Another proof for C^1 stability conjecture for flows*

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Abstract The C^1 structural stability conjecture for flows by C^1 connecting lemma and obstruction sets is proved.

Keywords: axiom A, strong transversality, obstruction set, stability conjecture.

Let M be a compact n-dimensional C^∞ Riemannian manifold without boundary ($n>1$). Denote by χ all C^1 vector fields on M. Denote by χ^* all the systems $X\in\chi$ satisfying the following property: X has a neighborhood μ in χ such that each $Y\in\omega$ has only finitely many singularities and at most countably many periodic orbits (or equivalently, all singularities and periodic orbits of each $Y\in\omega$ are hyperbolic (see refs. [1,2]). Denote by $\chi^\#$ all the systems $X\in\chi^*$ satisfying the following property: X has a neighborhood ω in χ^* such that for each $Y\in\omega$, Y is a Kupka-Smale system. In particular, the stable manifolds and unstable manifolds of singularities and periodic orbits of Y are all transversal. Denote by $\mathrm{Ob}(X)$ the obstruction set of $X\in\chi$ (see ref. [3] or the following for the definition of $\mathrm{Ob}(X)$). The main results of this paper are the following theorems.

Theorem A. *If* $S\in\chi^\#$, *then* $\mathrm{Ob}(S)=\emptyset$.

As a consequence, we generalize a result of Liao in reference [3].

Theorem B. *If* $S\in\chi$, *then the following conditions are equivalent*:

(i) S *is* C^1 *structurally stable*.

(ii) $S\in\chi^\#$.

(iii) $\mathrm{Ob}(S)\cup I(S)=\emptyset$. ($I(S)$ *denotes the interior of the set of all singularities of* S).

(iv) S *satisfies the Axiom A and the strong transversality condition*.

Proof. (i)⇒(ii) is proved by Robinson[4]. For $S\in\chi^\#$, obviously, $I(S)=\emptyset$. Therefore, according to Theorem A we have (ii)⇒(iii). For (iii)⇒(iv) see ref. [3]. (iv)⇒(i) is the classical structural stability theorem (for a proof see reference [5]).

If $\dim M=2$, the above theorem is a consequence of ref. [6]. If $\dim M=3,4$ and S has no singularities, the theorem is shown in Chapter 8 of reference [3].

The implication (i)⇒(iv) is the famous stability conjecture for flows, which is a long standing problem. It was first formulated in ref. [7], and many people have contributed to the problem. Liao proved the conjecture under some restricted condition for $\dim M=3$ in ref. [2]. As a consequence he proved the conjecture for two-dimensional diffeomorphisms, which is reproved in ref. [8] by a relatively different method. Along this line Mañé at last proved the conjecture for diffeomorphisms of general dimensions in ref. [9]. Sannami[10] also proved this result in $\dim M=$

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2. Starting from the extended form of the technique used in ref. [9], Hu proved the conjecture for $\dim M = 3$ in ref. [11]. Also, Liao proved the conjecture for $\dim M = 3$ in ref. [12] by using obstruction sets, a method created by himself in ref. [13], and developed in ref. [14] (see also ref. [3]). A great breakup in proving the conjecture came up recently with the C^1 connecting lemma of Hayashi[15]. Wen and Xia1) have a nice simple proof for the C^1 connecting lemma. Consequently, Hayashi and Wen proved this conjecture in refs. [15, 16], respectively, in a similar way by extending the method in ref. [9]. In this paper, we prove the conjecture by following the way in reference [12].

