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Hybrid state estimation and model-set design of invariable-structure semi-ballistic reentry vehicle

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Abstract Multiple-model approach is one of the main streams for hybrid estimation. The difficulty of this approach to estimate the hybrid state of the semi-ballistic reentry vehicle (SBRV) is model-set design. This paper proposes a quasi-Monte Carlo model set that can ensure the estimator near-optimal in the sense of minimum mean square error (MMSE). The SBRV has a high nonlinearity and its mode is spanned by multiple parameters with known bounds. The design methods and characteristics of the quasi-Monte Carlo model set are given. The proposed model set has a higher accuracy than the model-set generated by the Monte Carlo method. The theoretical analysis and simulation results show the effectiveness and reasonability of the newly designed model set.

Keywords semi-ballistic reentry vehicle, hybrid estimation, model-set design, Monte Carlo method, quasi-Monte Carlo method

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1 Introduction

Real-time state estimation of the reentry vehicle (RV) based on radar measurements is an important problem in the field of target tracking, where the study of the RV has traditionally focused on the ballistic reentry vehicle (BRV) [1–4]. The BRV has an balanced structure and its trajectory is determined by its state at the reentry point. However, the structure of a RV is usually unbalanced, which induces a complex reentry trajectory and difficulties in its control, estimation, and prediction [5–10]. The RV of this type is called semi-ballistic reentry vehicle (SBRV), such as space plane, space craft, debris, wreckage, and some kinds of satellites and missiles. During the reentry process, the SBRV has an invariable or controllable structure, and the state estimation of the former is the base of the latter. In this paper, giving no rotation, we consider the state estimation of the invariable-structure SBRV.

The state of the SBRV is composed of a continuous part and a discrete part. The former, with the name base state, is the state of a common system, and the latter is the mode of a system. Thus the state estimation of the SBRV, in essence, is a hybrid estimation [11, 12]. A main stream of the hybrid estimation is the multiple-model (MM) approach, for which a set of models are designed to cover the

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possible modes, and its output is the combination of each model-based output. The performance of the MM approach depends largely on the set of models used, and model-set design is one of the most important tasks in the applications of the MM approach [13]. In this paper, we mainly discuss the model-set design for the MM-SBRV estimator.

Model-set design is not only important but also difficult. Its design is still largely in the kingdom of black magic or the domain of art [13]. A model set has a fixed or variable structure. The former is most widely used, which is applied to the autonomous multiple-model (AMM) approach or the cooperative multiple-model (CMM) approach [11, 14, 15]. Each model of the AMM estimator works individually and independently. The advantage of the AMM estimator over the non-MM estimator stems from its superior processing of the outputs of all these models to generate the overall estimate [11]. In this paper, we use the AMM approach to estimate the hybrid state of the invariable-structure SBRV.

Magill [14] used a model set that is equal to the mode space for optimal estimation in the sense of minimum mean square error (MMSE). In fact, the mode space is usually much larger than the model set [16, 17]. In this case, there are few general model-set design methods. Given bounds of parameters, [18] proposed a model set design method for linear systems. Based on the known bounds of a parameter, [4] designed model sets in the one-dimensional mode space of a nonlinear system. With prior information of the distribution of the mode, Li et al. [17] presented a theoretical description of model-set design, where the true mode and the model are viewed as random variables, and then the random model is designed to approximate the distribution of the mode. Based on this description, Li et al. proposed three model-set design approaches. As a further development of [17], [19] designed model set for the multi-dimensional mode space in the minimum-mismatch sense.

Although the MM approach was widely used for hybrid estimation, it is seldom used for the reentry problem. The difficulty lies in the design of the model set, and also none of the model-set design approaches mentioned above fits the characteristics of the SBRV. The vehicle has a high nonlinearity, and the model set in [18] cannot be used; its mode space is spanned by multi-parameters, then the model set in [4] cannot be used; its mode is easily known for bounds and its distribution is often unknown, thus the model set in [17] and [19] cannot be used. The model-set design approach proposed in this paper fits the characteristics of the SBRV, which is distributed in the mode space, easier to use, and ensures the estimator near-optimal in the sense of minimum mean square error (MMSE).

This paper is organized as follows: The SBRV model with characteristics of its motion and mode are introduced in section 2. The near-optimal hybrid state estimator of the SBRV in the MMSE sense is designed in section 3. The quasi-Monte Carlo model set is proposed in section 4, along with its theoretical accuracy, design methods, and characteristics. Simulation results are provided in section 5. Conclusions are given in section 6.

2 Modeling of SBRV and analysis of its motion and mode

The SBRV has an unbalanced structure or form. The unbalanced form can be looked upon as the unbalanced structure. During the reentry process, the vehicle is mainly affected by the gravity force and the aerodynamic force. The gravity force imposes on mass center O, and the aerodynamic force imposes on point O' shown in Figure 1. The aerodynamic force can be decomposed into the lift force and the drag force. If O is on the line AB, the lift force is zero, and if O is not on the line AB, the lift force exists. The lift force originates from the unbalanced structure of the SBRV and can be decomposed into the climbing force and the turning force. For a SBRV, the former controls the climb and the dive, and the latter controls the left turning and the right turning. The climbing force is vertical to the drag force and the turning force. The turning force is parallel to the local level and vertical to the drag force. The drag force is opposite to the velocity vector, and if the lift force exists, there is a lift induced drag force generated in the drag force. The triple unit vectors of the turning force, the climbing force, and the velocity force comprise an orthogonal coordinate system (CS) with the name velocity-turn-climb (VTC)-frame [1]. The relations of these forces can be seen in Figure 1.

