THE TRAVEL OF ASTRONOMICAL TABLES FROM THE ISLAMIC WORLD TO JOSEON KOREA

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Abstract: In the thirteenth century, there were extensive astronomical exchanges between the Islamic world and the Mongol Yuan Empire (1271–1368). The Islamic calendar system, based on the *Almagest*, was first introduced into China, and the Islamic *Jamal al-Din Zij* was compiled in 1276 by Jamal al-Din and Muslim astronomers active in the Islamic Astronomical Bureau of Yuan. Subsequently, the Chinese–Islamic *zij*, *Huihui-lifa*, based on the *Jamal al-Din Zij*, was compiled in 1396 by the co-operation of Chinese and Muslim astronomers of the Ming Dynasty (1368–1644). It was transmitted to Joseon Korea in the King Sejong period (1418–1450). However, Korean astronomers found serious mistakes in the calendrical epoch and date conversion method of the *Huihui-lifa*. They corrected all the errors and applied newly revised algorithm, compiling the Korean version of Islamic *zij*, *Chiljeongsan-Oepyeon* in 1444.

This study particularly investigates the parallax correction table in the *Chiljeongsan-Oepyeon*. Interestingly, however, we find that the parallax theory of the Islamic *Jamal al–Din Zij* and Chinese *Shoushi-li* have a common origin from Hindu astronomy in the sixth to seventh centuries. Also, we find out that there are differences between the parallax correction tables in the *zijes* of Jamal al–Din type (i.e., originated from *Jamal al–Din Zij*) and Theon type (i.e., originated from Theon's handy table). Therefore, this study traces the travels of astronomical tables by comparing the parallax correction tables included in the Islamic and Chinese calendar systems. We discuss the routes of the parallax theory transmitted from ancient Greece and India to Joseon Korea via the Islamic world and China, respectively.

Keywords: *Almagest*; *Jamal al–Din Zij*; *Huihui-lifa*; *Chiljeongsan-Oepyeon*; *Shoushi-li*; Theon's handy table; parallax correction table; astronomical table; parallax theory.

1 INTRODUCTION

A *zij* is actually a compilation of astronomical–mathematical tables for calculating the positions of the Sun, the Moon and planets. *Zijes* are used for predicting the occurrence of solar and lunar eclipses. Although in early centuries of the Islamic Middle Age, *zijes* was written in Arabic, later Persian became the medium of compilation. All *zijes* in Arabic, Persian, Hebrew and Turkish, contain tables for functions of spherical astronomy and trigonometrical functions (Ansari, 2017). Therefore, it is considered that the transmission of astronomical tables in Islamic *zijes* played a crucial role in the astronomical exchanges and development between different civilizations.

Meanwhile, the parallax theory in Islamic astronomy was an important subject of moment to the Medieval astronomer, who had to take account of solar and lunar parallax in the computation of solar eclipses. For the historian of mathematics also this was a matter of interest, since the computation of the parallax in altitude and its resolution into applicable components involved trigonometry (Kennedy, 1956).

In this study, we examine and compare the parallax correction tables in the Medieval Islamic *zijes* of the Theon and Jamal al-Din types. As a result, we find that there are differences in the parallax methods of the two types of *zijes* with different origins. Also, we investigate the similarity and disparity of the parallax correction tables in the Islamic *Jamal al-Din Zij* and Chinese *Shoushi-li* which had a common origin from Hindu astronomy. From this research, we confirm that the transmission of ancient Hindu astronomy to the Arab world and China began in the eighth century, when Sanskrit works of Aryabhata and Brahmagupta were translated into Arabic and Chinese, respectively.[1]

2 DEFINITION OF PARALLAX

An observer observing a celestial body is located on the Earth's surface rather than at its center, causing a difference between the apparent and true positions of the celestial body. This difference is called 'parallax'. If the zenith distance of the celestial body and the distance from the Earth's center to the celestial body are known, the parallax can be easily calculated.

In Figure 1, let celestial body *P* be the Moon, the Earth's radius be R_e, and the parallax be p. The parallax $p\,(=\zeta' - \zeta)$ can be determined using the following equation by applying the Law of Sines to triangle ΔOEP, given the Moon's polar distance ζ', and the distance from the Earth's center to the Moon *d*.

$$sin\, p = \frac{sin(180-\zeta)}{d} R_e \qquad (1)$$

$$p = sin^{-1}(\frac{R_e}{d} sin\, \zeta') \qquad (2)$$

The altitude parallax $p\,(=\zeta'-\zeta)$ can be easily calculated using Equations (1) and (2) above. However, while the calculation of parallax is theoretically simple, in the case of solar and lunar eclipse calculations, multiple parallax factors must be corrected simultaneously, making it difficult to obtain values that match observed results.

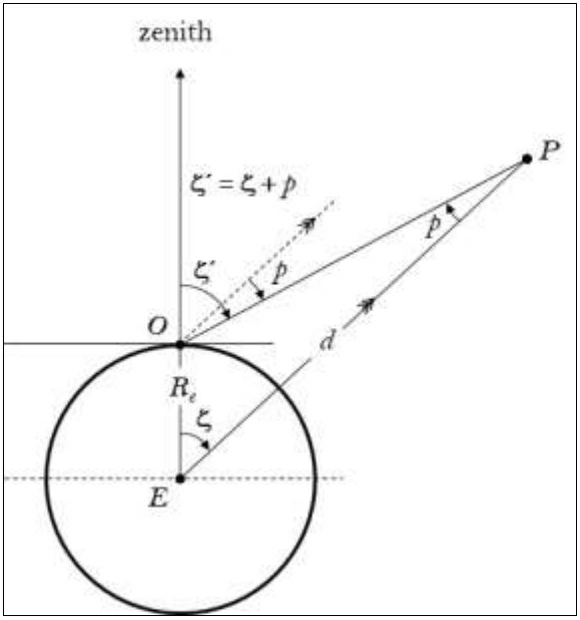

Figure1: The lunar parallax (after Neugebauer, 1975: 1235).

3 TRANSMISSION OF THE PARALLAX CORRECTION METHODS

The parallax correction method is a key factor necessary for accurately predicting the location and timing of a solar eclipse. Historically, it is not clear exactly when parallax was first considered in eclipse calculations. However, in the *Almagest* of Ptolemy, which inherited Hellenistic astronomy, there are detailed explanations regarding the parallax of the Sun and Moon, including parallax tables for the altitude circle and methods for determining parallax (Toomer, 1998).

Ptolemy's *Handy Table*, the parallax table in the *Almagest*, was later revised by Theon of Alexandria (335–405), an astronomer of Alexandria. Unlike Ptolemy's table, Theon's *Handy Table* divided parallax into longitude and latitude components. In the ninth century, Theon's *Handy Table* was transmitted to the Arabs along with *Almagest* and translated into Arabic, becoming widely known among Islamic astronomers. From the ninth century onward, most *zijes* compiled in the Islamic world adopted Theon's *Handy Table* for parallax calculations (Kennedy, 1956).

Meanwhile, Hellenistic astronomy was transmitted not only to Alexandria after the death of Alexander the Great (356–323 BC) but also through another route from Macedonia to India. In the sixth century, the Indian astronomical text, the *Aryabhatiya*, introduced theories and calculation methods for parallax, along with trigonometric and spherical astronomical formulas (Shukla and Sarma, 1976). The calculation methods in the *Aryabhatiya* were later partially revised by Brahmagupta,[2] and transmitted through his book *Brahma Sphuta Siddhanta* (in 628). His astronomical texts also reached the Islamic world, significantly influencing Islamic astronomy. Regarding this, Al-Biruni (973–1048) mentioned Brahmagupta in his book *Indology* as follows:

> Brahmagupta occupies an important place in the history of oriental culture. Brahmagupta taught astronomy to the Arabs before they came to know of Ptolemy's works, since, reference to the works, *Sindhind and Al-Arkand* frequently occur in Arabic literature; these are Arabic translations of Brahmagupta's works, *Brahma (Sphuta) Siddhanta and Khandakhadyaka.* (Rao, 2009: 103).

Brahmagupta's astronomical knowledge was also transmitted to China through the *Jiuzhi-li* (九執曆),[3] which was compiled in 718 by the Indian astronomer Gautama Siddhartha (瞿曇悉達, 670–730) during the Tang Dynasty. The *Jiuzhi-li* later influenced the *Dayan-li* (大衍曆), compiled by Yi Xing (一行, 683–727) in 729, which in turn had a significant impact on the *Xuanming-li* (宣明曆) of 822. The *Xuanming-li*, regarded as the most advanced calendar system after the *Dayan Calendar*, remained largely the same as its predecessor, except for modifications in timekeeping methods and some calculations related to solar and lunar eclipses (Chen, 1988; Lee, 2019).

