

# Scaling effect of breakthrough character in porous media

HU Xuejiao, DU Jianhua, LIU Xiang  
& WANG Buxuan

Department of Thermal Engineering, Tsinghua University, Beijing 100084, China

Correspondence should be addressed to Hu Xuejiao (e-mail: huxj@te.tsinghua.edu.cn)

**Abstract** Breakthrough phenomenon during fluids percolating through wet saturated porous layer is widely met in the study of heat and mass transfer in porous media. Breakthrough pressure (BP) is a characteristic pressure that indicates the intrinsic properties of seepage within porous media. Measuring results of BP for saturated narrow-sieved sand are reported here. The curve of BP varying with the height of porous layer was obtained. Experiment and analysis indicate that BP is independent of the height of particle packed layer if thick enough; however, when the height is less than a certain critical value, BP diminishes with the decrease of the height according to a universal scaling law.

**Keywords:** breakthrough pressure, porous media, percolation, boiling enhanced porous surface, scaling law.

Porous media are widely seen in nature and in the industrial applications. When fluids flow in porous media, they have to break through the resistance at the inlet/outlet under complex interfacial actions between phases<sup>[1]</sup>. Breakthrough character was used to interpret the classical capillary hysteresis phenomenon when multiphase fluids flow in porous media<sup>[2]</sup>. And the breakthrough pressure (BP) was taken as a micro property of porous media, which can be used to predict the permeability of porous rock<sup>[3]</sup>, unify fundamental experimental data of porous media such as the relationship of porosity and permeability<sup>[4]</sup>, capillary hysteresis<sup>[5]</sup>. Besides, it was also regarded as an index for wettability and diagenesis<sup>[6]</sup>. All these researches are based on a presupposition that the BP is a characteristic constant independent on the geometrical shapes and sizes of porous samples. For most application cases, in which the size of porous media is much larger than that of pores, the presupposition can be tenable. But we should notice that the BP is very small when the porous layer is thin enough. It is ambiguous to say whether the variation of BP with the size of porous media should be considered. Especially in the cases that the scale in the direction of fluid displacement is small, it is crucial to examine the variation of BP carefully. For sintered boiling enhanced porous surfaces, for example, gas bubbles

should break through the porous layer when bubbles can be observed and the boiling begins. In other words, the BP of the porous layer determines the degree of superheat for boiling. Therefore, it is important to study the variation of the BP with the height of porous layer for boiling heat transfer on porous surfaces. However, there are little reports on the relationship between the BP and the height of porous sample.

The BP of moisture saturated narrow-sieved sand was measured and a correlation curve of BP and the height of porous layer was obtained. Results indicate that BP is independent on the height if the particle packed layer is thick enough, however, when the height of the porous layer is less than the critical value, BP will decrease sharply with the height. The variation presents the effect of scale and follows the scaling law for continuous phase change.

## 1 Experiments

(i) Experiment method. The experimental equipment is illustrated in fig. 1. In all the test cases, fluids are air (non-wetting) and water (wetting) and the test porous sample is naturally packed sand. The sand should be added into the test cup with a known quantity. At the beginning, the whole experiment set should be filled with water. It must be ensured that there is no visible air bubbles in the discharge tube, the U-tube or the test cup. The water surface is beyond the sand surface with the height of about 10 mm. Churn up the sand in water and let it naturally sediment with a fairly flat surface. Open the discharge tube, the water level inside the test cup falls until it reaches the sample surface. If the discharge tube is still kept open, only the water level in the graduated tube drops because the air cannot pass through the saturated porous layer immediately. Thus, a negative pressure occurs and then increases in the buffer pool with the further flow of water from the discharge tube. When the water level in the graduated tube reached such a certain low level that the pressure difference is strong enough to let air

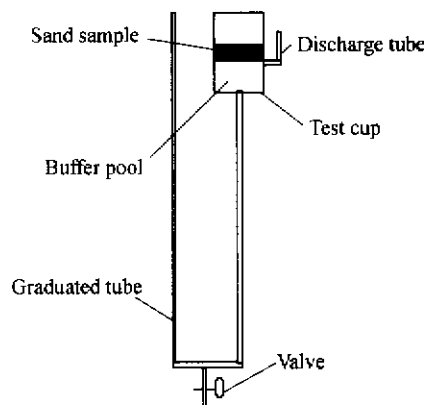


Fig. 1. Scheme of experiment set-up.

1) Hu, X. J., Zhou, L., Du, J. H. et al., Investigation on the relationship of the capillary hysteresis for unconsolidated porous media, *Transport in Porous Media* (submitted).

penetrate through the test sample into the buffer pool, the water level in the graduated tube rise rapidly. The negative pressure at this moment is recorded as the breakthrough pressure threshold  $p_{th}$ .

