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An Elementary Study of the Unearthed Mathematics Book, *Suanshu Shu*[1]

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Abstract: The transcription of the *Suanshu Shu* 算數書 (a bamboo book of mathematics) in simplified Chinese characters offers a new opportunity to explore the history of Chinese mathematics in ancient times. This paper analyzes the style and structure of the *Suanshu Shu* and makes comparisons with the *Nine Chapters on Mathematical Procedures* and a number of other texts in various social contexts. It will be shown that the *Suanshu Shu* was compiled from at least two sources, and that no direct textual interplay exists between the *Suanshu Shu* and the *Nine Chapters*, although both share the same origins in the Pre-Qin period when the major mathematical methods in the *Nine Chapters* came into being. It will also be shown that the *Suanshu Shu* was accomplished with the methods used in certain mathematical books in the Pre-Qin period or their results, which later led to the *Nine Chapters*, and by accommodating the actual conditions of the lower government administration. The *Suanshu Shu* is significant for establishing the evolution of algorithmic mathematics from the Warring States period to the Han dynasty.
Keywords: *Suanshu Shu*, *Nine Chapters on Mathematical Procedures*, mathematics from the Pre-Qin period to the Han dynasty

1 Introduction

A batch of bamboo slips were excavated from tomb no. 247 in Zhangjiashan, Jiangling County, Hubei Province at the turn of 1983–1984. Among these slips was a mathematical treatise, as well as legal documents, medical books, books on the art of war, a *rishu* 日書 (almanac),[2] a *lipu* 曆譜 (a calendar with records of events), and a *qiance* 遣冊 (a list of funerary objects). On the back of one slip in this

[1] The Chinese version of this paper was published as "Chutu *Suanshu Shu* chutan" 出土《算数书》初探 (A preliminary study of the mathematical treatise, *Suanshu Shu*) in *Ziran kexueshi yanjiu* 自然科学史研究 (Studies in the History of Natural Sciences) 20(3):193–205 in 2001, and has been revised in English translation for this journal. The translation was made by Lü Xin 吕昕.

[2] The *rishu* 日書 was unearthed from another tomb in Zhangjiashan and not from tomb no. 247.

mathematical treatise are the three characters "*suan shu shu* 算數書," giving its title. Since archaeologists have inferred that this tomb was sealed in 186 BCE or slightly later, the *Suanshu Shu* must have been written before that date. Rich in content, the *Suanshu Shu* is the earliest known Chinese mathematical treatise. Earlier research on this treatise was impeded by the limited number of publications available on the *Suanshu Shu* before 2000. Now that a transcription of the full text of the *Suanshu Shu* has been made available (Research group of the Zhangjiashan Han bamboo slips from Jiangling 2000),[3] we are able to ascertain, to a large degree, what the original version of the *Suanshu Shu* would have looked like. It was completed at least one and a half centuries before the extant version of the *Nine Chapters*. This transcription will doubtlessly give tremendous impetus to research on mathematics in early China and to reflections on research methods for the history of the early sciences.

The major arguments in this paper were presented at a symposium in 2000.[4] Several years later, Christopher Cullen, Joseph W. Dauben, and Karine Chemla published English treatises pertaining to the *Suanshu Shu*, including translations into English (Cullen 2004; Dauben 2004, 151–168; Cullen 2007; Dauben 2008; Chemla 2010, 253–285). The agreements between this paper and the above-mentioned works concern the following aspects: the *Suanshu Shu* is not systematically organized; it was compiled by combining materials from different sources, and demonstrates that the origins of the extant edition of the *Nine Chapters* are at least partly earlier than the Han dynasty. On the other hand, with regard to the relations between the *Nine Chapters* and the *Suanshu Shu*, there are some differences of opinion among the above three scholars, as well as with the author of this paper in terms of their views, perspectives, and methods for argumentation. This paper contains viewpoints different from the above-mentioned works and therefore merits republication in English. In reviewing his original work, the author has found only a few minor errors and a small number of view-points that have required adjustment, for which supplementary notes have been provided. When quoting passages from the *Suanshu Shu*, the translator has referred to the afore-mentioned treatises, especially Dauben's translation (Dauben 2008). In addition, the English translation of paragraphs in the *Nine Chapters* draws references from the *Nine Chapters on the Art of Mathematics* (Guo et al. 2013), as well as Chemla's work (Chemla 2010, 253–285). When quoting directly from the original text of the *Suanshu Shu*, this paper relies on the first published transcription of the text that varies slightly from later editions. Usually minor variations between these versions of the texts do not

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[3] The citations from the *Suanshu Shu* given in this paper are all based on the transcription by the research group of the Zhangjiashan Han bamboo slips from Jiangling in 2000, and notes are omitted to avoid redundancy. Additional annotations are also given for amended citations.

[4] The "Symposium for Commemorating the 1500th Anniversary of the Death of Zu Chongzhi 祖冲之" was held at Zu Chongzhi Middle School, Laishui County, Hebei Province, on October 9–14, 2000.

make much difference to the arguments made here.

2 The nature of the *Suanshu Shu*

Peng Hao 彭浩 argues, in my view correctly, that most of the computational problems in the *Suanshu Shu* were written no later than the Qin dynasty (221–206 BCE), and some in the Warring States period (475–221 BCE) (Peng 2000). In the *Suanshu Shu*, there are the characters "*Wang yichou* 王已讎" and "*Yang yichou* 楊已讎" at the end of two problems respectively, meaning that a person surnamed Wang 王 and another surnamed Yang 楊 proofread them. "Wang 王" and "Yang 楊" also show up in multiple places elsewhere in the text, and these two characters are so inconsistent with the context that it is only readable with them removed. Hence, they are the equivalent of "王已讎" and "楊已讎." This indicates that the *Suanshu Shu* must be a copy derived from somewhere else, and that it was not authored by the tomb owner during his lifetime, otherwise, it would not have required others to proofread it. In addition, this version of the *Suanshu Shu* cannot be an original edition compiled by the tomb owner himself from several different books (although this is a compilation made up of excerpts from other books), as there is little likelihood that he would have stopped in the middle of proofreading one problem instead of finishing it. Moreover, the paragraph, "5 people divide 7 and 1/3 and 1/2 *qian* … also take 6 times the amount of *qian* as the dividend," should not be placed in the section *hefen* 合分 (addition of fractions), as it actually belongs to the category *jingfen* 經分 (division of fractions) that is already included as the section *jingfen* 徑 (經) 分 in the *Suanshu Shu*. This clear error is improbably ascribed to the compiler, perhaps demonstrating that this version might not be the original *Suanshu Shu*, but a copy from other ones. Therefore, although this version could have been completed as a copy as late as 186 BCE prior to interment, the *Suanshu Shu* was likely complied years earlier, so it might have been finished in the Qin dynasty or slightly earlier.

Based on the transcription by archaeologists, the *Suanshu Shu* contains the following 68 headings:

> 1. *xiang cheng* 相乘 (mutual multiplication), 2. *fen cheng* 分乘 (multiplication of fractions), 3. *cheng* 乘 (multiplication), 4. *zeng jian fen* 矰 (增) 減分 (increasing and decreasing fractions), 5. *fen dang ban zhe* 分當半者 (fractions to be halved), 6. *fen ban zhe* 分半者 (halving fractions), 7. *yue fen* 約分 (reduction of fractions), 8. *he fen* 合分 (addition of fractions), 9. *jing fen* 徑分 (division of fractions), 10. *chu jin* 出金 (taking out gold), 11. *gong mai cai* 共買材 (buying wood together), 12. *hu chu guan* 狐出關 (fox goes through customs), 13. *hu pi* 狐皮 (fox furs), 14. *nü zhi* 女織 (woman weaving), 15. *bing zu* 并租 (combined taxes), 16. *fu mi* 負米 (carrying husked grain), 17. *jin jia* 金賈 (價) (the price of gold), 18. *chong su* 舂粟 (hulling unhusked grain), 19. *tong hao* 銅秏 (耗)

> (copper wastage), 20. *chuan ma* 傳馬 (post horses), 21. *fu zhi* 婦織 (women weaving), 22. *yu shi* 羽矢 (feathered arrows), 23. *qi qian* 桼 (漆) 錢 (the cost of lacquer), 24. *zeng fu* 繒幅 (pieces of silk), 25. *xi qian* 息錢 (interest), 26. *yin qi* (飲) 桼 (漆) (pouring [water] into lacquer), 27. *shui tian* 稅田 (taxing farmland), 28. *cheng zhu* 程竹 (norms for bamboo), 29. *yi* 醫 (doctors), 30. *shi lü* 石 (率) (ratios in *shi*), 31. *gu yan* 賈鹽 (buying or selling salt), 32. *ru zhi* 挐脂 (paste made from grain), 33. *qu cheng* 取程 (adopting the norms), 34. *hao zu* 秏 (耗) 租 (tax on wastage), 35. *cheng he* 程禾 (norms for grain), 36. *qu xi cheng* 取枲程 (adopting the norms for hemp), 37. *wu quan* 誤券 (mistaken bill), 38. *zu wu quan* 租吳 (誤) 券 (mistaken tax bill), 39. *bai hui* 粺毀 (毇) (refined and polished millet), 40. *hao* 秏 (wastage), 41. *su wei mi* 粟為米 (grain hulled to husked grain), 42. *su qiu mi* 粟求米 ([given] grain, find [the equivalent of] husked grain), 43. *mi qiu su* 米求粟 ([given] husked grain, find [the equivalent of] grain), 44. *mi su bing* 米粟并 (husked grain and grain combined), 45. *su mi bing* 粟米并 (grain and husked grain combined), 46. *fu tan* 負炭 (carrying charcoal), 47. *lu tang* 盧唐 (tube-shaped bamboo vessel), 48. *yu shi* 羽矢 (feathered arrows), 49. *si lian* 絲練 (raw and processed silk), 50. *xing* 行 (traveling), 51. *fen qian* 分錢 (distributing money), 52. *mi chu qian* 米出錢 (the cost of husked grain), 53. *fang tian* 方田 (square fields), 54. *chu* 除 ([volume of a] tunnel), 55. *yun du* 鄆都 (a wedge-shaped object), 56. *chu* 芻 (hay [piled in the shape of a frustum with two parallel rectangular bases]), 57. *xuan su* 旋粟 ([volume of a] cone of grain), 58. *qun gai* 囷蓋 ([volume of a conical] granary cover), 59. *yuan ting* 睘 (圜) 亭 (circular pavilion), 60. *jing cai* 井材 (a timber shaped like a well), 61. *yi yuan cai fang* 以睘 (圜) 材方 (from a circle cut a square), 62. *yi fang cai yuan* 以方材睘 (圜) (from a square cut a circle), 63. *huan cai* 睘 (圜) 材 (circular wood), 64. *qi guang* 啟廣 (finding the width), 65. *qi cong* 啟從 (縱) (finding the length), 66. *shao guang* 少廣 ([increasing lengths by] shortening widths), 67. *da guang* 大廣 (general widths), 68. *li tian* 里田 (rectangular fields [measured in] *li*).