1 Obstruction set and minimal rambling set

In the following we briefly introduce the concept of obstruction sets (for the details, see ref. [3]). Let $S \in \chi$. Then S induces one-parameter transformation groups on M, the tangent bundle TM and the conjugate bundle $\mathscr{D}$ of S, respectively:

$$\phi_t : M \to M, \quad -\infty < t < \infty,$$
$$\Phi_t : TM \to TM, \quad -\infty < t < \infty,$$
$$\psi_t : \mathscr{D} \to \mathscr{D}, \quad -\infty < t < \infty.$$

For any $u \in \mathscr{D}$, $\| u \| \neq 0$, denote by $\psi_t(u)$ the orthogonal projection of u on $\mathscr{D}$. For any $u \in \mathscr{D}_x (x \in M' = M - \mathrm{Sing}(S))$, denote by $\otimes u$ the $(n-2)$-dimensional linear subspace of $\mathscr{D}_x$, consisting of all the vectors orthogonal to u, i.e.

$$\otimes u = \{ v \in \mathscr{D}_x : \langle u, v \rangle = 0 \}.$$

For any $t \in \mathbb{R}$, there exists a unique vector $\Psi_t(u) \in \mathscr{D}_{\phi_t(x)}$, $\Psi_t(u) \perp \psi_t(\otimes u)$ and

$$\psi_t(u) - \Psi_t(u) \in \psi_t(\otimes u).$$

This gives a one-parameter transformation group

$$\Psi_t : \mathscr{D} \to \mathscr{D}, \quad t \in \mathbb{R}.$$

This transformation group can be uniquely extended to (still denoted by Ψ_t)

$$\Psi_t : \mathscr{D}^* \to \mathscr{D}^*,$$

where $\mathscr{D}^*$ is the closure of $\mathscr{D}$ in TM. The obstruction set of S is defined as

$$\mathrm{Ob}(S) = \{ x \in M : \exists\, u \in \mathscr{D}^* \cap TM_x \text{ such that } \| u \| = 1 = \inf_{t \in (-\infty, \infty)} \| \Psi_t(u) \| \}.$$

This is a closed subset of M.

Definition 1.1. A subset Λ of M is called the rambling set of S, if Λ is closed in M, invariant under $\phi_t (-\infty < t < \infty)$ and

$$\Lambda \cap \mathrm{Ob}(S) \neq \emptyset.$$

A rambling set of S is called minimal, if each of its proper subset is never a rambling set of S.

According to Zorn's lemma, the following is easily deduced[3].

Proposition 1.1. *Every rambling set of S contains at least one minimal rambling set.*

For the convenience of discussion, minimal rambling sets are classified into two classes.

Definition 1.2. A minimal rambling set Λ is called simple, if it satisfies the following condition (i) or (ii):

(i) Λ contains no ordinary point of S, or

(ii) Λ contains an ordinary point $a \in \mathrm{Ob}(S)$ such that both the ω-limit set $\omega(a)$ and the α-

1) Wen, L., Xia, Z., The C^1 connecting lemma, Preprint, 1995.

limit set $\alpha(a)$ of the orbit $\mathrm{Orb}(a) = \{\phi_t(a) \mid t \in (-\infty, \infty)\}$ are proper subsets of Λ. Otherwise, Λ will be called non-simple.

The following theorem gives a relatively complete description for simply minimal rambling sets.

Theorem 1.1. *Let Λ be a simply minimal rambling set of S. Then Λ consists of exactly one point if and only if this point is a nonhyperbolic singularity. Let Λ contain more than two points. Then S has an ordinary point $a \in \Lambda \cap \mathrm{Ob}(S)$ with the following properties:*

(i) *Both the ω-limit set $\omega(a)$ and the α-limit set $\alpha(a)$ of the orbit $P(a) = \{\phi_t(a) \mid t \in (-\infty, \infty)\}$ of S have hyperbolic structures.*

(ii) $\Lambda = P(a) \cup \omega(a) \cup \alpha(a)$ *and* $(\omega(a) \cup \alpha(a)) \cap \mathrm{Ob}(S) = \emptyset$. *Hence $\Lambda \cap \mathrm{Ob}(S)$ is contained in a finite arc-orbit of $P(a)$.*

(iii) $\dim(D_-(a) + D_+(a)) < n - 1$, *where*

$$D_-(a) = \{u \in \mathscr{D}_a \mid \lim_{t \to \infty} \psi_t(u) = 0\}, \quad D_+(a) = \{u \in \mathscr{D}_a \mid \lim_{t \to -\infty} \psi_t(u) = 0\}.$$

Hence $\dim((\otimes D_-(a)) \cap (\otimes D_+(a))) \geq 1$.