However, in the calculation of parallax observed at conjunction time, the *Xuanming-li* followed the method of the *Dayan-li* in the calculation of the time difference (*shicha*, 時差) between the conjunction and the center of the eclipse. In other words, while there are some differences in the calculation methods of solar and lunar eclipses, the *Xuanming-li* followed the *Dayan-li* precisely in considering *shicha* (時差, time difference) in its calculations. The parallax correction method of the *Xuanming-li* later influenced the *Chongxiu Daming-li* (重修大明曆) of the Jin (金) Dynasty and the *Shoushi-li* (授時曆) of the Yuan Dynasty (元). Furthermore, the *Datong-li* (大統曆) of the Ming Dynasty (明), and the *Chiljeongsan Naepyeon* (七政算內篇) of Joseon Dynasty (朝鮮, 1392–1910) continued to adopt this method (Lee, 2024).

What is particularly noteworthy is that the Islamic *Jamal al-Din Zij* also considers the *shicha* (時差, time difference) in parallax calculations, similar to the *Xuanming-li* and the *Shoushi-li*.

Most medieval Islamic *zijes* used Theon's table, which only considered longitudinal and latitudinal parallax. However, despite being an Islamic *zij*, the *Jamal al-Din Zij* incorporated an Indian-origin calculation method by taking the *shicha* into account in addition to longitudinal and latitudinal parallax (Lee, 2024). Moreover, both of the *Jamal al-Din Zij* and *Shoushi-li* share a unique characteristic: they use the terms *Rahu* (羅睺) and *Ketu* (計都) to denote the ascending and descending nodes of the ecliptic and lunar orbit—terminology found only in Indian astronomical texts. However, while *Rahu* represents the ascending node in the *Shoushi-li*, the *zijes* of *Jamal al-Din* type use *Ketu* for the ascending node instead (Lee, 2018). Furthermore, in these texts, the lunar distance from the ascending node, known as the 'angular distance of the Moon from Ketu (i.e., 月離計都宮度)', is used as a parameter to determine the Moon's ecliptic latitude, which represents its displacement from the ecliptic (Lee, 2024; Yu et al., 1974).

As mentioned above, the origins of parallax theory can be divided into two main traditions: one rooted in ancient Greek astronomy, as seen in Ptolemy's *Almagest* and Theon's methods, and the other derived from Indian astronomical texts such as *Aryabhatiya* and the works of Brahmagupta. Accordingly, the following sections will examine the Greek-origin parallax correction methods of Ptolemy and Theon, as well as the Hindu origin methods adopted in the *Jamal al-Din Zij* and *Shoushi-li.*

3.1 Parallax Correction Methods of Ptolemy and Theon

Most Medieval Islamic *zijes* based on the *Almagest* used *Theon's Handy Tables*, which incorporated Ptolemy's parallax table for the parallax calculations necessary for lunar and solar eclipse predictions. However, for the calculation of longitude and latitude parallax, they used trigonometry instead of *Menelaus' theorem*, which was traditionally used in the *Almagest*.

Figure 2 represents the lunar parallax p, which affects the predictions of solar eclipses, decomposed into its components: p_λ (longitude parallax), which is parallel to the ecliptic, and p_β (latitude parallax), which is the perpendicular component.

In Figure 2, Ptolemy, determined the parallax p of the Moon, *M* (λ, β), specifically the vector *MM′*, by taking *ZL* as the Moon's zenith distance ζ instead of *ZM* (i.e., $\zeta = ZL$). Also, to simplify the calculations, the following assumptions were made (Neugebauer, 1975: 116):

1. *ΔMNM′* and *ΔMLL′* are plane triangles.
2. The altitude circle, *ZLA* and *ZL′A′* through *L* and *M* are parallel.
3. Parallax p calculated at *L* has the same value at *M*.

Under the above assumptions, in Figure 2 we obtain $\angle MM'N = \angle ML'L = \gamma$ so the longitude component (p_λ) and latitude component (p_β) of the parallax can be calculated using the following equations.

$$p_\lambda = p\cos\gamma,\quad p_\beta = p\cos\gamma \qquad (3)$$

After determining the altitude parallax (p) using Equation (3) above, the longitude and latitude parallax between the true and apparent positions of the celestial body must be calculated.

Table 1 below provides an example of the calculation process using Equation (3). It shows the calculation results of longitude and latitude parallax observed in Rhodes (Clime IV, 36°) on the day when the Sun enters Scorpio (♏) 0° at an ecliptic longitude of 240°.

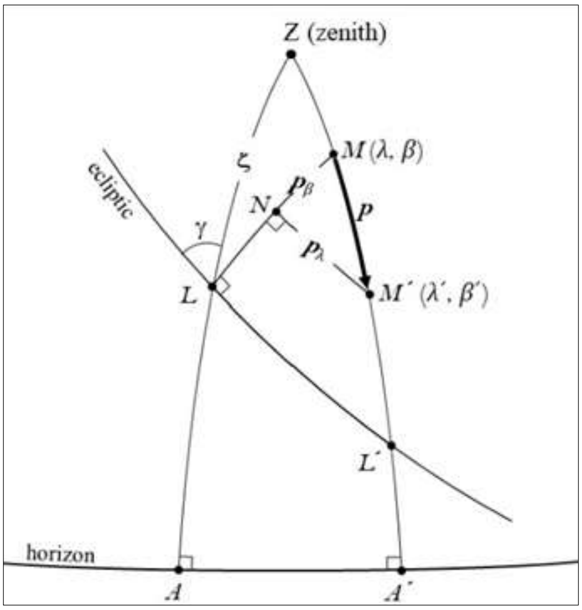

Figure 2: Longitude and latitude parallax of the Moon (after Neugebauer, 1975: 1238).

Table 1 is a Handy Table that calculates the longitude and latitude parallaxes at the time of conjunction from sunrise to sunset on the day the Sun enters Scorpio (♏) 0°. The table is divided into two parts—before and after noon—based on the zenith distance of the Sun at noon, showing the variations in longitude and latitude parallax accordingly.

From the Table 1, it can be observed that the zenith distance of the Sun is greatest at sunrise and sunset when the Sun is on the horizon (i.e., ζ = 90°), and smallest at noon when the Sun reaches the meridian (ζ = 47°40′). Meanwhile, the latitude parallax continuously increases from sunrise to sunset, whereas the longitude parallax

Table 1: Calculation table of longitude and latitude parallaxes for the day when the Sun enters Scorpius (♏) 0° in the region of Rhodes (after Neugebauer, 1975: 992).[4]

Cl.IV ♏°	z [°]	p [°]	γ [°]	$\cos\gamma$	$\sin\gamma$	$p_\lambda = -p\cos\gamma$ [°]†	$p_\beta = -p\sin\gamma$ [°]†	Handy Tables p_λ [°]	Handy Tables p_β [°]
–5;25^{h}	90;00	0;50,43	E.164;07	–0;57,43	0;16,25	+0;48,47	–0;13,53	0;49	0;14
–5	85;05	50,35	162;28	57,13	18,05	48,14	15,14	48	15
–4	73;55	48,59	157;51	55,34	22,37	45,22	18,28	45	18
–3	63;48	45,51	150;34	52,15	29,29	39,56	22,32	40	22
–2	55;26	42,08	140;20	46,11	38,18	32,26	26,54	32	27
–1	49;42	39,05	126;50	35,58	48,01	23,26	31,17	23	31
n	47;40	37,54	111;00	21,30	56,01	13,35	35,23	14	35
+1	49;42	39,05	W.95;10	5,24	59,45	3,31	38,55	4	39
2	55;26	42,08	81;40	+0;08,42	59,22	–0;06,06	41,41	6	42
3	63;48	45,51	71;26	19,06	56,53	14,36	43,28	15	44
4	73;55	48,59	64;09	26,10	54,00	21,21	44,05	21	44
5	85;05	50,35	59;32	30,25	51,43	25,39	43,37	26	44
5;25	90;00	50,43	57;53	31,54	50,49	26,58	42,57	27	43

† Red numbers in Table 1: Corrected numbers by the recalculation in this study

decreases from sunrise until noon and then increases again after noon. However, this result is particularly applicable on the day the Sun enters Scorpio, and the values change depending on the solar position.

