声学计算超材料

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摘要 尽管数字信号处理器已广泛用于执行复杂计算任务, 但仍存在诸多不足, 譬如模-数转换器成本高昂、运算复杂度高、低速和高功耗等。出于这个原因, 最近人们对基于波的模拟计算产生了浓厚的兴趣。这种计算避免了模-数转换并可以执行大规模并行运算, 特别是借助人工设计的超材料, 一些对声波进行模拟计算的新方案陆续被提出。这种被称为计算超材料的计算系统, 其计算速度可以与波速一样快, 尺寸与波长一样小, 可以对传入的波包进行复杂的数学运算, 甚至可以执行积分、微分方程的计算。这些功能有望实现基于声波传播的新一代超快速、紧凑和高效的计算硬件。本文讨论了声波计算超材料领域的最新进展, 调研了用于执行模拟计算的最新的超结构。接着, 进一步介绍了声波计算超材料的应用, 包括图像处理、边缘检测、方程求解和机器学习。最后, 对研究的关键问题和未来可能的方向进行了展望。

关键词 声学超材料, 模拟计算, 超材料计算电路, 声学超材料计算网络

声波在日常生活中无处不在, 比如语言和音乐的产生和传播, 以及医学和工业超声成像等。然而, 控制声波存在各种挑战。虽然电子设备能够放大和操纵声音信号, 但只有在将声信号转换成电信号之后才能进行控制。声学超材料对声波的操纵和控制能够实现传统材料不能实现的功能, 如声学斗篷、声学完美透镜等。因此, 研究声学超材料具有重要的理论价值和应用前景。

超材料通过设计人工结构, 可在亚波长范围内对入射波进行调控, 达到自然材料不能实现的功能, 如光波高吸收^[1,2]、光波^[3]/电磁波^[4]/声波^[5]负折射、微波^[6]/热^[7]/声波^[8]隐身、光波非对称传播^[9,10]、光学^[11,12]/声学^[13]全息等。近年来, 计算超材料(包括光学^[14]、电磁

波^[15,16]、声学^[17]计算超材料)的提出为模拟计算开辟了新研究方向, 可以高通量、低能耗地实现一些特殊的计算任务, 譬如空间积分、导数和卷积^[16]、时域积分、导数和Hilbert变换^[18]。虽然在过去的数百年里, 已有一些机械、电子或混合模拟计算机系统可以实现简单的数学运算^[19,20], 但这些计算机系统尺寸较大、反应较迟钝, 而计算超材料将有助于实现全波模拟计算系统的小型化。目前, 光学/电磁波计算超材料已实现数学运算^[16]、深度学习^[21]、量子搜索^[15]、图像的边界提取处理^[22]、DNA测序^[23]等工作, 如图1所示。

本文旨在介绍声学计算超材料领域当前的发展动态与现有成果, 同时, 对未来发展潜力和趋势、存在问题进行综合分析 with 展望。

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图1 (网络版彩色)光学/电磁波计算超材料已实现多种运算功能。(a)光子晶体电路用于深度学习。Reprinted with permissions from Ref. [24]. (b)电磁波超材料实现数学运算。Reprinted with permissions from Ref. [16]. (c)电磁波超材料实现量子搜索。Reprinted with permissions from Ref. [15]

Figure 1 (Color online) Optical/electromagnetic wave computational metamaterials have realized a variety of computing functions. (a) Photonic crystal circuits are used for deep learning. Reprinted with permissions from Ref. [24]. (b) Electromagnetic wave metamaterials realize mathematical operations. Reprinted with permissions from Ref. [16]. (c) Electromagnetic wave metamaterials realize quantum search. Reprinted with permissions from Ref. [15]