The SBRV is treated as a point mass that is at the point of its mass center, as shown in Figure 1. The

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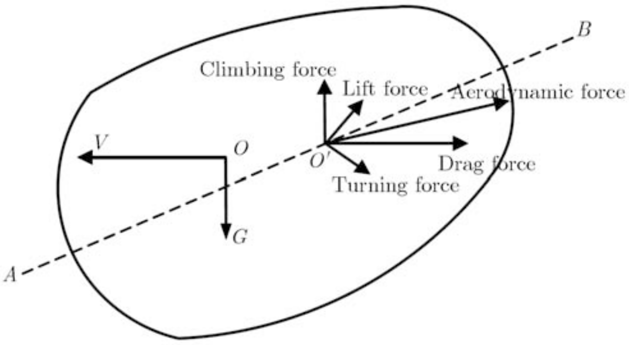

Figure 1 SBRV and geometry of forces.

state of the SBRV is described in the Earth-North-Up (ENU)-CS. Ignoring the effect of the centrifugal force and the coriolis force, the model of the SBRV can be obtained by converting the aerodynamic force from the VTC-frame to the ENU-CS:

$$X_{(k+1)} = FX_k + Gf(k, X_k) + w_k, \tag{1}$$

where $X_{(k+1)} = [x_{(k+1)}\ \dot{x}_{(k+1)}\ y_{(k+1)}\ \dot{y}_{(k+1)}\ z_{(k+1)}\ \dot{z}_{(k+1)}]'$, $w_{(k)} = [w_x\ \dot{w}_{vx}\ w_y\ w_{vy}\ w_z\ w_{vz}]'$,

$$F = \begin{bmatrix} 1 & T & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & T & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & T \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad G = \begin{bmatrix} \frac{T^2}{2} & 0 & 0 \\ T & 0 & 0 \\ 0 & \frac{T^2}{2} & 0 \\ 0 & T & 0 \\ 0 & 0 & \frac{T^2}{2} \\ 0 & 0 & T \end{bmatrix},$$

$$\begin{aligned} f(k, X_k) = & \\ & \begin{bmatrix} -\frac{1}{2}\alpha_d\rho\sqrt{\dot{x}_k^2+\dot{y}_k^2+\dot{z}_k^2}\dot{x}_k - \frac{1}{2}\alpha k_{td}\rho(\dot{x}_k^2+\dot{y}_k^2+\dot{z}_k^2)\frac{\dot{y}_k}{\sqrt{\dot{x}_k^2+\dot{y}_k^2}} - \frac{1}{2}\alpha k_{cd}\rho\sqrt{\dot{x}_k^2+\dot{y}_k^2+\dot{z}_k^2}\frac{\dot{x}_k\dot{z}_k}{\sqrt{\dot{x}_k^2+\dot{y}_k^2}} \\ -\frac{1}{2}\alpha_d\rho\sqrt{\dot{x}_k^2+\dot{y}_k^2+\dot{z}_k^2}\dot{y}_k + \frac{1}{2}\alpha k_{td}\rho(\dot{x}_k^2+\dot{y}_k^2+\dot{z}_k^2)\frac{\dot{x}_k}{\sqrt{\dot{x}_k^2+\dot{y}_k^2}} - \frac{1}{2}\alpha k_{cd}\rho\sqrt{\dot{x}_k^2+\dot{y}_k^2+\dot{z}_k^2}\frac{\dot{y}_k\dot{z}_k}{\sqrt{\dot{x}_k^2+\dot{y}_k^2}} \\ -\frac{1}{2}\alpha_d\rho\sqrt{\dot{x}_k^2+\dot{y}_k^2+\dot{z}_k^2}\dot{z}_k + \frac{1}{2}\alpha k_{cd}\rho\sqrt{\dot{x}_k^2+\dot{y}_k^2+\dot{z}_k^2}\sqrt{\dot{x}_k^2+\dot{y}_k^2} \end{bmatrix} \\ & - \frac{\mu}{(\dot{x}_k^2+\dot{y}_k^2+(\dot{z}_k+a)^2)^{3/2}}\begin{bmatrix} x_k \\ y_k \\ z_k+a \end{bmatrix} - \omega^2\begin{bmatrix} -x_k \\ -\sin(\omega_k)^2 y_k + \cos(\omega_k)\sin(\omega_k)(z_k+a) \\ \cos(\omega_k)\sin(\omega_k)y_k - \cos(\omega_k)^2(z_k+a) \end{bmatrix} \\ & - 2\omega\begin{bmatrix} -\sin(\phi_k)\dot{y}_k + \cos(\phi_k)\dot{z}_k \\ \sin(\phi_k)\dot{x}_k \\ -\cos(\phi_k)\dot{x}_k \end{bmatrix}, \end{aligned}$$