(iv) *Either $\omega(a)$ consists of only one singularity of S, or $\omega(a) \subset M$. In the latter case, there exist numbers $\mu < 0$ and $d > 0$ such that*

$$\| \psi_{s+t}(u) \| \leq \| \psi_s(u) \| \exp(\mu t) \text{ for } u \in D_-(a),\ s \geq 0 \text{ and } t \geq d,$$

$$\| \psi_{s+t}(u) \| \geq \| \psi_s(u) \| \exp(-\mu t) \text{ for } u \in \otimes D_-(a),\ s \geq 0 \text{ and } t \geq d,$$

and $\dim D_-(x) = \dim D_-(a)$ *for* $x \in \omega(a)$.

(v) *Also, either $\alpha(a)$ consists only one singular point of S, or $\alpha(a) \subset M$. In the latter case, there exist numbers $\mu' < 0$ and $d' > 0$ such that*

$$\| \psi_{s+t}(u) \| \leq \| \psi_a(u) \| \exp(-\mu' t) \text{ for } u \in D_+(a),\ s \leq 0 \text{ and } t \leq -d',$$

$$\| \psi_{s+t}(u) \| \geq \| \psi_a(u) \| \exp(\mu' t) \text{ for } u \in \otimes D_+(a),\ s \leq 0 \text{ and } t \leq -d',$$

and $\dim D_-(x) = \dim(\otimes D_+(a))$ *for* $x \in \alpha(a)$.

Definition 1.3. Let Λ be a connected hyperbolic set of S. Denote

$$\mathrm{Ind}\Lambda = \mathrm{Ind}_S\Lambda = \dim F_-(x),$$

where $x \in \Lambda$, and

$$F_-(x) = \{u \in TM_x : \lim_{t \to \infty} \Phi_t(u) = 0\}.$$

Obviously, the value $\mathrm{Ind}\Lambda$ is independent of x.

2 C^1-connecting lemma and the proof of the main result

To prove Theorem A, we also need Hayashi's C^1 connecting lemma[15]. The following strong version is taken from reference [16]1).

Theorem 2.11). *Let M be a compact n-dimensional C^∞ Riemannian manifold without boundary. Let $X \in \chi$, which induces a flow $\phi_t = \phi X_t$. Assume $z \in M$, which is not a singularity of X or a point on any periodic orbit of X. For any C^1 neighbourhood u of X in χ, there exist $\rho > 1$, $T > 1$ and $\delta_0 > 0$ such that for any $0 < \delta \leq \delta_0$ and any two points x, y outside the tube $\Delta = \bigcup_{t \in [0, T]} B(\phi_{-t}(z), \delta)$ if the positive X-orbit of x and the negative X-orbit of y hit $B(x, \delta/\rho)$, then there exists $Y \in u$ such that $Y \equiv X$ outside Δ and y is on the positive Y-orbit of x.*

This theorem is summarized from the linear version below. We state it as follows.

1) See the footnote on page 1077.

Theorem 2.2[1] **(The linear version of the C^1 connecting lemma).** *Let $\{V_n\}_{n=0}^{\infty}$ be a sequence of inner product spaces and let $T_n: V_n \to V_{n-1}$ be a sequence of linear isomorphisms. For any $\varepsilon > 0$ there exist $\rho \geqslant 1$ and $L \in N$ such that: For any sequences $\{x_i\}_{i=1}^{s}$ and $\{y_j\}_{j=1}^{t}$ in V_0, with an order $<$ defined on the union*

$$X = \{x_i, y_j: i = 1,2,\cdots,s; j = 1,2,\cdots,t\}$$

as

$$x_1 < x_2 < \cdots < x_s < y_t < y_{t-1} < \cdots < y_1,$$

there exist two points $x \in \{x_i\}_{i=1}^{s} \cap B(x_s, \rho|x_s - y_t|)$, $y \in \{y_i\}_{i=1}^{t} \cap B(x_s, \rho|x_s - y_t|)$, $k \geqslant 0$ together with some ordered pairs $\{p_i, q_i\} \subset X \cap B(x_s, \rho|x_s - y_t|)$ say, k of them, with the order

$$x_1 \leqslant p_1 \leqslant q_1 < p_2 \leqslant q_2 < \cdots < p_{k'} \leqslant q_{k'}$$
$$< x < y < p_{k'+1} \leqslant q_{k'+1} < \cdots p_k \leqslant q_k \leqslant y_1,$$

$(0 \leqslant k' \leqslant k)$ satisfying the following conditions.

