固体力学

各向异性复合材料强度失效判据综述[1)]

贾 斐[*]　杨成鹏[†, 2)]　宋远翔[†]

[*] (西安电子科技大学机电工程学院, 西安 710071)

[†] (西北工业大学力学与土木建筑学院, 西安 710072)

摘要　失效判据在复合材料的渐进损伤模拟和极限强度预测中具有极其重要的意义. 针对各向异性复合材料的损伤-破坏问题, 根据理论架构与发展脉络, 对现有失效判据进行了分类汇总和全面综述, 重点总结了破坏模式无关判据和破坏模式相关判据, 分析了基于破坏面假设的相关理论成果, 阐述了基于应力不变量进行判据构建的基本理论、方法和模型, 并论述了基于应变能和基于损伤参量的两类失效判据. 研究表明, 各向异性复合材料强度理论发展的基础和主线仍然是二次应力判据, 高阶唯象失效判据的合理性和适用性尚需验证; 能量判据和损伤判据对于非线性-准脆性复合材料显示出了合理性和适用性, 但表达式较为复杂. 在深入分析研究的基础上, 对复合材料强度理论的发展趋势进行了展望, 提出了"损伤-断裂协同理论"这一重点与难点方向, 以期对复合材料及其结构的设计、评估与应用提供更有价值的参考.

关键词　复合材料, 强度理论, 损伤演化, 失效判据, 断裂模式

中图分类号: O34, TB332　**文献标识码**: A　doi: 10.6052/0459-1879-23-507

REVIEW ON STRENGTH FAILURE CRITERIA OF ANISOTROPIC COMPOSITE MATERIALS[1)]

Jia Fei[*]　Yang Chengpeng[†, 2)]　Song Yuanxiang[†]

[*] (*School of Mechano-Electronic Engineering*, *Xidian University*, *Xi'an* 710071, *China*)

[†] (*School of Mechanics*, *Civil Engineering and Architecture*, *Northwestern Polytechnical University*, *Xi'an* 710072, *China*)

Abstract　Failure criteria are of great importance in progressive damage simulation and ultimate strength prediction of composite materials. Towards the damage-failure problem of anisotropic composites, the existing failure criteria are comprehensively classified and summarized according to model architecture and development venation, with focus on the failure-mode-independent and failure-mode-dependent failure criteria. Meanwhile, the theoretical models based on the action-plane assumption were analyzed. Moreover, the basic theories and methods for constructing stress-invariant-based criteria were described, and the resultant models were presented. Finally, the other two types of failure criteria, i.e., strain energy criteria and damage-based criteria were discussed. The investigation shows that the rationality and applicability of the higher-order phenomenological failure criteria still need to be verified, and the foundation and main direction of the strength theory development for anisotropic composites is still the quadratic stress criterion; the energy criterion and the damage criterion show rationality and applicability for nonlinear and quasi-brittle composites, but their expressions are more complicated. On the basis of in-depth analysis and research, the development trend of strength

2023–10–25 收稿, 2023–12–26 录用, 2023–12–27 网络版发表.

1) 国家自然科学基金资助项目 (12072274).

2) 通讯作者: 杨成鹏, 副教授, 主要研究方向为先进复合材料及其结构的力学行为. E-mail: yang@mail.nwpu.edu.cn

引用格式: 贾斐, 杨成鹏, 宋远翔. 各向异性复合材料强度失效判据综述. 力学学报, 2024, 56(4): 1006-1024

Jia Fei, Yang Chengpeng, Song Yuanxiang. Review on strength failure criteria of anisotropic composite materials. *Chinese Journal of Theoretical and Applied Mechanics*, 2024, 56(4): 1006-1024

theory of composites is prospected, and an important and difficult direction of "damage-fracture synergistic theory" is put forward, with a view to providing more valuable references for the design, evaluation and application of composites and their structures.

Key words composite materials, strength theory, damage evolution, failure criterion, fracture mode

引言

强度是结构设计的关键问题之一[1]. 随着航空航天技术的发展, 为了提高结构效率, 使用先进复合材料进行结构减重势在必行. 然而, 复合材料具有结构复杂性, 如层合结构和编织结构, 其力学行为具有各向异性, 甚至非线性和单边效应 (拉压性能不等); 损伤演化过程复杂, 失效模式多样化, 破坏机制难以表征, 给强度性能评估带来极大难度.

复合材料的强度问题是一个具有挑战性的力学课题, 一个多世纪以来, 研究者针对正交各向异性单向板、高度各向异性层压板以及织物增强复合材料开展了大量试验、理论及模型研究, 建立了许多强度分析的模型和方法, 有效表征了各类复合材料的失效特性, 并实现了承载性能的分析预测.

强度分析中强度理论的应用十分重要. 狭义上的强度理论指的是失效判据, 不包括本构关系与刚度退化准则. 经典失效判据针对正交各向异性线弹性单向板而提出, 只关注破坏所满足的应力条件, 不考虑材料的应力-应变响应历程以及失效机理、破坏过程和断裂模式, 大多属于宏观唯象模型. 随着强度研究的深入, 经典失效判据不断被使用、拓展与改进, 并获得了新的应用途径及领域.

本文针对各向异性复合材料的渐进损伤模拟及强度预测问题, 按照强度理论的基础架构和发展脉络, 对现有失效判据进行分类归纳与总结, 广泛而深入剖析各类失效判据的模型构建思路、基本适用范围及其优缺点, 在此基础上对复合材料强度理论的发展方向进行展望.

1 破坏模式无关判据

不考虑具体破坏模式的失效判据多为宏观唯象模型. 有学者指出[2], 该类模型必须满足一些基本要求: 其一, 失效判别式应具有坐标转换的不变性; 其二, 判别式在应力空间任何给定的径向载荷路径下拥有唯一解; 其三, 在全部应力空间判别式应给出凸的失效曲面; 其四, 在不考虑失效机理的条件下可以灵活给出任意形状的破坏包络面. 能同时满足上述要求的通常为应力张量多项式判据, 如高阶张量判据和二次应力判据等.

1.1 高阶张量判据

各向异性复合材料在多应力作用下的破坏机制十分复杂, 材料失效时的应力条件隐晦不明. 有学者建议使用高阶张量多项式来描述复合材料在复杂应力下的强度特征, 并指出高阶多项式模型具有强的表征能力, 有潜力描述复杂多变的强度包面或包线. Goldenblat 等[3] 试图考虑所有的应力耦合作用模式, 于 1965 年针对各向异性材料提出了一个高阶张量函数强度失效判据, 并讨论了平面应力情形, 其表达式为

$$\left.\begin{array}{l}(F_i\sigma_i)^\alpha+(F_{ij}\sigma_i\sigma_j)^\beta+(F_{ijk}\sigma_i\sigma_j\sigma_k)^\gamma+\cdots=1\\ \quad (i,j,k=1,2,\cdots,6)\end{array}\right\} \tag{1}$$

其中, $\sigma_i\,(i=1,2,\cdots,6)$是材料主方向应力分量; α, β和 γ 等是材料常数, 可取$\alpha=1$, $\beta=1/2$, $\gamma=1/3$, 以此类推; F_i, F_{ij} 和 F_{ijk} 等是强度张量. 该模型虽然在理论层面具有一般性和普适性, 但由于待定系数繁多、参数识别困难而很少被采用. Ashkenazi[4] 针对高度各向异性材料, 基于 4 阶强度张量分析, 提出了四次多项式准则, 该准则曾在美国玻璃纤维增强复合材料的设计中被广泛应用, 其中涉及偏轴强度和双轴强度参量.