1 声学计算超材料的发展历程

声学计算超材料的发展经历了从对单一数学运算的计算模拟,到融合多种运算模拟的可调设计和计算电路、计算网络的提出,以及针对计算鲁棒性提出的拓扑绝缘体计算超材料等过程。单一数学运算的计算模拟可以分为空间域和时间域计算超材料。空间域声学计算超材料和时域声学计算超材料的不同主要来源于空间和时间维度的不同。在概念上,空间域声学计算超材料对空间分布的稳态声学信号进行处理,而时域声学计算超材料则对瞬态的声学时间信号进行处理。在设计原理上,空间域声学计算超材料主要对材料的空间分布进行设计以实现所需计算的Green函数,时域声学计算超材料则需要对材料的频谱进行设计以实现所需计算的传递函数。空间域和时间域声学计算超材料都实现了积分、导数、微分方程等运算。可调声学计算超材料的提出,针对的是超材料的结构固定则功能不能改变的问题。其通过在超材料中引入可调参数,实现在同一块超材料上融合积分、导数等功能。声学超材料计算电路、计算网络的提出,将超材料计算的复杂度又提升了一个维度。通过引入声学开关等声学电路元件的设计,控制声波在不同的回路中传播,实现了多个不同超材料块的串联、并联等的复杂运算。声学超材料计算网络结合声全息和卷积神经网络的设计思路,将超材料每个像素单元等效为卷积神经网络中

的神经元,将不同超材料层间声波在介质中的传播等效为神经元间的信号传播。通过训练学习,完成了手写体数字识别等机器学习任务。针对计算鲁棒性提出的拓扑绝缘体计算超材料,通过引入拓扑保护的边界模式,使得声学计算超材料在结构尺寸发生扰动的情況下,依然可以保证计算的精度。未来,非线性计算超材料的研究将越来越受到重视,将有可能应用到非线性方程求解、非线性滤波、图像处理方法等领域。

2 空间域声学计算超材料

在空间域,最先提出的是电磁波/光学计算超材料。Silva等人^[16]在2014年提出了计算超材料的概念,并给出了两种类型的计算超材料设计:基于Fourier变换和基于Green函数的计算超材料设计。Fourier变换设计基于

$$g(x) = \mathcal{F}^{-1}\{H(k_x)\mathcal{F}[f(x)]\}, \quad (1)$$

其中, $f(x)$ 和 $g(x)$ 为输入和输出信号, $H(k_x)$ 为传递函数, $\mathcal{F}$ 为Fourier变换。Fourier变换可以通过设计透镜实现,传递函数可以由超表面实现。而Green函数的设计基于卷积定理:

$$G(k_x) = F(k_x)H(k_x), \quad (2)$$

其中, $F(k_x)$ 和 $G(k_x)$ 分别为 $f(x)$ 和 $g(x)$ 的空间Fourier变换。通过在波数域设计超材料Green函数 $H(k_x)$ 来实现

对输入声波的数学运算。

受空间域电磁波/光学计算超材料启发,空间域声波/弹性波计算超材料也被提出。Zuo等人^[25~27]在2017年基于分层迷宫超表面,在2018年基于相位幅度解耦的反射超表面,设计了基于Fourier变换型的空间域声

学计算超材料,实现了Fourier变换、导数、积分、卷积、常微分方程求解运算,如图2(a)和(b)所示。2021年,Lü等人^[28]在板波中设计了条带型空间域 SH_0 波计算超材料,实现了分数阶Fourier变换运算,如图2(c)所示。他们运用了两步的优化策略,优化过程稍显复杂。2018年,

图 2 (网络版彩色)空间域声学计算超材料。(a) 分层迷宫超表面。Reprinted with permissions from Ref. [25]. a.u., arbitrary unit。(b) 相位幅度解耦的反射超表面。Reprinted with permissions from Ref. [27]。(c) 弯曲条带型 SH_0 波超材料。Reprinted with permissions from Ref. [28]. n.u., normalized unit。(d) 高折射率声学超材料。Reprinted with permissions from Ref. [29]

