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Symbolic learning of material constitutive laws

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Since the introduction of Hooke's Law, the development of material constitutive laws has progressed rapidly over the past century. However, their establishment remains reliant on phenomenological models that often inadequately describe material responses [1]. Symbolic learning, characterized by its high interpretability and flexibility driven by data, offers a novel approach to discovering these laws, significantly impacting key challenges such as capturing complex nonlinear material relationships and accelerating the discovery of constitutive laws.

The methods for discovering material constitutive laws using symbolic learning can be categorized into sparse regression, genetic programming, symbolic neural networks, Monte Carlo tree search (MCTS), and other hybrid approaches [2]. Sparse regression focuses on identifying constitutive law expressions from a pre-constructed library. Genetic programming combines and updates common analytical functions to fit given datasets. As universal function approximators, neural networks impose constraints of constitutive models via weak-form methods, suitable for large datasets. Additionally, combining these methods provides more flexible options for uncovering constitutive laws.

Figure 1 illustrates the process of discovering solid constitutive laws using symbolic learning methods and its primary approaches. Among these methods, Bomarito et al. [3] proposed a symbolic regression framework based on genetic programming for plasticity models, utilizing automatic multi-scale computational homogenization techniques to identify plastic yield potential from representative volume element (RVE) response data. For hyperelastic material models, Abdusalamov et al. [4] developed an automatic generation method for constitutive laws based on strain energy function. This method mitigates the high computational costs associated with RVE-based homogenization and can be extended to include additional mechanical parameters like temperature. Subsequently, the EUCLID method developed by Flaschel et al. [5] expanded the types of materials constitutive laws that symbolic learning can handle, encompassing elasticity, viscosity, plasticity, and arbitrary combinations thereof. The EUCLID method requires only the full-field displacement (or strain) data under various loading conditions and the net reaction forces at the boundaries as input, using sparse regression algorithms like LASSO to select simple subsets from a large candidate feature library established based on the theory of generalized standard materials to characterize Helmholtz free energy and dissipation potential, from which constitutive relationships and internal variable evolution can be derived.

With advancements in computational power, Fuchs et al. [6] combined traditional MCTS with deep reinforcement learning to enhance the exploration capability of high-dimensional parameter spaces while reducing the search space. For symbolic neural networks, KAN 2.0 method es-

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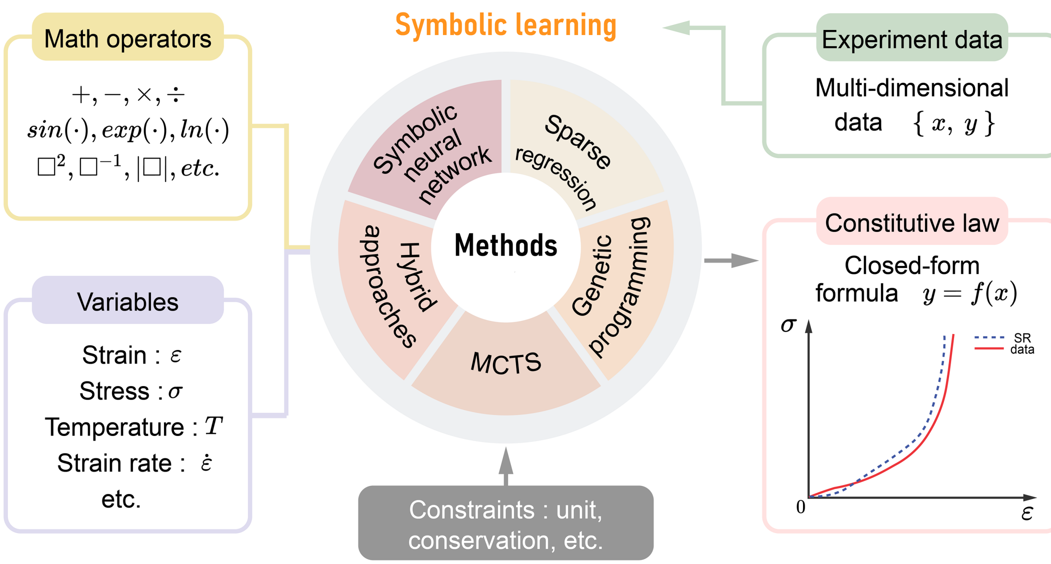

Figure 1 (Color online) Illustration of the methodology for discovering solid constitutive laws through symbolic learning techniques. Experimental data $\{x, y\}$ is input into these methods, including symbolic neural networks, genetic programming, sparse regression, Monte Carlo tree search (MCTS) and hybrid approaches. These methods utilize basic arithmetic operations (addition, subtraction, multiplication, division) and functions such as sine, exponential, natural logarithm, square, inverse, and absolute value to predict variables like strain ε, stress σ, temperature T, and strain rate $\dot{\varepsilon}$. The goal is to derive concise expressions for constitutive equations. Constraints such as unit consistency and conservation laws are applied throughout this prediction process to ensure physical validity.

tablished by Liu et al. [7] posits that any multivariate continuous function can be decomposed into combinations of univariate functions through addition operations. This approach allows for more flexible and direct incorporation of prior knowledge. However, it also faces challenges when dealing with noisy or sparse datasets due to its high dependence on empirical pruning strategies. In contrast, the Symbolic Physics Learner method proposed by Sun et al. [2] utilized physical prior knowledge to guide the search process by combining MCTS with context-free grammar. This approach reduces the need for manual tuning of specific pruning strategies and threshold parameters thereby enhancing the adaptability to noisy or sparse datasets.

Despite promising results achieved in certain scenarios, three main challenges persist: (1) ensuring discovered laws conform to physical principles; (2) minimizing human intervention; and (3) reducing the training difficulty while maintaining prediction accuracy. In the design of algorithms, it is crucial to incorporate hard constraints that ensure the predicted constitutive laws adhere to physical requirements. Early consideration of physical principles in the development of these algorithms is equally important. While human expertise can simplify the complexity of algorithm training and inference, it often limits the expressive power of the algorithms, affecting their ability to generalize to more complex and unknown material characteristics. Moreover, due to the inherent high computational cost of symbolic learning, existing methods still face significant training costs for complex expressions.

Developing efficient symbolic learning algorithms for material constitutive laws has become a key research area in establishing material constitutive models. However, current methods are predominantly data-driven. Future research should face the challenge of integrating physical principles such as dimensional constraints, symmetry, and conservation laws, reducing human involvement, and enhancing training efficiency, thereby advancing the application and development of symbolic learning in material science.

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