鉴于式 (1) 的复杂性, Malmeister[5] 于 1966 年将其中的 α, β 和 γ 等材料常数取值为 1. 此外, Huang 等[6] 提出了另一种简化方法, 即忽略式 (1) 中 6 阶以上的强度张量, 给出

$$(F_i\sigma_i)^\alpha+(F_{ij}\sigma_i\sigma_j)^\beta+(F_{ijk}\sigma_i\sigma_j\sigma_k)^\gamma=1 \tag{2}$$

平面应力下 Osswald 等[7] 令式 (2) 中的 6 阶强度张量为 0, 且令 $\alpha=1$, $\beta=1/2$, 并根据失效曲面与不同应力轴之交点的斜率确定了应力交互项的系数. 不失一般性, Tennyson 等[8] 令式 (2) 中 $\alpha=\beta=\gamma=1$, 给出了三次多项式模型, 并证明其对于玻璃/环氧和石墨/环氧圆筒试件的强度预测值与试验值相吻合, 而且其强度失效曲面的封闭性条件也在随

后的研究中得到了分析论证[9].

整体上, 高阶张量型强度准则在强度分析预测的理论和工程实践中适用性较差, 原因是其强度系数很难通过简单加载试验悉数确定, 而且其预测的强度包面之封闭性尚需深入论证, 且模型反映的应力耦合作用机制也不一定就与实际相符. 事实上, 大量试验数据表明, 二次应力多项式对绝大多数加载模式下的强度数据的拟合优度通常较好. 因此, 二次应力多项式准则的发展、论证与应用成为了后续研究的重点方向.

1.2 二次应力判据

最具普适性和一般性的二次应力判据, 实际上是高阶张量判据的简化模型, 即

$$F_i\sigma_i + F_{ij}\sigma_i\sigma_j = 1 \quad (i,j = 1,2,\cdots,6) \tag{3}$$

上式习惯上称为张量判据或应力空间判据, 包含了全部的应力一次项和二次项. 应该指出, 多数经典失效判据都可以转化为式 (3) 的形式; 同时, 由于强度系数的识别方法和结果不同, 式 (3) 衍生出了多种强度理论.

Tsai 等将 Hill 于 1948 年针对金属提出的一个正交各向异性屈服准则[10] 拓展用于复合材料的破坏强度预测, 提出了著名的 Tsai-Hill 准则[11], 该准则也可归并于式 (3), 其具有歪形能的形式, 忽略了应力的一次项以及正应力与剪应力的交互项, 因此只需要通过轴向简单加载试验就能确定所需的强度系数; 但是其判别式中的轴向强度须分别根据应力分量的正负号取相应拉伸或压缩强度, 从而实现拉压应力下破坏强度的粗略预测.

Tsai-Hill 准则给出了正应力交互项系数的一种表达. 也有学者针对特定材料忽略正应力的交互项, 在平面应力下给出 $F_{12} = 0$ 的结果[12]. 可见, 由于研究思路的差异, 导致强度系数的识别结果并不唯一; 当然, 其深层次的原因是不同材料在强度失效时的应力耦合作用机制不同. Fischer[13] 针对复合材料层压板, 考虑变形协调性, 给出了 F_{12} 的另一种表达, 其中包含了材料主方向的工程弹性常数. Chamis[14] 则考虑材料细观结构的影响, 引入试验-理论关联参数 K 对 F_{12} 进行了变更, 用于表征拉压强度不等效应以及正应力耦合效应, 参数 K 在应力空间的 4 个象限中取不同值.

表征拉压强度不等的另一途径是在判别式中考虑应力的一次项. Marin[15] 提出的广义强度理论中即包含应力的一次项, 从而在判别式中引入了主方向拉伸和压缩强度值; 但其针对的是三向应力, 判别式中不包含剪应力相关项, 适用性受限. Hoffman 判据[16] 则针对任意应力状态均具有较好的表征能力; 该模型协同考虑了各主方向拉压强度不等对材料整体失效曲面或包线的影响效应, 强度系数确定简便, 预测精度较高, 应用较为广泛.

Franklin[17] 同样考虑拉压性能不等的单边效应, 通过双轴试验增加每个应力象限的数据点, 将 Marin 广义强度理论[15] 进行变更以提高强度预测的准确性和可靠性, 并给出了平面应力下的失效条件, 其中 F_{12} 即通过双轴加载试验确定; 该判据的预测精度通常要高于 Hoffman 准则. Huang 等[6] 则通过偏轴剪切试验确定了正应力交互系数. 此外, Theocaris[18] 提出了 EPFS 准则, 其中正应力交互项系数不通过组合加载试验确定, 而是采用与 Hoffman 准则相同的系数值, 剪切相关的强度系数则考虑了正负剪切效应. Osswald 等[7] 也对式 (3) 进行了研究, 不仅分析了正应力之间的耦合作用, 同时对正应力和剪应力之间的耦合影响效应进行了表征.

一些经典且重要的失效判据及其强度系数全部列于表 1. 分析认为, 所有二次应力失效判据所考虑的应力耦合作用模式均存在局限性, 虽然 Tsai 等[19] 同时考虑拉压强度不等以及正负剪切效应给出了式 (3) 的一般形式, 但仍然无力描述更复杂的失效曲面. 应该指出, 多数情况下剪应力的正负号并不影响破坏性能, 因此式 (3) 中剪应力的一次项通常被忽略, 且正应力与剪应力的交互项也被忽略, 而正应力交互项系数 F_{12} , F_{13} 和 F_{23} 往往需要通过偏轴或双轴加载试验确定, 以增加试验数据点, 提高可靠性.

为了避免进行难度较大的双轴加载试验, Tsai[22] 通过物理分析的方法给出了 F_{12} 的取值范围, 即

$$-\frac{1}{2}\sqrt{\frac{1}{S_1^+S_1^-S_2^+S_2^-}} \leqslant F_{12} \leqslant 0 \tag{4}$$

而 DeTeresa 等[23] 则通过横观各向同性静水压缩与横向压缩分析, 给出

$$F_{12} = -\frac{1}{4S_1^+S_1^-},\ F_{23} = -\frac{1}{S_2^+S_2^-} \tag{5}$$

Li 等[24] 同样针对复合材料单向板, 分析得出

表 1　各向异性复合材料经典失效判据的强度系数

Table 1　Strength coefficients of classical failure criteria for anisotropic composite materials