where k_{cd} is the ratio of the climbing force to the drag force, k_{td} is the ratio of the turning force to the drag force, μ is Earth's gravitational constant, ω is the module of the Earth's angular velocity vector, ϕ_k is the latitude of the RV, a is the distance from the Earth center to the original point of the ENU-CS with a value of the averaged Earth radius added by the sea level elevation of the original point of the ENU-CS. Then the distance from the RV to the Earth center is $(x_k^2 + y_k^2 + (z_k + a)^2)^{1/2}$, ρ is the air density, the drag coefficient is $\alpha_d = (1 + ck^2)\alpha$, c is the coefficient related to the lift induced drag, the lift-drag ratio k is the ratio of the lift force to the drag force, $k^2 = k_{td}^2 + k_{cd}^2$, α=$(SC_D)/m$ is determined by the vehicle's mass m, the reference area S, and the drag coefficient C_D. C_D depends on the viscosity of air and the RV's velocity, the altitude, and the ratio of its length to its diameter [5]. If $k_{td} = 0$ and $k_{cd} = 0$, the SBRV model turns to be the BRV model.

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The trajectory of a invariable-structure SBRV is affected by the lift-drag ratio whose value is mainly determined by the structure of the SBRV. The bounds of the lift-drag ratio are always known. The value of a space plane can be larger than 1, the value of a rotational body cannot be larger than 0.8, the value of a space craft will not exceed 0.5, and the value of a slender body will be much smaller. The drag coefficient of the BRV is usually assumed to be a constant [3, 4]. Accordingly, the trajectory of the SBRV is affected by k_{cd}, k_{td}, and the lift induced drag. For a SBRV, these parameters represent its characteristics, and their values represent the vehicle's mode. The possible modes comprise the mode space of the SBRV. Thus the uncertain trajectory of a SBRV is mainly determined by the space spanned by k_{td}, k_{cd}, and α_d, that is, the mode space of the SBRV.

Figure 2 shows some trajectories of an invariable-structure SBRV, and Figure 3 shows the corresponding modes of these trajectories, the related parameters are x_0=232000 m, y_0=0 m, z_0=88000 m, $\gamma_0 = 190°$, v_0 =2290 m/s, α_d=$\frac{9.8}{40000}$ m^2/kg, $k_{cd} \in (-0.8, 0.8)$, $k_{td} \in (-0.8, 0.8)$. Ignoring the process noise, each trajectory corresponds to a mode in the mode space. The two figures show that the trajectory of the BRV is a special case of the SBRV, and its mode is fixed in the mode space of the SBRV. Given the initial state of a BRV, the length of the trajectory, the maximum overload, and the impact point are all determined. Under the constant assumption of the drag coefficient, the mode of a invariable-structure SBRV is fixed during the reentry process. Given the initial state and with the selection of the initial structure of a SBRV, the length of the trajectory, the maximum overload, and the impaction point can be designed.

3 Estimator design

The hybrid state estimator is composed of the base state estimator and the mode estimator. For the optimal base state estimator, a basic idea is to minimum the mean square error. For Bayesian estimation, this idea leads to the MMSE estimator:

$$\hat{X}^{\text{MMSE}} = \arg_{\hat{X}(Z^k)}^{\min} E\{(X - \hat{X}(Z^k))'(X - \hat{X}(Z^k))|Z^k\} = \int_{\Omega} X f(X|Z^k) dX, \tag{2}$$

where Ω is the state space, Z^k=$\{Z_1, \ldots, Z_k\}$ is the set of measurements from the initial time until k, $f(X|Z^k)$ is the conditional probability density function (PDF) of the state vector X given Z^k. Considering the complexity of the state estimation of a SBRV induced by the uncertain mode, (2) can be expressed as

$$\begin{aligned}\hat{X}^{\text{MMSE}} &= \int_{\Omega} X \int_{\alpha} \int_{k_{cd}} \int_{k_{td}} f(X, \alpha, k_{cd}, k_{td}|Z^k) dk_{td} dk_{cd} d\alpha dX = \int_{\Omega} X \int_{\text{S}} f(X, \Theta|Z^k) d\Theta dX \\ &= \int_{\text{S}} f(\Theta|Z^k) \int_{\Omega} X f(X|\Theta, Z^k) dX d\Theta = \int_{\text{S}} \hat{X}(\Theta) f(\Theta|Z^k) d\Theta,\end{aligned} \tag{3}$$

where $\Theta = \{\alpha_d, k_{cd}, k_{td}\}$ and the estimator is integrated in the mode space S. (3) can be approximated as

$$\hat{X}^{\text{MMSE}} \approx \sum\nolimits_i \hat{X}(\Theta_i) p(\Theta_i|Z^k), \tag{4}$$

where S is the M-dimensions that does not change during the tracking process. The mode space can be divided into a series of subspaces $\text{S} = \bigcup_{i=1}^{R} \text{S}_i$, $\forall i \neq j$ and $i, j \in \{1, \ldots, R\}$, so $\text{S}_i \cap \text{S}_j = \phi$, Θ_i lies in the center of S_i, model m_i is constructed on the value of Θ_i, $\hat{X}(\Theta_i)$ is the estimated result of model m_i, $p(\Theta_i|Z^k)$ is the posterior probability of m_i. The model set in (4) is M=$\{m_1, \ldots, m_R\}$.