The transcription presents the above 68 headings in *Heiti* 黑体 (a font of simplified Chinese characters). Following the 41st section, *su wei mi* 粟為米 (grain hulled to husked grain), there is a parallel section, *su qiu mi* 粟求米 ([given] grain, find [the equivalent of] husked grain), which is temporarily marked as 41a, and this heading is not in *Heiti*. It remains unknown whether this resulted from typographical errors or from mistakes in the original slips. There would be a total of 69 headings if the *su qiu mi* were to be counted as another heading.[5]

These 69 titles head all parts of the *Suanshu Shu*, problems which can be categorized into different types in the *Nine Chapters on Mathematical Procedures*, such as *fang tian* 方田 (rectangular fields), *su mi* 粟米 (grain and husked grain), *cui fen* 衰分 (proportional distribution), *shao guang* 少廣 ([increasing lengths by] shortening widths), *shang gong* 商功 (consultations on works), *jun shu* 均輸 (equitable transport), and *ying bu zu* 盈不足

[5] A newly added note by the author: according to the photograph of the slip, this "*su qiu mi*" is truly a heading.

(excess and deficit). The *Suanshu Shu* is concerned with contemporary integer and fractional arithmetic, various proportions, area, volume, negative numbers, and the rule of double false position. Despite the fact that its scope and depth are slightly overshadowed by the *Nine Chapters*, the *Suanshu Shu* embodies a wide spectrum of knowledge. That being the case, is the *Suanshu Shu* rigorously systematic?

Firstly, let us take a look at these headings, which are designated by various naming methods, as summarized in the following table:

Table 1 Classification of naming methods

Number	1	2	3	4
Categories	Mathematical methods	Excerpts of words (or summaries of words and sentences)	Descriptions of problems	Others
Headings	2, 7, 8, 9, 30, 64–68	4, 5, 10, 12–16, 20–22, 24, 26–29, 32–40, 44–50, 53–60, 63	1, 3, 11, 17–19, 23, 25, 31, 41–43, 51–52, 61–62	6
Subtotal	10	41	17	1

It is difficult to be precise for the above classification in view of divided opinions and especially many overlaps between the second category and the third. There are a large number of omissions and errors in the sixth section *fen ban zhe* 分半者 (halving fractions). Its category is very hard to ascertain based on the texts available so far, and hence this section is listed separately for the time being. Furthermore, the 30th, the 66th–68th, and the 64th–65th sections that could be included in the second category are classified into the first. This is because the 30th involves *shi lü zhi shu* 石率之術 (the method for ratios in *shi*), corresponding names of methods in the 66th–68th are found in the *Nine Chapters*, and the 64th–65th are named similarly to the 67th–68th. As a matter of fact, the standard for the first category is loose. Notwithstanding certain classification errors in the above table, it offers sufficient testimony to the fact that the vast majority of the headings in the *Suanshu Shu* are not designated in accordance with mathematical methods, but randomly. In particular, the most widely adopted naming method falls into the second category, namely the excerpts of words, and the third and the fourth provide no evidence of adequate systematic consideration on the part of the author. Hence, there is little evidence of a systematic approach demonstrated in the headings of the *Suanshu Shu*.

Relations within each section and among all of them in the *Suanshu Shu* can also be summed up as follows: it contains an abundance of computational methods and some conversion standards, most of which are coupled with corresponding examples and answers; these methods and standards are mostly universal, and some methods themselves in the *Suanshu Shu* even contain theoretical foundations for algorithms such as the method in the 46th section *fu tan* 負炭 (carrying charcoal). This indicates that mathematics at that time was indeed grounded in theoretical foundations.

In the *Suanshu Shu*, sections 1–3 and 5–9 are concerned with problems or methods regarding numerical computation (mainly focusing on fractions). Section 4 discusses the relation between the increase or decrease in fraction value and the change in the numerator or the denominator. Sections 10–53 primarily consist of practical problems or solutions using fractional arithmetic, various proportions and the rule of double false position. Moreover, there are also a few sections merely comprised of conversion standards instead of computational problems and methods, such as sections 35 and 41. Sections 54–60 are mainly concerned with the issue of volume, and sections 61–68 are aimed at geometry problems. These sections are up to a point associated, regardless of the errors in terms of the order they were arranged in by the research group of the Zhangjiashan Han bamboo slips in 2000. For instance, the conversion ratios provided in sections 35 and 41 are mostly applied in sections 39, 40, 41a and 43–45; sections 61–62 are mutually inverse and so forth. The above-mentioned implies that the compiler may still have intended to classify these sections.

However, the majority of the sections in the *Suanshu Shu* appear to be loosely connected, and are mainly arranged in parallel. Furthermore, methods of various sorts are not organized systematically, and there is a lack of categories each of which heads a group of similar problems. As a consequence, the whole book, loose and disorganized, seems to be a stack of problems, methods and standards. This feature is all the more palpable when comparison with the *Nine Chapters* is made.[6] An example can be found in section 53 *fang tian* 方田 (square field), which does not belong to the problems in the "*fang tian* 方田" chapter in the *Nine Chapters*, but pertains to the practical problems solved through *ying bu zu shu* 盈不足術 (method for excess and deficit) converted from the rule of double false position. After a list of multiplication computations in great quantities, section 1 *xiang cheng* 相乘 (mutual multiplication) offers *cheng fen zhi shu* 乘分之術 (method for multiplication of fractions) at its end. Afterwards, section 2 *fen cheng* 分乘 (multiplication of fractions) specifically lists *fen cheng fen shu* 分乘分術 (method for multiplication of fractions), which is the same as that in the previous section. These methods are apparently the rules for fractional multiplication, which is the equivalent of *cheng fen shu* 乘分術 (method for multiplication of fractions) that in the *Nine Chapters* comes after the 21st question in the chapter *fang tian* 方田 (rectangular fields) (the *Suanshu Shu* is missing "Divide the dividend by the divisor 實如法而一"). In the *Suanshu Shu*, section 7 *yue fen* 約分 (reduction of fractions) describes the universal method for the reduction of fractions, and the answers to most questions in the *Suanshu Shu* are reduced. However, some answers in section 14 *nü zhi* 女織 (woman weaving) and section 44 *mi su bing* 米粟并 (husked grain and grain combined) are not reduced. This indicates that the *Suanshu Shu* as a treatise was not carefully

[6] In this paper, the citations from the *Nine Chapters on Mathematical Procedures* are all based on Guo 1998, and notes are omitted to avoid redundancy.

compiled. What is particularly noteworthy is that in the *Suanshu Shu*, section 35 *cheng he* 程禾 (norms for grain), quoting the Qin Laws unearthed in Shuihudi 睡虎地, Yunmeng 云梦 County (see details below), sets the conversion rates as follows: *su* 粟 (millet) : *li mi* 糲米 (husked millet) : *zuo mi* 糳米 (refined millet) : *hui mi* 毇米 (polished grain) = 16 2/3 : 10 : 9 : 8. Nonetheless, problems concerning *zuo mi* 糳米 (refined millet) make no appearance anywhere in the rest of the book, while sections 39, 41 and 52 take *bai mi* 粺米 (refined millet) as the example instead of *zuo mi* 糳米 (refined millet), and the rates for *bai mi* 粺米 (refined millet) are all set as 9 (namely the rate for *zuo mi* 糳米 [refined millet]), thus achieving consistency. The above case implies that section 35, citing the Qin Laws, and sections 39, 41, and 52 are derived from at least two separate sources. Furthermore, there are two facts that warrant attention. On the one hand, computational errors occur in some answers in the *Suanshu Shu,* such as section 27 *shui tian* 稅田 (taxing farmland), section 31 *gu yan* 賈鹽 (buying or selling salt), and the last three answers in section 40 *hao* 秏 (wastage). On the other hand, in comparison with the *Nine Chapters*, certain types of methods are adopted in the *Suanshu Shu* such as proportion, proportional distribution and *ying bu zu* 盈不足 (excess and deficit), but they give no complete expressions of the following methods: *jin you shu* 今有術 (rule of three), *cui fen shu* 衰分術 (method for proportional distribution) and *fan cui fen shu* 返衰術 (method for inversely differential distribution), and *ying bu zu shu* 盈不足術 (method for excess and deficit). Meanwhile, these cases should not be regarded as one common method summarized from various circumstances, but should be viewed as applications or modifications of *jin you shu*, *cui fen shu*, *fan cui fen shu* and *ying bu zu shu*.

For the above reasons, I hold the view that the *Suanshu Shu* is not a systematically planned mathematical monograph, but is in the nature of a collection of problems, methods and standards, and so on. Even if the *Suanshu Shu* itself is derived from a principal source, this must also have been a compilation. Various parts of this book were obtained from other more systematic works, and were the products of derivations and applications achieved with a combination of the actual conditions of socio-economic management, thereby serving the demands of lower government officials.

3 The relationship between the *Suanshu Shu* and the *Nine Chapters on Mathematical Procedures*

Attention is easily drawn to the relationship between the *Suanshu Shu* and the *Nine Chapters*. The fact that the date of the *Suanshu Shu* emerged at least one and a half century earlier than the *Nine Chapters* has naturally resulted in the conjecture that the former is the origin, at least in part, of the latter. Peng Hao 彭浩 has also stated that the *Suanshu Shu* "had a direct impact on the formation of another mathematical masterpiece, the *Nine Chapters on Mathematical Procedures*" (Peng 2000). In reality, however, the situation is not nearly so straightforward.

At the beginning of the chapter *su mi* 粟米 (grain and husked grain) in the *Nine Chapters*, the *su mi zhi fa* 粟米之法 (rule for exchanges of grain and husked grain) offers a list of conversion rates among various grains:

The *su mi zhi fa* 粟米之法 (rule for exchanges of grain and husked grain):

rate of *su* 粟 (millet): 50	*li mi* 糲米 (husked millet): 30
bai mi 粺米 (refined millet): 27	*zuo mi* 糳米 (highly refined millet): 24
yu mi 御米 (imperial millet): 21	*xiao zhi* 小䵂 (refined wheat): 13 ½
da zhi 大䵂 (coarse wheat): 54	*li fan* 糲飯 (cooked coarse rice): 75
bai fan 粺飯 (cooked refined millet): 54	*zuo fan* 糳飯 (cooked highly refined millet): 48
yu fan 御飯 (cooked imperial millet): 42	*shu* 菽 (soy beans), *da* 荅 (small beans), *ma* 麻 (sesame seeds), *mai* 麦 (wheat): 45
dao 稻 (fresh rice): 60	*chi* 豉 (fermented beans): 63
sun 飧 (cooked rice mixed with water): 90	*shu shu* 熟菽 (cooked beans): 103 ½
nie 糵 (fermented grain): 175	

The conversion regulations for an assortment of grains are also given in section 35, *cheng he* 程禾 (norms for grain) in the *Suanshu Shu*:[7]

程曰：禾黍一石為粟十六斗泰（大）半斗，舂之為糲米一石，糲米一石為糳米九斗，糳米（九）斗為毀（毇）米八斗。王程曰：稻禾一石為粟廿斗，舂之為米十斗，為毀（毇）粲米六斗泰（大）半斗。麥十斗，䵂三斗。程曰：麥、菽、荅、麻十五斗一石，稟毀（毇）糳（糳）者，以十斗為一石。

The norm says: 1 *shi* of *he shu* 禾黍 (unhusked millet or unhusked broomcorn millet) is equivalent to 16 2/3 *dou* of *su* 粟 (unhusked millet); milling it gives 1 *shi* (in volume, 10 *dou*) of *li mi* 糲米 (husked millet); 1 *shi* (10 *dou*) of husked millet is hulled to 9 *dou* of *zuo mi* 糳米 (refined millet); (9) [8] *dou* of refined millet is hulled to 8 *dou* of *hui mi* 毇米 (polished millet). (Wang 王, the surname of the proofreader) The norm says: 1 *shi* of *dao* 稻 (fresh rice) is equivalent to 20 *dou* of *su* 粟 (unhusked rice); grinding it produces 10 *dou* of *hui mi* 毇米 (polished rice), which is hulled to 6 and 2/3 *dou* of *can mi* 粲米

7 A newly added note by the author: Though this paper does not offer a detailed explanation of the original bamboo slips quoted, according to the author's later research, this paragraph reflects the fact that from the Qin dynasty to the early Han dynasty, *shi* was used as a special unit for measuring foodstuffs in the storage department of the government, and its magnitude varies with different kinds of foodstuffs (Zou 2009).