Theory	F_{11}	F_{22}	F_{33}	F_1	F_2	F_3	$2F_{12}$	$2F_{13}$	$2F_{23}$	F_{44}	F_{55}	F_{66}
Ashkenazi[4]	$\frac{1}{S_1^2}$	$\frac{1}{S_2^2}$	—	0	0	—	$\frac{4}{\sigma_{45}}-\frac{1}{S_1^2}-\frac{1}{S_2^2}-\frac{1}{S_6^2}$	—	—	—	—	$\frac{1}{S_6^2}$
Malmeister[5]	$\frac{1}{S_1^+S_1^-}$	$\frac{1}{S_2^+S_2^-}$	—	$\frac{1}{S_1^+}-\frac{1}{S_1^-}$	$\frac{1}{S_2^+}-\frac{1}{S_2^-}$	—	$\frac{1}{\tau_{45}^-}\left(\frac{S_1^- - S_1^+}{S_1^+S_1^-}-\frac{S_2^- - S_2^+}{S_2^+S_2^-}\right)+\left(\frac{1}{S_1^+S_1^-}+\frac{1}{S_2^+S_2^-}\right)^2-\frac{1}{(\tau_{45}^-)^2}$	—	—	—	—	$\frac{1}{S_6^2}$
Tsai-Hill[10]	$\frac{1}{S_1^2}$	$\frac{1}{S_2^2}$	$\frac{1}{S_3^2}$	0	0	0	$\frac{1}{S_3^2}-\frac{1}{S_2^2}-\frac{1}{S_1^2}$	$\frac{1}{S_2^2}-\frac{1}{S_3^2}-\frac{1}{S_1^2}$	$\frac{1}{S_1^2}-\frac{1}{S_2^2}-\frac{1}{S_3^2}$	$\frac{1}{S_4^2}$	$\frac{1}{S_5^2}$	$\frac{1}{S_6^2}$
Fisher[13]	$\frac{1}{S_1^2}$	$\frac{1}{S_2^2}$	—	0	0	—	$\frac{E_1(1+\nu_{21})+E_2(1+\nu_{12})}{2S_1S_2\sqrt{E_1E_2(1+\nu_{21})(1+\nu_{12})}}$	—	—	—	—	$\frac{1}{S_6^2}$
Chamis[14]	$\frac{1}{S_1^2}$	$\frac{1}{S_2^2}$	—	0	0	—	$\frac{KE_1(1-\nu_{23})+KE_2(1+4\nu_{12}-\nu_{13})}{S_1S_2\sqrt{E_1E_2(2+\nu_{21}+\nu_{23})(2+\nu_{12}+\nu_{13})}}$	—	—	—	—	$\frac{1}{S_6^2}$
Marin[15]	$\frac{1}{S_1^+S_1^-}$	$\frac{1}{S_1^+S_1^-}$	—	$\frac{1}{S_1^+}-\frac{1}{S_1^-}$	$\frac{1}{S_2^+}-\frac{S_2^+}{S_1^+S_1^-}$	—	$\frac{1}{S_1^+S_1^-}-\frac{1}{2S^2}\left[1-S\left(\frac{1}{S_1^+}-\frac{1}{S_1^-}-\frac{1}{S_2^+}+\frac{S_2^+}{S_1^+S_1^-}\right)\right]$	—	—	—	—	0
Hoffman[16]	$\frac{1}{S_1^+S_1^-}$	$\frac{1}{S_2^+S_2^-}$	$\frac{1}{S_3^+S_3^-}$	$\frac{1}{S_1^+}-\frac{1}{S_1^-}$	$\frac{1}{S_2^+}-\frac{1}{S_2^-}$	$\frac{1}{S_3^+}-\frac{1}{S_3^-}$	$\frac{1}{S_3^+S_3^-}-\frac{1}{S_1^+S_1^-}-\frac{1}{S_2^+S_2^-}$	$\frac{1}{S_2^+S_2^-}-\frac{1}{S_3^+S_3^-}-\frac{1}{S_1^+S_1^-}$	$\frac{1}{S_1^+S_1^-}-\frac{1}{S_2^+S_2^-}-\frac{1}{S_3^+S_3^-}$	$\frac{1}{S_4^2}$	$\frac{1}{S_5^2}$	$\frac{1}{S_6^2}$
Franklin[17]	$\frac{1}{S_1^+S_1^-}$	$\frac{1}{S_2^+S_2^-}$	—	$\frac{1}{S_1^+}-\frac{1}{S_1^-}$	$\frac{1}{S_2^+}-\frac{1}{S_2^-}$	—	$\frac{K_1}{S_1^+S_1^-}$	—	—	—	—	$\frac{1}{S_6^2}$
Theocaris[18]	$\frac{1}{S_1^+S_1^-}$	$\frac{1}{S_2^+S_2^-}$	$\frac{1}{S_3^+S_3^-}$	$\frac{1}{S_1^+}-\frac{1}{S_1^-}$	$\frac{1}{S_2^+}-\frac{1}{S_2^-}$	$\frac{1}{S_3^+}-\frac{1}{S_3^-}$	$\frac{1}{S_3^+S_3^-}-\frac{1}{S_1^+S_1^-}-\frac{1}{S_2^+S_2^-}$	$\frac{1}{S_2^+S_2^-}-\frac{1}{S_3^+S_3^-}-\frac{1}{S_1^+S_1^-}$	$\frac{1}{S_1^+S_1^-}-\frac{1}{S_2^+S_2^-}-\frac{1}{S_3^+S_3^-}$	$\frac{1}{S_4^+S_4^-}$	$\frac{1}{S_5^+S_5^-}$	$\frac{1}{S_6^+S_6^-}$
Tsai-Wu[19]	$\frac{1}{S_1^+S_1^-}$	$\frac{1}{S_2^+S_2^-}$	$\frac{1}{S_3^+S_3^-}$	$\frac{1}{S_1^+}-\frac{1}{S_1^-}$	$\frac{1}{S_2^+}-\frac{1}{S_2^-}$	$\frac{1}{S_3^+}-\frac{1}{S_3^-}$	non-unique	non-unique	non-unique	$\frac{1}{S_4^+S_4^-}$	$\frac{1}{S_5^+S_5^-}$	$\frac{1}{S_6^+S_6^-}$
Cowin[20]	$\frac{1}{S_1^+S_1^-}$	$\frac{1}{S_2^+S_2^-}$	$\frac{1}{S_3^+S_3^-}$	$\frac{1}{S_1^+}-\frac{1}{S_1^-}$	$\frac{1}{S_2^+}-\frac{1}{S_2^-}$	$\frac{1}{S_3^+}-\frac{1}{S_3^-}$	$2\sqrt{\frac{1}{S_1^+S_1^-}\frac{1}{S_2^+S_2^-}}-\frac{1}{S_6^2}$	—	—	$\frac{1}{S_4^2}$	$\frac{1}{S_5^2}$	$\frac{1}{S_6^2}$
Goldenblat et al.[3]	$\left(\frac{S_1^- - S_1^+}{2S_1^+S_1^-}\right)^2$	$\left(\frac{S_2^- - S_2^+}{2S_2^+S_2^-}\right)^2$	—	$\frac{S_1^- - S_1^+}{2S_1^+S_1^-}$	$\frac{S_2^- - S_2^+}{2S_2^+S_2^-}$	—	$\frac{1}{8}\left[\left(\frac{S_1^- - S_1^+}{S_1^+S_1^-}\right)^2+\left(\frac{S_2^- - S_2^+}{S_2^+S_2^-}\right)^2-\left(\frac{\tau_{45}^- - \tau_{45}^+}{\tau_{45}^+\tau_{45}^-}\right)^2\right]$	—	—	—	—	$\frac{1}{S_6^2}$
Liu et al.[21]	$\left(\frac{S_1^+ + S_1^-}{2S_1^+S_1^-}\right)^2$	$\left(\frac{S_2^+ + S_2^-}{2S_2^+S_2^-}\right)^2$	$\left(\frac{S_3^+ + S_3^-}{2S_3^+S_3^-}\right)^2$	$\frac{S_1^- - S_1^+}{2S_1^+S_1^-}$	$\frac{S_1^- - S_1^+}{2S_1^+S_1^-}$	$\frac{S_3^- - S_3^+}{2S_3^+S_3^-}$	$\left(\frac{S_1^+ + S_1^-}{2S_1^+S_1^-}\right)^2+\left(\frac{S_2^+ + S_2^-}{2S_2^+S_2^-}\right)^2-\left(\frac{S_3^+ + S_3^-}{2S_3^+S_3^-}\right)^2$	$\left(\frac{S_1^+ + S_1^-}{2S_1^+S_1^-}\right)^2+\left(\frac{S_3^+ + S_3^-}{2S_3^+S_3^-}\right)^2-\left(\frac{S_2^+ + S_2^-}{2S_2^+S_2^-}\right)^2$	$\left(\frac{S_2^+ + S_2^-}{2S_2^+S_2^-}\right)^2+\left(\frac{S_3^+ + S_3^-}{2S_3^+S_3^-}\right)^2-\left(\frac{S_1^+ + S_1^-}{2S_1^+S_1^-}\right)^2$	$\frac{1}{S_4^2}$	$\frac{1}{S_5^2}$	$\frac{1}{S_6^2}$

Note: The strengths in 1-, 2-, 3- and 45° off-axis directions are respectively denoted by S_1, S_2, S_3 and σ_{45}, where the superscripts "+" and "−" stand for tension and compression respectively. The shear strengths in 2-3, 1-3 and1-2 planes and in 45° off-axis directions are respectively denoted by S_4, S_5, S_6 and τ_{45}, where the superscripts "+" and "−" stand for positive shear and negative shear respectively. Beside, E_1, E_2, ν_{12}, ν_{13} and ν_{23} are engineering elastic constants.

$$F_{12} = -\frac{1}{2}\sqrt{4 - \frac{S_2^+ S_2^-}{S_4^2}}\sqrt{\frac{1}{S_1^+ S_1^- S_2^+ S_2^-}} \tag{6}$$

式 (4) ~ 式 (6) 中, $S_i (i = 1,2,\cdots,6)$ 为轴向强度, 上标 “ + ”和“–”分别代表拉伸和压缩.