The following Table 2 is a comparison table of the longitude and latitude parallaxes at a latitude of 30° in Theon's Handy Table, Ulugh Beg's *Zij* (1440), al-Kashi's *Khaqani Zij* (1420), and *Surya Siddanta* (773),[5] which are astronomical texts from the eighth to the fifteenth centuries (Kennedy, 1956).

In Table 2 below, the vertical axis represents the conjunction time (divided into before and after noon times). The horizontal axis on the top and bottom depicts the position of the Sun along the ecliptic in relation to the 12 zodiac signs. Below the horizontal axis, it shows from left to right the signs from Aries (♈, 0 sign) at the spring equinox to Cancer (♋, 9th sign) and Capricorn (♑, 3rd sign). Above the horizontal axis, it shows from right to left the signs from Capricorn (♑, 3rd sign) to Cancer (♋, 9th sign) at the autumn equinox (♎, 6th sign). Directly beneath each zodiac sign in the upper part of the table are cells that record the corresponding parallax and longitude for each sign, with the longitude and latitude for the day of the solar eclipse corresponding to the time of the conjunction for that sign.

Kennedy (1956), who made the parallax comparison table (Table 2) by investigating Theon's Handy Table and Medieval Islamic *zijes*, explained that the sine theorem, which was introduced through the *Sabi Zij* (900) by the Muslim astronomer al-Battani (858–929), replaced spherical trigonometry with Ptolemy's plane parallax triangle. He argued that the parallax theory system, which originated from Hellenistic astronomy, was more precisely refined through the sine theorem invented by Muslims, and thus gained an advantage over the competing Hindu theory.

In addition, among the *zijes* presented above, the *Khaqani Zij* is a typical Islamic astronomical handbook that follows the direct tradition of Ptolemy and Theon's Handy Tables, and it used the Law of Sines and spherical trigonometry for longitude and latitude parallax calculations. After the *Khaqani Zij*, *Ulugh Beg Zij*, based on observations from Ulugh Beg Observatory, used similar methods to the *Khaqani Zij* but differed in the parameters used for the calculations. These parameters include the distance from the Earth to the Sun and the Moon, the obliquity of the ecliptic (ε), the orbital eccentricity (e), and the longitude of the Apogee (λ_A). It is also evaluated that the accuracy of the parallax tables in the *Ulugh Beg Zij* is much higher than that of the *Khaqani Zij*.

In summary, the values of longitude and latitude parallax presented in Table 2 are parallax values for the same latitude of 30°, but they show different results due to the use of different calculation methods and parameters in each *zij* (Lee, 2024).

3.2 The Parallax Correction Tables of Jamal al-Din type

Jamal al-Din was an astronomer from Bukhara, who served as the first Director of the Islamic Observatory established in Shangdu (the summer capital) in 1271. He was commissioned by Kublai Khan to compile the Persian *zij* in 1276. Although this *zij* has not survived, the tables for 'planetary mean motion' in the *Sanjufini Zij* (1366) records that the table was based on the observations of Jamal, and thus the *zij* is referred to as the *Jamal al-Din Zij* (van Dalen, 2002: 338).

The *Jamal al-Din Zij* is an Islamic *zij* compiled in the early Yuan Dynasty for Muslims from the

Table 2: Comparison table of longitude and latitude parallaxes for regions of $\phi = 30°$ in Theon's Handy Table and Medieval Islamic *zijes* (after Kennedy, 1956: 33–35).

Hr	*Zij*†	Cancer (♋)		Leo (♌)		Virgo (♍)		Libra (♎)		Scorpius (♏)		Sagittarius (♐)		Capricorn (♑)	
		p_λ	p_β	p_λ	p_β	p_λ	p_β	p_λ	p_β	p_λ	p_β	p_λ	p_β	p_λ	p_β
7	Ulugh Beg	43;04	28;00												
	Theon	43	28												
	al-Kashi	50	28	54	20	57	09								
	Sur. Siddn.														
6	Ulugh Beg	44;36	22;37	48;42	14;06	50;53	07;52	51;02	05;49						
	Theon	44	23	48	14	51	08	51	06						
	al-Kashi	50	27	54	18	57	09	56	04	57	06	54	12		
	Sur. Siddn.	42;29	22;23												
5	Ulugh Beg	43;15	19;05	46;33	10;36	48;31	06;17	49;48	06;26	49;58	10;34	47;53	18;33	44;02	26;27
	Theon	43	19	46	11	48	06	49	07	49	10	48	19	43	28
	al-Kashi	50	26	53	16	56	08	55	04	57	07	54	13	53	22
	Sur. Siddn.														
4	Ulugh Beg	38;51	14;49	43;27	08;18	43;41	05;52	45;53	08;15	46;46	13;58	44;32	22;57	38;33	32;30
	Theon	38	15	41	08	42	06	45	08	46	14	45	23	38	32
	al-Kashi	49	21	51	13	54	07	55	05	56	08	54	16	50	26
	Sur. Siddn.	36;36	13;44			41;10	05;17			44;13	13;08			36;32	30;58
3	Ulugh Beg	31;49	11;08	33;57	06;19	36;28	06;40	39;26	11;06	41;05	17;55	38;25	27;40	31;39	36;19
	Theon	31	11	33	06	35	07	39	11	40	18	38	28	31	36
	al-Kashi	44	16	47	09	48	06	54	06	54	10	53	20	45	30
	Sur. Siddn.														
2	Ulugh Beg	22;32	08;18	24;22	05;54	27;19	08;38	27;58	14;43	32;42	23;07	29;55	32;05	22;25	39;13
	Theon	22	08	24	06	27	08	31	14	33	23	30	32	22	39
	al-Kashi	34	11	36	07	40	06	47	08	48	15	45	26	36	35
	Sur. Siddn.	21;07	07;25			25;34	07;52			32;17	21;40			21;07	37;05
1	Ulugh Beg	11;40	06;31	13;23	06;42	13;51	11;38	21;50	19;12	22;55	27;48	19;40	35;56	11;36	41;01
	Theon	11	07	13	07	17	11	21	19	23	28	19	36	11	41
	al-Kashi	19	07	21	06	25	08	34	13	38	22	34	31	20	38
	Sur. Siddn.														
0 [noon]	Ulugh Beg	00;00	05;55	01;52	08;41	05;49	15;27	10;23	23;51	12;08	32;12	08;21	38;57	00;00	41;39
	Theon	00	06	02	09	06	15	11	24	12	32	08	39	00	42
	al-Kashi	00	05	04	08	08	14	16	20	17	29	15	37	00	41
	Sur. Siddn.	00;00	03;32			05;30	14;17			11;31	30;18			00;00	39;25
1	Ulugh Beg	11;40	06;31	09;24	11;39	05;00	19;52	00;19	28;31	01;07	36;01	02;59	40;51	11;36	41;01
	Theon	11	07	09	11	05	20	00	28	01	36	03	41	11	41
	al-Kashi	19	07	14	12	07	20	00	28	00	36	06	40	20	38
	Sur. Siddn.														
2	Ulugh Beg	22;32	08;18	19;43	15;29	14;50	24;31	10;16	32;50	09;20	38;57	13;54	41;36	22;25	39;13
	Theon	22	08	20	16	14	24	10	33	09	39	14	42	22	39
	al-Kashi	34	11	29	19	21	26	15	35	15	37	23	40	36	35
	Sur. Siddn.	21;07	07;25			15;36	22;59			08;24	36;43			21;07	37;05
3	Ulugh Beg	31;49	11;08	28;15	19;51	23;00	29;04	18;46	36;25	19;03	40;34	23;27	41;07	31;39	36;19
	Theon	31	11	28	20	22	29	18	37	18	40	23	41	31	36
	al-Kashi	44	16	38	24	29	32	26	39	27	38	36	39	45	30
	Sur. Siddn.														
4	Ulugh Beg	38;51	14;49	34;27	25;36	28;28	33;14	25;15	39;11	25;56	41;52	30;59	39;22	38;38	32;30
	Theon	38	15	34	24	28	33	25	39	25	41	30	40	38	32
	al-Kashi	49	21	39	29	34	36	33	40	34	38	41	36	50	26
	Sur. Siddn.	36;36	13;44			26;41	31;30			24;07	39;22			36;32	30;58
5	Ulugh Beg	43;15	19;05	37;57	28;59	32;18	36;44	29;15	40;49	30;34	40;54	35;51	36;46	44;02	26;27
	Theon	43	19	37	29	32	36	29	41	29	40	35	37	43	28
	al-Kashi	50	26	41	33	35	38	34	41	35	39	42	34	53	22
	Sur. Siddn.														
6	Ulugh Beg	44;36	22;37	38;27	33;03	32;48	39;15								
	Theon	44	23	37	33	32	40	30	41						
	al-Kashi	50	27	42	35	36	39	34	42	36	39	42	33		
	Sur. Siddn.	42;29	22;23												
7	Ulugh Beg	43;04	28;00												
	Theon	43	28												
	al-Kashi	50	28	42	36	36	40								
	Sur. Siddn.														
		(Cancer, ♋)		(Gemini, ♊)		(Taurus, ♉)		(Aries, ♈)		(Pisces, ♓)		(Aquarius, ♒)		(Capricorn, ♑)	