8 The original transcription lacks the character "*jiu* 九 (nine)" which is supplemented in accordance with the Qin bamboo slips at Shuihudi. Peng Hao 彭浩 has proofread this (Peng 2000).

> (highly polished rice). 10 *dou* of *mai* 麥 (wheat) is crushed and makes 3 *dou* of *zhi* 䵂 (refined wheat). The norm says: for *mai* 麥 (wheat), *shu* 菽 (soy beans), *da* 荅 (small beans), and *ma* 麻 (sesame seeds) 1 *shi* is equivalent to 15 *dou*; given *hui* 毀 or *zuo* 糳 (polished millet or refined millet), 1 *shi* is equivalent to 10 *dou*.

The character "程 *cheng*" here refers to "norm" or "regulation." This paragraph is reminiscent of the "*cang lü* 仓律 (granary law)" in the *Qin lü shiba zhong* 秦律十八種 (18 laws of the Qin dynasty), possibly completed before the first year of the First Emperor of the Qin dynasty (King Zheng of Qin 秦王政, who was later called the First Emperor), namely before 246 BCE. It was unearthed from the Qin tomb in Shuihudi, Yunmeng County (the tomb owner passed away in 217 BCE):

> 1 *shi* and 6 2/3 *dou* of *su* 粟 (millet) is milled to 1 *shi* of *li mi* 糲米 (husked millet); 1 *shi* (10 *dou*) of husked millet is hulled to 9 *dou* of *zuo mi* 糳米 (refined millet); 9 *dou* of refined millet is hulled to 8 *dou* of *hui mi* 毇米 (polished millet). 1 *shi* of *dao he* 稻禾 (fresh rice) is equivalent to 20 *dou* of *su* 粟 (unhusked rice); grinding it yields 10 *dou* of *hui mi* 毇米 (polished rice), which is hulled to 6 and 2/3 *dou* of *can mi* 粲米 (highly polished rice). 10 *dou* of *mai* 麥 (wheat) is crushed and makes 3 *dou* of *zhi* 䵂 (refined wheat). 15 *dou* of *mai* 麥 (wheat), *shu* 菽 (soy beans), *da* 荅 (small beans) and *ma* 麻 (sesame seeds) are equivalent to 1 *shi*; given *hui* 毀 or *zuo* 糳 (polished or refined millet), 1 *shi* is equivalent to 10 *dou*. (Research group of the Shuihudi Qin bamboo slips 1978, 44–45)

Peng Hao takes these two paragraphs as almost identical, and that the *Suanshu Shu* drew on quotations from the Qin Laws (Peng 2000). According to *su mi zhi fa* 粟米之法 (rule for exchanges of grain and husked grain) in the *Nine Chapters*, *su* 粟 (millet) : *li mi* 糲米 (husked millet) : *bai mi* 粺米 (refined millet) : *zuo mi* 糳米 (highly refined millet)[9] = 50 : 30 : 27 : 24. Rates in the Qin Laws turn out to be: *su* 粟 (millet) : *li mi* 糲米 : *zuo mi* 糳米 (refined millet) : *hui mi* 毇米 (polished millet) = 16 2/3 : 10 : 9 : 8, which equals 50 : 30 : 27 : 24. The *Nine Chapters* set the rate for *zuo mi* 糳米 (highly refined millet) as 8, while in the Qin Laws it was specified as 9. Thus, the *Nine Chapters* mistook this rate. However, in other respects the *Nine Chapters* is a well-organized, consistent book. It is almost flawless, with the exception of a few inaccuracies or errors in the following cases: π (the numerical value of the ratio of the circumference of a circle to its diameter) taken as 3, the area formula for *wan tian* 宛田 (bowl-shaped field), the volume formula for a sphere, and using the rule of double false position to address nonlinear problems. Computations with respect to *zuo mi* were all based on this false value, attesting to the conscientiousness and rigorousness of the compilers, Zhang Cang 張蒼 and Geng Shouchang 耿壽昌.[10] If they

[9] A newly added note: "*zuo mi* 糳米" is translated differently in the *Suanshu Shu* and the *Nine Chapters* because the ratios of *zuo mi* to *su* are different.

[10] I share the view of Liu Hui 劉徽 that the *Nine Chapters* was compiled by Zhang Cang and

had taken note of the *Suanshu Shu*, they would have naturally rectified the false data on the basis of the quoted Qin Laws, or set problems by means of correct data. Therefore, the only reasonable explanation is that Zhang Cang and Geng Shouchang had not read the *Suanshu Shu* when they composed the *Nine Chapters*, and thus unknowingly they adopted the previously mistaken data.

Since small amounts of the texts of the *Suanshu Shu* and the *Nine Chapters* overlap, the assumption has been made that the *Suanshu Shu* may have served as one origin of the present *Nine Chapters*. Be that as it may, various options present themselves when we consider factors that could have led to the parallels in these two books. First, that the *Suanshu Shu* was finished earlier, and the *Nine Chapters* took the text from the *Suanshu Shu*. Second, that the *Nine Chapters* was finished earlier, and the *Suanshu Shu* drew references from the *Nine Chapters*. And third that parts of both works share the same source. Now, conventional wisdom has it that the current *Nine Chapters* was finished in the Han dynasty, so the second possibility is out of the question, while the first is also highly improbable in view of the argument just made above. As a consequence, the only feasible conclusion is that these two books have a common origin, and neither directly adopts text from the other. In fact, a comparison of the similarities makes evident that there are also quite a few differences, and that the resemblances are relative.

Table 2 Comparison of the partial similarities between the *Suanshu Shu* and the *Nine Chapters on Mathematical Procedures*

Sections	The *Suanshu Shu*	The *Nine Chapters on Mathematical Procedures*	Explanations on similarities, differences and relations
Multiplication of fractions	*fen cheng fen shu* 分乘分術 (method for multiplication of fractions) states that mutually multiply the denominators as the divisor, and mutually multiply the numerators as the dividend; dividing the dividend by the divisor. (Section 2 *fen cheng* 分乘 [multiplication of fractions]) ... *cheng fen zhi shu* 乘分之術 (method for multiplication of fractions) states that mutually multiply the denominators as the divisor, and mutually multiply the numerators as the dividend. (Section 1 *xiang cheng* 相乘 [mutual multiplication])	*cheng fen shu* 乘分術 (method for multiplication of fractions) states that mutually multiply the denominators as the divisor, and mutually multiply the numerators as the dividend; dividing the dividend by the divisor. (The chapter *fang tian* 方田 [rectangular fields])	These two documents slightly differ from each other in terms of the method names. "Dividing the dividend by the divisor 實如法而一" is absent from the *Suanshu Shu* which is less complete than is the *Nine Chapters*.

Geng Shouchang. For the credibility of Liu Hui's statement, see Guo 1992, 94–105.

Sections	The *Suanshu Shu*	The *Nine Chapters on Mathematical Procedures*	Explanations on similarities, differences and relations
Division of fractions	*jing fen* 徑分 (division of fractions) takes 1 person and determines his share, therefore it says: five people share 3 and 1/2 and 1/3, each receives 23/30. This method says: when the denominator has 1/3, taking 1 as 6, taking 1/2 as 3,[11] and taking 1/3 as 2, add them together as 23 (as numerator). Now put down (on the counting board) the number of people,[12] then multiply this by 6 in order to name the denominator for the corresponding numerator. Another rule says: if below (on the counting board) there is 1/2, then double; if below (on the counting board) there is 1/3, then triple; if below (on the counting board) there is 1/4, then quadruple. (Section 9 *jing fen* 徑分 [division of fractions])	Suppose there are 7 people dividing 8 1/3 *qian*. The question: how much should each person get? The answer: each person gets 1 4/21 *qian*. Again, suppose there are 3 1/3 people ... The answer: each person gets 2 1/8 *qian*. *jing fen shu* 經分術 (method for division of fractions): use the number of people as the divisor, and the amount of *qian* as the dividend; divide the dividend by the divisor. If there are (mixed) fractions, interconnect them. (The chapter *fang tian* 方田 [rectangular fields])	*jing fen* 徑分 (division of fractions) is the equivalent of *jing fen* 經分. The *Nine Chapters* offers a universal method, while that of the *Suanshu Shu* is only aimed at special cases. Titles of these two excerpts are also different. It cannot be inferred that the author of the *Suanshu Shu* has no knowledge of the general *jing fen shu* 經分術 (method for division of fractions); the author seems to illustrate how to *tong zhi* 通之 (interconnect it) when doing the fractional division.
Addition of fractions	*he fen shu* 合分術 (method for addition of fractions): if the denominators are of the same kind (equal), add the numerators together; if the denominators are not of the same kind, but (some) can be doubled (to make the denominators equal), then double (them); if (some) can be tripled (to make the denominators equal), then triple (them); if (some) can be quadrupled (to make the denominators equal), then quadruple (them); if (some) can be quintupled (to make the denominators equal), then	*he fen shu* 合分術 (method for addition of fractions) states that cross-multiply the numerators and denominators, combining them as the dividend; mutually multiply all of the denominators together as the divisor. Divide the dividend by the divisor. If it does not divide completely, then use the divisor to name the fraction (the results). If their denominators are the same, then directly add them together. (The	They both offer general methods, but the *Suanshu Shu* additionally provides specific instructions for common scenarios, with multiple citations from *he fen shu*. Evidence makes clear that the computations in the *Suanshu Shu* are carried out using existing methods.

11 "*san* 三 three" in the source text is corrected as "*yi* 一 one" according to computational principles.

12 A newly added note by the author: the "number of people" is the English translation of "*ren shu* 人數," which was the correction of "*yi shu* 一數 (one number)" in the transcription text given by the research group of the Zhangjiashan Han bamboo slips in 2000. Guo Shuchun was the first to make this correction (Guo 2001).