Similar to (2), the uncertain model can be estimated as

$$\hat{s}^{\text{MMSE}} = \int_{\text{S}} \Theta f(\Theta|Z^k) d\Theta. \tag{5}$$

Eq. (5) can be approximated as

$$\hat{s}^{\text{MMSE}} \approx \sum\nolimits_i \Theta_i p(\Theta_i|Z^k). \tag{6}$$

The mode can be estimated as $\hat{s} = [\hat{\theta}_1 \ \ldots \ \hat{\theta}_M]'$, and the mode of the SBRV is $\hat{s} = [\hat{\alpha}_d \ \hat{k}_{cd} \ \hat{k}_{td}]'$.

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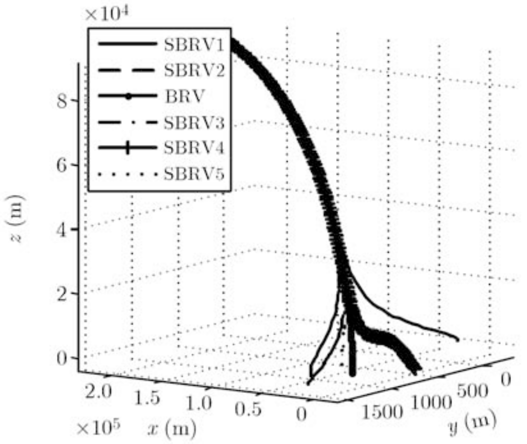

Figure 2 Three-dimensional trajectories of an invariable-structure SBRV.

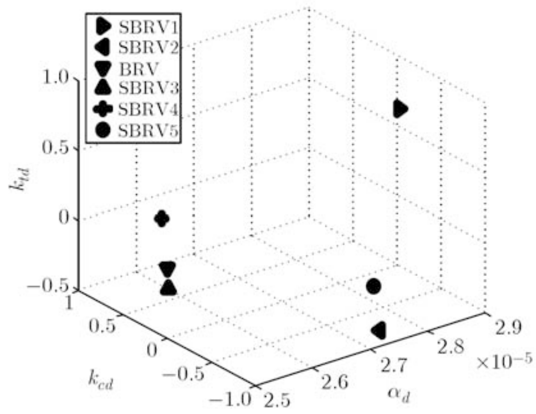

Figure 3 The corresponding mode of each trajectory.

A special case was discussed in [14] where the mode space is one-dimensional and only contains a finite set of modes. Thus a model set M that is equal to the mode space S is applied, M=S. For the general case (S>>M), the model-set design methods proposed in [17] and [19] can be adopted with the prior information of the distribution of mode. With the bounds of each parameter, the model-set design approach in [18] only fits for linear systems. Then for the nonlinear system such as the SBRV, a new approach is needed for the model-set design. (4) and (6) show that the MM estimator is nearly optimal in the MMSE sense, and the estimation error is mainly determined by the model set in the mode space [13].

The hybrid estimator in (4) and (6) shows that its accuracy is affected not only by the model set, but also by the filter of each model and its posterior probabilities on the assumption that the model in consideration is the true model. The comparison between many filters for the RV has been done in [1–4], and the posterior probability of each model can be approximated by the Bayesian formula [11, 14]. Thus we focus on the model-set design of the MM-SBRV estimator.

4 Quasi-Monte Carlo model set

A MMSE estimator is provided in [14], however, the approximations of (3) and (5) to (4) and (6) show that the performance of the hybrid estimator of the SBRV is mainly determined by the model set in the mode space. The mode space of the SBRV is spanned by the drag coefficient α_d, the turn-drag ratio k_{td}, and the climb-drag ratio k_{cd}. With the independence assumption of these parameters, the mode space of the SBRV turns to be a rectangular parallelepiped.

The Monte Carlo method is commonly used for numerical integration, and the discrete points can be uniformly sampled from the parametrized uncertain region. Model set generated by this method can be called Monte Carlo model set. It uniformly samples R models from the mode space. Consequently, the errors of the MMSE estimators in (3) and (5) can be $O(R^{-1/2})$ [20]. Due to the random sampling, the accuracy of the Monte Carlo model set is probabilistic. To overcome this defects, model set generated by the number theoretic method is proposed, for which the estimation error is not only smaller but also determined.

4.1 Uniformity measure of model set

Lemma 1. For any model set M designed in an M-dimensional mode space, the errors of the estimators (4) and (6) will be larger than $C\ 2^M D^\star(\mathrm{M})$ [21], and will not be less than $O(R^{-1})$ [22], where $D^\star(\mathrm{M})$ is the star discrepancy of the model set. For a model set generated by the number theoretical method, the errors of (4) and (6) will not be less than $C(M)\frac{(\log R)^{(M-1)/2}}{R}$ [23].

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Lemma 1 for numerical integration has been proved in [21–23]. Ignoring the approximations in the filter of each model and its Bayesian posterior probability, Lemma 1 fits for the estimators (4) and (6). The theoretical error bounds for the hybrid estimation are provided in Lemma 1. Also, Lemma 1 shows that the star discrepancy is effective for the uniformity measure of a model set; the smaller the star discrepancy, the higher uniformity of the model set. Based on Lemma 1, Wang et al. [24] concluded that $\forall\epsilon > 0$, the uniform points generated by the number theoretical method can ensure the errors of the numerical integrations in (4) and (6) to be $O(R^{-1+\epsilon})$. Thus the quasi-Monte Carlo model set generated by the number theoretical method can ensure the estimators (4) and (6) to be near-optimal. Therefore, the model set generated by the number theoretical method is applied in this paper, which has a lower estimation error than the Monte Carlo model set with the same cardinality.