It is difficult to measure the thickness of the sand layer accurately, especially when the layer is thin, because the boundary of packed sand is far from being determined. The error is no less than 1 mm, or in relative quantity, the error can be as large as 30% when the height is measured directly. We notice that the porosity deposited by unconsolidated sand is determined by the size of particles, as illustrated in fig. 2, and independent of the thickness of the sand layer if the section of the test cup is large enough to affect the dynamic character little during the sediment process<sup>1)</sup>. An indirect method was adopted to determine the height of the sand layer. If the diameter of the test cup  $D$ , the density of the sand  $\rho$  and the mass of the adding time during each measurement  $m$  can be known, the height of the sand layer  $H$  is calculated according eq. (1):

$$H = \frac{4m}{\pi D^2 \rho \phi} \quad (1)$$

The relative error can be controlled within 1%.

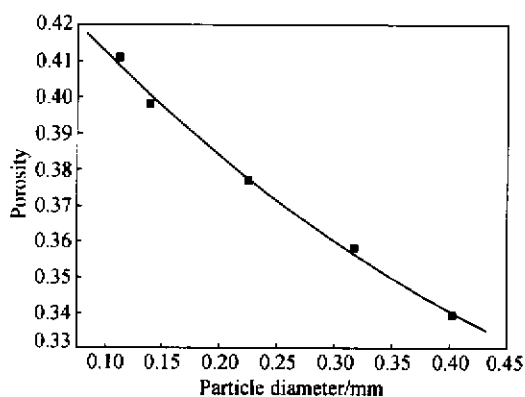


Fig. 2. Porosity of freely settled sand in water.

(ii) Experimental results. Fig. 3 shows the measured BP for different sands, whose particle diameters are 0.280–0.355 mm and 0.125–0.154 mm. When the height of the sample is large enough, BP almost maintains constant. In this case, BP is determined by the surface tension between fluids and the structural character of porous media and the whole sample can be taken as homogeneity. However, when the height of the sample is less than a certain critical value, BP diminishes with the decrease of the height obviously. It can be inferred that the superheated degree of incipient boiling, for sintered boil-

ing enhanced porous surfaces, rises with the increase of the thickness of the porous layer until the layer is thicker than a critical value.

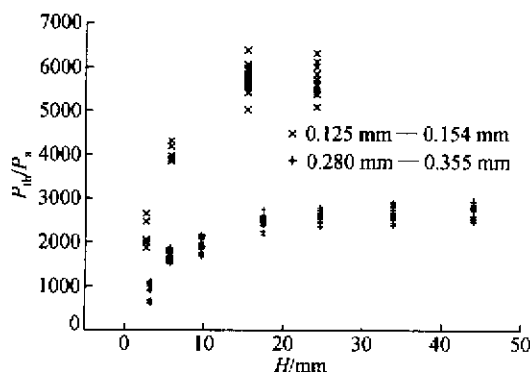


Fig. 3. Results of breakthrough experiments: variation of BP vs. height of porous layer.

## 2 Analysis and discussion

(i) Scaling effect and scaling law. BP is a kind of spontaneous resistance when non-wetting fluids press wetting fluids in the wet saturated porous media from a given direction, which occurs at the meniscus. According to the Yang-Laplace equation, the pressure jump at the interface is

$$\Delta p = \frac{2\sigma \cos \theta}{r_e} \quad (2)$$

where  $r_e$  is equivalent radius of the receding front,  $\sigma$  is the surface tensor and  $\theta$  the contact angle. When the outer pressure increases, the interface moves towards the pore throats, and the receding front radius decreases as well, therefore, the pressure jump has more powerful ability to maintain the pressure difference between the two phases. Until it reaches the pore throat, the interface is unable to resist more pressure, which results in non-wetting fluid passing through pore throats into the interior of porous media. Once a pore throat is penetrated, the corresponding pore body is also automatically occupied by non-wetting fluid. Similar processes pistol-like advance with the support of outer pressure, until a sub-path of non-wetting fluid is constructed in the whole sample, when the wet saturated porous media are broken through and the corresponding pressure is called "breakthrough pressure". The size of pore throat follows a determined probability distribution. For given pressure, according to eq. (2), it is easier for non-wetting fluid to pass through larger pore throats. Non-wetting fluid passable paths are formed as a result of the connection of passable pore throats and their pore bodies, which grow up continuously with the increase of pressure, and in the end, breakthrough occurs.

Researches for two-phase displacement in porous media by using the percolation method were introduced

1) Zhou, L., Experimental investigation on the relationship of the capillary hysteresis and flow characteristics of unconsolidated porous media (in Chinese), Master Thesis, Tsinghua University, 1999.

by monographs<sup>[6,7]</sup>, which lead us to study the potential scaling law. When the porous layer is so thin that the length of breakthrough correlation is mainly determined by the thickness of the layer, a dimensionless height is defined as the sequential parameter, which is

$$\zeta = \frac{H}{\sqrt{K}}, \quad (3)$$

where  $K$  is absolute permeability of porous media. The measured permeability values for the porous media formed freely by sedimentary sand as mentioned above, are 38.14 and 18.48  $\mu\text{m}^2$  respectively. Define a dimensionless BP as the control parameter during the breakthrough process, which is

$$\pi = \frac{P_{\text{th}}(\zeta) - P_{\text{th}}^{\infty}}{P_{\text{th}}^{\infty}}, \quad (4)$$

where  $p_{\text{th}}(\zeta)$  is BP with the height of  $\zeta$  and  $p_{\text{th}}^{\infty}$  is BP when the height of the porous layer is large enough that the value of BP is independent of the height.