Sections	The *Suanshu Shu*	The *Nine Chapters on Mathematical Procedures*	Explanations on similarities, differences and relations
	quintuple (them); if (some) can be sextupled (to make the denominators equal), then sextuple (them); likewise, the numerator should be doubled, (so) double it; when multiplied by 3, 4, or 5 like the denominators, and if the denominators are the same amount, (then) add the numerators together. If they (the denominators) are (still) not of the same kind, then mutually multiply (all of) the denominators together as the divisor, and (after) cross-multiplying the numerators with the denominators, add them together as the dividend; and then divide. Another rule says: multiply the denominators together as the divisor, cross-multiply the numerators with the denominators. This one is (another) rule: if (some) need to be multiplied by 10, multiply by 10; if (some) need to be multiplied by 9, multiply by 9; if (some) need to be multiplied by 8, multiply by 8; if (some) need to be multiplied by 7, multiply by 7; if (some) need to be multiplied by 6, multiply by 6; if (some) need to be multiplied by 5, multiply by 5; if (some) need to be multiplied by 4, multiply by 4; if (some) need to be multiplied by 3, multiply by 3; if (some) need to be doubled, double. Stop when the denominators are all the same. When the denominators are all the same, add the numerators together. (Section 8 *he fen* 合分 [addition of fractions])	chapter *fang tian* 方田 [rectangular fields])	
Reduction	*yue fen shu* 約分術 (method for	*yue fen shu* 約分術	The *Suanshu Shu*

Sections	The *Suanshu Shu*	The *Nine Chapters on Mathematical Procedures*	Explanations on similarities, differences and relations
of fractions	reduction of fractions): take the numerator and subtract it (successively) from the denominator; also take the denominator and subtract it (successively) from the numerator; (when) the amounts of the numerator and denominator are equal, this will simplify it (the fraction will be simplified). Another method for reduction of a fraction says: if it can be halved, halve it; if it can be divided by a certain number, divide by it. Yet another method says: using the numerator of the fraction, subtract it (successively) from the denominator; when the (remainder of the denominator) is smaller than the numerator, using the (remainder of) the denominator, subtract it (successively) from the numerator; use what is equal to (both the remainders of) the numerator and denominator as the divisor; then it is possible to divide both the numerator and denominator by this number. If it is not possible (lit. if there is not enough) to subtract but it can be halved, halve the denominator and also halve the numerator. (Section 7 *yue fen* 約分 [reduction of fractions])	(method for reduction of fractions): if they (both the numerator and the denominator) can be halved, halve them; if they cannot be halved, put down (on one side of the counting board) the numerator and the denominator separately, subtract the smaller from the larger, and subtract alternately, seeking equality. Use the equal number to reduce the fraction. (The chapter *fang tian* 方田 [rectangular fields])	proposes three methods for *yue fen* 約分 (reduction of fractions). The third is the same as that in the *Nine Chapters*, but they are expressed in different manners. The second method is only applicable in the case where both the numerator and denominator are even numbers. The first and the third provide universal approaches. In the latter, "Yet another method says: ..." may come from a source similar to that of the *Nine Chapters*.
Problems on *nü zhi* 女織 (woman weaving)	There is a woman in the neighborhood who is very happy to weave: every day she doubles her weaving. In five days (she) weaves five *chi*. How much does she weave in the first day, and how much every day thereafter? The answer: the first day she weaves 1 38/62 *cun*; then 3 and 14/62 *cun*; then 6 and	Suppose there is a woman good at weaving, and every day (the amount she weaves) doubles. In 5 days, she weaves 5 *chi*. The question: how much does she weave each day? The answer: the first day she weaves 1 19/31 *cun*; the next day she weaves 3	Despite the values set in these two problems being the same, in the *Suanshu Shu* the answers and successive differences in the methods are not reduced, while those in the *Nine Chapters* are. The successive differences in

Sections	The *Suanshu Shu*	The *Nine Chapters on Mathematical Procedures*	Explanations on similarities, differences and relations
	28/62 *cun*; then 1 *chi* 2 and 56/62 *cun*; then 2 [13] *chi* 5 and 50/62 *cun*. The method says: put down the values 2, 4, 8, 16, 32; add these together as the divisor; taking the 5 *chi*, multiply this by each of them (2, 4, 8, 16, 32) as the dividend; dividing the dividend by the divisor gives the (amount of) *chi*. When (the quotient or remaining quotient) is not as much as 1 *chi*, multiply the dividend by 10, dividing it by the divisor gives the (amount of) *cun*. If (the remaining quotient) is not as much as 1 *cun*, use the divisor to name a fraction (to express the remaining quotient). (Section 14 *nü zhi* 女織 [woman weaving])	7/31 *cun*; the next day she weaves 6 14/31 *cun*; the next day she weaves 1 *chi* 2 and 28/31 *cun*; the next day she weaves 2 *chi* 5 and 25/31 *cun*. The method: put down (on the counting board) 1, 2, 4, 8 and 16 as proportions, on the side combine these as the divisor; multiplying the 5 *chi* by the (individual proportions) not yet combined, each respectively becomes a dividend. Dividing the dividends by the divisor gives the (amount of) *chi* per day. (The chapter *cui fen* 衰分 [proportional distribution])	the former are double those in the latter. In *Sunzi suanjing* 孫子算經 (The mathematical classic of Sunzi), problem 27 involves this question as well, and the answers are also reduced, while its method employs successive differences identical to those in the *Nine Chapters*.
shao guang shu 少廣術 (method for [increasing lengths by] shortening widths)	*qiu shao guang zhi shu* 救（求）少廣之術 (method for [finding increasing lengths by] shortening widths): first put (down on the counting board) the width, that is to say: in the lower place, there is a certain number of *bu*; letting one stand for this number, letting 1/2 stand for (1/2) this number, letting 1/3 stand for (1/3) this number, letting all the fractions given in the width stand for certain numbers to be accumulated, combine these numbers together as the divisor. Then put down the field of 240 (square) *bu* (on the counting board), and also letting 1 stand for the certain number (of *bu*; i.e., multiply by this number); taking the product as the accumulated *bu*. Divide the accumulated *bu*	*shao guang shu*: put down (on the counting board) the whole number of *bu* and the numerators and denominators of (any additional) fractions, multiply all of the numerators of the fractions and the whole number of *bu* by the denominator of the fraction at the bottom (of the counting board). Divide each of the numerators by their corresponding denominators, (and if the result is a whole number) put them on the left. To interconnect all of the fractions, again using the denominator at the bottom (of the counting board), multiply all of the	Obviously, these two documents give different expressions for the *shao guang shu*. The *Nine Chapters* offers general procedures, and is theoretically complete. In contrast, in the *Suanshu Shu*, *shao guang zhi shu* provides no explanations on how to determine "letting one stand for this number, letting 1/2 stand for (1/2) this number, letting 1/3 stand for (1/3) ..." through "in the lower place there is a certain number of *bu*," which is exactly what *shao guang shu* in the *Nine Chapters* can resolve. In the *Suanshu Shu*,

[13] The original text mistook "*er* 二 two" as "*yi* 一 one," which has been corrected based on computational principles.

Sections	The *Suanshu Shu*	The *Nine Chapters on Mathematical Procedures*	Explanations on similarities, differences and relations
	by the divisor: if the dividend (or the remaining dividend) is as much as the divisor, (then the quotient gains one,) thus the operation of dividing gives the length in *bu*. If the quotient is not as much as one *bu*, use the divisor to name its fraction. (The method) goes on to say, to check this (lit. going back), then multiply the width by the length, and check that it is a field of 240 (square) *bu* or 1 *mu*. If its length has no fractional part, and the combined divisor no parts, to check (the result) multiply it by what stands for "little ten."[14] *shao guang* 少廣 ([increasing lengths by] shortening widths): (If the width is) 1 *bu* and 1/2 *bu*, taking 1 (the unit) as 2 and 1/2 as 1, combining them use 3 as the divisor; then put (down on the counting board) 240 (square) *bu*, and again taking 1 (the unit) as 2 (to obtain 480 as dividend). When dividing, (if the dividend or the remaining dividend) is as much as the divisor, (then) this gives one *bu* of the length, thus this operation gives a length of 160 *bu*. As a result of multiplying (this) by 1 *bu* and one-half *bu*, (there is a field of 1 *mu*). Down (on the counting board) there is 1/3 (added to the 1 and 1/2); taking 1 (the unit) as 6, 1/2 as 3, and 1/3 as 2, combining them use 11 (as the divisor; then put down on the counting board the field of 240 square *bu*; again taking 1 as 6 to obtain dividend; dividing) gives a length of 130	numerators and what have been interconnected, and make them all interconnected and equalized. (When all of them are interconnected and equalized), combine them as the divisor. Put down (on the counting board) the number of *bu* which is sought (i.e., 240 square *bu*), and multiply it by the accumulated parts of the whole number of *bu* (i.e., 1 *bu*) (by the product of the denominators of the fractions) as the dividend. 1. Suppose there is a field of width 1 *bu* and 1/2 (*bu*). To find a field of 1 *mu*, the question: what is the length? The answer: 160 *bu*. ... 2. Suppose there is a field of width 1 *bu* and 1/2 (*bu*) and 1/3 *bu*. To find a field of 1 *mu*, the question: what is the length? The answer: 130 10/11 *bu*. ... 3. Suppose there is a field of width 1 *bu* and 1/2 (*bu*) and 1/3 *bu* and 1/4 *bu*. To find a field of 1 *mu*, the question: what is the length? The answer: ... 4. Suppose there is a field of width 1 *bu* and 1/2 (*bu*) and 1/3 *bu* and 1/4 *bu* and 1/5 *bu*. To find a field of 1 *mu*, the question: what is the length? The answer: ... 5. Suppose there is a	what is below "少廣：廣一步半步 (if the width is) 1 *bu* and 1/2 *bu*" is the equivalent of problems 1–9 in the chapter *shao guang* in the *Nine Chapters*. However, the expressions in these two documents are significantly different. This is the case not only in terms of textual content, but also in structure: the former as a whole directly depicts the computation process, while the latter gives questions, answers and methods respectively. Again, the fifth problem in the *Nine Chapters* writes that "At the bottom (of the counting board) is a sixth. Taking 1 as 120, 1/2 as 60, 1/3 as 40, 1/4 as 30, 1/5 as 24 and 1/6 as 20," and does not give a more simplified value as in "taking 1 as 60 ..." using the afore-mentioned *shao guang shu*. In contrast, it is in the *Suanshu Shu* that the figures are more simplified as follows: "Down (on the counting board) there is 1/6; taking 1 (the unit) as 60, 1/2 as 30, 1/3 as 20, and 1/4 as 15, 1/5 as 12, and 1/6 as 10." Hence, *shao guang shu* in the *Nine Chapters* must already

[14] A newly added note by the author: Following this is a sentence "有分步者，以廣乘分子，如廣步數，得一步" in the first edition of the transcription, which has been omitted in the English translation because this sentence is omitted in later versions of the transcription (Research group of the Zhangjiashan Han bamboo slips from tomb no. 247 2001; idem 2006).

Sections	The *Suanshu Shu*	The *Nine Chapters on Mathematical Procedures*	Explanations on similarities, differences and relations
	10/11 *bu*. (As a result, taking 1 *bu*, 1/2 *bu*, and 1/3 *bu*), multiplying (by 130 10/11 *bu*) gives a field of 1 *mu*. Down (on the counting board) there is 1/4 (added to the 1, 1/2, and 1/3); taking 1 (the unit) as 12, 1/2 as 6, 1/3 as 4, and 1/4 as 3, combining them use 25 (as the divisor; then put down on the counting board the field of 240 square *bu*; again taking 1 (the unit) as 12 to obtain the dividend; dividing) gives a length of 115 5/25 *bu*. (As a result, taking 1 *bu*, 1/2 *bu*, 1/3 *bu*, and 1/4 *bu*), multiplying (by 115 5/25 *bu*) gives a field of 1 *mu*. Down (on the counting board) there is 1/5 (added to the 1, 1/2, 1/3, and 1/4); taking 1 (the unit) as 60, 1/2 as 30, 1/3 as 20, 1/4 as 15, and 1/5 as 12, combining them use 137 (as the divisor; then put down on the counting board the field of 240 (square) *bu*; again taking 1 (the unit) as 60 to obtain the dividend; dividing gives a length of 105 15/137 *bu*. (As a result, taking 1 *bu*, 1/2 *bu*, 1/3 *bu*, 1/4 *bu*, and 1/5 *bu*), multiplying (by 105 15/137 *bu*) gives a field of 1 *mu*. Down (on the counting board) there is 1/6 (added to the 1, 1/2, 1/3, 1/4, and1/5); taking 1 (the unit) as 60, 1/2 as 30, 1/3 as 20, and 1/4 as 15, 1/5 as 12, and 1/6 as 10, combining them use 147 (as the divisor; then put down on the counting board the field of 240 square *bu*; again taking 1 (the unit) as 60 multiply both together as the dividend; dividing) gives a length of 97 141/147 *bu*. (As a result, taking 1	field of width 1 *bu* and 1/2 (*bu*) and 1/3 *bu* and 1/4 *bu* and 1/5 *bu* and 1/6 *bu*. To find a field of 1 *mu*, the question: what is the length? The answer: 97 47/49 *bu*. The method: at the bottom (of the counting board) is 1/6. Taking 1 as 120, 1/2 as 60, 1/3 as 40, 1/4 as 30, 1/5 as 24, 1/6 as 20, and combining them gives 294, to be used as the divisor. Put down (on the counting board) the field of 240 *bu*, and again, using 1 as 120 to multiply it (240 *bu*); this is the dividend. Divide the dividend by the divisor to obtain the length in *bu*. 6. Suppose there is a field of width 1 *bu* and 1/2 (*bu*) and 1/3 *bu* and 1/4 *bu* and 1/5 *bu* and 1/6 *bu* and 1/7 *bu*. To find a field of 1 *mu*, the question: what is the length? The answer: ... 7. Suppose there is a field of width 1 *bu* and 1/2 (*bu*) and 1/3 *bu* and 1/4 *bu* and 1/5 *bu* and 1/6 *bu* and 1/7 *bu* and 1/8 *bu*. To find a field of 1 *mu*, the question: what is the length? The answer: ... 8. Suppose there is a field of width 1 *bu* and 1/2 (*bu*) and 1/3 *bu* and 1/4 *bu* and 1/5 *bu* and 1/6 *bu* and 1/7 *bu* and 1/8 *bu* and 1/9 *bu*. To find a field of 1 *mu*, the question: what is the length? The answer: ... 9.[15] Suppose there is a field of width 1 *bu* and 1/2	have existed in the era of the *Suanshu Shu*.