Korobov, Hua, Wang, Hlawka, Halton, and Niederreiter have done a lot of work to prove the possibility and existence of the uniform model set. Hua and Wang studied how to generate the uniform points in multi-dimensions, and a series of uniform points have been provided [20, 25–27]. Wang and Fang provided a series of uniform points with a smaller cardinality [28, 29].

Model set generated according to the uniformity measure to take the place of the Monte Carlo model set in the mode space is called quasi-Monte Carlo model set. In this paper, the uniformity of a model set is measured by the star discrepancy which is most widely used in the number theoretical method (see [20, 30]):

$$D^{\star}(\mathrm{M}) = \mathrm{Sup}_{\Theta\in \mathrm{S}}\left|\frac{N(\mathrm{M},\Theta)}{R} - \frac{\mathrm{Vol}(0,\Theta)}{\mathrm{Vol}(\mathrm{S})}\right|, \tag{7}$$

where M=$\{m_1,\ldots,m_R\}$, R is the cardinality of the model set, M means the number of parameters included in Θ, the parameters are mutually independent, $[\mathbf{0},\Theta) = [0,\theta_1)\times\cdots\times[0,\theta_M)$ is the rectangle spanned by the original point $\mathbf{0}$ and the point Θ, $\mathrm{Vol}(\mathbf{0},\Theta)$ is the volume of rectangle $[\mathbf{0},\Theta)]$, $N(\mathrm{M},\Theta)$ represents the number of models included in $[\mathbf{0},\Theta)]$, Vol(S) is the volume of mode space S. The star discrepancy represents the non-uniformity of model set M, the smaller star discrepancy is, the higher uniformity of the model set has.

The matrix constructed by the values of each model's parameters is called model set matrix. Each element of the matrix is represented by an index of a parameter's discretized levels. Each row of the matrix corresponds to the values of the parameters of a model, and each column corresponds to an order of the indices of the set of discretized levels of a parameter. For M_1 parameters with N_1 levels and M_2 parameters with N_2 levels, the model set is denoted as $\mathrm{M}(R;N_1^{M_1},N_2^{M_2})$, and so on.

4.2 Construction of quasi-Monte Carlo model set

To construct a quasi-Monte Carlo model set by the number theoretical method, each parameter should be discretized at first, then the models can be uniformly constructed in the mode space. For a model set designed in the M-dimensional mode space with a cardinality R, the discretized levels of any $\theta_i \in \Theta$ can be obtained by the following formula:

$$\theta_{ij} = \theta_i^- + \frac{2j-1}{2N_i}(\theta_i^+ - \theta_i^-), j = 1,\ldots,N_i, \tag{8}$$

where N_i is the number of discretized levels of θ_i, $\mathrm{mod}(R,N_i) = 0, i\in\{1,\ldots,M\}$, $r_i = R/N_i$ means the number of times for each discretized level of θ_i to appear in the model set, θ_i^+ and θ_i^- are the bounds of θ_i. If the combinations of all these discretized levels are included, the cardinality of the model set is $N_1N_2\ldots N_M$, and the value is usually too large. To overcome this defect, the model set with a smaller cardinality should be uniformly selected from these models.

Two methods are mainly used to construct the quasi-Monte Carlo model set. The first method was pioneered by Korobov [31] and Hlawka [32], and promoted by Hua and Wang [20] etc. It can be generated by the following formula:

$$P_N(\mathrm{M}) = (ka_1, ka_2,\ldots,ka_{E(N)})(\mathrm{mod}\ N), k = 1,\ldots,N, \tag{9}$$

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where $P_N(\mathrm{M})$ is the model-set matrix, the number of discretized levels of these parameters are $N_1 = \cdots = N_M = N$, and the cardinality of the model set is $R = N$, $(a_1, \ldots, a_M)$ is selected based on the uniformity of the designed model set, $a_1 = 1$, a_i is an integer, $(a_i, N) = 1$ and $1 < a_i < N$ $(2 \leqslant i \leqslant N)$, $a_i \neq a_j$ $(i \neq j)$. $E(N)$ means the largest number of parameters for the model set generated by (11):

$$E(N) = N \prod\nolimits_{i=1}^{s} \left(1 - \frac{1}{p_i}\right), \tag{10}$$

where $p_1 < p_2 < \cdots < p_s$ are distinct prime numbers, $E(N) \leqslant N - 1$. If N is a prime number, $E(N) = N - 1$. For any $M \leqslant E(N)$, the number of possible model sets for (11) is $C_{E(N)}^{M}$, and the computation is always too heavy.

If N is a prime number, a second method was proposed by Korobov [33], and was greatly developed by Hua and Wang [20], Wang and Fang [28], and Niederreiter [30]

$$P_N(\mathrm{M}) = (kb^0, kb^1, \ldots, kb^{M-1})(\mathrm{mod}\ N), k = 1, \ldots, N, \tag{11}$$

where $N_1 = \cdots = N_M = N$, $E(N) = N - 1$, b is an integer and $1 < b < N$, $b^i \neq b^j(\mathrm{mod}\ N)(i \neq j)$, b is always a primitive and determined by N and M, the cardinality of the model set is $R = N$. Compared with (10), this method is easier to use. If N is large, the computation of (12) will be greatly decreased, but the discrepancy is slightly decreased. This method is most often used to generate the uniform points. Thus (12) is one of the main methods to generate quasi-Monte Carlo model sets.

