

Error rate performance of Hybrid QAM-FSK in OFDM systems exhibiting low PAPR

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Multicarrier transmission systems like orthogonal frequency division multiplexing (OFDM) support high data rate and generally require no equalization at the receiver, making them simple and efficient. This paper studies the design and performance analysis of a hybrid modulation system derived from multi-frequency and MQAM signals, employed in OFDM. This modulation scheme has better bit error rate (BER) performance and exhibits low PAPR. The proposed hybrid modulator reduces PAPR while keeping the OFDM transceiver design simple, as it does not require any side information or a little side information (only one bit) to be sent and is efficient for arbitrary number of subcarriers. The results of the implementations are compared with those of conventional OFDM system.

orthogonal frequency division multiplexing (OFDM), hybrid MQAM/LFSK modulator (HQFM), bit error rate (BER), peak to average power ratio (PAPR)

1 Introduction

The mobile radio channel is characterized by multipath reception, i.e. in addition to direct LOS signal, the received signal may contain more than one replica of the transmitted signals arriving at different delays, causing ISI and degrading the overall system performance. One method to combat the ISI problem is to design an efficient adaptive equalizer, but this makes the receiver complex. To avoid the complexity of the receiver, the use of multicarrier modulation (MCM) has been explored by researchers in the last two decades.

MCM divides the entire data stream into a small number of low data-rate subcarriers, thus reducing the effect of frequency selective fading. One of the

candidates for these MCM schemes is orthogonal frequency division multiplexing (OFDM). OFDM is easy to implement because it makes efficient use of discrete Fourier transforms^[1,2].

The attractive feature of OFDM, i.e., the principle of orthogonality makes the entire system robust against multipath frequency selective fading and Doppler spread^[3]. Nevertheless, this highly spectrally efficient and robust against ISI, OFDM technique faces certain problems. For example^[3]:

- It is very sensitive to carrier frequency offsets (CFO) caused by frequency differences between oscillators in the transmitter and receiver. Several methods are proposed to overcome this problem.
- It has a high peak-to-average power ratio

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(PAPR) that calls for high power amplifier (HPA) of a very large linear region. This high PAPR forces the signal peaks to get into a nonlinear region of HPA which distorts the signal by introducing inter-modulation among the subcarriers and out-of-band radiation.

Conventionally OFDM uses either QAM or PSK as a baseband modulator. In 2002, CPM was proposed in place of QAM/PSK as a baseband modulator^[4], which causes a reduction in PAPR^[5]. Also, using FSK (a special case of CPM) in OFDM makes the signal to reduce PAPR^[6]. But, FSK (if orthogonality is maintained) is bandwidth inefficient while QAM/PSK is bandwidth efficient. If compared in terms of BER performance, QAM/PSK is power inefficient and FSK is power efficient. Comparing QAM and PSK, we know that QAM is more power efficient than PSK. In this paper, therefore, a hybrid of bandwidth efficient QAM and power efficient FSK is used and found that when applied in OFDM, PAPR is reduced and is referred here as hybrid QAM-FSK modulator (HQFM). It is to be noted that many available hybrid modulation schemes, like Q²PSK^[7], QFPM^[8], JPFM^[9] for single-carrier transmission and one described in ref. [10] for multicarrier systems like OFDM, make use of hybridization of orthogonal LFSK and MPSK/MDPSK, but neither of these consider the combination of orthogonal LFSK with MQAM.

The modified OFDM transceiver makes use of multilevel QAM constellations, where the level of QAM is decided by specific number of bits chosen from a group of bits to be encoded in the QAM symbol. The simulation results show that BER performance is improved while PAPR is considerably reduced at the cost of decreased bandwidth efficiency and little detection complexity^[11]. Like PTS^[12,13], it works with arbitrary number of subcarriers but needs no side information to be transmitted.