†Ulugh Beg: *Ulugh Beg Zij* (1440), Theon: *Theon's Handy Table* (450), al-Kashi: *Khaqani Zij* (1420), Sur. Siddn.: *Surya Siddanta* (773)

Western Regions. However, after the fall of the Yuan Dynasty and the rise of the Ming Dynasty, Emperor Taizu (太祖) (Zhu Yuanzhang, 1328–1398, reigned 1368–1398) ordered the translation of Jamal al-Din's Persian *zij* into Chinese. In 1383, Mashayihei (馬沙亦黑), who was then the Director of the Islamic Astronomical Bureau (回回欽天監), along with his younger brother Mahama (馬哈麻) and other Muslim scholars, translated Jamal al-Din's Persian *zij* into Chinese. However, Emperor Taizu later issued another decree, stating that the Islamic calendrical system in the *Jamal al-Din Zij* differed from the Chinese calendrical system and had to be unified to ensure compatibility.

Following this order, Chinese astronomers learned about the calculation methods from the Islamic astronomers of the *Huihui Qintianjian* (回回欽天監) for several years. After mastering Islamic methods, they revised and adapted the *Jamal al-Din Zij* into a Chinese–Islamic calendric-cal system. This revised version, compiled in 1396, is the *Huihui-lifa* (回回曆法) of the Ming Dynasty (Lee et al., 2018; Lee, 2020).[6]

During the King Sejong reign (1432–1450), the Chinese *Huihui-lifa*, was transmitted to Joseon Korea, and recompiled as the *Chiljeongsan Oepyeon* (七政算外篇) which included a sunrise and sunset time table adjusted for the latitude of Seoul. Therefore, the Chinese *Huihui-lifa*, Korean *Chiljeongsan Oepyeon* and Tibetan *Sanjufini Zij* were all considered *zijes* originated from the Persian *zij* of Jamal al-Din. As evidence, they commonly include a parallax correction table corresponding to the latitude of Nanjing (φ = 32°). Also, recent research has shown that, apart from typographical errors in the printed versions of *zijes* of the Jamal al-Din type, the contents of the entries for the parallax for longitude, latitude, and time difference within this latitude (φ = 32°) are entirely identical (Li, 2016).

3.2.1 Parallax Correction Table in the *Chiljeongsan-Oepyeon*

The parallax correction table contains parallax values for the position of the Moon observed at conjunction time, which appear differently depending on the season and time (i.e., the parallax correction values for longitude, latitude, and time difference at conjunction time). Figure 3 is a copy of a parallax correction table (φ = 32°) in the *Chiljeongsan Oepyeon*. Table 3 is a parallax correction table translated into English, based on the annotated version of the *Chiljeongsan Oepyeon* (Yu et al., 1974).

The parallax correction table in Figure 3 and Table 3 presents the parallax values for longitude, latitude, and conjunction time, using the zodiac sign representing the solar longitude (λ) and the time from midnight to conjunction time as two input parameters (ibid.). Here, the vertical axis represents the parameter indicating the solar position at conjunction time in zodiac signs, while the upper and lower parts of the horizontal axis represent the parameter indicating the conjunction time occurring between sunrise and sunset.

Since the sunrise and sunset times, which indicate the length of the day, are symmetrical around the vernal and autumnal equinoxes, the left vertical axis provides correction values for longitude, latitude, and time difference (時差) for the seven zodiac signs around the vernal equinox, while the right vertical axis provides corresponding values for the seven zodiac signs centered on the autumnal equinox. If the solar longitude falls within the right-side seven zodiac signs, the conjunction time should be taken from the upper row of the horizontal axis. Conversely, if it falls within the left-side seven zodiac signs, the conjunction time should be taken from the lower row of the horizontal axis.

Thus, by knowing the longitude and latitude of the Sun at conjunction time as well as the time interval from midnight to conjunction time, one can easily determine the actual position and timing of a solar eclipse using the parallax table by applying the correction values for longitude, latitude, and time difference recorded at the corresponding position.

Yu et al. (1974) noted that such a double-entry table was not present in the *Almagest* and concluded that it was instead developed by the Islamic astronomers, who inherited its tradition.

3.2.2 The Parallax Correction Tables of Jamal al-Din Type and their Common Characteristics

The parallax correction tables of Jamal al-Din type show the following same characteristics as the parallax tables in the *Chiljeongsan-Oepyeon* (七政算外篇) presented in Figure 3 and Table 3:

1. Parallax correction values for the solar longitude and latitude, and time difference represent a double argument for the solar longitude (of zodiac signs) and conjunction time (counted from the midnight).
2. The correction values for solar longitude and latitude are symmetrically with respect to noon in Cancer (3rd sign) and Capricorn (9th sign).
3. All *zijes* of the Jamal al-Din type include parallax correction tables corresponding to the latitude of Nanjing (32°).

Figures 4 and 5 present extant parallax correction tables (especially corresponding to latitude of 32°) preserved in the *Sanjufini Zij* (Arabic 6040) from the Bibliothèque Nationale de France and in manuscript MS C 2460 from the Institute of Oriental Studies in Russia (Li, 2016).

These distinctive features of the parallax tables are commonly found in the Jamal al-Din-type *zijes* such as the Chinese *Huihui-lifa*, the Korean *Chiljeongsan-Oepyeon*, and the Tibetan *Sanjufini Zij*, etc. As shown in Figures 3, 4, and 5, Jamal al-Din-type *zijes* have the same tabular layout. However, when comparing the parallax values among them, misprinted parallax values are found in each *zij.* By comparing the color versions of the parallax correction tables in Figures 3 and 5, it is found that not only the tabular layout of the tables but also the colors of the recorded values are identical.