与式 (3) 不同, Daniel 等[25] 基于能量原理, 考虑歪形能和膨胀能的耦合影响, 针对复合材料单层提出了更为复杂的屈服与失效判别条件, 即

$$\varSigma_1^2 + \varSigma_2^2 + \varSigma_1\varSigma_2 - \varSigma_3^2 + 2\varSigma_3 = 1 \tag{7}$$

式中采用等效应力分量, 其表达式分别为

$$\left.\begin{aligned} \varSigma_1 &= [a_1(\sigma_1-\sigma_2)^2 + a_2(\sigma_2-\sigma_3)^2 + \\ &\quad a_3(\sigma_3-\sigma_1)^2 + a_4\sigma_4^2 + a_5\sigma_5^2 + a_6\sigma_6^2]^{1/2} \\ \varSigma_2 &= b_1\sigma_1 + b_2\sigma_2 + b_3\sigma_3 \\ \varSigma_3 &= c_1\sigma_1 + c_2\sigma_2 + c_3\sigma_3 \end{aligned}\right\} \tag{8}$$

模型包含的 12 个待定系数可根据轴向简单加载、偏轴加载和物理分析确定.

上述复合材料的各个失效判据均采用了材料的主方向应力进行失效判别. Tan[26] 基于主应力分量, 提出了一个与式 (3) 形式相似的失效判据, 即

$$f_i\sigma_i + f_{ij}\sigma_i\sigma_j = 1 \quad (i,j = \mathrm{I,II,III}) \tag{9}$$

其中 σ_{I} , σ_{II} 和 σ_{III} 为 3 个主应力. 对于复合材料单向板, 应用式 (9) 时涉及到偏轴强度的分析测试, 以及主应力方向强度的求解, 以确定判别式当中的强度张量分量. Tan[26] 将材料的偏轴强度表达为正弦函数, 并指出模型无需剪切强度性能, 其系数可根据轴向和偏轴向拉压强度进行标定.

虽然主应力在各向同性材料的失效判据中经常采用, 但由于主应力的方向在不同应力状态下具有可变性, 因此将其用于各向异性材料的失效判别可行性较差. 式 (9) 作为主应力判据, 具有一般性, 其缺点在于参数识别比较繁杂, 优点在于其预测精度可通过增加试验数据点进行插值而得以提高, 但整体上效率太低而很少被采用.

上述各个准则均由一个方程确定复合材料的整体失效曲面或包线, 很难合理表征材料在不同工况下的强度特征, 且一般不用于多向层压板整体失效判别. Puppo 等[27] 在式 (3) 的基础上, 使用 3 个二次多项式协同构建失效曲面, 其中引入了 3 个强度交互因子, 并给定了各个强度系数; 模型对多向层压板具有适用性. 实际上, 正交各向异性材料具有 3 个主轴和相对于主轴的 3 个弹性对称平面. 因此, 相对于材料的 3 个主轴, 在材料的 3 个参考平面内进行失效应力状态的校核也是一种具有可行性的失效曲面构建方法. 有学者在应力空间的不同象限, 基于二次唯象判据实现了强度包络面的分区划片预测.

Norris[28] 针对正交各向异性材料的 3 个弹性对称面, 即 1-2 面、2-3 面和 1-3 面, 通过以下 3 个并列条件构建失效曲面

$$\left.\begin{aligned} \left(\frac{\sigma_1}{S_1}\right)^2 - \frac{\sigma_1\sigma_2}{S_1S_2} + \left(\frac{\sigma_2}{S_2}\right)^2 + \left(\frac{\sigma_6}{S_6}\right)^2 &= 1 \\ \left(\frac{\sigma_2}{S_2}\right)^2 - \frac{\sigma_2\sigma_3}{S_2S_3} + \left(\frac{\sigma_3}{S_3}\right)^2 + \left(\frac{\sigma_4}{S_4}\right)^2 &= 1 \\ \left(\frac{\sigma_3}{S_3}\right)^2 - \frac{\sigma_1\sigma_3}{S_1S_3} + \left(\frac{\sigma_1}{S_1}\right)^2 + \left(\frac{\sigma_5}{S_5}\right)^2 &= 1 \end{aligned}\right\} \tag{10}$$

上式处理拉压加载的方法与 Tsai-Hill 准则相似, 只是在应力空间中分象限进行失效判别; 该理论所提供的失效曲面构造方法在后续研究中得到了一定程度的应用. Yeh 等[29] 同样针对单向复合材料的 3 个弹性对称面, 提出了广义 Y-S 准则, 即

$$\left.\begin{aligned} \frac{\sigma_1}{S_1} + \frac{\sigma_2}{S_2} + B_{12}\sigma_1\sigma_2 + \frac{\sigma_6^2}{S_6^2} &= 1 \\ \frac{\sigma_2}{S_2} + \frac{\sigma_3}{S_3} + B_{23}\sigma_2\sigma_3 + \frac{\sigma_4^2}{S_4^2} &= 1 \\ \frac{\sigma_1}{S_1} + \frac{\sigma_3}{S_3} + B_{13}\sigma_1\sigma_3 + \frac{\sigma_5^2}{S_5^2} &= 1 \end{aligned}\right\} \tag{11}$$

注意到, 在 σ_1-σ_2-σ_3 应力空间中共有 8 个象限, 其中每个象限涉及 3 个失效判别式, 因此, 总共需要通过 24 个方程来确定完整失效曲面. 此后, Yeh[30] 考虑了更复杂的应力交互模式, 包括正应力与剪应力的耦合, 提出了二次曲面准则, 其中相对于 1-2 面的破坏条件为

$$\begin{aligned} &a\left(\frac{\sigma_1^2}{S_1^2} + \frac{\sigma_2^2}{S_2^2} + \frac{\sigma_6^2}{S_6^2}\right) + c\left(\frac{\sigma_1}{S_1} + \frac{\sigma_2}{S_2} + \frac{\sigma_6}{S_6}\right) + \\ &\quad b\left(\frac{\sigma_1\sigma_2}{S_1S_2} + \frac{\sigma_1\sigma_6}{S_1S_6} + \frac{\sigma_2\sigma_6}{S_2S_6}\right) = 1 \end{aligned} \tag{12}$$

式中, a , b 和 c 为待定系数. 该理论被认为对复合材料单向板及角铺设层压板均适用.

此外, 根据破坏实验现象, 也可将正交各向异性材料的 2-3 面、1-3 面和 1-2 面视为潜在断裂面. 图 1 描述了各个断面的形成机理及其应力分量, 可见沿 2-3 面断裂, 起关键作用的应力分量为 σ_1 , σ_5 和 σ_6 ;

而对于 1-3 断面, σ_2, σ_4 和 σ_6 起主导作用; 形成 1-2 断面则有 σ_3, σ_4 和 σ_5. 因此, 可针对上述 3 个方位的断面, 给出如下并列失效条件

$$\left.\begin{aligned}\left(\frac{\sigma_1}{S_1}\right)^2+\left(\frac{\sigma_5}{S_5}\right)^2+\left(\frac{\sigma_6}{S_6}\right)^2=1\\ \left(\frac{\sigma_2}{S_2}\right)^2+\left(\frac{\sigma_4}{S_4}\right)^2+\left(\frac{\sigma_6}{S_6}\right)^2=1\\ \left(\frac{\sigma_3}{S_3}\right)^2+\left(\frac{\sigma_4}{S_4}\right)^2+\left(\frac{\sigma_5}{S_5}\right)^2=1\end{aligned}\right\} \tag{13}$$

此即所谓的 Hashin-Rotem 准则[31], 该准则一定程度上表征了破坏模式和机理, 预测精度较高, 适用的前提条件是材料的真实断面方位与图 1 相吻合; 其缺点是需要分区/分段描绘失效包线, 且在多应力耦合作用下, 材料的真实断面方位可能与图 1 大相径庭, 导致判据应用的物理前提产生偏差.