[15] The sequence numbers are added in citations.

Sections	The *Suanshu Shu*	The *Nine Chapters on Mathematical Procedures*	Explanations on similarities, differences and relations
	bu, 1/2 *bu*, 1/3 *bu*, 1/4 *bu*, 1/5 *bu*, and 1/6 *bu*), multiplying (by 97 141/147 *bu*) gives a field of 1 *mu*. Down (on the counting board) there is 1/7 (added to the 1, 1/2, 1/3, 1/4, 1/5, and 1/6); taking 1 (the unit) as 420, 1/2 as 210, 1/3 as 140, and 1/4 as 105, 1/5 as 84, 1/6 as 70, and 1/7 as 60, combining them use 1089 (as the divisor; then put down on the counting board the field of 240 square *bu*; again taking 1 (the unit) as 420 to obtain the dividend; dividing) gives a length of 92 (68) /541 *bu*. (As a result, taking 1 *bu*, 1/2 *bu*, 1/3 *bu*, 1/4 *bu*, 1/5 *bu*, 1/6 *bu*, and 1/7 *bu*), multiplying (by 92 68/541 *bu*) gives a field of 1 *mu*. Down (on the counting board) there is 1/8 (added to the 1, 1/2, 1/3, 1/4, 1/5, 1/6, and 1/7); taking 1 (the unit) as 840, one-half as 420, 1/3 as 280, and 1/4 as 210, 1/5 as 168, 1/6 as 140, 1/7 as 120, and 1/8 as 105, combining them use 2283 二千二百八[16] (as the divisor; then put down on the counting board the field of 240 square *bu*; again taking 1 (the unit) as 840 multiply both together as the dividend; dividing) gives a length of 88 696/2283 *bu*. (As a result, taking 1 *bu*, 1/2 *bu*, 1/3 *bu*, 1/4 *bu*, 1/5 *bu*, 1/6 *bu*, 1/7 *bu*, and 1/8 *bu*), multiplying (by 88 696/2283 *bu*) gives a field of 1 *mu*. Down (on the counting board) there is 1/9 (added to the 1, 1/2, 1/3, 1/4, 1/5, 1/6, 1/7, and 1/8);	(*bu*) and 1/3 *bu* and 1/4 *bu* and 1/5 *bu* and 1/6 *bu* and 1/7 *bu* and 1/8 *bu* and 1/9 *bu* and l/10 *bu*. To find a field of 1 *mu*, the question: what is the length? The answer: 81 6939/7381 *bu*. The method: at the bottom (of the counting board) is 1/10. Taking 1 as 2520, 1/2 as 1260, 1/3 as 840, 1/4 as 630, 1/5 as 504, 1/6 as 420, 1/7 as 360, 1/8 as 315, 1/9 as 280, 1/10 as 252, and combining them gives 7381, to be used as the divisor. Put down (on the counting board) the field of 240 *bu*, and again using 1 as 2520 to multiply it (240 *bu*); this is the dividend. Divide the dividend by the divisor to obtain the length in *bu*. (The chapter *shao guang* 少廣 [increasing lengths by shortening widths])	

[16] “*er qian er bai ba* 二千二百八” (lit. two, thousand, two, hundred, eight) was originally taken as “*er bai* 二百□” (lit. two, hundred □), and it is restored based on computational principles.

Sections	The *Suanshu Shu*	The *Nine Chapters on Mathematical Procedures*	Explanations on similarities, differences and relations
	taking 1 (the unit) as 2520, 1/2 as 1260, 1/3 as 840,[17] and 1/4 as 630, 1/5 as 504, 1/6 as 420, 1/7 as 360, 1/8 as 315, and 1/9 as 280, combining them use 7129 (as the divisor; then put down on the counting board the field of 240 (square) *bu*; again taking 1 (the unit) as 2520 (to obtain the dividend; dividing) gives a length of 84 5764/7129 *bu*. (As a result, taking 1 *bu*, 1/2 *bu*, 1/3 *bu*, 1/4 *bu*, 1/5 *bu*, 1/6 *bu*, 1/7 *bu*, 1/8 *bu*, and 1/9 *bu*), multiplying (by 84 5764/7129 *bu*) gives a field of 1 *mu*. Down (on the counting board) there is 1/10 (added to the 1, 1/2, 1/3, 1/4, 1/5, 1/6, 1/7, 1/8, and 1/9); taking 1 (the unit) as 2520, 1/2 as 1260, 1/3 as 840, and 1/4 as 630, 1/5 as 504, 1/6 as 420, 1/7 as 360, 1/8 as 315, 1/9 as 280, and 1/10 as 252, combining them use 7381 as the divisor; (then put down on the counting board the field of 240 square *bu*; again taking 1 (the unit) as 2520 to obtain the dividend; dividing) gives a length of 81 6939[18] / 7381 *bu* (As a result, taking 1 *bu*, 1/2 *bu*, 1/3 *bu*, 1/4 *bu*, 1/5 *bu*, 1/6 *bu*, 1/7 *bu*, 1/8 *bu*, 1/9 *bu*, and 1/10 *bu*), multiplying (by 81 6939/7381 *bu*) gives a field of 1 *mu*.		
da guang shu 大廣術 (method for general widths)	*da guang shu* says: put (down on the counting board) the width and length, and respectively using their fractions' denominators, multiply them	*da guang tian shu* says: denominators of the fractions are respectively multiplied by their whole numbers, to which the	Expressions in the *Nine Chapters* are more concise, while those in the *Suanshu Shu* are more colloquial, appearing to

[17] "40 卌" was originally written as "30 卅十" and it is corrected according to computational principles.

[18] In the source text, "*jiu bai san shi jiu* 九百三十九" (lit. nine, hundred, three, ten, nine) was written as "八百□ □ □ (lit. eight, hundred, □ □ □)," it is restored based on computational principles.

Sections	The *Suanshu Shu*	The *Nine Chapters on Mathematical Procedures*	Explanations on similarities, differences and relations
	together with their integer parts which are above (on the counting board); add the fractions' numerator (to each, respectively), and mutually multiply the results together as the dividend; again let each of the fractions' denominators be mutually multiplied together as the divisor; dividing gives the result in (square) *bu*; if (the remaining quotient) is not as much as 1 (square) *bu*, use the divisor (as denominator) to name a fraction (to express the remaining quotient).	numerators of the fractions are added; mutually multiplying (these) together gives the dividend. Mutually multiplying the denominators of the fractions together gives the divisor. Dividing gives the result. (The chapter *fang tian* 方田 [rectangular fields])	facilitate the readers' understanding.
li tian shu 里田術 (method for measuring farmland in *li*)	*li tian shu* says: *li* times *li* is (square) *li*. If the width and length are both 1 *li*, then put (down on the counting board) 1 and times 3, also multiply by 5 three times; the result is a field of 3 *qing* 75 *mu*. If its width and length are not equal, first multiply the (amounts of) *li* together, then multiply (the result) by 3, again multiply by 5 three times, which completes it.	*li tian shu* says: mutually multiplying the number of *li* of the length and width together gives the product in *li*. Multiplying this (product) by 375 gives the number of *mu*. (The chapter *fang tian* 方田 [rectangular fields])	Distinctions are evident between these two texts. The *Nine Chapters* figures out the area of a field in square *li* and then directly multiplies this value by 370 to convert the unit of measurement to *mu*, whereas the *Suanshu Shu* describes the conversion process.
Algorithm for *xi qian* 息錢 (interest)	100 *qian* are loaned with interest for 1 (lunar) month at 3 (*qian*). If 60 *qian* are now loaned and repaid before the end of the month in 16 days, how much is the interest? The answer says: 24/25 *qian*. The method says: calculate (the product of) 100 *qian* and 1 month (30 days), the total number of *qian* serves as the divisor (100 × 30); put down (on the counting board) the money loaned multiplied by the amount of interest on 100 *qian* for 1 month (60 × 3); and also multiplying by the number of days (16) as the dividend; dividing the dividend by the divisor gives the interest in *qian*.	Suppose there is a loan to someone for 1000 *qian*, with monthly interests of 30 (*qian*). Now suppose a loan is made to someone for 750 *qian*, which is repaid in nine days. The question: how much is the interest? The answer: 6 3/4 *qian*. The method: using the month of 30 days multiplied by the 1000 *qian* as the divisor, use the interest of 30 (*qian*) multiplied by the amount of *qian* which is now loaded, and again multiply (the result) by 9 days as the dividend. Dividing the	Although the monthly interest rates are the same, discrepancies are found in the promises and the expression of the methods. The methods in the *Suanshu Shu* tend to be general, and those in the *Nine Chapters* are more specific.