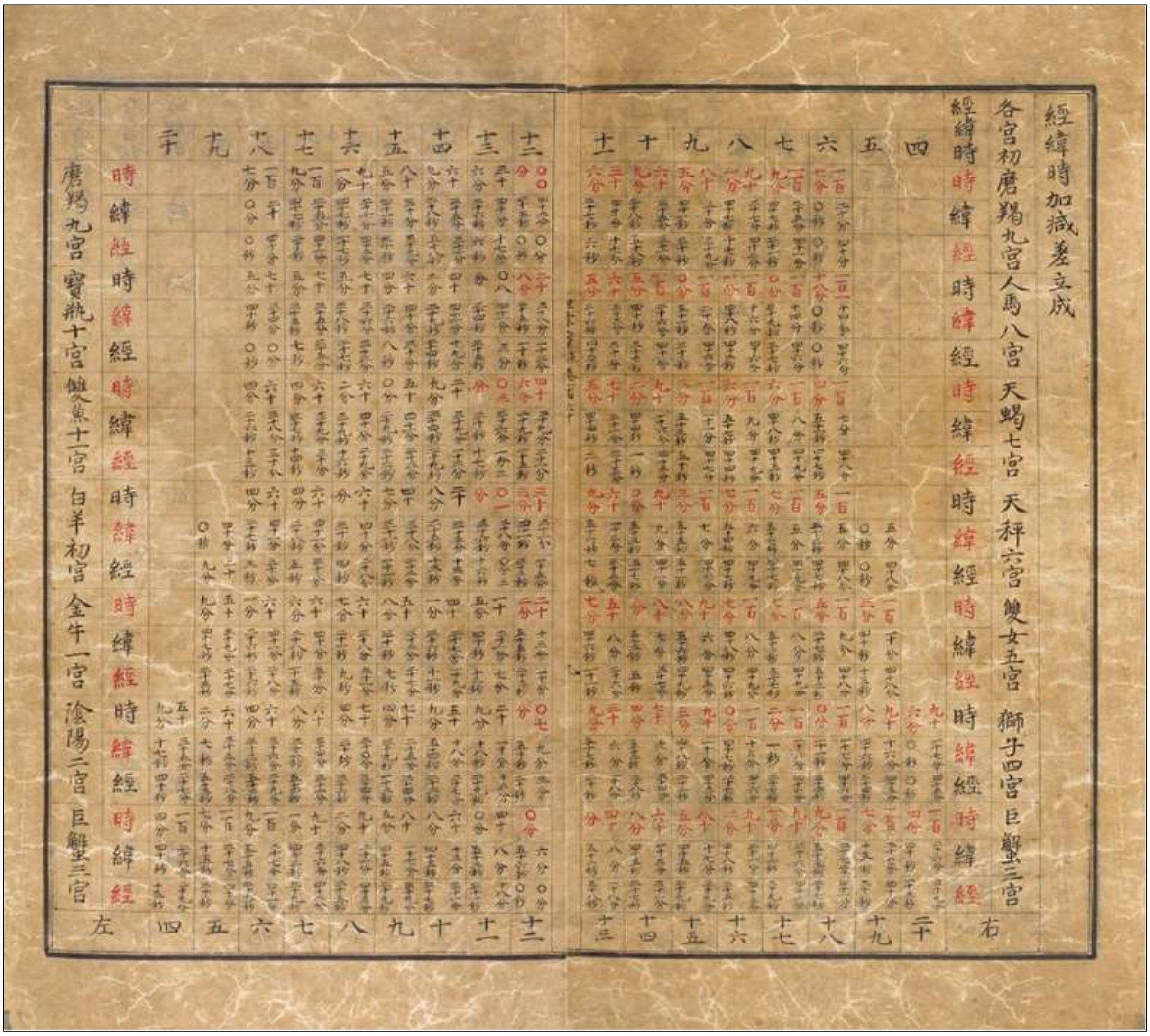

Figure 3: Manuscript of the parallax correction table (φ = 32°) in the *Chiljeongsan-Oepyeon* preserved in the Gyujanggak (奎章閣) Institute for Korean Studies, Seoul National University (七政算外篇, 鼎足山本: 奎 12722).

The misprinted values in each parallax table, presented in Table 4, were identified using the following method:

1. In the case of Cancer (♋, 3rd sign) and Capricorn (♑, 9th sign), where the parallax values at the conjunction time are symmetric before and after noon with respect to the meridian, the parallax values are found to be asymmetric.
2. In the case of signs where the parallax values tend to decrease or increase according to the conjunction time, the values do not follow the expected trend of decrease or increase, and instead, different values are recorded.
3. In cases like Aquarius (♒ 10th sign) at 6 o'clock, where the parallax value is recorded as 0 minutes 0 seconds, which has no relation to the sign or time, it is considered a misprint.
4. The misprints identified in the above three cases are ultimately confirmed as misprints by comparing them with the parallax values recorded in each *zij*.

Table 4 is a comparison table that collects and compares the misprinted values for the longitude, latitude and time difference recorded in each *zij* of the Jamal al-Din type. As seen in Table 4, the parallax correction values recorded in the *zijes* of Jamal al-Din type contain several printing errors in each *zij*. However, a printing error recorded at the same position with the same value across all *zijes* is the value of the time difference for "Aquarius, 10^s (6^h)". The original value is 35 minutes, but instead, the incorrect value of 00′00 is recorded. Such misprints are found in all *zijes* compiled at different times and places.

Table 3: Parallax correction table (φ = 32°) in the *Chiljeongasn-Oepyeon* (after Lee, 2024; Yu et al., 1974).

Zodiac / Parallax		20^h	19^h	18^h	17^h	16^h	15^h	14^h	13^h	12^h	11^h	10^h	9^h	8^h	7^h	6^h	5^h	4^h	Zodiac / Parallax	
Capricorn (9) ♑	Time			107^m	109^m	91^m	85^m	69^m	36^m	00^m	36^m	69^m	85^m	91^m	109^m	107^m			Time	Capricorn (9) ♑
	Lat.			20;00	25;47	27;47	30;41	35;28	40;26	42;25	40;27	35;28	30;41	27;47	25;47	20;00			Lat.	
	Long.			40;00	41;20	42;17	39;30	31;58	17;06	00;00	17;06	31;58	39;30	42;17	42;21	40;00			Long.	
Aquarius (10) ♒	Time			75^m	75^m	75^m	64^m	42^m	08^m	28m	65^m	105^m	100^m	101^m	100^m	118^m			Time	Sagittarius (8) ♐
	Lat.			34;40	35;33	36;38	40;26	41;29	41;24	38;25	32;36	26;40	20;51	16;48	143;9	14;00			Lat.	
	Long.			00;00	35;07	33;27	30;08	19;24	03;35	13;10	30;43	40;37	46;30	46;42	46;37	46;00			Long.	
Pisces (11) ♓	Time			64^m	64^m	62^m	50^m	29^m	03^m	46^m	75^m	92^m	103^m	106^m	106^m	104^m			Time	Scorpio (7) ♏
	Lat.			38;26	39;37	40;39	41;29	39;34	36;30	29;29	21;44	15;44	11;53	09;57	08;48	07;51			Lat.	
	Long.			30;13	30;14	29;16	23;22	12;29	01;27	21;15	35;02	43;01	47;50	49;44	49;45	48;47			Long.	
Aries (0) ♈	Time			64^m	64^m	60^m	47^m	28^m	01^m	33^m	69^m	90^m	103^m	107^m	107^m	105^m			Time	Libra (6) ♎
	Lat.		40;00	41;27	41;28	40;30	38;31	35;25	28;36	21;41	13;56	09;53	07;55	06;54	05;57	05;53	05;00		Lat.	
	Long.		29;00	30;03	30;05	28;04	22;11	13;13	00;36	15;32	32;07	41;57	47;51	49;47	49;47	48;47	48;00		Long.	
Taurus (1) ♉	Time		59^m	61^m	66^m	67^m	58^m	41^m	15^m	22^m	57^m	80^m	98^m	107^m	107^m	105^m	103^m		Time	Virgo (5) ♍
	Lat.		39;47	41;26	40;28	38;21	32;30	27;36	21;40	13;55	08;46	07;52	06;56	08;48	08;52	09;27	10;40		Lat.	
	Long.		27;33	28;26	30;15	31;09	27;07	19;11	07;13	10;23	26;11	37;05	45;49	49;46	49;57	48;53	48;13		Long.	
Gemini (2) ♊	Time	59^m	62^m	64^m	68^m	74^m	74^m	59^m	29^m	07^m	39^m	74^m	93^m	100^m	102m	100^m	98^m	96^m	Time	Leo (4) ♌
	Lat.	35;17	36;07	36;34	34;31	30;30	25;39	18;54	10;18	09;50	06;57	07;52	10;48	13;47	16;01	17;00	16;46	17;00	Lat.	
	Long.	27;40	28;53	29;52	31;54	34;36	34;12	27;13	13;29	03;20	18;20	34;38	43;22	46;27	47;27	46;34	45;34	45;00	Long.	
Cancer (3) ♋	Time	104^m	107^m	109^m	91^m	92^m	85^m	68^m	40^m	00^m	40^m	68^m	85^m	92^m	91^m	109^m	107^m	104^m	Time	Cancer (3) ♋
	Lat.	26;40	27;15	27;55	26;42	21;48	17;45	13;45	08;52	06;56	08;52	13;45	17;45	21;48	26;42	27;55	27;15	26;40	Lat.	
	Long.	29;13	40;20	41;27	42;29	43;25	39;29	31;37	18;31	00;00	18;38	31;37	39;29	43;25	42;29	41;27	40;20	39;13	Long.	
Left		4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	12^h	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	Right	

* The incorrect values recorded in the original text are highlighted in yellow cells, and the true values are as follows ; (Cancer 3^s, 4^h ; p_λ= 29;13 → 39;13), (Aquarius 10^s, 6^h; p_λ= 00;00 → 35;00), (Taurus 1^s, 9^h; p_β= 32;20 → 33;30), (Cancer 3^s, 11^h; p_λ= 18;31 → 18;38).