Figure 2 (Color online) Computational metamaterials in space domain acoustics. (a) Layered labyrinth metasurface. Reprinted with permissions from Ref. [25]. a.u., arbitrary unit. (b) Phase-amplitude decoupled reflective metasurface. Reprinted with permissions from Ref. [27]. (c) Curved strip type SH_0 wave metamaterial. Reprinted with permissions from Ref. [28]. n.u., normalized unit. (d) High refractive index acoustic metamaterial. Reprinted with permissions from Ref. [29]

Zangeneh-Nejad和Fleury^[29]基于高折射率声学超材料^[30]设计了基于Green函数的空间域声学计算超材料,实现了导数、积分运算,并探索了其在图像边缘检测领域的应用,如图2(d)所示.此外,2021年,Lü等人^[31]在空气声波计算超材料中实现了偏微分方程的运算.

3 时域声学计算超材料

现有的时域计算超材料同样可以划分为基于Fourier变换的和基于传递函数的.2020年,Lü等人^[32]基于“U”形跑道声波管超材料,实现了时域Fourier变换,如图3(a)所示.设计的“U”形跑道声波管超材料的一阶色散系数在整个带宽 $\Delta\omega$ 范围内是恒定的,即

$$\ddot{\Phi} = -\frac{\partial^2 \Phi(\omega)}{\partial \omega^2} = \frac{\partial \tau_g(\omega)}{\partial \omega} \quad (3)$$

为常数,其中, Φ 是超材料的相位响应, τ_g 是群速度延迟.那么,输入信号 $a_i(t)$ 和输出信号 $a_o(t)$ 近似有

$$a_o(t) = C e^{i\frac{t^2}{2\ddot{\Phi}}} \mathcal{F}[a_i(t)] \Big|_{\omega=\frac{t}{\ddot{\Phi}}}, \quad (4)$$

其中, C 为常数,即输出信号 $a_o(t)$ 为输入信号 $a_i(t)$ 的时域Fourier变换.

同年,Lü等人^[33]基于传递函数,通过遗传算法优化设计弯曲条带型 SH_0 波超材料,实现了对时域信号的包络的导数,如图3(b)所示.所得实验结果与理论结果吻合较好,但实验信号的信噪比不好,实验测试过程需要精心设计.同时,Lü等人^[34]设计空气声超材料实现了时域的分阶导数、积分和高斯滤波计算,如图3(c)所示.入射脉冲声波经过超材料调制之后,透射脉冲声波包络 $g(t)$ 形式如下:

$$g(t) = \int_{-\infty}^{+\infty} \tilde{T}(\omega + \omega_0) \tilde{f}(\omega) e^{-i\omega t} d\omega, \quad (5)$$

其中, $\tilde{T}(\omega)$ 是作为频率函数的复透射系数(透射复振幅), $\tilde{f}(\omega)$ 为输入脉冲包络 $f(t)$ 的频谱.那么,入射脉冲包络进行透射相当于信号 $f(t)$ 通过具有传输函数(transfer function, TF)的线性时间不变量(linear and time-invariant, LTI)系统 $\tilde{H}(\omega) = \tilde{T}(\omega + \omega_0)$.在实验上,由于没有成熟的商业仪器可以提供声学材料的时域测试,2020年,Lü等人^[32]搭建了空气声波时域计算超材料测试环境,解决了反射信号干扰和频散的问题.他们编写了测试空气声波时域计算超材料LabVIEW代码,直接得到测试结果和理论解的对比.所得实验结果与理

图3 (网络版彩色)时域声学计算超材料. (a) “U”形跑道声波管超材料. Reprinted with permissions from Ref. [32]. a.u., arbitrary unit. (b) 弯曲条带型 SH_0 波超材料. Reprinted with permissions from Ref. [33]. (c) 北京大学校徽和九叠篆“北京大学”组成的超材料. Reprinted with permissions from Ref. [34]. n.u., normalized unit

