1.

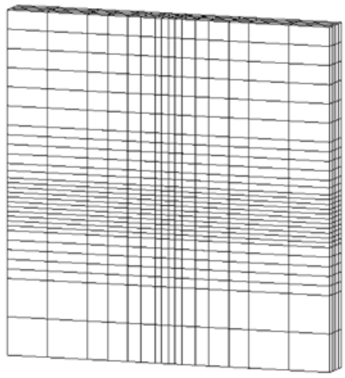

Figure 2 FEM mesh of the specimen.

Table 1 Material parameters

Elastic modulus (GPa)	Poisson's ratio	Tensile strength (MPa)	Fracture energy (N/m)
33.4	0.197	2.79	202.75

4.2 Results and analysis

Table 2 is the results from the experiment and numerical simulation. From the results, the numerical results are close to the experimental ones. Table 3 shows that the numerical and experimental values of the various variables are involved in the crack propagation process.

Using the load p and displacement δ at the loading point, the external work W_P can be calculated. By subtracting external work W_P from the total system strain energy U, the dissipative energy ΔU of crack propagation can also be

Table 2 Comparison of experimental and numerical results

	P_{max} (kN)	Δa (m)	P_s/P_{max}
Experimental results	83.53	0.21	0.42
Numerical results	85.50	0.20–0.22	0.38–0.41

Note: P_{max} is the maximum load, Δa is the crack propagation length, P_s is the load at crack initiation.

Table 3 Values of the various variables involved in the crack propagation

Incremental step	Displacement at loading point δ(m)	External work W_P (J)	Total strain energy U(J)	Dissipative energy $\triangle U$ (J)	Entropy H
1	6.31×10^{-6}	1.86×10^{-2}	1.85×10^{-2}	1.39×10^{-4}	6.21
2	1.26×10^{-5}	7.45×10^{-2}	7.44×10^{-2}	1.55×10^{-4}	6.21
3	1.89×10^{-5}	1.68×10^{-1}	1.67×10^{-1}	3.50×10^{-4}	6.21
4	3.53×10^{-5}	2.98×10^{-1}	2.97×10^{-1}	1.22×10^{-3}	6.21
5	3.16×10^{-5}	4.66×10^{-1}	4.64×10^{-1}	1.47×10^{-3}	6.21
6	3.79×10^{-5}	6.71×10^{-1}	6.69×10^{-1}	1.51×10^{-3}	6.21
7	4.42×10^{-5}	9.13×10^{-1}	9.12×10^{-1}	8.07×10^{-4}	6.21
8	5.05×10^{-5}	1.19	1.18	8.89×10^{-3}	6.21
9	5.68×10^{-5}	1.51	1.50	1.13×10^{-2}	6.21
10	6.31×10^{-5}	1.86	1.85	1.39×10^{-2}	6.21
11	6.95×10^{-5}	3.25	3.24	1.68×10^{-2}	6.21
12	7.60×10^{-5}	2.69	2.66	2.78×10^{-2}	6.23
13	8.30×10^{-5}	3.18	3.13	4.99×10^{-2}	6.26
14	8.98×10^{-5}	3.71	3.64	7.03×10^{-2}	6.28
15	9.67×10^{-5}	4.28	4.19	9.07×10^{-2}	6.3
16	1.04×10^{-4}	4.89	4.78	1.11×10^{-1}	6.31
17	1.11×10^{-4}	5.55	5.41	1.37×10^{-1}	6.32
18	1.18×10^{-4}	6.27	6.09	1.87×10^{-1}	6.34
19	1.26×10^{-4}	7.05	6.81	3.40×10^{-1}	6.36
20	1.33×10^{-4}	7.87	7.58	2.88×10^{-1}	6.37
21	1.41×10^{-4}	8.74	8.40	3.55×10^{-1}	6.38