(i) *There exists an ε-kernel transition from x to y of length L, supported in $B(x_s, \rho|x_s - y_t|)$.*

(ii) *For any $i = 1,2,\cdots,k$ there exists an ε-kernel transition from p_i to q_i of length L, supported in $B(x_s, \rho|x_s - y_t|)$.*

(iii) *Thesee $(k+1)$ transitions avoid $X - [x_1, y_1] - [p_1, q_1] - \cdots - [p_k, q_k]$.*

(iv) *These $(k+1)$ transitions are mutually disjoint.*

Remark 2.1. The existence of ρ and L in the above theorem is only dependent on ε, $\{T_n\}$ and $\{V_n\}$. In other words in Theorem 2.1 ρ, L and δ_0 are only dependent on the tangent maps of f at $\mathrm{Orb}(z)$ and the size of ω. This fact is very useful for the later application of Theorem 2.1.

Write $M' = M - \mathrm{Sing}(S)$. For $x \in M'$ denote $\mathcal{D}_x = \{u \in TM_x: \langle u, S(x)\rangle = 0\}$. Assume that $\Lambda \subset M'$ is a compact invariant subset of S. Denote by $\mathcal{D}_\Lambda$ the restriction of the conjugate bundle $\mathcal{D} = \bigcup_{x \in M'} \mathcal{D}_x$, i.e. $\mathcal{D}_\Lambda = \bigcup_{x \in \Lambda} \mathcal{D}_x$. For any Whitney sum $\mathcal{D}_\Lambda = \Delta_- \oplus \Delta_+$ with Δ_- and Δ_+ invariant under S, we say S is contracting on Δ_- if there exist $\eta > 0$, $T > 0$ such that $\eta_-(t, \Delta_-(x)) \leqslant -t\eta$, for any $x \in \Lambda$ and $t \geqslant T$, where

$$\eta_-(t, E) = \begin{cases} \sup_{\|u\|=1} \ln \|\psi_t(u)\|, & \text{if } \dim E > 0, \\ -\infty, & \text{if } \dim E = 0, \end{cases}$$

for any linear subspace $E \subset \mathcal{D}_x$. If $-S$ is contracting on Δ_+ then S is called expanding on Δ_+.

In the following we fix two numbers $\tilde{\eta}$ and $\tilde{T}$ and a neighbourhood $\tilde{\omega}$ for each $S \in \chi^*$ as in reference [3].

A sequence $\{X_i, P_i\}$ is called a fundamental p-sequence (of S), where $X_i \in \tilde{\omega}$, P_i is a periodic orbit of X_i and p an integer $(0 \leqslant p \leqslant n-1)$, if

$$\lim_{i \to \infty} \|X_i - S\|_1 = 0 \text{ and } \mathrm{Ind}_{X_i} P_i = p, \quad i = 1,2,3,\cdots.$$

Let a number $\zeta \in (0, \tilde{\eta})$ and an integer $p \in \langle 0, n-1\rangle$ be given. For $S \in \chi^*$, a point $a \in M$ is called a right (ζ, p)-hunched point of S, if there exist a fundamental p-sequence $\{X_i, P_i\}$ and points x_i on P_i satisfying

1) See the footnote on page 1077.

$$\lim_{i\to\infty} x_i = a,$$

$$\frac{1}{kT}\sum_{j=0,1,\cdots,k-1} \eta_-(X_i, D_-(X_i, \phi_{X_i(j\tilde{T})}(x_i)); \tilde{T}) \geqslant -\zeta$$

and

$$\frac{1}{kT}\sum_{j=-k,-k+1,\cdots,-1} \eta_-(X_i, D_-(X_i, \phi_{X_i(j\tilde{T})}(x_i)); \tilde{T}) \leqslant -\zeta$$

for $k = 1, 2, \cdots, q_i$; $i = 1, 2, 3, \cdots$.