Figure 3 (Color online) Computational metamaterials in space domain acoustics. (a) “U” shape runway acoustic tube metamaterial. Reprinted with permissions from Ref. [32]. a.u., arbitrary unit. (b) Curved strip type SH_0 wave metamaterial. Reprinted with permissions from Ref. [33]. (c) Metamaterial with the PKU logo and Bei Jing Da Xue characters written in Jiu Die Zhuan. Reprinted with permissions from Ref. [34]. n.u., normalized unit

论结果吻合较好,但也存在测试仪器所需空间较大等问题,不利于器件的小型化、集成化。

4 可调声学计算超材料和超材料计算电路

在过去的10年中超材料尽管发展迅速,但主要集中于单一、线性和被动超材料设计,限制了超材料的适用性和一般性。一旦超材料的结构固定,被动超材料的功能将不能改变,无法适应环境变化的需求。被动线性声学材料的频率色散受Kramers-Kronig型色散关系^[35]的约束,限制了超材料器件的工作频谱。因此,人们提出了可调超材料设计,如可调电磁超材料^[36-39]等。可调声学超材料可以实现对声音的频率可调的滤波^[40]、可调的通断^[41,42]等功能。2016年, Babaee等人^[41,42]提出了胞元散射特性可调的声学超材料,实现了声子晶体带隙的调节。他们基于折纸工艺实现了变构型的可调超材料设计。相比之下,现有的计算超材料功能单一,没有可调设计。材料固定,相应的计算功能就固定,单一功能的器件满足不了复杂计算任务和集成化设计的需求。因此,发展功能可调的计算超材料具有科研意义和工程价值。同时,实现复杂的函数运算,难以通过设计单一的声波计算超材料来实现,需要声波计算电路的集成化、模块化等设计方法,同时需要克服声波计算电路中的声波反射干扰、声波定向传播的难题。因此,如何设计实现计算超材料电路、网络,实现复杂、并行、高通量计算功能,更好地应用到信号处理、机器学习等领域,这是一个极具挑战性的科学问题。

围绕以上存在的科学问题,2020年,Lü等人^[34]提出了可调计算超材料设计方法,将优化参数分类为可调参数和共用参数分别优化,解决了不同计算功能设计参数不通用的问题。通过遗传算法优化设计制作了可调计算超材料,实验验证了在同一个材料上可实现分数阶积分、导数,如图4(a)所示。同时,他们设计了声学超材料计算电路^[34],如图4(b)所示,提出了声学开关的设计,使得声波有定向传播的特性。若开关的联结发生变化,则只需改变橡胶材料的位置即可实现声波传播回路的切换。为了解决声波反射干扰问题,他们提出了回环电路设计。这样,声波脉冲在波导回路中有足够的时间传播,而折叠的设计使得模型可尽可能节约空间和制作成本。通过超材料计算电路实现了导数、积分及其串联、并联运算,为集成复杂超材料计算网络设计了必需的元器件。但是,由于受到电路中声波传播

路径较长、不同功能的超材料块的透射信噪比不一等因素的影响,声学超材料计算电路的实验验证工作比较难进行。

5 声学超材料计算网络和声学拓扑绝缘体计算超材料

分析散射波并进行物体的识别在声学中具有重要意义。最近,深度学习技术在解析声方面取得了阶段性成果,例如超声无损检测和疾病诊断,但通常需要耗时的计算机后处理或体积庞大的衍射元件。针对这些问题,Weng等人^[43]从理论上提出并通过实验证明了一种纯被动的、小尺寸的声学计算超材料网络,通过分析声学散射来实时识别复杂的物体,如图5(a)所示。他们证明声学计算超材料网络可以模仿标准神经网络,通过手写数字识别和辨别错位轨道角动量涡旋的两个例子,展示了执行所需任务的计算能力,并有可能克服当前耗时等限制。基于超材料的深度学习使得超材料能够自动分析信号,对声学和相关领域具有深远的影响。