2 Hybrid MQAM/LFSK (HQFM) OFDM system model

In a typical OFDM system, the bit rate per sub-carrier (not the total bit rate) is reduced by con-

verting binary serial bit stream into large number of parallel streams. Then these bits are mapped through any modulation technique making N subcarriers and transformed into OFDM symbols by applying N -point IFFT. In order to avoid ISI and ICI, the transmitted signal is made periodic by appending the last part of OFDM symbol. The signal is then D/A converted to produce the analog baseband signal, up-converted to RF and then transmitted. The reception is the converse and is self-explanatory. Mathematically, an OFDM symbol is expressed as

$$s_{i,k} = \text{IFFT}\{A_{i,k}\} \\ = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} A_{i,k} e^{j2\pi kn/N}, 0 \leq k \leq N, \quad (1)$$

where $s_{i,k} = [s_{i,0}, s_{i,1}, \dots, s_{i,N-1}]$ is the time domain representation of vectors associated with the i th OFDM symbol. $A_{i,k} = [A_{i,0}, A_{i,1}, \dots, A_{i,N-1}]$ is a set of alphabet taken from any suitable modulation scheme like MQAM.

MQAM is a conventional 2D modulator, where the centre frequency f_c , used for modulating the information signals is fixed so the notion of 3rd dimension does not exist. Now if the modulating frequency, $f_c + f'_c$, changes occasionally, according to some information available from $(n-k)$ source bits out of n bits, a new dimension can be defined in the signal space diagram. In this way QAM signal with higher dimensions can be split into QAM of lower dimension with modulating frequencies taken from FSK respectively. This type of modulator, termed as hybrid MQAM/LFSK modulator (HQFM), is very similar to that described in refs. [7–9] but the modulator described there employs DPSK not QAM. Generally these FSK frequencies are orthogonal to each other; therefore the points lying in the HQFM signal space can be viewed as points lying in a smaller QAM with non interfering (orthogonal) planes, where each plane is distinguished by its corresponding FSK frequencies (Figure 1).

In HQFM, instead of modulating the $n = \log_2 ML$ information bits using a single frequency f_c , the $n - k = \log_2 L$ bits, where the choice of bits is arbitrary, are used to select the modulating frequency $f_c + f'_c$, $f'_c \ll f_c$ from an LFSK according to $f'_c = m\Delta f$, $m = 0, 1, 2, \dots, L^{[14]}$, Δf is the fre-

quency separation between two adjacent LFSK frequencies such that $\Delta f = 1/T_s$, T_s being HQFM symbol duration. The remaining $k = \log_2 M$ bits are modulated using ordinary MQAM. This hybrid MQAM/LFSK (HQFM) can be depicted in Figure 2.

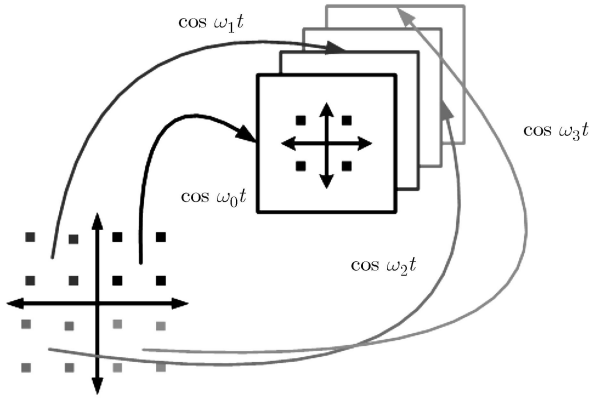


Figure 1 Decomposition of 16-QAM into 4-QAM using 4-FSK for HQFM.

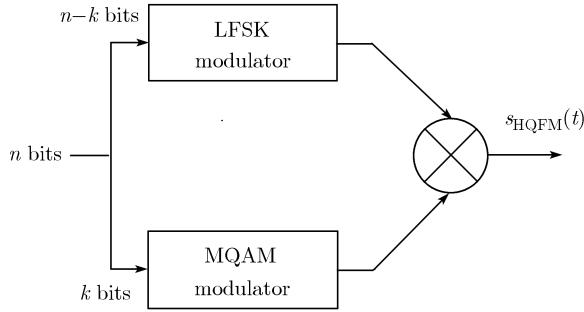


Figure 2 Hybrid MQAM/LFSK modulator.