$$\lim_{i\to\infty} q_i = \infty.$$

Denote by $R(\zeta, p)$ the set of all right (ζ, p)-hunched points of S. A point $a \in M$ is called a left (ζ, p)-hunched point of $S \in \chi^*$ if a is a right $(\zeta, n-1-p)$-hunched point of $-S$ (obviously, $S \in \chi^*$ implies $-S$ also $\in \chi^*$). Denote by $L(\zeta, p)$ the set of all left (ζ, p)-hunched points. It is easy to know that $R(\zeta, p)$ and $L(\zeta, p)$ are both closed subsets in M.

The following theorem is a reformulation of Theorem 8.3.2 in reference[3].

Theorem 2.3[3]. *Let $S \in \chi^\#$. If Λ is a nonsimply minimal rambling set without singularities, $\{X_i, P_i\}$ is a fundamental p-sequence and P_i converges to Λ in the sense of Hausdorff, which gives a dominated splitting $\mathscr{D}_\Lambda = \Delta_-^p \oplus \Delta_+^p$, then S cannot be contracting on Δ_- and S cannot be expanding on Δ_+.*

Also from ref. [3] we have

Theorem 2.4[3]. *Let $S \in \chi^\#$. Then S has no simply minimal rambling sets.*

Now we use C^1 connecting lemma to establish the following lemma.

Lemma 2.1. *Let $S \in \chi^\#$. If Λ is a nonsimply minimal rambling set, then it contains no singularities.*

Proof. Suppose on the contrary that Λ is a nonsimply minimal rambling set and contains a singularity x. Assume $b \in \Lambda \cap \mathrm{Ob}(S)$ and $x \in \omega(b) = \Lambda$ without loss of generality. Obviously, x is a saddle. Since $x \in \omega(b)$ we can take $x_s \in W^s(x) \cap \Lambda - \{x\}$, $x_u \in W^u(x) \cap \Lambda - \{x\}$.

Choose a neighbourhood u of S in $\chi^\#$. Using Theorem 2.1 twice we see that there exists $X \in u$ such that x is a singularity of X and $\alpha(y) = \omega(y) = x$ for some $y \in M$, in other words y is a homoclinic point of x with respect to X which contradicts the fact that X is a Kupka-Smale system.

Lemma 2.2. *Let $S \in \chi^*$ and Λ be a compact invariant set of S with no singularities and there exists a fundamental p-sequence $\{X_i, P_i\}$ such that P_i converges to Λ in the sense of Hausdorff, which gives a dominated splitting $\mathscr{D}_\Lambda = \Delta_-^p \oplus \Delta_+^p$. If there exists $\zeta > 0$ such that $\zeta \leqslant \tilde{\eta}$ and $\Lambda \cap R(\zeta, p) = \emptyset$ then S is contracting on Δ_-. Similarly if $\Lambda \cap L(\zeta, p) = \emptyset$ then S is expanding on Δ_+.*

Proof. Assume $\Lambda \cap R(\zeta, p) = \emptyset$. According to Lemma 6.4.8 in ref. [3] there exist a number $T > 0$, a neighbourhood U of Λ in M and a neighbourhood u of S in χ^* such that for any $X \in u$, if P is a periodic orbit of X with $\mathrm{Ind}P = p$ and $x \in P$ then

$$\eta_-(X, D_-(X, x), T) \leqslant -\frac{\zeta}{2}T.$$

Therefore for any $x \in \Lambda$, since P_i converges to Λ in the sense of Hausdorff we can take $x_i \in P_i$ such that $\lim_{i\to\infty} x_i = x$. Now we have

$$\eta_-(X_i, D_-(X_i, x_i), T) \leqslant -\frac{\zeta}{2}T, \ \forall i \geqslant 1.$$

By taking subsequence we may assume that $D_-(X_i, x_i)$ converges to Δ_-^p in the sense of Hausdorff. Letting i tend to ∞, by the continuity of η_- with respect to X_i and D_- we have

$$\eta_-(T, \Delta_-(x)) = \eta_-(S, \Delta_-(x), T) \leqslant -\frac{\zeta}{2}T,$$

from which it is easily deduced that S is contracting on Δ_-. The other half of the lemma can be proved similarly.