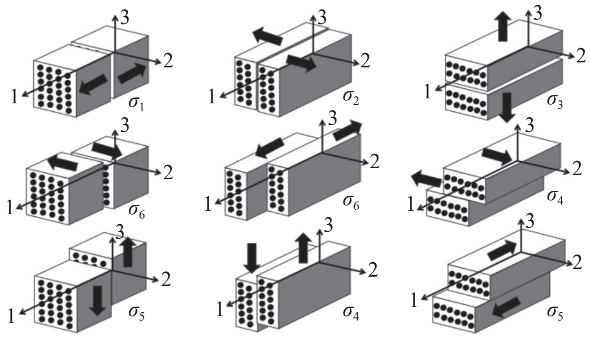

图 1　3 个潜在断裂面上的应力

Fig. 1　Stress components on the three possible fracture planes

1.3　一次应力判据

除了大量使用的二次应力判据, 研究者还提出了一次应力判据和准线性判据. 最著名且应用最为广泛的当属最大应力判据[32-33], 其表达式为

$$|\sigma_i| = S_i,\ i=1,2,\cdots,6 \tag{14}$$

该判据并非针对破坏模式而提出, 但应用表明其在一定程度上可以表征破坏模式; 此外, 该判据具有原理易懂、表达式简单和方便应用等优点, 但没有考虑各个应力分量对破坏行为的耦合影响机制, 所以预测值往往偏高.

考虑应力分量之间的耦合影响时, Cabrero 等[34] 在平面应力下提出了如下线性准则

$$\frac{\sigma_1}{S_1}+\frac{\sigma_2}{S_2}+\frac{\sigma_6}{S_6}=1 \tag{15}$$

该准则也可以拓展至三维应力状态[35], 但其表征的应力相互作用属于强耦合, 往往低估了复合材料的承载能力, 预测值偏于保守.

另外, 在式 (2) 中令 $\alpha=1$, $\beta=1/2$, 并忽略三次应力项, 可以给出[7, 21]

$$F_i\sigma_i+\sqrt{F_{ij}\sigma_i\sigma_j}=1 \tag{16}$$

上式称为准线性判据. Goldenblat 等[3] 给出了式 (16) 在平面应力下的具体表达式, 强度系数见表 1, 并基于玻璃纤维圆筒试件受双向应力作用的强度试验值对模型进行了验证. Liu 等[21] 则给出了式 (16) 的三维应力具体表达式, 强度系数也列在表 1 中.

研究表明, 一次应力判据和准线性判据可能会错误估计复合材料的承载能力, 因此在实际工程和科学问题中应该谨慎使用. 从本质上讲, 不论是高阶张量判据、二次应力判据, 还是一次应力判据, 一旦其强度系数和形状参数确定后, 各自所反映的应力耦合作用模式就具有了既定性和唯一性. 然而, 对于不同的复合材料体系, 由于组分材料及其细观结构的差异, 应力分量对其破坏过程的耦合影响机制并不相同, 因此, 各个判据的适用范围都会有不同程度的局限性, 不可能具有完全的普适性.

1.4　参数准则

除了上述的极限强度准则和多项式准则外, 还有学者提出了参数准则. 该类准则的应用需要一定的试验数据进行失效曲面或包线的插值建模. Labossiere 等[36] 针对平面应力状态, 在球面坐标系下将失效曲面方程表示为

$$\rho(\alpha,\ \phi)=(\sigma_1^2+\sigma_2^2+\sigma_6^2)^{1/2}= \sum_m\sum_n B_{mn}\{1+[\sin(2n-1)\alpha]\cdot[\sin(2m-1)\phi]\} \tag{17}$$

其中, 几何参数 α 和 ϕ 与失效应力分量的关系为

$$\left.\begin{aligned}&\alpha=\arctan(\sigma_2/\sigma_1)\quad (0<\alpha<2\pi)\\ &\phi=\arctan[(\sigma_1^2+\sigma_2^2)/\sigma_6^2]^{1/2}\quad (0<\phi<2\pi)\end{aligned}\right\} \tag{18}$$

根据几何关系给出失效应力分量为

$$\left.\begin{aligned}\sigma_1&=\rho(\alpha,\ \phi)\cos\alpha\sin\phi\\ \sigma_2&=\rho(\alpha,\ \phi)\sin\alpha\sin\phi\\ \sigma_6&=\rho(\alpha,\ \phi)\cos\phi\end{aligned}\right\} \tag{19}$$

该方法并未明确指出参数 m 和 n 如何取值, 但无疑需要额外实测数据才能准确模拟形状复杂的失效包面, 因此, 其适用性较差. 类似地, 还有基于应变的双轴参数失效判据[37]. Echaabi 等[38] 则针对复合

材料层压板应用 Kriging 插值提出了失效曲面的参数化建模方法, 该方法可根据基本强度数据、物理机制和失效模式给出任意形式的插值函数, 实现复杂失效曲面或包线的模拟预测.

不难看出, 参数准则本质上也具有唯象性, 且数学处理程序更为复杂, 模型更抽象, 但可操作性强, 因此适合描述复杂多变的失效曲面或包线, 但是其预测精度取决于插值样本数量, 即对试验数据同样具有较强的依赖性, 不便于理论和工程应用.

2 应力不变量理论

应力不变量是应力分量的某种组合, 基于应力不变量的优势在于所提出的判别式具有坐标转换的不变性, 能够反映应力不变量对材料的整体破坏效应. 对于横观各向同性单向板而言, 在材料主方向坐标系下, 绕纤维轴向旋转的应力不变量有

$$\left.\begin{aligned} &I_1=\sigma_1\\ &I_2=\sigma_2+\sigma_3\\ &I_3=\sigma_4^2-\sigma_2\sigma_3 \text{ or } (\sigma_2-\sigma_3)^2/4+\sigma_4^2\\ &I_4=\sigma_5^2+\sigma_6^2\\ &I_5=2\sigma_4\sigma_5\sigma_6-\sigma_2\sigma_5^2-\sigma_3\sigma_6^2 \end{aligned}\right\} \tag{20}$$

这些不变量在复合材料单向板的力学性能表征模型中经常使用.

著名学者 Hashin[39] 认为, 复合材料单向板的失效判别式在坐标系绕纤维轴旋转的情形下须是应力不变量, 在忽略三次应力项的条件下, 提出了一个二次函数准则, 即

$$\begin{aligned} &A_1I_1+A_2I_2+A_3I_3+A_4I_4+\\ &\quad B_1I_1^2+B_2I_2^2+C_{12}I_1I_2=1 \end{aligned} \tag{21}$$

刘方龙等[35] 分析指出不包含 I_5 的强度准则在理论层面不够合理, 但未就引入 I_5 的合理性进行深入论证, 他们仍然沿用了式 (21), 并根据莫尔理论的断裂面假设给出沿 2-3 面断裂的判别式为

$$A_1I_1+A_4I_4+B_1I_1^2=1 \tag{22}$$

而对于沿 1-2 面或 1-3 面的断裂行为, Vogler 等[40] 给出如下判别式

$$A_2I_2+A_3I_3+A_4I_4+B_2I_2^2=1 \tag{23}$$

其中, I_3 取式 (20) 中的后者. 式 (22) 和式 (23) 实际上粗略表征了纤维失效和基体失效两种模式.