声学计算超材料提供了高速执行专门计算任务的可能性,但它们有一个重要的限制,可能会阻碍它们在大规模应用中的适用性。与数字信号处理(digital signal processing, DSP)对输入信号二值化或离散化的性质不同,基于波的模拟计算机通常容易受到噪声和扰动的影响。特别是由几何缺陷引起的误差会在串联运算中累积,从而降低模拟计算机的性能。为了解决这个问题,研究人员引入了一类特殊的具有非平凡拓扑结构的拓扑绝缘体超材料^[44-53]。这种超材料的带隙中无法传播体波,但是频带拓扑结构保证了受拓扑保护的边界模式的存在。由于边界模式的存在是系统能带结构拓扑的结果,因此不会受到由局部引入小缺陷或几何缺陷引起的连续变形的影响。相反,拓扑结构只能通过系统的全局改变来改变。因此,只要缺陷不破坏拓扑序所依赖的对称性,拓扑结构的边界模式对低、中等水平的缺陷具有很强的免疫力。基于此,Zangeneh-Nejad和Fleury^[17]提出了一类利用鲁棒拓扑边界模式的计算超材料,通过一维拓扑波绝缘体的边界模式共振隧穿效应来实现格林函数方法,对大范围类型的缺陷具有很强的鲁棒性,如图5(b)所示。

6 总结与展望

本文讨论了计算超材料领域的最新进展,概述了其在现代工程实际应用中的几个例子,包括方程求

图4 (网络版彩色)可调时域声学计算超材料(a)和时域声学超材料计算电路(b). Reprinted with permissions from Ref. [34]. n.u., normalized unit
 Figure 4 (Color online) Tunable time-domain acoustic computational metamaterials (a) and time-domain acoustic metamaterial computing circuits (b). Reprinted with permissions from Ref. [34]. n.u., normalized unit

解、图像处理和机器学习等。最近,研究者提出了基于波的多功能模拟计算^[54-56],其中多个计算任务在不同的输入通道上同时执行。这种运算提供了并行处理信息的可能性,大大提高了计算速度,而且还可以节省大量电能,避免使用大型电子系统以及随之而来的散热问题。本文中的大多数计算超材料都是基于线性运算

功能。随着基于机器学习方法的发展,非线性计算超材料可能是未来研究的方向。除此之外,非线性计算超材料可以有其他应用,例如复杂的非线性方程求解、非线性滤波、图像处理方法。与光学计算超材料相比,声学计算超材料不如光学超材料运算速度快,但声学信号需要使用声学计算超材料来处理。声学计算超材料

图 5 (网络版彩色)声学超材料计算网络和声学拓扑绝缘体计算超材料. (a) 声学计算超材料网络用于深度学习. Reprinted with permissions from Ref. [43]. (b) 声学拓扑绝缘体计算超材料. Reprinted with permissions from Ref. [17]

Figure 5 (Color online) Acoustic metamaterial computing network and acoustic topological insulator computational metamaterial. (a) Acoustic computational metamaterial for deep learning. Reprinted with permissions from Ref. [43]. (b) Acoustic topological insulator computational metamaterial. Reprinted with permissions from Ref. [17]

可以允许以并行方式执行模拟运算任务来加速声学成像和检测. 同时, 随着表面声波技术的发展, 未来声学计算超材料在小型化方面具有巨大的潜力. 另外, 声学计算超材料结构设计的复杂程度相对于商业应用而言仍偏高, 目前针对不同计算目标的特定声人工结构, 基本都需要具有一定经验的科研人员手动设计, 具体的材料参数与几何参数往往非常复杂, 从而带来一系列可能的生产时效、生产成本等问题. 不仅如此, 缺陷对声学计算超材料计算精度有较大影响, 超材料结构尺