Proof of Theorem A. Let $S \in \chi^*$. Assume $\mathrm{Ob}(S) \cup I(S) \neq \emptyset$. Obviously $I(S) = \emptyset$. Hence S has a minimal rambling set, say, Λ. From Theorem 2.4 and Lemma 2.1 we know that Λ is nonsimple and contains no singularities. Assume $\{X_i, P_i\}$ to be a fundamental p-sequence such that P_i converges to Λ in the sense of Hausdorff, which gives a dominated splitting $\mathscr{D}_\Lambda = \Delta_-^p \oplus \Delta_+^p$. From Theorem 2.4 S is not contracting on Δ_-^p. Therefore according to Lemma 2.2 $\Lambda \cap R(\zeta, p) \neq \emptyset$ for any $0 < \zeta < \tilde{\eta}$. Thus by the Theorem 6.5.7 in ref. [3] there exists a hyperbolic set $\Lambda_<$ contained in Λ with index less than p. Similarly, there exists a hyperbolic set $\Lambda_>$ contained in Λ with index greater than p. Since Λ is nonsimple, we can take $b \in \Lambda \cap \mathrm{Ob}(S)$ and may assume $\omega(b) = \Lambda$. Take a compact neighbourhood $\overline{U}_1$ of $\Lambda_<$ and a compact neighbourhood $\overline{U}_2$ of $\Lambda_>$. We may assume that U_i, $i = 1, 2$ is small enough such that $\overline{U}_1 \cap \overline{U}_2 = \emptyset$ and

$$\Lambda_i = \bigcap_{t \in \mathbb{R}} \phi_t(\overline{U}_i), \ i = 1, 2$$

is a hyperbolic set. And take an open neighbourhood V_1 of $\Lambda_<$ such that $V_1 \subset U_1$. Since $\Lambda_< \subset \Lambda = \omega(b)$, there exist $y_i = \phi_{t_i}(b)$, $z_i = \phi_{s_i}(b) (i \geqslant 2) t_i < s_i$ and $y_i, z_i \in \partial V_1$ and $\phi_{(t_i, s_i)}(b) \subset V_1$. Since $\Lambda_<$ is a proper subset of Λ, it is easily proved that $\lim_{i \to \infty}(s_i - t_i) = \infty$. By taking subsequence we may assume that $\lim_{i \to \infty} y_i = y_1 \in \partial V_1$. Then it is easily seen that $\omega(y_1) \subset \overline{V}_1$. Obviously $y_1 \in \Lambda$. Similarly we have Λ_2, $y_2 \in \partial V_2 \cap \Lambda$ and $\alpha(y_2) \subset \overline{V}_2$.

Now, using the C^1 connecting lemma twice, we get $X \in \chi^\#$ and $x \in M$ such that $\omega(x)$ is a hyperbolic set with index equal to $\mathrm{Ind}\Lambda_< = q < p$ and $\alpha(x)$ is a hyperbolic set with index equal to $\mathrm{Ind}\Lambda_> = r > p$. It is easily deduced that

$$\dim D_-(x) = \mathrm{Ind}\Lambda_< = q,$$
$$\dim D_+(x) = n - 1 - \mathrm{Ind}\Lambda_> = n - 1 - r.$$

Thus

$$\dim D_-(x) + \dim D_+(x) = q + n - 1 - r < n - 1 + p - p = n - 1.$$

From Corollary 4.4.13 in ref. [3] $\overline{\mathrm{Orb}(x)}$ is an obstruction set of X. Obviously, it is a simply minimal rambling set, which contradicts Theorem 2.4. This proves Theorem A.

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