应该指出, 在弹塑性理论框架内, 还有两个应力不变量具有很重要的物理意义, 即

$$\left.\begin{aligned} &J_1=\sigma_{\rm I}+\sigma_{\rm II}+\sigma_{\rm III}=\sigma_1+\sigma_2+\sigma_3\\ &J_2=[(\sigma_{\rm I}-\sigma_{\rm II})^2+(\sigma_{\rm II}-\sigma_{\rm III})^2+(\sigma_{\rm I}-\sigma_{\rm III})^2]/6 \end{aligned}\right\} \tag{24}$$

其中, J_1 为球应力第一不变量, J_2 为偏应力第二不变量, $\sigma_{\rm I}$, $\sigma_{\rm II}$ 和 $\sigma_{\rm III}$ 为 3 个主应力. 研究认为材料的体积改变与 J_1 有关, 而形状改变与 J_2 有关; 并且材料的体积改变和形状改变最终导致了材料破坏. 因此, 基于上述两个不变量进行失效判据的构建也具有合理性. 根据 Roetsch 等[41] 提出的失效模型, 可以给出基于 J_1 和 J_2 的失效判据一般形式为

$$A_1'J_1+A_2'J_1^2+B_1'J_2+\cdots=1 \tag{25}$$

为了表征简单, 通常不考虑三次以上的应力项. 式 (25) 又可以分为静水拉伸和静水压缩两种情况进行讨论, 并分别给出失效判别式[42]. Rolfes 等[43] 同样考虑静水压力的影响, 引入了与式 (20) 不完全等同的 6 个应力不变量, 提出了更为复杂的宏观正交各向异性失效判据.

文献研究表明, 应力不变量理论在复合材料基体的失效判别中经常使用, 而对于复合材料本身使用较少. 其原因是, 基体具有各向同性性质, 其失效特性与坐标选择无关, 因此基于应力不变量构造失效判别式是自然选择. 而对于各向异性体, 其各个方向的失效机理不同, 即各个应力的作用效应不同, 此时使用应力不变量进行整体建模, 将不足以反映出不同应力分量对失效行为的不等同作用和影响机制. 对于横观各向同性材料, 由于其具有各向同性面, 各向异性程度得以降减, 所以上述缺陷并没有凸显出来.

3 破坏模式相关判据

因为复合材料由力学性能迥异的纤维和基体构成, 所以材料在不同荷载下通常以不同模式发生破坏. Hashin 等[39,44] 将复合材料单向板的失效分为纤维拉伸、纤维压缩、基体拉伸和基体压缩 4 种破坏模式, 并基于应力不变量理论分别给出了失效判别式. Hashin 在该项研究中忽略了纤维-基体界面的剪切破坏, 即第 5 种失效模式, 但其定义了基体断裂面[39], 如图 2 所示, 并结合莫尔理论提出了断裂面假设, 即复合材料的失效始于某一应力作用面, 且存在于该作用面上的应力对失效起作用, 而那些不存在于该作用面上的应力则不影响破坏. 在此基础上, Hashin

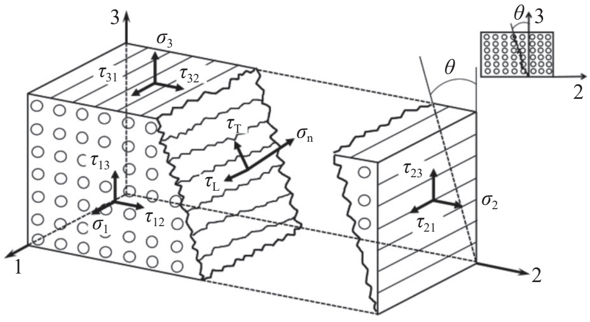

图 2　基体断裂面

Fig. 2　Matrix fracture plane

区分拉压破坏机制, 给出了作用面上正应力 $\sigma_n > 0$ 时的基体失效判据为

$$\left(\frac{\sigma_n}{S_A^+}\right)^2 + \left(\frac{\tau_T}{S_T}\right)^2 + \left(\frac{\tau_L}{S_L}\right)^2 = 1 \tag{26}$$

其中, S_A^+ 为作用面抗拉强度, S_L 和 S_T 分别为作用面沿纤维方向和垂直纤维方向的剪切强度. 遗憾的是 Hashin 并没有实现对断面方位的分析预测, 因此也就无法推导出用材料主方向应力表示的断裂面应力分量, 也就无法基于式 (26) 进行失效判别.

在 Hashin 理论的基础上, Puck 等[45] 为了提高强度预测精度, 对式 (26) 进行了改进, 给出如下基体拉伸失效判别式

$$c_2\left(\frac{\sigma_n}{S_A^+}\right)^2 + c_1\frac{\sigma_n}{S_A^+} + \left(\frac{\tau_T}{S_T}\right)^2 + \left(\frac{\tau_L}{S_L}\right)^2 = 1 \tag{27}$$

其中, c_1 和 c_2 为待定系数. 而当 $\sigma_n < 0$ 时, 基体会产生剪切主导的破坏模式, 其失效判别式为

$$\left(\frac{\tau_T}{S_T - \eta_T\sigma_n}\right)^2 + \left(\frac{\tau_L}{S_L - \eta_L\sigma_n}\right)^2 = 1 \tag{28}$$

其中, η_T 和 η_L 为两个类摩擦系数. 在式 (27) 和式 (28) 的基础上, Puck 等[45] 根据断裂面的不同方位分别建立了纤维间 3 种失效模式的判别条件, 包括断面与 σ_2 垂直的基体拉伸判据和压缩判据, 以及断面与 σ_2 呈倾斜角度的基体压缩判据, 表达式较为复杂. Dávila 等[46-48] 考虑到失效判据在多向层压板中的应用问题, 故采用单层的就位强度对式 (28) 中的强度系数进行了替换. Catalanotti 等[49] 在式 (27) 的基础上, 考虑了更一般的应力交互模式, 给出

$$\left(\frac{\sigma_n}{S_A^+}\right)^2 + \left(\frac{\tau_T}{S_T}\right)^2 + \left(\frac{\tau_L}{S_L}\right)^2 + c_2\left(\frac{\sigma_n}{S_A^+}\right)\left(\frac{\tau_L}{S_L}\right)^2 + c_1\left(\frac{\sigma_n}{S_A^+}\right) = 1 \tag{29}$$

根据该断裂面模型, Catalanotti 等[49] 提出了更为复杂的基体失效和纤维皱折失效判别式. Li 等[50] 和 Guo 等[51] 则区分断裂面法向受拉和受压两种模式, 分别提出了新的纤维间失效判别式.

在延续 Hashin 准则采用应力不变量的建模思路方面, Mayes 等[52] 分别给出了纤维和基体的横观各向同性应力不变量, 并结合多连续体理论提出了准确计算组分材料细观应力/应变场的有限元模型和方法, 建立了预测复合材料单向板纤维和基体拉伸或压缩失效的细观力学判据. Cuntze 等[53] 借鉴 Puck 理论的研究方法, 首先将单向复合材料的破坏分为纤维拉伸断裂、纤维压缩屈曲、基体拉伸开裂、纤维-基体剪切开裂和基体压缩破坏 5 种模式, 最后基于横观各向同性应力不变量分别给出了 5 种模式的失效判别式. Li 等[54] 则基于式 (24) 所示应力不变量 J_1 和 J_2 给出了基体材料的体积膨胀失效判据和形状畸变失效判据. Chen 等[55] 将 Tsai-Wu 判据基于横观各向同性应力不变量进行了改进, 使其适用于单向板的基体拉、压失效判别.