Sections	The *Suanshu Shu*	The *Nine Chapters on Mathematical Procedures*	Explanations on similarities, differences and relations
		dividend by the divisor gives the result in *qian*. (The chapter *cui fen* 衰分 [proportional distribution])	
Volume of a circular cone	*xuan su* 旋粟 (cone of millet) ... Its method says: multiply the lower circumference by itself, multiply it by the height, divide by 36. (Section 57 *xuan su* 旋粟 [cone of millet]) *qun gai* 囷蓋 (conical granary cover) ... The method of multiplying says: put (down on the counting board) the number the same as its circumference and let it be multiplied by itself, again multiply by the height and divide by 36. (Section 58 *qun gai* 囷蓋 [conical granary cover])	*yuan zhui shu* 圓錐術 (method for a circular cone) says: multiply the lower circumference by itself, and multiplying by the height, divide by 36. *wei su shu* 委粟術 (method for finding the volume of a pile of millet) says: multiply the lower circumference by itself, and multiplying by the height, divide by 36. (The chapter *shang gong* 商功 [consultations on works])	Both documents take π (the numerical value of the ratio of the circumference of a circle to its diameter) as 3, and are slightly different in expression, but they are distinct from each other in terms of the premises and names.
Volume of a circular platform	*yuan ting* 圜亭 (circular pavilion) … The method says: multiply the lower circumference by the upper circumference, multiply (each of) the circumferences by themselves, add them all together, multiply by the height, and divide by 36 (卅六成一).[19] (Section 59 *yuan ting* 圜亭 [circular pavilion])	*yuan ting* 圓亭 (circular pavilion) ... The method says: mutually multiply together the upper and lower circumferences, and again self-multiply each of these, adding them together and multiplying by the height, divide by 36. (The chapter *shang gong* 商功 [consultations on works])	Both documents take π (the numerical value of the ratio of the circumference of a circle to its diameter) as 3, and are slightly different in expression, but they differ from each other in the premises.
Volume of a column	To find the contents (volume) of a round timber, well-pit or other similar objects: … The method says: take the circumference multiplied by itself, multiply (the product) by the depth, and divide by 12. Another (method) says: multiply the circumference by the diameter (*yi zhou cheng jing* 以[周][20]乘徑), and divide by	Suppose there is a *yuan bao dao* 圓垛壔 (circular city wall or cylinder-shaped city wall) ... The method says: multiply the circumferences by itself, and multiplying by the height, divide by 12. (The chapter *shang gong* 商功 [consultations on	The names are different. Moreover, the *Suanshu Shu* includes one further method, and the first method is identical to that in the *Nine Chapters*, but the expressions differ slightly from each other.

[19] The character "*yi* 一 one," absent from the original text, is added based on the meaning.
[20] The character "*zhou* 周 circumference," which is presented as a gap in the original transcription, is supplemented based on the meaning.

Sections	The *Suanshu Shu*	The *Nine Chapters on Mathematical Procedures*	Explanations on similarities, differences and relations
	4. (Section 60 *jing cai* 井材 [a timber shaped like a well])	works])	
Volume of a quadrangular frustum	(The dimensions of a) *chu tong* 芻童 (piled hay in the shape of a frustum with two parallel rectangular bases) and a *fang que* 方闕 (building near the city gate with two parallel rectangular bases) ... The method says: multiply together the upper width and length and the lower width and length (*shang guang mao, xia guang mao ge hu cheng* 上廣袤、下廣袤各互[21]乘), and again (take the) upper length added to the lower length, multiplied by the upper width, and (take the) lower length added to the upper length multiplied by the lower width, add them all together, multiplying by the height (*gao cheng zhi* [高][22]乘之), divide by 6.	(To find the volumes of) a *chu tong* 芻童 (piled hay in the shape of a frustum with two parallel rectangular bases), *qu chi* 曲池 (frustum with two parallel circular ring bases), *pan chi* 盤池 (tray pool in the shape of a frustum with two parallel rectangular bases) and *ming gu* 冥谷 (grave, frustum with two parallel rectangular bases), all employ the same method. The method: double the upper length, add the lower length, also double the lower length, add the upper length; multiply each (of the sums) by their respective widths; combine (the results) and multiplying by the height or the depth, divide everything by 6. (The chapter *shang gong* 商功 [consultations on works])	Both similarities and differences occur in the names. The methods are equivalent but differ from each other. Designate the upper width and length as a_1 and b_1, the lower width and length as a_2 and b_2, and the height as h respectively. The corresponding formula in the *Suanshu Shu* can be expressed as $[a_1b_1 + a_2b_2 + (b_2 + b_1)\, a_1 + (b_1 + b_2)\, a_2] \times h \div 6$, whereas that in the *Nine Chapters* is $[(2b_1 + b_2)\, a_1 + (2b_2 + b_1)\, a_2] \times h \div 6$.
Volume of *chu meng* 芻甍 (a special wedge with a rectangular base and an edge parallel to one side of the base) and *yun du* 鄆都	The lower thickness *yun du* is 4 *chi*, the upper thickness is 2 *chi*, the height is 5 *chi*, and the length is 2 *zhang*; the volume is 133 1/3 (cubic) *chi*. The method says: double the upper thickness, add the lower thickness, multiply by the height and length, and divide by 6. (Section 55 *yun du*	Suppose there is a *chu meng* of lower width 3 *zhang* and length 4 *zhang*; upper length 2 *zhang* and no width; and height 1 *zhang*. The question: how much is the volume? The answer: 5000 *chi*. The method: double the lower	According to the pronunciation, "*yun du* 鄆都 (a wedge-shaped object)" in the *Suanshu Shu* should be regarded as *qian du* 壍堵[23] (a *qian du*-prism [prism with the base of a right-angled triangle]) in the *Nine*

21 The character "*zi* 自 by itself" in the source text is corrected to "*hu* 互 mutually" based on the meaning.
22 The character "*gao* 高 height," which is missing in the original transcription, is added based on the meaning.
23 Mr. Peng Hao said "'*yun du*' should probably be read as '*qian du*'" (Peng 2000). A newly added note by the author: the character "*yun* 鄆" was changed to "*zhan* 斬" in the revised transcription (Research group of the Zhangjiashan Han bamboo slips from tomb no. 247 2006, 151).

Sections	The *Suanshu Shu*	The *Nine Chapters on Mathematical Procedures*	Explanations on similarities, differences and relations
(a wedge-shaped object)	郪都 (a wedge-shaped object])	length, add the upper length, multiply by the width, and again multiply by the height, divide by 6. (The chapter *shang gong* 商功 [consultations on works])	*Chapters*, but its shape is quite distinct from that of *qian du* and identical to *chu meng*. Equations for *yun du* and *chu meng* are the same, but both are different in their expressions and terms.
ying bu zu shu 盈不足術: (method for excess and deficit)	Cross-multiply the excess and deficit by the denominators (and combine the products) as the dividend, add the numerators together as the divisor; if there is always an excess or likewise a deficit, cross-multiply the numerators and denominators and put each down (on the counting board) separately; subtract the smaller numerator from the larger numerator; the remainder is the divisor; use the deficit as the dividend. (Section 51 *fen qian* 分錢 [dividing coins]) (The method) says: combine the excess and deficit as the divisor; (taking) the deficiency numerator multiplied by the excess denominator and the excess numerator times the deficiency denominator, combine them as the dividend. Check this (lit. going back), as in *qi guang zhi shu* 啟廣之術 (method for finding the width). (Section 53 *fang tian* 方田 [rectangular fields])	*ying bu zu shu* 盈不足術 (method for excess and deficit): put down (on the counting board) the payment rates, and the excess and the deficit, each in their respective positions under them (i.e., each of rates, respectively, on the counting board). Cross-multiply these (the excess and deficit) with the payment rates, and combine them as the dividend; add the excess and deficit as the divisor. Divide the dividend by the divisor (to obtain the price of the goods or items in question). If there are fractions, interconnect them. (If) the excess and deficit both appear for buying goods or an item in common, put down (on the counting board) the payment rates, subtract the smaller from the larger, and use the remainder to simplify the dividend and the divisor. The (simplified) dividend is the price of the items bought, and the (simplified) divisor is the number of people. *liangying liangbuzu shu* 兩盈兩不足術 (method for	The *Nine Chapters* provides complete methods, by which computations can be carried out directly, in the cases of one excess and one deficit, and two excesses or two deficits. In the *Suanshu Shu*, section 51 "Cross-multiply the excess and deficit by the denominators (and combine the products) as the dividend, add the numerators together as the divisor 赢不足互乘母為實，子相從為法," and section 53 "combine the excess and deficit... as the dividend 并赢……為實" correspond to *ying bu zu shu* 盈不足術 (method for excess and deficit) in the *Nine Chapters*, but some of the subsequent steps have obviously been omitted by the author to whom the *ying bu zu shu* was already familiar. Section 51 "if there is always an excess or likewise a deficit, ... as the dividend. 皆赢若不足，……為實" is also a simplified version of *liangying liangbuzu shu* 兩盈兩不足術 (method

Sections	The *Suanshu Shu*	The *Nine Chapters on Mathematical Procedures*	Explanations on similarities, differences and relations
		two excesses or two deficits): put down (on the counting board) the payment rates, and the excesses or the deficits, both in their respective positions under them (i.e. each of the payment rates, respectively, on the counting board). Cross-multiply these (the excesses or the deficits) with the payment rates, and subtracting the smaller from the larger, the remainder is the dividend. Of the two excesses, or the two deficits, subtract the smaller from the larger, and the remainder is the divisor. Divide the dividend by the divisor (to obtain the price of the goods or items in question). (If) there are fractions, interconnect them. (If) two excesses or two deficits appear for buying goods or an item in common, put down (on the counting board) the payment rates, subtract the smaller from the larger, and use the difference to simplify the dividend and the divisor. The (simplified) dividend is the price of the item bought, and the (simplified) divisor is the number of people. (The chapter *ying bu zu* 盈不足 [excess and deficit])	for two excesses or two deficits). Although errors may occur in the process of making hand-written copies, it can be concluded that the author had already familiarized himself with the *liangying liangbuzu shu*.

The above table lists the similarities between the *Suanshu Shu* and the *Nine Chapters* with respect to their methods and texts (not strictly the same), and there would be further similarities, if the criteria for comparison were loosened. Section 16 in the *Suanshu Shu*, *fu mi*

負米 (carrying husked grain), and the 27th question in the chapter *jun shu* 均輸 (equitable transport) in the *Nine Chapters,* both involve the issue of levying a tax when carrying husked grain through three customs stations. In the former, the tax is 1/3 of the grain at three stations, while in the latter, at the furthest station, the middle station and the nearest station, 1/3, 1/5 and 1/7 of the grain are taken as the tax, respectively, which is more inconvenient for calculation. On the whole, the aforesaid comparisons and discussions, which focus on the similarities and differences of the resemblances between the two documents, do not support the hypothesis that the *Suanshu Shu* had a direct impact on the *Nine Chapters*. On the contrary, the methods applied in the *Suanshu Shu* are evidently not the ones in their original form, namely not those of universality based on the summary or generalization of certain examples. Some brief and simplified descriptions attest that the *Suanshu Shu* provides lists of existing methods and computational problems or resolves practical problems using existing methods. In other words, the methods from these parts of the *Suanshu Shu* already existed long before it was compiled, and some of them are noticeably more primitive in nature than those in the *Nine Chapters*.

The majority of the *Suanshu Shu* is different from the *Nine Chapters*. As a matter of fact, these different problems in the *Suanshu Shu* are settled by means of existing mathematical methods, mainly involving direct and inverse proportion, proportional distribution, and continued proportion, which can all be found in the *Nine Chapters*. In the *Suanshu Shu*, practical problems are solved on the basis of a large quantity of standards, such as *shi lü* 石 (率) (ratios in *shi*), *qi yin shui* 漆飲水 (pouring water into lacquer), *cheng zhu* 程竹 (norms for bamboo), and existing methods, such as *jin you shu* 今有術 (rule of three). Some problems in the *Suanshu Shu* can be addressed through the direct use (or even the mechanical application) of certain methods found in the *Nine Chapters*. Examples of this can be found in section 11 *gong mai cai* 共買材 (buying wood together), section 12 *hu chu guan* 狐出關 (fox goes through customs), and section 13 *hu pi* 狐皮 (fox furs), to which the *cui fen shu* 衰分術 (method for proportional distribution) is directly applicable. In the *Suanshu Shu*, there are even problems that are the same as those in the *Nine Chapters*, if their underlying natures are abstracted. For instance, in section 12 *hu chu guan*: "a fox, raccoon, and hound go through customs, and (together) pay tax of 111 *qian*. The hound says to the raccoon, and the raccoon says to the fox: since your fur is worth twice as much as mine, then the tax you pay should be twice as much! How much should each one pay?" Another example is given in the second question in the chapter *cui fen* 衰分 (proportional distribution) in the *Nine Chapters*: "suppose there are a cow, a horse, and a sheep eating someone's crops, the owner of the crops demands 5 *dou* of millet. The owner of the sheep says: 'My sheep eats half (as much as) the horse.' The owner of the horse says: 'My horse eats half (as much as) the cow.' Now, wishing to pay proportionally, the question: how much should each give (in compensation)?" The quoted examples all take 1, 2 and 4 as the successive differences, and employ *cui fen shu* to resolve the problems, taking into consideration the obvious differences between these two problems.