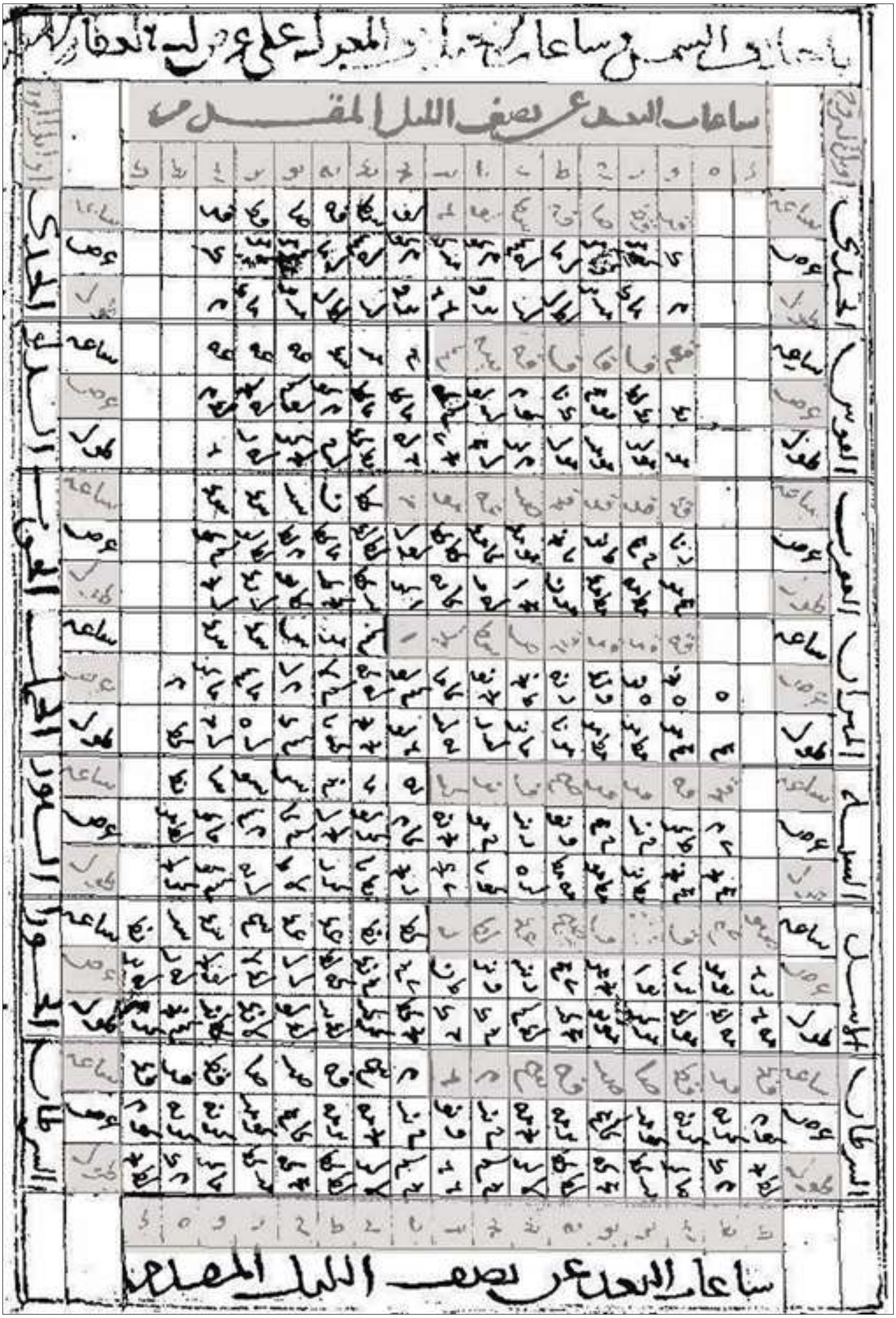

Figure 4: Manuscript of the parallax correction table (φ = 32°) in the *Sanjufini Zij* (Bibliothèque Nationale de France, Arabic 6040, fol.29.) (after Li, 2016: 33).

Figure 5: Persian manuscript of the parallax correction table (φ = 32°) as known to be titled 24 folios of Astronomical Tables 6 (Institute of Oriental Studies, St. Petersburg, MS C 2460) (after Li, 2016: 34).

Table 4: Comparison table of incorrect parallax values for the longitude, latitude, and time difference recorded in each *zij* of Jamal al-Din type (yellow cells indicate incorrect values recorded in the original *zij*) (after Lee, 2024: 61).

Parallax	Zodiac Signs (Conjunction)	True value	*CJOP*†	*Sanjufini Zij*	MS C 2460	*HHLF*‡
p_λ	Cancer 3^s (04^h)	39′ 13″	29′ 13″	39′ 13″	39′ 13″	39′ 13″
	Cancer 3^s (11^h)	18′ 38″	18′ 31″	18′ 38″	18′ 38″	18′ 31″
	Leo 4^s (08^h)	46′ 27″	46′ 27″	46′16″	46′ 27″	46′ 27″
	Leo 4^s (14^h)	27′ 22″	27′ 22″	27′ 22″	27′ 13″	27′ 22″
	Libra 6^s (08^h)	49′ 47″	49′ 47″	49′ 47″	04′ 47″	49′ 47″
	Aquarius 10^s (6^h)	35′ 00″	00′ 00″	00′ 00″	00′ 00″	00′ 00″
p_β	Taurus 1^s (09^h)	33′ 30″	32′ 30″	33′ 30″	33′ 30″	33′ 30″
	Cancer 3^s (06^h)	27′ 55″	27′ 55″	27′ 55″	27′ 35″	27′ 55″
	Libra 6^s (08^h)	06′ 54″	06′ 54″	06′ 54″	66′ 54″	06′ 54″
p_τ	Leo 4^s (13^h)	29^m	29^m	29^m	59^m	29^m

† *CJOP*: *Chiljeongsan-Oepyeon* (七政算外篇)
‡ *HHLF*: *Huihu-lifa* (回回曆法) of *Qizheng-tuibu* (七政推步) version

There is a similar case where a peculiar irregularity in Mercury's first equation in Al-Biruni's *Masudi Qānūn* (i.e., *Masudi Zij*, 1030) is identically found in all *zijes* of the Jamal al-Din type (Yano, 2013).[7] Although the *Jamal al-Din Zij* no longer exists, the Korean *Chiljeongsan Oepyeon*, Chinese *Huihui-lifa* and Tibetan *Sanjufini Zij* (Arabic 6040) and *MS* C 2460 all serve as strong evidence that they belonged to the *zijes* of Jamal al-Din type.

3.2.3 Comparison of Parallax Correction Tables in the *Zijes* of Theon and Jamal al-Din Type

In the Islamic literature, the *zij* is interpreted as an astronomical table or an astronomical handbook. In other words, the characteristic of Islamic *zijes* is that they are all written in a tabular layout, where the necessary values for calculations are presented in tables, and calculations are performed using the values listed in these tables. However, the layout and content of the tables differ among the *zijes*, each having its own characteristics and origins. Earlier, we examined the parallax correction tables following the tabular format of Theon's Handy Tables and the Jamal al-Din type. If we refer to these as the Theon and Jamal al-Din series, the characteristics of the *zijes* belonging to each series are as follows.

1) The *zijes* of Theon type consider the parallax for longitude and latitude, while the *zijes* of Jamal al-Din type consider the parallax for longitude, latitude and time difference.
2) The *zijes* of Theon type represent the time from noon to conjunction (中前中後分) on the vertical axis, and represents solar longitude on the horizontal axis. In contrast, the *zijes* of the Jamal al-Din type represent the solar longitude on the vertical axis and time from midnight to conjunction (子正之合朔分) on the horizontal axis.

In other words, in the parallax correction tables, the *zijes* of the Jamal al-Din type include the parallax for time difference (時差) in addition to longitude and latitude, whereas the Theon type only considers the parallaxes for longitude and latitude. Also, the content of the vertical and horizontal axes for the Sun's ecliptic longitude and conjunction time shows a difference in position between the Theon and Jamal al-Din types, with the axes being placed in opposite positions. Additionally, for the time of solar eclipses, in the Theon type, the conjunction time is divided into before and after noon based on the noon time, whereas in the Jamal al-Din type, the time is presented as the interval from midnight to conjunction.

The Figure 6 is a flow chart showing that the parallax theory of Greek and Hindu astronomy and derived from Hellenistic astronomy were transmitted to the Islamic world and China etc., and subsequently divided into *zijes* of the Theon and Jamal al-Din type.