Fig. 4 shows a log-log plot of  $\zeta$  vs.  $\pi$ . All the measured data presented in fig. 3 are almost located in a line, which indicates that the breakthrough process follows surely the scaling law:

$$\pi \propto \zeta^{-1/\gamma}. \quad (5)$$

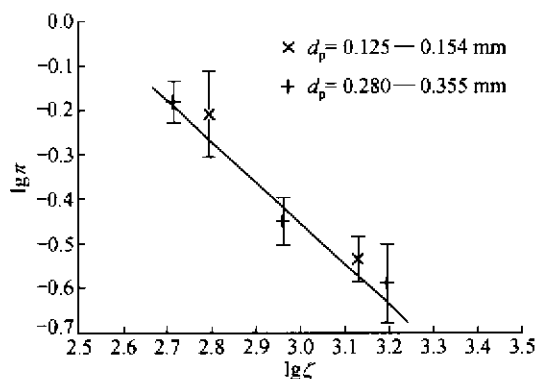


Fig. 4. Universal scaling law followed by different porous media.

The linear fit gives  $\gamma = 1.09 \pm 0.15$ , which approximately agrees with the theoretical value  $\gamma = 4/3$  and is close to the prediction value  $\gamma = 1.35 \pm 0.02$  by using the percolation method<sup>[8]</sup>, although there is still some differences in quantity. Klemm et al.<sup>[9]</sup> also found the difference between the experiment results and the prediction of percolation when studying the dimensional character of non-wetting fluid backbone in infinite porous layer with NMR. It is possible that the difference comes from the preciseness issues of measurement, however, it may also indicate the limitation of researching multiphase flows in porous media from a purely geometrical aspect, such as the percolation model.

(ii) Critical thickness. It is assumed that BP tends

to be constant after the ratio of  $p_{\text{th}}(\zeta)$  and  $p_{\text{th}}^{\infty}$  up to 90% (i.e.  $\lg \pi = -1.0$ ). Then the critical height, above which BP is independent of the height of porous media, can be approximately determined by fig. 4.

$$\lg \zeta_c \approx 3.5 \text{ or } \zeta_c \approx 3000. \quad (6)$$

After the values of measured absolute permeability are substituted into eq. (6), the practical critical heights of two kinds of sand samples are obtained as 12.9 and 18.5 mm, which agree with the experimental results presented in fig. 3.

## 3 Conclusions

The results of breakthrough experiments indicate that BP is independent of the height of porous layer if the height is beyond the critical value, otherwise, BP diminishes obviously with the decrease of the height. It is recommended that the critical height  $\zeta_c = 3000$ , based on the measurement and analysis results of freely settled sand layer in the present experiment.

It is necessary to pay enough attention to the effect of porous layer thickness on BP, and the effect follows a universal scaling law:

$$\pi \propto \zeta^{-1/\gamma},$$

where the critical exponent  $\gamma = 1.09 \pm 0.15$ . Theoretical predictions agree with the measured results reasonably.

**Acknowledgements** This work was supported by the National Natural Science Foundation of China (Grant Nos. 59995550-3 and 59776048).

## References

1. Dullien, F. A. L., *Porous Media: Fluid Transport and Pore Structure*, 2nd edition, London: Academic Press, 1992.
2. Leverett, M. C., Capillary behavior in porous solids, *Trans. Am. Inst. Min. Metall. Pet. Eng.*, 1941, 142: 152.
3. Katz, A. J., Thompson, A. H., Quantitative prediction of permeability in porous rock, *Phys. Rev. B*, 1986, 34: 8179.
4. Lei, S. Y., Jia, L. Q., Zheng, G. Y. et al., Experiments of pressure threshold and permeability of sand, *Chinese J. Engineering Thermophysics* (in Chinese), 1998, 19: 80.
5. Boukadi, F., Bemani, A., Rumhy, M. et al., Threshold pressure as a measure of degree of rock wettability and diagenesis in consolidated Omani limestone cores, *Marine and Petroleum Geology*, 1998, 15: 33.
6. Sahimi, M., *Flow and Transport in Porous Media and Fractured Rock: From Classical Methods to Modern Approaches*, New York: VCH, 1995.
7. Selyakov, V. I., Kadet, V. V., *Percolation Models for Transport in Porous Media With Applications to Reservoir Engineering*, Boston: Kluwer Academic Publishers, 1996.
8. Adler, P. M., Thovert, J.-F., *Fractures and Fracture Networks*, London: Kluwer Academic, 1999.
9. Klemm, A., Muller, H.-P., Kimmich, R., Evaluation of fractal parameters of percolation model objects and natural porous media by means of NMR microscopy, *Physica A*, 1999, 266: 242.

(Received December 1, 2000)