The modulated HQFM signal then becomes

$$s_{\text{HQFM}}(t) = \sqrt{\frac{2E_s}{T_s}} a_i \cos(2\pi(f_c + m\Delta f)t) \pm \sqrt{\frac{2E_s}{T_s}} b_i \sin(2\pi(f_c + m\Delta f)t) = \text{Re}[(a_i \mp jb_i)u_m(t)e^{j2\pi f_c t}], \quad (2)$$

where $0 \leq t \leq T_s$, $u_m(t) = e^{j2\pi m\Delta f t}$, $i = \{0, 1, 2, \dots, M-1\}$ (from QAM), $m = \{0, 1, 2, \dots, L-1\}$ (from FSK).

The complete OFDM symbol is transformed by passing these HQFM modulated signals through IFFT and pre-pending cyclic prefix (CP).

At the receiver side, these signals are recovered after removal of CP and application of FFT. A two-stage demodulator is designed to extract the n information bits. In the early stage of demodulation

process, coherent detection of FSK frequencies is carried out through the bank of L correlators or matched filters. As all the FSK frequencies are orthogonal to each other, the bank of correlators gives maximum value for the frequency matched to that particular frequency and zero for all other frequencies. After detecting these frequencies ($n-k$ bits), the remaining k bits are detected using conventional demodulation process. Figure 3 shows a complete picture of the proposed demodulator.

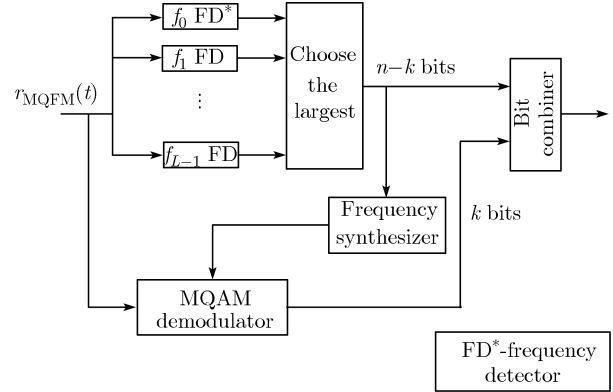


Figure 3 HQFM Demodulator.

3 Error rate performance of HQFM in AWGN

In this section, the BER and SER for the HQFM is compared with conventional QAM (both QAMs, i.e. individual QAM employed in HQFM and MQAM when all bits are injected to a single QAM, and FSK using Monte Carlo simulations. These results are obtained in AWGN channel using non-coherent detection of FSK frequencies. Each FSK and 16 QAM/16 FSK HQFM used for simulation have 2 numbers of symbols per sample.

From Figure 4(a), it is evident that the required E_b/N_0 for the 16 QAM/16 FSK HQFM to be detected correctly at $P_e \approx 10^{-4}$ is ~ 13 dB while for 16 FSK and 16 QAM it is ~ 7.5 dB and ~ 16 dB respectively. Also, HQFM system performs much better than 256 QAM (more than 10 dB). Comparing Figures 4(a) and 4(b), it is evident that the relationship between BER and SER is not in the same way as in the case of BER and SER of LFSK or MQAM^[14,15]. This can be described below:

There are two different types of symbols errors that can occur. For the first type of symbol errors, the symbol is detected incorrectly if the number of