3.2.4 Comparison of Parallax Theory Adopted in the *Zijes* of Jamal al-Din and Shoushi-li Type

The parallax correction method introduced in the *Jamal al-Din Zij* is based on the parallax theory transmitted through the Greek *Almagest*, while also incorporating calculation methods originated from Hindu astronomy. On the other hand, the *Shoushi-li* uses the parallax calculation methods of Hindu origin transmitted during the Tang Dynasty (唐) but employs interpolation methods such as the Xianggeng Xiangsheng (相減相昇) Method, which is used in traditional Chinese astronomical texts, rather than trigonometric functions or spherical trigonometry. Additionally, the *Shoushi-li* uses the 365.2425 du(度) method and has its calculation epoch point at the winter solstice, not the spring equinox. The Chinese *Shoushi-li* is a calendrical system that adopted the parallax cor-

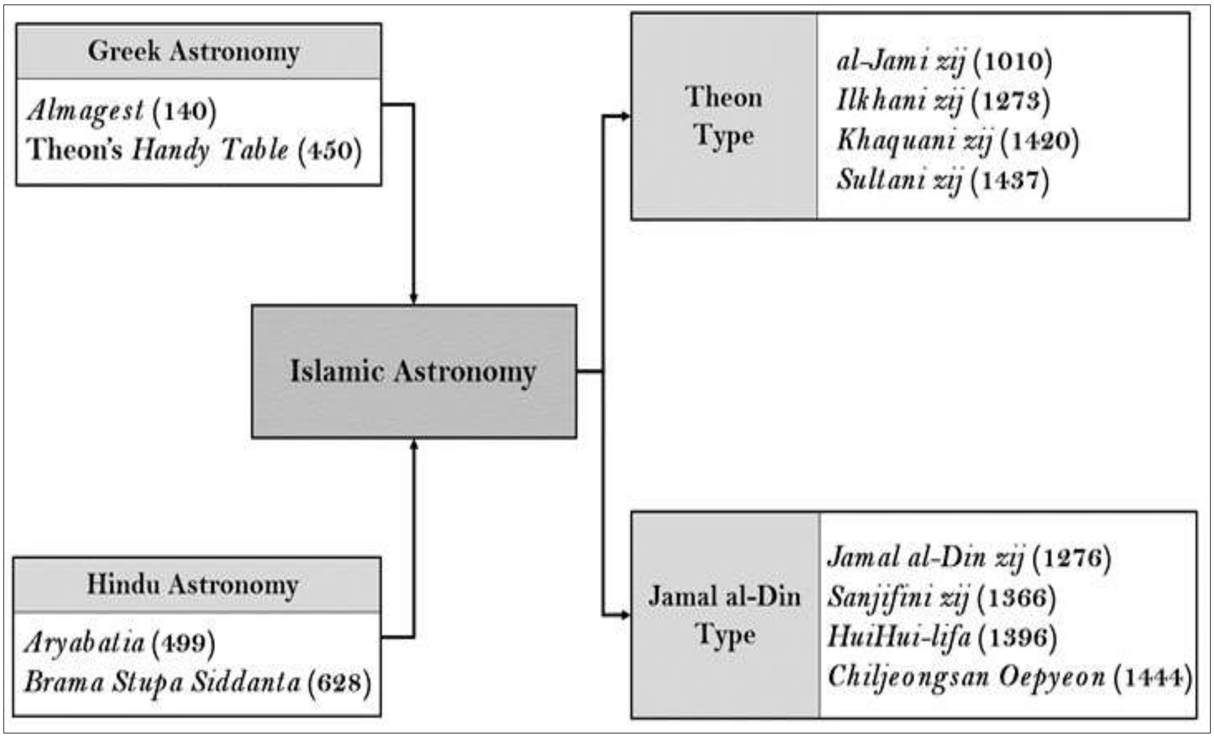

Figure 6: The transmission of parallax theory through the astronomical handbooks from the 2nd to 15th centuries.

rection method of the Tang Dynasty's *Xuanming-li* (宣明曆). While there are differences in terminology and the expression of formulae, it is fundamentally the same as the calculation method of the *Xuan-ming-li*. The parallax correction methods of the *Xuanming-li* were transmitted to China through the *Jiuzhi-li* (九執曆), which was compiled in 718 by the Indian astronomer Gautama Siddhartha (瞿曇悉達). The *Xuanming-li* divided the parallax which varies with the seasons from the parallax that changes according to the conjunction time, such as Qicha (氣差, p_λ) and Kecha (刻差, p_β), respectively. However, in the *Mingtien-li* (明天曆) compiled by Zhou Chong (周琮) of the Song(宋) Dynasty, it was renamed as an East–West difference (東西差, p_λ) and a North–South difference (南北差, p_β). Later the *Shoushi-li* also used East–West and North–South differences to represent longitude and latitude parallaxes, respectively. Figure 7 represents Qicha (氣差) and Kecha (刻差) for 24 solar term (i.e., 24 氣氣差 and 24 氣刻差) presented in the tables of the *Xuan-ming-li*.

Figure 8 is a flow chart showing that the parallax theory of Hindu origin was transmitted to Central Asia, Tibet, China, and Joseon through the *zijes* of the Jamal al-Din and Shoushi-li types compiled in Arabic (or Persian) and Chinese.

4 CONCLUDING REMARKS

Historically, Hellenistic astronomy was transmitted to Alexandria in Greece and India after the death of Alexander the Great in 323 BC. Ancient Hindu astronomy was introduced to the Arab world and to China in the eighth century, when Sanskrit works of Aryabhata and Brahmagupta were translated into Arabic and Chinese. In the ninth century, Theon's *Handy Table* was transmitted to Arabic as was the *Almagest*, which became widely known among Islamic astronomers. Parallax theories in the Medieval Islamic *zijes*, were influenced by both Greek and Hindu astronomy, and were classified into the Theon type and Jamal al-Din type. Meanwhile, parallax theory of Hindu origin was transmitted to China during the Tang Dynasty through the compilation of *Jiuzhi-li* based on the Brahmagupta's works. After the Tang Dynasty, the parallax theory of Hindu origin had a great influence on the parallax calculations of the Chinese calendar system.

Given the above historical background, this study examines and compares the parallax correction tables in the Medieval Islamic *zijes* of the Theon and Jamal al-Din types. Meanwhile, we investigate the parallax theories and their origins adopted in the Islamic *Jamal al-Din Zij* and Chinese *Shoushi-li* and compiled in the Mongol (Yuan) Empire other.

The *Jamal al-Din Zij* compiled in 1276 is an Islamic *zij* based on the *Almagest* of Ptolemy. However, it also contains mathematical and astronomical knowledge transmitted from India to the Arabs from the eighth century. In 1396, the Chinese–Islamic Calendar the *Huihui-lifa*, based on the *Jamal al-Din Zij*, was compiled by the Chin-

二十四氣刻差

冬至夏至	損益差	刻差積
初日	益二十	空
一日	益二十	二
二日	益二十	四
三日	益二十	六
四日	益二十	八
五日	益二十	十
六日	益二十	十二

小寒小暑	損益差	刻差積
初日	益二十	三十一
一日	益二十	三十三
二日	益二十	三十五
三日	益二十	三十七
四日	益二十	三十九
五日	益二十	四十二
六日	益二十	四十四

二十四氣氣差

冬至夏至	損益差	氣差積
初日	損卄六	二千三百五十
一日	損卄六	二千三百二十三
二日	損卄六	二千二百九十七
三日	損卄六	二千二百七十一
四日	損卄六	二千二百四十五
五日	損卄六	二千二百一十九
六日	損卄六	二千一百九十三

小寒小暑	損益差	氣差積
初日	損卄六	一千九百七十八
一日	損卄六	一千九百五十二
二日	損卄六	一千九百六
三日	損卄六	一千八百八十
四日	損卄六	一千八百五十三
五日	損卄六	一千八百二十七
六日	損卄六	一千八百一

Figure 7: Parallax correction tables in the *Waseda Xuanming-li* (after Li, 2016:31).