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Incremental step	Displacement at loading point δ(m)	External work W_P (J)	Total strain energy U(J)	Dissipative energy $\triangle U$ (J)	Entropy H
22	1.50×10^{-4}	9.72	9.28	4.42×10^{-1}	6.4
23	1.58×10^{-4}	1.07×10^{1}	1.02×10^{1}	5.37×10^{-1}	6.42
24	1.67×10^{-4}	1.18×10^{1}	1.12×10^{1}	6.35×10^{-1}	6.43
25	1.77×10^{-4}	1.31×10^{1}	1.23×10^{1}	8.00×10^{-1}	6.45
26	1.88×10^{-4}	1.44×10^{1}	1.34×10^{1}	9.46×10^{-1}	6.46
27	2.00×10^{-4}	1.59×10^{1}	1.47×10^{1}	1.20	6.49
28	3.15×10^{-4}	1.77×10^{1}	1.62×10^{1}	1.49	6.5
29	3.42×10^{-4}	2.07×10^{1}	1.86×10^{1}	2.09	6.53

obtained. Theoretically, before cracking, the external work will be fully converted to the internal work of the system, that is, ΔU=0. But in the numerical simulation, there exist certain calculation errors due to the effect of spring elements. Therefore, ΔU>0. However, the maximum error of the total strain energy is only 0.74%, which can meet engineering requirements.

For the physical quantities with different dimensions in Table 3, to analyze and compare them, normalization is necessary. Formula (12) can be used for this purpose

$$r_{ij} = (x_{ij} - x_{i\max}) / (x_{i\max} - x_{i\min}), \quad (12)$$

where r_{ij} is the normalized value of the jth incremental step of the ith variable. $x_{i\max}$ and $x_{i\min}$ are the maximum and minimum values of the ith variable, respectively. Figure 3 shows the curves of the normalized total strain energy, dissipative energy, displacement, entropy versus loading step.

From Figure 3, the following conclusions can be drawn:

(1) The total strain energy U and the dissipative energy ΔU gradually increase with the increase of the external load P. When the external load P reaches its maximum $P_{\max}$, the internal energy of the system reaches the critical state. During the crack propagation process, the lines of the normalized total strain energy and dissipated energy show no jumps at the time of the crack initiation and propagation, so it is difficult to judge the initiation and propagation of cracks from the variation of the total strain energy and dissipative energy.

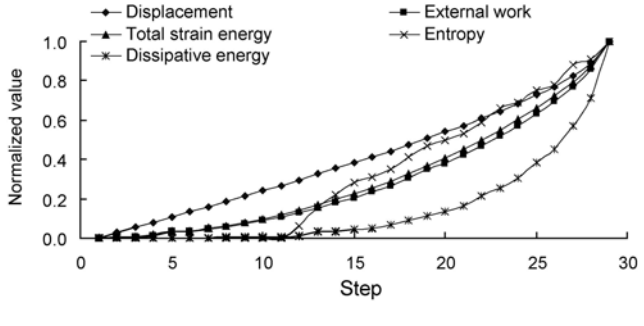

Figure 3 The curves of the normalized variables.

(2) The displacement δ at the loading point reflects the opening variation during the crack propagation process. The line of the normalized displacement is close to being a straight one at the initial stage of loading and then shows no obvious nonlinearity. Therefore, it is also difficult to judge the initiation and propagation of cracks from the displacement variation.

(3) From the line of the normalized entropy, at the 12th incremental step, the entropy shows an obvious sharp change, which indicates that the crack initiates between the 11th and 12th steps. After the crack initiation, some of the external work is consumed by the slow crack propagation in the fracture process zone, and the other is converted into the elastic strain energy stored in the concrete structure, which causes the structure change to be in a high internal energy state. With the crack propagation, the strain energy in the fracture process zone will be redistributed. As a result, the energy concentration at the crack tip region will mitigate to a certain extent, and the entropy generated in this process is positive. If the load does not increase, no new cracks will appear, and the state of the energy distribution will remain unchanged. Therefore, the entropy will remain unchanged. If the loading continues, the entropy of the structure will increase with the increase of the external load, which indicates that the crack continues to propagate.

5 Conclusions

In this paper, basic concepts of entropy are first introduced, and then the concrete structure is taken as a dissipative structure while the structural information entropy of concrete structures is defined. Then a compact tension experiment and numerical simulation are carried out to study the law of entropy variation in the crack propagation process. The results show that the structural information entropy increases with the propagation of cracks and that the crack propagation process can be described by the structural information entropy.

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