另外需要指出, 式 (14) 所示最大应力判据也可用于判定材料在多轴应力状态下的拉伸、压缩和剪切破坏, 因此也被归类于考虑破坏模式的应力准则, 而且该准则广泛用于复合材料单向板的纤维拉压破坏判别. 同时, 基于最大应力准则, 研究者还提出了一些改进模型. 黄争鸣等[56] 分别采用纤维和基体的细观主应力进行纤维和基体的拉伸、压缩失效判别. Zhao 等[57] 针对基体的拉压破坏模式, 根据单向板横观各向同性面内的主应力提出了改进的最大应力判据, 用于单向复合材料的损伤预测. Wang 等[58] 则考虑纤维初始位错, 基于细观力学方法给出了纤维和界面应力计算式, 并提出了纤维弯曲破坏的极限应力判据和纤维间剪切破坏判据.

其他常用的宏观准则还有 Chang-Chang 准则[59]、Christensen 准则[60] 和 NU 准则[61] 等. Chang 等[59] 针对层压板单层的基体开裂、纤维-基体剪切和纤维断裂, 考虑了剪切应力-应变行为的非线性, 采用剪切应变比能作为参量, 所提出的失效判据被植入了有限元软件, 较为常用. Christensen[60] 则考虑了拉压强度不等效应, 将拉压模式并入了同一判别式, 提出的纤维和基体失效判据在形式上类似于 Hoffman 判据. NU 准则针对的是纤维间和层间失效判别, 适用于单向和织物增强复合材料, 并可改进用于动态加载下的纤维间或层间失效预测[62]. 此外, 备受关注的

还有 LaRC02 准则[46]、LaRC03 准则[47] 和 LaRC04 准则[48], 这些模型均基于断裂面假设、不包含拟合参数、采用就位强度参数, 且考虑纤维皱折失效模式; 其中 LaRC02 准则和 LaRC03 准则可处理平面应力问题; 而 LaRC04 准则注重三维应力状态分析, 并引入了剪切非线性的影响.

根据破坏模式的 5 种类型, 本文将一些常用的判据分别汇总在表 2 ~ 表 6 中. 可以看出, 即使针对

表 2 复合材料单向板拉伸纤维模式破坏判据

Table 2 Failure criteria for tensile fiber mode of unidirectional composites

Contributor(s)	Expression of criterion	Model description
Jenkin[32]	$\sigma_1 = S_1^+,\ \sigma_1 > 0$	Maximum stress criterion, where σ_1 is apparent axial stress, $\sigma_{1\mathrm{f}}$ is axial fiber stress, $S_{1\mathrm{f}}^+$ is axial tensile strength of fiber.
Stowell et al.[33]	$\sigma_{1\mathrm{f}} = S_{1\mathrm{f}}^+,\ \sigma_{1\mathrm{f}} > 0$	
Waddoups et al.[63]	$\varepsilon_1 = e_1^+,\ \varepsilon_1 > 0$	Maximum strain criterion, where ε_1 is axial strain and e_1^+ is axial tensile failure strain.
Puck et al.[45]	$\frac{1}{e_1^+}\left(\varepsilon_1 + \frac{\nu_{\mathrm{f}12}}{E_{\mathrm{f}1}} m_{\sigma\mathrm{f}}\sigma_2\right) = 1$	Modified maximum strain criterion, where $\nu_{\mathrm{f}12}$ is fiber Poisson ratio, $E_{\mathrm{f}1}$ is fiber axial modulus and $m_{\sigma\mathrm{f}}$ is a magnification factor.
Mayes et al.[52]	$\left(\frac{\sigma_{1\mathrm{f}}}{S_{1\mathrm{f}}^+}\right)^2 + \frac{\sigma_{5\mathrm{f}}^2 + \sigma_{6\mathrm{f}}^2}{(S_{6\mathrm{f}})^2} = 1$	Micromechanical criterion, where $\sigma_{5\mathrm{f}}$ and $\sigma_{6\mathrm{f}}$ are fiber shear stress components, $S_{6\mathrm{f}}$ is fiber shear strength.
Huang et al.[56]	$\frac{\sigma_{1\mathrm{f}} + \sigma_{2\mathrm{f}}}{2} + \frac{1}{2}\sqrt{(\sigma_{1\mathrm{f}} - \sigma_{2\mathrm{f}})^2 + 4\sigma_{6\mathrm{f}}^2} = S_{1\mathrm{f}}^+$	Modified maximum stress criterion in which the subscript “f” stands for fiber. The model involves isotropic tensile strength of fiber.
Hashin[39]	$\left(\frac{\sigma_1}{S_1^+}\right)^2 + \frac{1}{S_{\mathrm{L}}^2}(\sigma_5^2 + \sigma_6^2) = 1,\ \sigma_1 > 0$	Characterize tension-shear coupled rupture of fiber. Parameter S_{L} is shear strength along fiber direction in the fracture plane. Yen[64] replaced the stress components by corresponding strain ones.
Hou et al.[65]	$\left(\frac{\sigma_1}{S_1^+}\right)^2 + \frac{1}{S_6^2}(\sigma_5^2 + \sigma_6^2) = 1,\ \sigma_1 > 0$	Similar to Hashin criterion, used to simulate impact damage, where S_6 is in-plane shear strength involving fiber tearing fracture.
Chang et al.[59]	$\left(\frac{\sigma_1}{S_1^+}\right)^2 + \frac{2 + 3\alpha E_6\sigma_6^2}{2 + 3\alpha E_6 S_6^2}\left(\frac{\sigma_6}{S_6}\right)^2 = 1,\ \sigma_1 > 0$	Characterize fiber failure dominated by stress σ_1 . Parameter S_6 is in-situ shear strength of lamina which can be measured by symmetric cross-ply laminate.
Christensen[60]	$\left(\frac{1}{S_1^+} - \frac{1}{S_1^-}\right)\sigma_1 + \frac{1}{S_1^+ S_1^-}\sigma_1^2 = 1,\ \sigma_1 > 0$	Similar to Hoffman criterion, do not distinguish tension and compression failure modes.