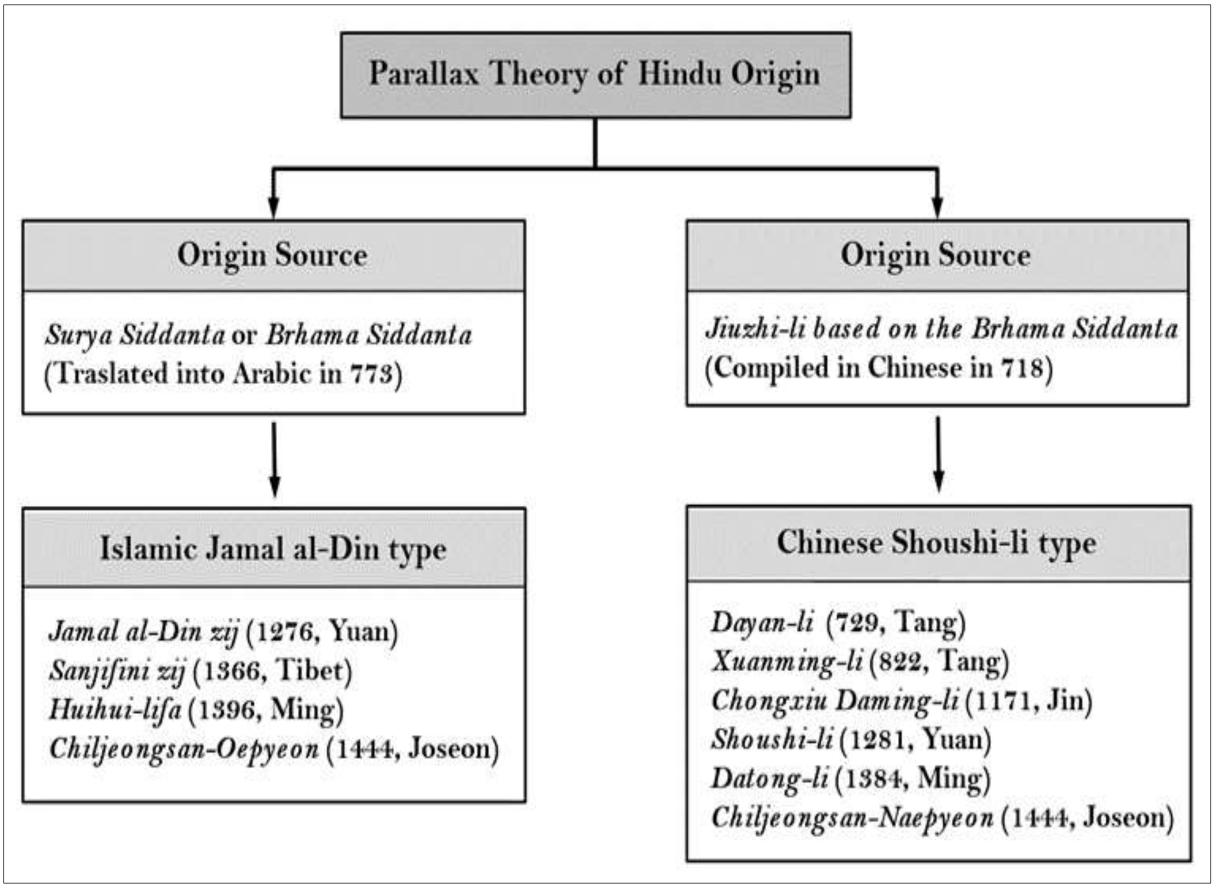

Figure 8: The transmission of parallax theory from Hindu origin through the astronomical handbooks of the *Jamal al-Din* and *Shoush-li* type.

Table 5: Similarities and disparities of the parallax correction methods adopted in the *Chiljeongsan-Oepyeon* of Jamal al-Din type and *Chiljeongsan-Naepyeon* of Shoushi-li Type (after Lee, 2024: 65).

	Jamal al-Din type CJOP(七政算外篇)	*Shoushi-li type* CJNP(七政算內篇)
Similarities		
Parallax' Components	- longitude p_λ - latitude p_β - time difference p_τ	- longitude p_λ - latitude p_β - time difference p_τ
Terms of Rahu & Ketu Meaning of Rahu Meaning of Ketu Usage of the Terms	- Descending node - Ascending node - Calculation of $\lambda_☽$ & $\beta_☽$	- Ascending node - Descending node - Calculation of nodal points
Disparities		
Calculation methods Calendrical epoch Angular system Solar position Time system Conjunction time	Spherical trigonometry Vernal equinox 360° 12 zodiac signs (12 宮) Midnight system Time from midnight (子正之合朔分)	Quadratic equation Winter solstice 365.2425 *du* (度) 24 solar terms (24 節氣) Midnight system Time from noon (中前中後分)

ese and Muslim astronomers at the Astronomical Bureau of the Ming Dynasty. In 1444, the Korean version of the Islamic calendar, the *Chiljeongsan-Oepyeon* based on the *Huihui-lifa*, was compiled by Royal astronomers of the Joseon Dynasty. Meanwhile, it is known that the *Sanjufini Zij* of Tibet, compiled in 1366, also originated from the *Jamal al-Din Zij*. A comparison of the parallax correction tables confirm that all these *zijes* of the Jamal al-Din type originated from Hindu astronomy.

On the other hand, the *Shoushi-li* compiled in 1281 is a traditional Chinese calendar, but the parallax correction method adopted the method of *Xuanming-li* (宣明曆). which originated from the *Jiuzhi-li* that was compiled by the Indian astronomer Gautama Siddhartha of the Tang Dynasty. In China, the parallax correction method of the *Xuanming-li* influenced the *Mingtien-li* (明天曆) of the Song Dynasty (宋), the *Chongxiu Daming-li* (重修大明曆) of the Jin Dynasty (金) and the *Shoushi-li* (授時曆) of the Yuan Dynasty. Furthermore, the *Datong-li* (大統曆) of the Ming Dynasty and the *Chiljeongsan Naepyeon* (七政算內篇) of the Joseon Dynasty continued to adopt this method. In this study, we also confirmed that all these calendars adopted the parallax theory of Hindu origin through a comparison of the parallax components used in the parallax correction tables.

Furthermore, Table 5 (which presents the similarities and disparities of the parallax correction methods of the Jamal al-Din and Shoushi-li Type) shows that the Islamic *Jamal al-Din Zij* and the Chinese *Shoushi-li* have a common origin in Hindu astronomy, but the characteristic of the parallax correction tables adopted in the Islamic and Chinese tradition calendars differ from one other.

This paper examined the transmission of astronomical tables by comparing the parallax correction tables included in the Medieval Islamic *zijes* and Chinese calendar systems. Also, we traced the routes of the parallax theory transmitted from ancient Greece and India to Joseon Korea via the Islamic world and China, respectively. Finally, we confirmed the astronomical exchanges and mutual influence between the East and the West.

5 NOTES

1. Arab astronomers were first introduced to Indian astronomy when Sanskrit works of Indian astronomers were translated into Arabic. In 773, the works of Aryabhata and Brahmagupta, along with the Sanskrit text of the *Surya Siddhanta*, were first translated into Arabic.
2. Brahmagupta is generally recognized as the Indian astronomer and mathematician par excellence. Brahmagupta states in his masterpiece, the *Brahma Sphuta Siddhanta*, that he compiled this work in Saka 550 (i.e., 628 A.D.) when King Vyaghramukha of the Capa Dynasty was ruling (Rao, 2009).
3. The *Jiuzhi-li* (九執曆): During the reign of Emperor Xuanzong (玄宗) of the Tang Dynasty, Gautama Siddhartha (who was a Director of the Imperial Astronomical Bureau) was ordered by the Emperor to translate Indian astronomical texts. As a result, he compiled the *Jiuzhi-li* in 718. Later, this Calendar was recorded as the 104th volume of the *Great Tang Kaiyuan Zhanjing* (大唐開元占經), a 120-volume treatise on divination and astronomy (Lee et al., 2014).
4. Table 1 is a recalculated version of the data calculated by Neugebauer (1975), with corrections to errors. Signs in the first column

(–5; 25^h) of $\sin\gamma$ and p_β were changed from – to +, and in the 8th column ($+1^h$), the signs were changed from + to –. In addition, the value 42.07 in the last column of line p_β was corrected to 42.57. Several unrounded values were also modified. The red numbers in Table 1 indicate the corrected errors.

5. A Persian manuscript with tables had been obtained in China and was described by A. Wagner (1882) when it was still in the library of the Pulkovo Observatory near St. Petersburg (van Dalen, 2002).

6. It is known that the *Huihui-lifa* was compiled in 1383 by Muslim astronomers, Mashayihei and Mahama from the Islamic Astronomical Bureau of the Ming Dynasty. However, 1383 is the year when they translated the Persian *zij* of Jamal al-Din into Chinese, while the year when it was revised and compiled into a Chinese–Islamic calendrical system was 1396 (epoch; the winter solstice of 1391). According to the records in the *Weidu Taiyang Tongjing* (緯度太陽通經, 1396) of Yuan Tong(元統) and the Chinese–Islamic Star table (i.e. the 黃道南北各像內外經緯度立成) included in the Korean *Chiljeongsan Oepyeon*, the *Huihui-lifa* was compiled in 1396 (Lee et al., 2018; Lee, 2020).

7. Although the *Masudi Qānūn* was unknown in Medieval Europe, the book was well read in the Eastern half of the Muslim world and further East. An example of its influence is the fact that a peculiar irregularity in Mercury's first equation table in the *Masudi Qānūn* is attested also in the Chinese text titled the *Huihui-lifa* (Yano, 2013).

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