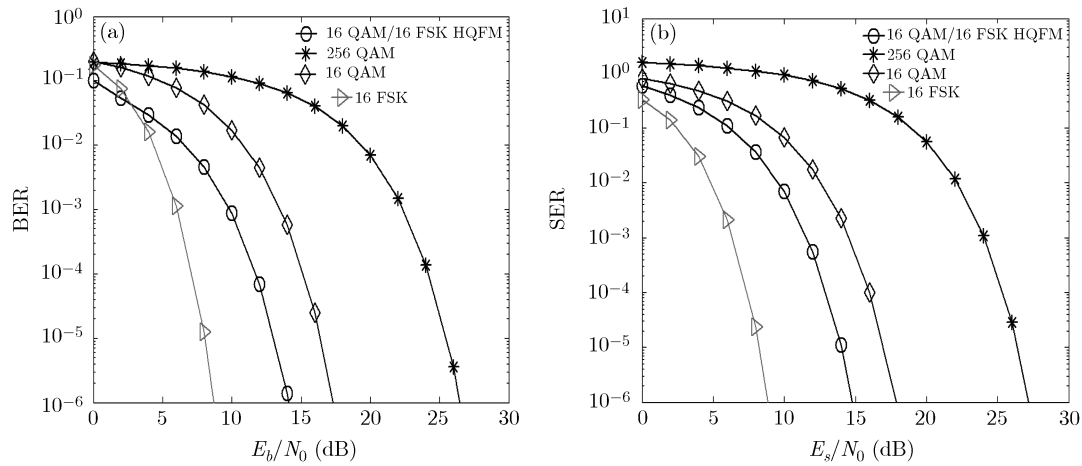


Figure 4 Error probability performance of hybrid 16QAM/ 16FSK compared with 256 QAM, 16 QAM and 16 FSK using non-Coherent detection of FSK frequencies. (a) Bit error rate; (b) symbol error rate.

FSK frequencies are detected incorrectly causing at least one bit in $\log_2 L + \log_2 M$ bits chosen incorrectly. The second type of SER occurs when FSK tones are detected correctly but QAM symbol is detected incorrectly causing at least one bit in error out of $\log_2 M$ bits.

4 BER performance of HQFM-OFDM in fading channel

The significance of HQFM when applied in OFDM can be well understood in some realistic channel like Rayleigh fading channel.

The channel model simulated for the results in this section uses 3 taps WSSUS channel, having Rayleigh distribution. The channel delays are specified to be $[0, 20, 40]$ ns with a gain vector of $[0, -3, -6]$ dB. Therefore r.m.s. delay spread is 0.632 ns. Other simulation parameters are listed in Table 1.

The simulation results shown in this section employs Reed-Solomon (RS) encoding and decoding algorithms, because of their good distance properties. These codes are selected because they are the most popularly used block codes, particularly useful for correcting burst errors^[15]. The OFDM link in presence of fading multipath is a very good application for this code. The BER performance of HQFM-OFDM compared with conventional QAM-OFDM, shown in Figure 5 in this section, was produced using Monte-Carlo simulations instead

of analysis. The required E_b/N_0 for the HQFM to be detected correctly at $P_e \simeq 10^{-3}$ is ~ 37 dB while for 256 QAM it is ~ 39 dB. It also shows that proper coding like Reed-Solomon reduces the required E_b/N_0 for HQFM-OFDM. But this gain is obtained at the cost of transmitter's throughput.

Table 1 Simulation parameters for HQFM-OFDM in rayleigh slow fading channel

N -OFDM symbol size	64
Number of Active subcarriers	52
$f_d T$	0.0064
n -total number of bits/subcarrier	8
M -QAM levels	4,8,16
L -FSK frequencies	4,8,16
f_c -Centre frequency	450 MHz
Data rate supported	50 Mbps
T -OFDM symbol period	8.32 μ s

5 Peak-to-average power ratio (PAPR)

Figure 6 shows a portion of an arbitrary 512-carrier OFDM symbol, when 256 QAM is employed and is compared with hybrid 16 FSK/16 QAM OFDM symbol. Figure 6 clearly shows that the peak of the OFDM symbol is drastically reduced when the hybrid signals are injected into the IFFT resulting in low PAPR.

Figure 7 plots the probabilities $\Pr(\text{PAPR})$ against a specified threshold PAPR_0 . The outermost line shows the $\Pr(\text{PAPR}_0)$ against a specified threshold PAPR_0 for conventional 256 QAM-

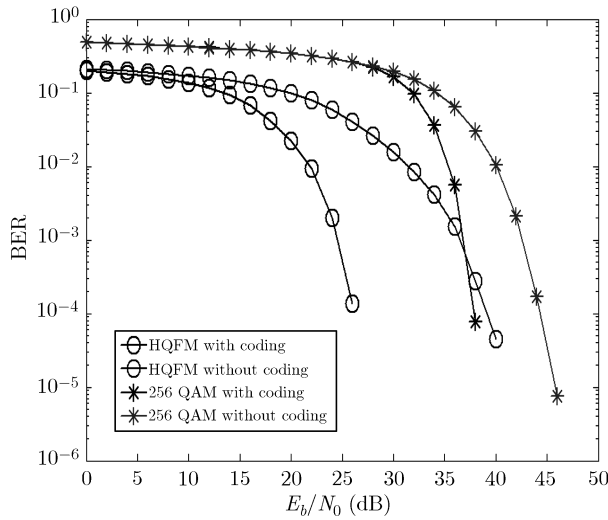


Figure 5 BER of 16QAM/16FSK, $N=64$ OFDM in rayleigh slow fading channel with normalized Doppler shift, $f_d T = 0.0064$. The coding applied is RS (13, 7).