寸等的变化可能引起运算结果的较大偏差. 缺陷对计算超材料的影响是未来研究的另一个重要课题. 拓扑计算结构对于杂质和缺陷等无序具有鲁棒性, 可能是未来研究的一个方向. 实现稳健计算的另一种可能方法是使用在非互易结构中传播的单向模式. 作为一种被动设计结构, 声学计算超材料一旦设计出来后便不可改变, 针对运算目标单一, 拓宽计算超材料的带宽对于某些应用很重要, 发展功能可调的计算超材料具有科研意义和工程价值. 基于与波长无关的设计, 如

布鲁斯特效应^[57], 或相对宽带效应, 如自旋霍尔效应^[58], 或者有源超材料设计可能同样是实现宽带模拟计算的方式. 此外, 声学计算超材料效能相对低下,

透射声波能量效率较低. 发展引入声波透射能量效率的声学计算超材料优化设计方法是未来重要的研究内容.

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Summary for “声学计算超材料”

Acoustic computational metamaterials

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Although digital signal processors have been widely used to perform complex computing tasks, they still suffer from some limitations, including the complexity, low speed, and high-power consumption caused by expensive analog-to-digital converters. For this reason, there has recently been a strong interest in wave-based analog computing, which avoids analog-to-digital conversion and can perform massively parallel operations, especially based on artificially designed metamaterials. A new analog computing scheme based on sound waves has been proposed. This computing system, called computational metamaterials, can be as fast as the wave speed and as small as the wavelength. It can perform complex mathematical operations on incoming wave packets, and can even perform calculations on integral and differential equations. These functions are expected to realize a new generation of ultra-fast, compact and efficient computing hardware based on sound wave propagation. The development of acoustic computing metamaterials has gone from the computation of a single mathematical operation, the tunable design and computing circuit and computational network that integrate multiple computing blocks, to the topological insulator computational metamaterial proposed for the robustness of computation. The computation of a single mathematical operation can be divided into space and time domain. The difference of main design principles is dimension of time and space. The spatial acoustic computational metamaterials mainly require the realization of spatial distribution of Green's function of required calculation in materials. And the time-domain acoustic computational metamaterial needs to design the frequency spectrum to realize the transfer function. Both the space domain and the time domain acoustic computing metamaterials realize operations, such as integration, derivative, and differential equations. The proposed tunable acoustic computational metamaterials are aimed at the problem that the function of the metamaterials cannot be changed while the structure is fixed. By introducing tunable parameters into the metamaterial, it realizes the integration of integral, derivative and other functions on the same metamaterial. The proposal of acoustic metamaterial calculation circuit and calculation network has raised the complexity of metamaterial calculation by another dimension. By introducing the design of acoustic circuit components, such as acoustic switches, the sound waves are controlled to propagate in different circuits, and complex operations, such as series and parallel connection of multiple different metamaterial blocks are realized. Acoustic metamaterial computing network combines the design ideas of acoustic holography and convolutional neural network (CNN). Each pixel unit of the metamaterial is equivalent to a neuron in the CNN, and the propagation of sound waves between different metamaterial layers in the medium is equivalent to the signal propagation between neurons in different layers. Through training and learning, machine learning tasks, such as handwritten number recognition, have been completed. In view of the computational robustness, the proposed topological insulator computational metamaterials, by introducing the boundary mode of topological protection, ensure that the acoustic computational metamaterials can still guarantee the accuracy of the calculation even when the structure parameter is disturbed. In the future, the research of nonlinear computational metamaterials will receive more and more attention, and it will be possible to apply to the fields of nonlinear equation solving, nonlinear filtering, image processing methods and so on. In this review, we discussed the latest developments in the field of acoustic computational metamaterials and studied the latest superstructures used to perform analog computing. We further introduced the applications of acoustic computational metamaterials, including image processing, edge detection, equation solving, and machine learning. Finally, we look forward to key research issues and possible future directions.

acoustic metamaterials, analog computing, metamaterial computing circuit, acoustic metamaterial computing network

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