As mentioned above, a quasi-Monte Carlo model set can be constructed by the following steps:

1. Get the discretized levels according to (9);
2. If N is a prime number, get b according to N and M [28]. Then design the model set by (12);
3. If $N + 1$ is a prime number, get b according to N and M and generate a model set by (12). Then delete the last line from the model-set matrix;
4. For other cases, get the vector $(a_1, \ldots, a_M)$ according to N and M [20]. Then, generate the model set by (10).

Based on the quasi-Monte Carlo model set generated above, the hybrid-level quasi-Monte Carlo model set can be constructed in two steps [29]. At first, the discretized levels for any $\theta_i \in \Theta$ can be decreased from N_i to N_i' by combination distinct levels, where mod $(R, N_i') = 0$. Then, for all possible combined model sets, the one with the smallest discrepancy is the hybrid-level quasi-Monte Carlo model set.

4.3 Characteristics of quasi-Monte Carlo model set

The construction of the quasi-Monte Carlo model set shows that its uniformity lies in the mode space's first-dimensional projection space and the M-dimensional mode space. The definition of the star discrepancy and the constructions of model sets show that the quasi-Monte Carlo model set designed in this paper is the minimax design. Thus the uniform model set is robust to the uncertain system.

According to Lemma 1 and the inference given by Wang, the accuracy of the quasi-Monte Carlo model set based estimator is determined by the number of models. Thus for a lower estimation error, the cardinality of the quasi-Monte Carlo model set should be increased. The quasi-Monte Carlo model set is generally used with the same discretized levels of each parameter. However, the large cardinality will induce a heavy computation. In order to have a small estimation error with a smaller cardinality, the model set should fit the characteristics of the system in consideration. For a linear system, the number of discretized levels should be increased for parameters with a large range. For a nonlinear system, the degree of each parameter to the system's output should be considered. If the parameters have the same effect to the system's output, each parameter should have the same discretized levels. If distinct parameters have different effects on the output of the system, from the viewpoint of numerical integration the model set with a large cardinality will have a small estimation error. However, the cardinality should be decreased for real-time estimation. Thus the hybrid-level quasi-Monte Carlo model set should be used.

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Table 1 Model set $\mathrm{M}(6;6^3)$

Model	Para. 1	Para. 2	Para. 3
1	1	3	2
2	2	6	4
3	3	2	6
4	4	5	1
5	5	1	3
6	6	4	5

Table 2 Model set $\mathrm{M}(12;12^3)$

Model	Para. 1	Para. 2	Para. 3
1	1	4	3
2	2	8	6
3	3	12	9
4	4	3	12
5	5	7	2
6	6	11	5
7	7	2	8
8	8	6	11
9	9	10	1
10	10	1	4
11	11	5	7
12	12	9	10

Table 3 Model set $\mathrm{M}(18;18,6^2)$

Model	Para. 1	Para. 2	Para. 3
1	1	8	7
2	2	16	14
3	3	5	2
4	4	13	9
5	5	2	16
6	6	10	4
7	7	18	11
8	8	7	18
9	9	15	6
10	10	4	13
11	11	12	1
12	12	1	8
13	13	9	15
14	14	17	3
15	15	6	10
16	16	14	17
17	17	3	5
18	18	11	12

5 Simulation

This section is about the simulation results. The initial state is Gaussian distributed with mean $X_0 = [232000\ 2290\cos(190^\circ\ 0\ 0\ 8800\ 2290\sin(190^\circ)]'$, and covariance $P_0 = \mathrm{diag}([1000^2\ 20^2\ 1000^2\ 20^2\ 1000^2\ 20^2])$. The drag coefficient is $\alpha_d = \frac{9.8}{40000}$ m^2/kg, and the ranges of the climb-drag ratio and the turn-drag ratio are $k_{cd} \in [-0.3\ 0.3]$, $k_{td} \in [-0.3\ 0.3]$. The radar is fixed on the Earth and its distance to Earth

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center is the averaged Earth radius. The sampling period is T=1 s, the measurement vector is composed of the range r, the bearing b, and the elevation e:

$$Z = \left[\begin{array}{ccc} r & b & e \end{array}\right]'. \tag{12}$$

The standard deviations of the measurement errors are $\sigma_r = 100$ m, $\sigma_b = 0.05$ rad, $\sigma_e = 0.05$ rad.

Three quasi-Monte Carlo model sets are used in the simulation, M$(6; 6^3)$, M$(12; 12^3)$, M$(18; 18^3)$ [29]. For 6, 12, and 18, the corresponding values $N + 1$ are all prime number, thus b is obtained with the corresponding value as 3, 4, and 8 according to $N + 1$ and M, and these model sets are generated by (12) at first, then the last row of these model set matrices are deleted and the model sets in Tables 1–3 are obtained. Corresponding to these quasi-Monte Carlo model sets, three Monte Carlo model sets are sampled from the mode space. These Monte Carlo model sets are sampled for each Monte Carlo run, and are kept unchanged during the tracking process.