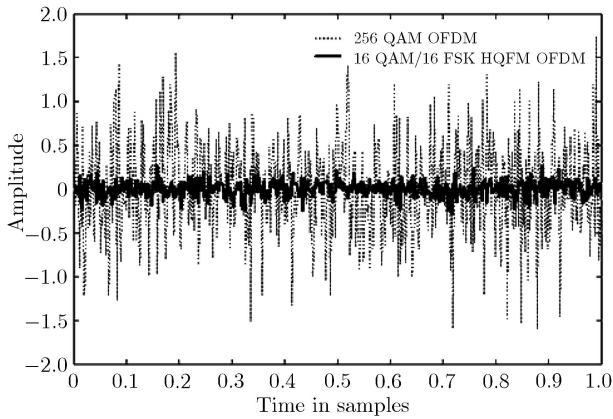


Figure 6 Amplitude and mean of a single 512-OFDM symbol, $\text{PAPR}_{256\text{QAM}} = 11.706$ dB, $\text{PAPR}_{16\text{QAM}/16\text{FSK HQFM}} = 8.784$ dB.

OFDM. This system is compared with different QAM levels (M) and different FSK frequencies (L). The following results can be inferred from this figure:

- 1) HQFM makes the probabilities decay faster, yielding a more desirable statistical behavior.
- 2) PAPR for HQFM cannot exceed ~ 13 dB while this value can take up a value of ~ 15.5 dB for a conventional one at $\text{Pr}(\text{PAPR}) = 10^{-6}$.
- 3) PAPR decays faster if the number of FSK frequencies, L , per HQFM symbols increases. But this is achieved at the cost of reduced bandwidth efficiency.

4) 16 FSK is enough to reduce OFDM's PAPR. Also there is no improvement in PAPR statistics if FSK frequencies are increased beyond 32.

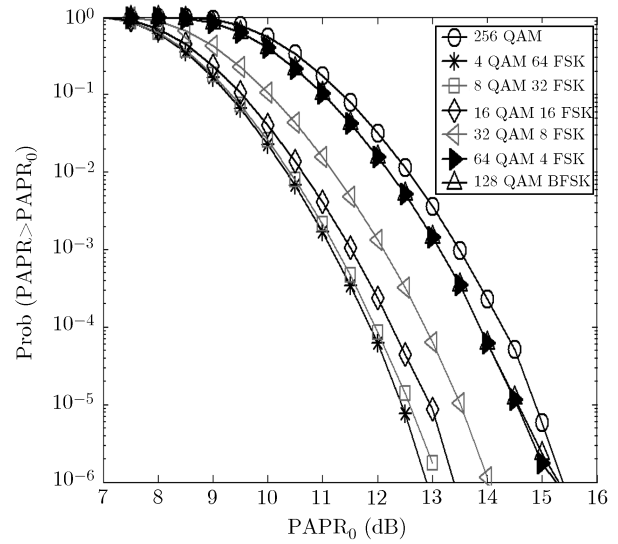


Figure 7 CCDFs of PAPR of an OFDM compared with the proposed transceiver with $N=512$.

It is well understood that statistics of PAPR is the function of number of subcarriers, N , only and does not depend on the size of baseband modulator, 2^n [12]. Therefore the above mentioned results can be extended to other hybrid combinations of HQFM.

6 Conclusions

In this paper, a novel OFDM transceiver is proposed showing lower PAPR than that of conventional system. The results are discussed and compared based upon different simulation results. In addition to better PAPR reduction capabilities, this system also shows better BER performance. Although, the HQFM system has poor bandwidth efficiency when compared with 256 QAM, this remains unchanged when employed in OFDM. 16 FSK/16 QAM is an appropriate solution to be replaced with conventional 256 QAM both in terms of better BER performance and PAPR reduction capability.

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