Furthermore, in section 15, *bing zu* 并租 (combined taxes):

> (The tax on) 3 (square) *bu* of *he* 禾 (millet) is 1 *dou*; (on) 4 (square) *bu* of *mai* 麥 (wheat) is 1 *dou*, (and on) 5 (square) *bu* of *da* 荅 (small beans) is 1 *dou*. If the combined tax (on all of them together) is 1 *shi*, then how much is the tax (on each one)? The result says: the tax on millet is 4 12/47 *dou*; the tax on wheat is 3 9/ (47) *dou*; (and) the tax on beans is 2 26/ (47) *dou*. The method says: put down (on the counting board) 3 *bu* for millet, 4 *bu* for wheat, and 5 *bu* for beans; let the product of the (amount for) millet multiplied by the (amount for) wheat be the dividend for the beans; the product of the (amount for) beans multiplied by the (amount for) millet be the dividend for the wheat; (and the product of the (amount for) wheat multiplied by the (amount for) beans be the dividend for the millet); for each of the different (amounts) put down (on the counting board) one *shi* multiplied by each (of the amounts for beans, wheat, and millet) as the dividend; (taking) 47 as the divisor gives the result in *dou*.[24]

These problems all adopt the *fan cui shu* 返衰術 (method for differences that are inversely distributed) as in the *Nine Chapters*. Some of the problems in the *Suanshu Shu* are similar to problems on non-equitable transport in the chapter *jun shu* 均輸 (equitable transport) in the *Nine Chapters*. For example, the 46th section, *fu tan* 負炭 (carrying charcoal):

> Carrying charcoal from a mountain, in 1 day it is possible to make 7 *dou* of charcoal and take it to a wagon. The next, in 1 day it is possible to (transport) 1 *shi* (volume) of charcoal that has been carried to the wagon to a government post (*guan* 官). Now wishing to (carry charcoal) to the government post, (one) goes to the (the mountain) where he carries charcoal, and carrying charcoal the long distance to the government post, the question is how much charcoal is delivered in 1 day?

Its mathematical significance is equivalent to the 22nd question in the chapter *jun shu* 均輸 (equitable transport) in the *Nine Chapters*:

> Suppose one person in one day can make 38 *pin wa* 牝瓦 (female roof tiles), and one person in one day can make 76 *mu wa* 牡瓦 (male tiles). Now let one person in one day make both male and female tiles, half of each respectively. The question: how many roof tiles are made?

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24 The character "*mai* 麥" of "*mai si bu* 麥四步 4 (square) *bu* of *mai* 麥 (wheat)" is written as "*li* 吏" in the transcription. "*Mai cheng da wei he shi* 麥乘荅為禾實 (let) the product of the (amount for) wheat multiplied by the (amount for) beans be the dividend for the millet" is absent from the transcription. The character "*fu* 副" and "*ge zi yi* 各自以" are signs that there are gaps in the transcription. These are corrected based on meaning derived from the context.

The prerequisite for the former is that the amounts of charcoal carried to a wagon equal those carried to a government post, and the premise for the latter resides in the same quantities of the two kinds of tiles. Section 47 *lu tang* 盧唐 (tube-shaped bamboo vessel) and section 48 *yu shi* 羽矢 (feathered arrows) in the *Suanshu Shu* have similar cases. Section 48 in the *Suanshu Shu* and the 23rd question in the chapter *jun shu* in the *Nine Chapters* all concern procedures for manufacturing arrows, with the 23rd question including one more step. Sections 46 to 49 in the *Suanshu Shu* can all be classified in the chapter *jun shu* in accordance with the categories of the *Nine Chapters*. Thus, it seems that the problems on non-equitable transport seen in the *Nine Chapters* already existed in the Pre-Qin period.

One can also argue that even the problems that are very different from those in the *Nine Chapters* can be figured out by modifying or directly applying the typical methods in the *Nine Chapters*. In view of reasons given above as to why the compilers of the *Nine Chapters* in the Han dynasty had never read the *Suanshu Shu*, why these sections in the *Suanshu Shu* are not present in the *Nine Chapters*, and why the *Suanshu Shu* is a work compiled by collecting excerpts from other books, we hold the belief that not only had the compilers of the *Nine Chapters* in the Han dynasty never read the *Suanshu Shu*, but also some authors of the *Suanshu Shu* finished the corresponding parts after having studied Pre-Qin mathematical works or their ramifications from which the *Nine Chapters* later originated.

Liu Hui states in the preface to his *Commentary on the Nine Chapters*:

> When the Duke of Zhou 周公 created the system of rites, the *jiu shu* 九數 (the nine parts of mathematics) came into being. The development (*liu*) of these nine parts, this is precisely what *The Nine Chapters* is. In the past, the brutal Qin dynasty burned the books, and the classic texts (as well as *the Nine Chapters*) were destroyed or damaged. From that time on, the (Western) Han dynasty Marquis of Beiping Zhang Cang and Deputy Grand Minister of Agriculture Geng Shouchang both are renowned (among those) of their generation for having excelled at computations. Cang and the other, based upon what survived of the old damaged works, made both excisions and completions. Therefore, upon comparing their contents, these (reconstructed works) and the old (texts) may differ. Furthermore, most of the words and terms (in the reconstructed works of Zhang and Geng) belong to times closer to ours. (Guo 1998, 196)

Thus, Liu Hui deemed that the *Nine Chapters on Mathematical Procedures*—which came into being in the Pre-Qin period based on the *jiu shu*, namely the nine parts of mathematics formulated by the Duke of Zhou when establishing the system of rites in the Western Zhou dynasty—was destroyed and lost as a consequence of the destruction of books in the Qin dynasty. The Han scholars Zhang Cang and Geng Shouchang reconstructed the later renowned *Nine Chapters on Mathematical*

Procedures by excising and supplementing the collected incomplete version on bamboo slips of the *Nine Chapters* dating from the Pre-Qin period. Liu Hui's opinion is well founded in light of the above analysis of the relationships between the *Suanshu Shu* and the *Nine Chapters*. I have conducted a detailed analysis of the primitive accumulation, theoretical foundations, and social conditions and demand for mathematics in early China, in order to demonstrate that the vast majority of mathematical methods in the current *Nine Chapters* were produced in the Warring States period.[25] In particular, the Qin Laws unearthed in Shuihudi, Yunmeng County, Hubei Province provide sufficient vindication that high-level mathematical knowledge, as was attained in the *Nine Chapters* in the Han dynasty, was definitely applied in the Pre-Qin period.[26] For this reason, I am convinced that a single mathematical work existed in the Pre-Qin period that served as the major source for the *Nine Chapters* in the Han dynasty, even though this may not have taken the form of a single book titled *The Nine Chapters on Mathematical Procedures*. This book (from which the Han *Nine Chapters* originates) is a classic, the earliest origin of which can be traced back to the *jiu shu* learned by the Western Zhou aristocrats. In the first volume of the *Zhoubi suanjing* 周髀算經 (Mathematical Classic of Zhou's Gnomon), Chen Zi 陳子 replies to Rong Fang 榮方's question, saying "this is what *suanshu* 算術 (mathematical procedures) can solve" (Qian 1963, 24). These "mathematical procedures 算術" may also have relevance to this book (the ancestor of the Han *Nine Chapters*). The *Suanshu Shu* was compiled by drawing experience from this book or ones derived from it, and applying such methods to economic and social life. It can be viewed as the result of the historical testing, selecting and eliminating of material among various classics and non-classics, and while the *Suanshu Shu* was lost, the *Nine Chapters* survived after the Han dynasty.

4 The significance of the *Suanshu Shu* in establishing the history of the evolution of algorithmic mathematics from the Pre-Qin period to the Han dynasty

Chinese mathematics in the Spring and Autumn and Warring States periods (770–221 BCE) is characterized by two tendencies: algorithmic mathematics and theoretical mathematics. After the Han dynasty, the former became the mainstream of Chinese mathematics. The *Nine Chapters on Mathematical Procedures*, completed during the Han dynasty, was the earliest treatise of this mainstream. Hence, the Han dynasty was frequently referred to in discussions on the achievements attained

[25] For details, see Zou 2001.
[26] For details, see Zou 2005 and Zou 2007.

in Chinese mathematics, while the evolution of mathematics from the Pre-Qin period to the Han dynasty tended to be skirted over. Qian Baocong 钱宝琮 divided the history of Chinese mathematics into two phases, before and after 221 BCE, the year the Qin unified China. On the one hand, Qian said:

> Although there is not a single Pre-Qin mathematical book passed on to later generations, it is undeniable that in the *Nine Chapters*, the content of the following chapters mostly emerged before the Qin dynasty, *fang tian* 方田 (rectangular fields), *su mi* 粟米 (grain and husked grain), *cui fen* 衰分 (proportional distribution), *shao guang* 少廣 ([increasing lengths by] shortening widths) and *shang gong* 商功 (consultations on works).

On the other hand, he regarded the period before the Qin unified China as the embryonic phase of Chinese mathematics (Qian 1981, 1, 14), which is inconsistent with his former statement. Guo Shuchun 郭书春 basically categorized the parts of the *Nine Chapters* involving methods for practical problems as coming from the Pre-Qin period, taking account of the structure of the *Nine Chapters*. He classed the remainder as complementary material dating from the Han dynasty (Guo 1992, 94–105; idem 1997, 112–121). This is a workable scheme for distinguishing the content of mathematics in the Qin and Pre-Qin periods from those in the Han dynasty. Prof. Guo and I have also shown that the time-span from the Warring States period to the Western Han dynasty was the formative phase for the framework of ancient Chinese mathematics (Guo et al. 1995, 14–59). This is preferable to making a distinction between the cases before and after Qin's unification of China, for which there are insufficient grounds. In recent years, I have made systematic use of extant primary and secondary sources to explore the mathematical knowledge of these early periods, especially against the background of social demands. From this I am confident that I have validated the point of view that the major mathematical methods in the *Nine Chapters* already existed in the Pre-Qin period, that Chinese mathematics first reached a high point during the Spring and Autumn and the Warring States periods, and that this Pre-Qin mathematics underwent sifting and integration in the Han dynasty.[27] Notwithstanding my own certainty about this standpoint, there still remains a gap between this argument and the available evidence, resulting from the absence of abundant relevant mathematical materials.