Due to the nonlinearity of the dynamic model and the measurement model, the unscented filter (UF) is used for state estimation [34, 3, 4]. UF is a method of approximating posterior moments (usually mean and covariance) of any nonlinear function by a set of deterministic designed points (sigma points) with weights. All simulation results were obtained by average over 100 independent Monte Carlo runs. Because of the uncertain initial state and parameters, from the reentry point until the impact point the reentry time varies from 107 to 158 s.

We compared the averaged normalized estimation error squared (ANEES) and the root mean square errors (RMSE) of the position, the velocity, and the parameters in the mode (see [15]):

$$ANEES = \frac{1}{Nn} \sum\nolimits_{i=1}^{N} (X_i - \hat{X}_i)' P_i^{-1} (X_i - \hat{X}_i), \tag{13}$$

where $X_i - \hat{X}_i$ and P_i are the state estimation error vector and the corresponding covariance matrix on the ith run. n is the dimension of the state vector, N is the number of Monte Carlo runs. The nearer this value is to one, the higher credibility of the estimator has.

The RMSE of any parameter θ_i in the mode s is (see [15])

$$||\theta_i - \hat{\theta}_i||^2 = \frac{1}{N} \sum\nolimits_{j=1}^{N} (\theta_{ij} - \hat{\theta}_{ij})' (\theta_{ij} - \hat{\theta}_{ij}), \tag{14}$$

where $\theta_{ij} - \hat{\theta}_{ij}$ represents the estimation error of θ_i in the jth Monte Carlo run. The RMSE of the position and velocity can be obtained in (15) through replacing the parameter estimation error by the position estimation error vector or the velocity estimation error vector.

Figure 4 shows that the credibilities of these MM estimators are quite similar during the initial period. After 70 s, the credibilities of the quasi-Monte Carlo model sets are better than the corresponding Monte Carlo model sets with the same cardinalities. The increase of the cardinality of each type induces the increase of the credibility. The credibilities of the quasi-Monte Carlo model set with the cardinality 6 or 12 nearly have the same credibilities as the corresponding Monte Carlo model sets with the cardinality 12 or 18.

It also can be seen from Figures 5 and 6 that the estimation errors of all these model sets are quite similar. Only after the initial period (70 s), these estimation error curves vary greatly. The estimation errors of these quasi-Monte Carlo model sets are smaller than those of the corresponding Monte Carlo model sets with the same cardinality. The estimation error of the former with the cardinality 6 has the similar estimation error of the latter with the cardinality 12, and the former with the cardinality 12 has the similar estimation error as the latter with the cardinality 18.

From the estimation results of the ANEES and the base state, it can be concluded that the credibility and estimation error are affected by model-set types and their cardinalities. The quasi-Monte Carlo model set has a higher credibility and a smaller estimation errors than the Monte Carlo model set with the same cardinality. For a given credibility and estimation errors, the quasi-Monte Carlo model sets save about one-third models than the Monte Carlo model sets.

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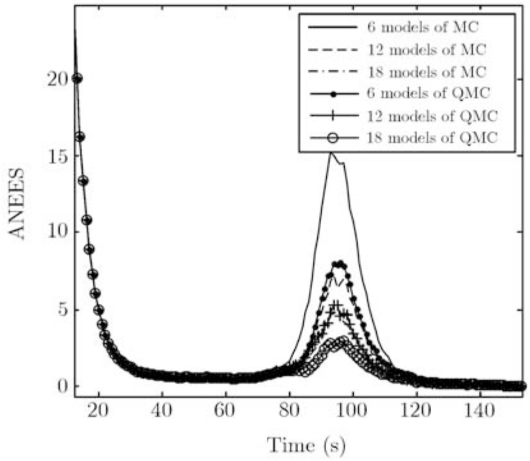

Figure 4 ANEES.

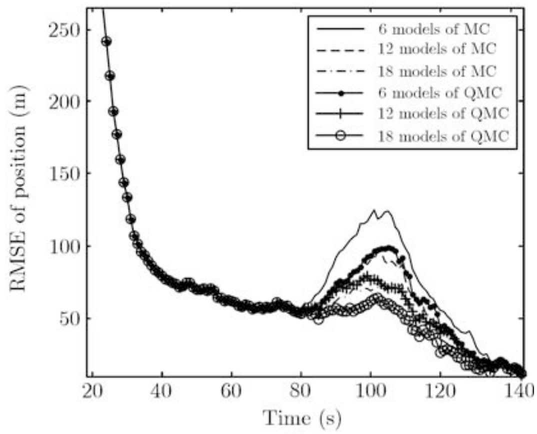

Figure 5 RMSE of position.

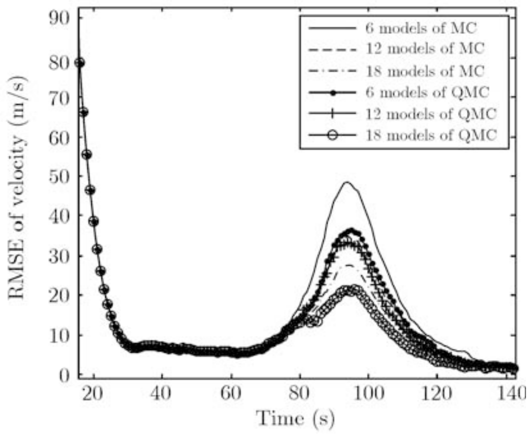

Figure 6 RMSE of velocity.

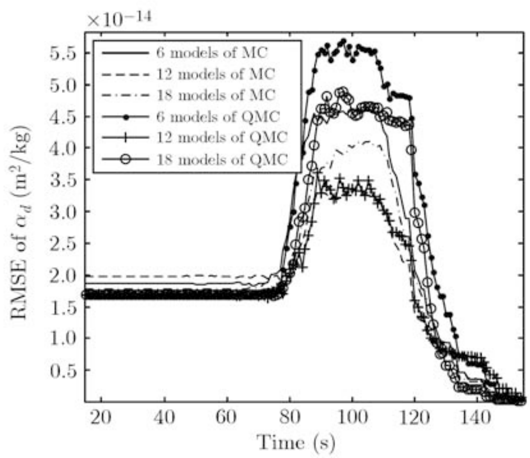

Figure 7 RMSE of α_d.

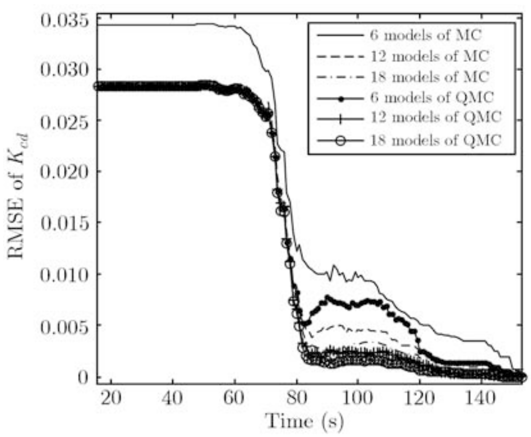

Figure 8 RMSE of k_{cd}.

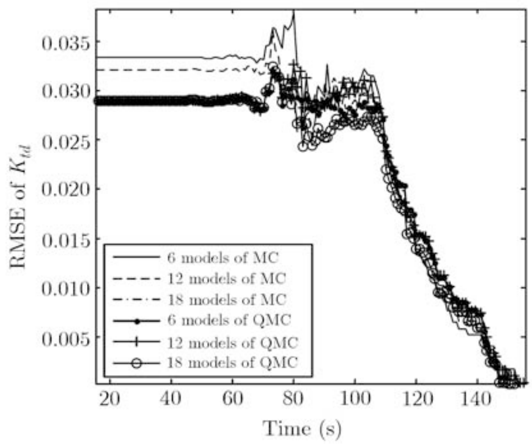

Figure 9 RMSE of k_{td}.

Figures 7–9 show that the differences of these model sets are small. Compared with the Monte Carlo model sets, the quasi-Monte Carlo model sets have smaller estimation errors, and their estimation errors are barely affected by their cardinalities. After 70 s, these mode estimation curves vary greatly. The estimation errors of the quasi-Monte Carlo model sets are smaller than the corresponding Monte Carlo model sets in general. Due to the complexity of the non-linear system, the mode estimation errors are not quite consistent with the base state estimation errors. The simulation results show that the mode

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estimation error of the uniform model sets is more consistent with its base state estimation error than those of the Monte Carlo model sets.

The simulation curves for different periods of the reentry process vary greatly. This is because of the variance of the air density and the speed of the RV. During the initial period, the air density is quite small and the RV's state is barely affected by the aerodynamic force. As time goes on, the air density increases and the speed of the RV is still high, then the aerodynamic force becomes greater and greater. Until a time after the reentry process (for this scenario the time is 70 s), the state of the RV is greatly affected by the aerodynamic force, and the estimation error curves vary greatly. The differences of these model sets mainly exist during this period. With the further development of the reentry process, the velocity of the RV is decreased, and its aerodynamic force will also be decreased. Then the state estimation curve will become smoother.

The simulation results fit the theoretical analysis. In fact, due to the uncertain mode of the invariable-structure SBRV, the uniform model set roughly ensures each model to represent the same number of modes. Thus the quasi-Monte Carlo model set is more representative to the unknown mode. The estimation result of the model set will not be bad for any possible reentry process.

The increase of the cardinality of the quasi-Monte Carlo model set will decrease the estimation error. For real-time estimation, it should be noticed that the cardinality of the model sets depends on the computing power of the processor and the estimation precise requirement. Since the drag coefficient has a large range, its discretized levels can be increased, and the hybrid-level uniform model set can be used, such as $\mathrm{M}(12;12,6^2)$, $\mathrm{M}(24;24,12^2)$. The model set generated from [28] often has a lower discrepancy than the one obtained from [29]. In addition, there is a filter for the base state estimator other than the mode estimator. Therefore, the robustness of the base state estimation is better than that of the mode estimation when the state varies greatly. The consistency of the cardinality of the model set to the base state estimation is better than that to the mode estimation.

6 Conclusions

We provide a near-optimal hybrid state estimator according to the MMSE estimator for the invariable-structure SBRV on the analysis of the vehicle's motion characteristics and mode characteristics. With the known bounds of the parameters of the mode, the quasi-Monte Carlo model sets are proposed. Also, the construction and the characteristics of the quasi-Monte Carlo model sets are discussed. The proposed model set fits for the system with known bounds of parameterized mode space, and also works well for the nonlinear system of the invariable-structure SBRV. The theoretical analysis and the simulation results show that the quasi-Monte Carlo model sets have a higher credibility and a smaller estimation error than the Monte Carlo model set with the same cardinality.

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