Fortunately, publication of the transcription of the *Suanshu Shu* helps to bridge this gap. The *Suanshu Shu* dates from the transition from the Pre-Qin period to the Qin and Han dynasties, serving as the foundation for establishing the history of the evolution of practical algorithmic mathematics from the Warring States to the Qin and Han. As elucidated above, the *Suanshu Shu*, a book compiled by collecting excerpts from other books, solves problems through extant mathematical methods, and thus there is no

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[27] For details, see Zou 2001.

dispute over the Pre-Qin origin of these methods in the *Nine Chapters*. The *Suanshu Shu* basically made no use of the methods in the two chapters *fang cheng* 方程 (the corresponding method for solving the problems of simultaneous linear equations) and *gou gu* 勾股 (base and altitude of a right triangle) in the *Nine Chapters*. Some examples of other methods that appear in the *Nine Chapters* but not in the *Suanshu Shu* can be summed up in the following table:

Chapter in the *Nine Chapters*	**Methods**
fang tian 方田 (rectangular fields)	*fang tian shu* 方田術 (method for rectangular fields); *gui tian shu* 圭田術 (method for isosceles triangular fields, or triangular fields); *xie tian shu* 邪田術 (method for right-angled trapezoid fields); *ji tian shu* 箕田術 (method for dustpan-shaped fields or isosceles trapezoid fields); *yuan tian shu* 圓田術 (method for circular fields); *huan tian shu* 環田術 (method for ring-shaped fields); *hu tian shu* 弧田術 (method for fields in the shape of a segment of a circle); *wan tian shu* 宛田術 (method for bowl-shaped fields).
su mi 粟米 (grain and husked grain)	*qi lü shu* 其率術 (method for the prices of a unit of item [with different sizes or quality] being two consecutive natural numbers); *fan qi lü shu* 反其率術 (method for the quantities of items [with different sizes or quality] that a unit of *qian* can buy which are two consecutive natural numbers).
shao guang 少廣 ([increasing lengths by] shortening the widths)	*kai fang shu* 開方術 (method for extracting square roots); *kai yuan shu* 開圓術 (method for finding the circumference of a circle given its area by extracting the square root); *kai li fang shu* 開立方術 (method for extracting cubic roots); *kai li yuan shu* 開立圓術 (method for finding the diameter of a sphere given its volume by extracting the cubic root).
shang gong 商功 (consultations on works)	*fang bao dao shu* 方堢壔術 (method for a cuboid with a square base); *fang zhui shu* 方錐術 (method for a square-based pyramid); *fang ting shu* 方亭術 (method for a square pavilion [frustum of a pyramid]); *qian du shu* 塹堵術 (method for a *qian du*-prism [prism whose base is a right-angled triangle]); *yang ma shu* 陽馬術 (method for a *yangma*-pyramid [pyramid with one edge perpendicular to its rectangular base]); *bie nao shu* 鱉臑術 (method for tetrahedron with the base of a right-angled triangle perpendicular to an edge).
jun shu 均輸 (equitable transport)	*jun shu ben shu* 均輸本術 (method for original problems of equitable transport).

On the other hand, most of these methods must have been applied in the Pre-Qin period. Examples are found in sections 55–60 of the *Suanshu Shu* adopting the volume formulae for *chu meng* 芻甍 (a special wedge with a rectangular base and an edge parallel to one side of the base), *chu tong* 芻童 (piled hay in the shape of a frustum with two parallel rectangular bases), *yuan zhui* 圓錐 (circular cone), *yuan tai* 圓臺 (circular platform, circular truncated cone-shaped platform), and *yuan zhu* 圓柱 (circular column), as well as in sections 64–68 using the rectangular area formulae and inverse operations. It can thus be inferred that the following methods in the *Nine Chapters* existed without a doubt in the Pre-Qin period:

fang tian shu 方田術 (method for rectangular fields);
gui tian shu 圭田術 (method for isosceles triangular fields, or triangular fields);
xie tian shu 邪田術 (method for right-angled trapezoid fields);
ji tian shu 箕田術 (method for dustpan-shaped fields or isosceles trapezoid fields);
yuan tian shu 圓田術 (method for circular fields);
huan tian shu 環田術 (method for ring-shaped fields);
fang bao dao shu 方堢壔術 (method for a cuboid with a square base);
fang zhui shu 方錐術 (method for a square-based pyramid);
fang ting shu 方亭術 (method for a square pavilion [frustum of a pyramid]);
qian du shu 塹堵術 (method for a *qian du*-prism [prism whose base is a right-angled triangle]);
yang ma shu 陽馬術 (method for a *yangma*-pyramid [pyramid with one edge perpendicular to its rectangular base]).

Furthermore, section 29 *yi* 醫 (doctors) states that:

> The norm says: (if) a doctor who cures sick people gains 60 (positive) *suan* 算 (a unit for measuring assessment) (positive points), while (he merely gains) 20 negative *suan* 算 (negative points), (he can be regarded as qualified). ... The norm ... not ... (Under a certain circumstance), if (the positive points) are converted to 60 *suan* 算, then what are the corresponding negative points (*suan* 算)? The answer: the negative points are 17 11/269 *suan* 算. The method says: take the current positive points as the divisor, and let the product of 60 multiplied by the current negative points be the dividend.[28]

Accordingly, there can be no question that rules of multiplication and division for positive and negative numbers are utilized herein, regardless of the fact that these incomplete texts require further study on the specific meanings of certain words. In the section *kao gongming* 考功名 (assessing the services of officials) in *Chunqiu fanlu* 春秋繁露 (the Luxuriant Dew of the Spring and Autumn Annals), Dong Zhongshu 董仲舒 writes that (Su 1992, 180–182):

[28] A newly added note by the author: for more detailed analysis and explanation, see Zou 2010.

> The method for assessing officials, ... (The results of the assessment) are divided into nine levels that are graded every three levels. The first level ranks the highest, the fifth ranks the middle and the ninth the lowest. Suppose all levels are categorized into the middle rank. Afterwards, classify the officials who gain positive points into the levels above the middle rank, and place those who receive negative points into the levels below the middle rank. For those who gain the least positive points, one positive point is added to their scores (so as to lift them above the middle rank, the fifth level), (the more the positive points are, the more points are added), until from one to four positive points (are added). For those whose negative points are the most, subtract four points from the middle point (the fifth level), (the less the absolute value of the negative points is, the less points are subtracted), until only one point (is subtracted). The ordinal number of a level is inversely proportional to its scoring. Three times four is twelve, and twelve *shi* 試 (test) make one *ji* 計 (assessment cycle).[29] After an assessment cycle is completed, the dismissal or the promotion of an official can be determined according to the scoring. Each and every *ji* should be carried out based on the previous level, in a bid to be coherent with the future scoring. The first period of assessment includes two *ji*, and the second consists of four *ji*. In addition, each *ji* should be consistent with the level in each preceding *ji*. Once the desired number of *ji* is reached, the discharge or the elevation of an official is dependent on the assessment result.

In comparison with the above texts, "gains (positive) points 得若干算" and "(gains) negative points 負若干算" should be considered as a kind of assessment and scoring method. In the *Nine Chapters*, negative numbers only make their appearance in the chapter *fang cheng* 方程 (the corresponding method for solving the problems of simultaneous linear equations), and are introduced when a smaller number in one equation was not large enough for a larger number in the other equation to be subtracted from the smaller number in the process of the elimination of elements. Therefore, it is certainly justifiable to apply the Pre-Qin concept of negative numbers to the appraisal of the doctors' treatments in the section *yi* 醫 (doctors) in the *Suanshu Shu*. Moreover, certain parts of the chapter *fang cheng* were presumably extant in the Pre-Qin period. Problems solved by the original procedures of equitable transport were not included in the *Suanshu Shu*, which could be ascribed to the fact that the *Suanshu Shu* was compiled in the Qin dynasty when Qin Laws prohibited having others transport goods (Research group of the Shuihudi Qin bamboo slips

[29] A newly added note by the author: "*san si shi er* 三四十二 three four twelve" belongs to the multiplication table, meaning that three times four is twelve. A basic assessment is called *shi* 試 (test). *Shi* 試 may refer here to the assessment of the local officials who were evaluated once a season (each evaluation is called *shi* 試). Four *shi* 試 in a year make one *kao* 考 (examination). The Emperor assessed the prefectures once a year through one *shi* 試. Three *shi* 試 make an assessment cycle, and local officials underwent twelve *shi* 試 in one cycle. Therefore, "*san si shi er, er cheng yu ji* 三四十二，而成于計 three times four is twelve, and twelve *shi* 試 (tests) make one *ji* 計 (assessment cycle)."

1978, 123).[30] Hu Pingsheng 胡平生 claims that *jun shu* 均輸 (equitable transport) dates back to ancient times, based on the incomplete slips of another *Suanshu Shu* 算術書 (Writings on Arithmetic) (transcribed no later than 165 BCE) found in the Shuanggudui 双古堆 in Fuyang, Anhui Province, which contains the remains of damaged slips of the first problem in the chapter *jun shu* in the *Nine Chapters* (Hu 1998, 12–30). This is in accordance with my viewpoint concluded from extant literature that the mathematical methods[31] for original problems of equitable transport were applied in the Pre-Qin period.

Despite the *Suanshu Shu* not proving conclusively that some mathematical methods found in the *Nine Chapters* already existed in the Pre-Qin period, it offers credible testimony to the presence of corresponding mathematical methods in the Pre-Qin period, within the context of the social background and demands of the times. Taking into account that the *Suanshu Shu* is an incomplete book compiled by collecting excerpts from other books and meant to cater directly to the management needs of lower officials, and that as a work compiled by Qin scholars,[32] it was unavoidably limited in its range of mathematical materials. Therefore, it is not surprising that it lacks certain content, in particular some relatively advanced methods. I thus confidently believe the viewpoint expressed above about the history of the evolution of practical algorithmic mathematics from the Pre-Qin period to the Han dynasty is essentially correct.

5 Concluding remarks

In brief, tailored to the management demands of lower level officials, the *Suanshu Shu* is a relatively unsystematic compilation completed just before or after the Qin unification, and originated from at least two sources. It is also mainly the product of the need to apply existing mathematical methods and socio-economic standards to real situations, although it does contain a small number of more abstract problems. In addition, the *Suanshu Shu* and the *Nine Chapters on Mathematical Procedures* do not have any direct textual relationships, but both stemmed from some shared sources. For a few methods, the expressions in the *Nine Chapters* are closer to the forms when these methods were first created than those in the *Suanshu Shu*, and the vast majority of its contents came into existence in the Pre-Qin period. There may even have already been a book similar to the *Nine Chapters* in scale in existence in the Pre-Qin period, later serving as the major

30 A newly added note by the author: in the original paper, two characters "讓人 having others" are missing before "委輸."

31 For details, see Zou 2001.

32 A newly added note by the author: the *Suanshu Shu* was probably finished at the beginning of the Han dynasty, but most of its contents date from the Qin dynasty or even earlier.

source for the *Nine Chapters*. This book or its derivatives acted as didactic and reference materials for the authors of the *Suanshu Shu*. The *Suanshu Shu* is thus valuable for establishing the history of the evolution of algorithmic mathematics from the Pre-Qin period to the Qin and Han dynasties.

Appendix (for the English version)

The units of measure of the Qin and Han dynasties mentioned in this article are as follows:

Length: 1 *chi* 尺 ≈ 23 centimeters; 1 *chi* = 10 *cun* 寸; 1 *zhang* 丈 = 10 *chi*; 1 *bu* 步 = 6 *chi*; 1 *li* 里 = 300 *bu*.

Volume (or capacity, in most cases): 1 *chi* = 1 cubic *chi*; 1 *sheng* 升 ≈ 200 milliliters; 1 *dou* 斗 = 10 *sheng*; 10 *shi* 石 (or *hu* 斛) = 10 *dou*.

Weight: 1 *jin* 斤 ≈ 250 grams; 1 *liang* 兩 = 24 *zhu* 銖; 1 *jin* = 16 *liang*; 1 *shi* = 120 *jin*.

Land area: 1 *bu* 步 = 1 bu^2; 1 *mu* 畝 = 240 *bu*; 1 *qing* 頃 = 100 *mu*; 1 *li* 里 = 1 li^2 = 375 *mu* = 3 *qing* 75 *mu*.

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