表 3 复合材料单向板压缩纤维模式破坏判据

Table 3 Failure criteria for compressive fiber mode of unidirectional composites

Contributor(s)	Expression of criterion	Model description
Jenkin[32]	$\sigma_1 = -S_1^-,\ \sigma_1 < 0$	Maximum stress criterion describing σ_1 or $\sigma_{1\mathrm{f}}$ dominated compressive fiber failure where $S_{1\mathrm{f}}^-$ is axial compressive strength of fiber.
Stowell et al.[33]	$\sigma_{1\mathrm{f}} = -S_{1\mathrm{f}}^-,\ \sigma_{1\mathrm{f}} < 0$	
Puck et al.[45]	$\frac{1}{e_1^-}\left\lvert\left(\varepsilon_1 + \frac{\nu_{\mathrm{f}12}}{E_{\mathrm{f}1}} m_{\sigma\mathrm{f}}\sigma_2\right)\right\rvert + (10\gamma_{21})^2 = 1,\ (\cdots) < 0$	Compression-shear coupled model, where γ_{21} is shear strain, e_1^- is longitudinal compressive failure strain, the introduced factor is used to characterize shear effect.
Mayes et al.[52]	$\left(\frac{\sigma_{1\mathrm{f}}}{S_{1\mathrm{f}}^-}\right)^2 + \frac{\sigma_{5\mathrm{f}}^2 + \sigma_{6\mathrm{f}}^2}{(S_{6\mathrm{f}})^2} = 1$	Characterize compressive stress dominated failure during compression-shear coupled loading.
Huang et al.[56]	$\frac{\sigma_{1\mathrm{f}} + \sigma_{2\mathrm{f}}}{2} - \frac{1}{2}\sqrt{(\sigma_{1\mathrm{f}} - \sigma_{2\mathrm{f}})^2 + 4\sigma_{6\mathrm{f}}^2} = S_{1\mathrm{f}}^-$	Micromechanical criterion, where fiber is assumed to be isotropic.
Yen[64]	$\left(\frac{\langle \sigma'_1 \rangle}{S_1^-}\right)^2 = 1,\ \sigma'_1 = -\sigma_1 + \left\langle -\frac{\sigma_2 + \sigma_3}{2} \right\rangle$	The model considers the effect of transverse compressive stress on longitudinal compressive failure.
Yen[64]	$\left(\frac{\langle \sigma_2 \rangle}{S_2^-}\right)^2 + \left(\frac{\langle \sigma_3 \rangle}{S_2^-}\right)^2 = 1$	Towards unidirectional laminates, characterize fiber failure by transverse compression, where S_2^- is transverse compressive strength.
Christensen[60]	$\left(\frac{1}{S_1^+} - \frac{1}{S_1^-}\right)\sigma_1 + \frac{1}{S_1^+ S_1^-}\sigma_1^2 = 1$	Similar to Hoffman criterion, do not distinguish tension and compression failure modes.
Dávila et al.[46]	$\frac{\left\langle \lvert\sigma_6^{\mathrm{m}}\rvert + \eta_{\mathrm{L}}\sigma_2^{\mathrm{m}} \right\rangle}{S_6} = 1,\ \sigma_2^{\mathrm{m}} < 0$ $\left(\frac{\sigma_2^{\mathrm{m}}}{S_2^+}\right)^2 + \left(\frac{\sigma_6^{\mathrm{m}}}{S_6}\right)^2 = 1,\ \sigma_2^{\mathrm{m}} \geqslant 0$	Plane-stress LaRC02 criterion characterizing fiber kinking failure, the superscript “m” represents local stress of the kink band. Mohr-Coulomb theory is used when the matrix is compressive, and tensile failure criterion is used when the matrix is tensile.
Dávila et al.[47]	$\frac{\left\langle \lvert\sigma_6^{\mathrm{m}}\rvert + \eta_{\mathrm{L}}\sigma_2^{\mathrm{m}} \right\rangle}{S_6} = 1,\ \sigma_2^{\mathrm{m}} < 0$ $(1-g)\frac{\sigma_2^{\mathrm{m}}}{S_2^+} + g\left(\frac{\sigma_2^{\mathrm{m}}}{S_2^+}\right)^2 + \left(\frac{\sigma_6^{\mathrm{m}}}{S_6}\right)^2 = 1,\ \sigma_2^{\mathrm{m}} \geqslant 0$	Plane-stress LaRC03 criterion improved from LaRC02 criterion, without fitting parameter. For single layer in laminates, in-situ strength should be used. The model has advantage over Hashin criterion in predicting compressive matrix and fiber failure.
Pinho et al.[48]	$\left(\frac{\sigma_6^{\mathrm{m}}}{S_{\mathrm{L}} - \eta_{\mathrm{L}}\sigma_2^{\mathrm{m}}}\right)^2 = 1,\ \sigma_2^{\mathrm{m}} < 0$ $(1-g)\frac{\sigma_2^{\mathrm{m}}}{S_2^+} + g\left(\frac{\sigma_2^{\mathrm{m}}}{S_2^+}\right)^2 + \frac{\Lambda_{23}\sigma_4^2 + \chi(\varepsilon_6)}{\chi(\varepsilon_{6\mathrm{u}})} = 1,\ \sigma_2^{\mathrm{m}} \geqslant 0$	3D stress-state LaRC04 criterion improved from LaRC03 criterion, considering shear nonlinearity. Towards single layer in laminates, in-situ strength is adopted.
Camanho et al.[66]	$\alpha_1 I_3^{\mathrm{m}} + \alpha_2 I_4^{\mathrm{m}} + \alpha_3^{\mathrm{t}} I_2^{\mathrm{m}} + \alpha_{32}^{\mathrm{t}}(I_2^{\mathrm{m}})^2 = 1,\ I_2^{\mathrm{m}} > 0$ $\alpha_1 I_3^{\mathrm{m}} + \alpha_2 I_4^{\mathrm{m}} + \alpha_3^{\mathrm{c}} I_2^{\mathrm{m}} + \alpha_{32}^{\mathrm{c}}(I_2^{\mathrm{m}})^2 = 1,\ I_2^{\mathrm{m}} \leqslant 0$	Stress-invariant based 3D failure criterion, the superscript “m” represents local stress of the kink band.

表 4　复合材料单向板拉伸基体模式破坏判据

Table 4　Failure criteria for tensile matrix mode of unidirectional composites

Contributor(s)	Expression of criterion	Model description
Jenkin[32] Stowell et al.[33]	$\sigma_2 / S_2^+ = 1,\ \sigma_2 > 0$	The model characterize tensile matrix failure dominated by stress σ_2.
Mayes et al.[52]	$\frac{\sigma_{2\,\mathrm{m}}^2 + \sigma_{3\,\mathrm{m}}^2 + 2\sigma_{4\,\mathrm{m}}^2}{(S_{2\,\mathrm{m}}^+)^2 + (S_{3\,\mathrm{m}}^+)^2} + \frac{\sigma_{5\,\mathrm{m}}^2 + \sigma_{6\,\mathrm{m}}^2}{(S_{6\mathrm{m}})^2} = 1,\ \sigma_2 > 0$	Micromechanical criterion including tensile and shear strengths of matrix, where the subscript "m" stands for matrix.
Huang et al.[56]	$\frac{\sigma_{1\mathrm{m}} + \sigma_{2\mathrm{m}}}{2} + \frac{1}{2}\sqrt{(\sigma_{1\mathrm{m}} - \sigma_{2\mathrm{m}})^2 + 4\sigma_{6\mathrm{m}}^2} = S_{1\mathrm{m}}^+$	Micromechanical criterion including isotropic tensile strength of matrix, where the subscript "m" stands for matrix.
Zhao et al.[57]	$\frac{\sigma_{\mathrm{m}}^{\max}}{S_2^+} = 1,\ \sigma_{\mathrm{m}}^{\max} = \frac{\sigma_2 + \sigma_3}{2} + \sqrt{\left(\frac{\sigma_2 - \sigma_3}{2}\right)^2 + \sigma_4^2} > 0$	The criterion assumes that tensile matrix cracking is determined by the maximum principal stress in transverse isotropic plane.
Hashin et al.[44]	$\left(\frac{\sigma_2}{S_2^+}\right)^2 + \left(\frac{\sigma_6}{S_6}\right)^2 = 1,\ \sigma_2 > 0$	Hashin-Rotem criterion and/or LaRC02 criterion, applicable to unidirectional laminates.
Hashin[39]	$\left(\frac{I_2}{S_2^+}\right)^2 + \frac{I_3}{S_{\mathrm{T}}^2} + \frac{I_4}{S_{\mathrm{L}}^2} = 1,\ \sigma_2 + \sigma_3 > 0$	3D failure criterion towards unidirectional laminates, with $S_{\mathrm{T}} = S_4$, $S_{\mathrm{L}} = S_5 = S_6$.
Cuntze et al.[53]	$\frac{I_2 + \sqrt{I_3}}{2S_2^+} = 1,\ \sigma_2 > 0$	The model characterize tensile matrix failure dominated by stress σ_2.
Chang et al.[59]	$\left(\frac{\sigma_2}{S_2^+}\right)^2 + \frac{2 + 3\alpha E_6 \sigma_6^2}{2 + 3\alpha E_6 S_6^2}\left(\frac{\sigma_6}{S_6}\right)^2 = 1,\ \sigma_2 > 0$	Towards laminates S_6 adopts in-situ shear strength of lamina, measured from cross-ply laminate.
Liu et al.[67]	$\frac{\sigma_2}{S_2^+} + \left(\frac{E_2}{2E_6}\right)^2\left(\frac{\sigma_6}{S_2^+}\right)^2 + \left(\frac{E_2}{2E_4}\right)^2\left(\frac{\sigma_4}{S_2^+}\right)^2 = 1$	NU criterion, predicting matrix failure dominated by transverse tensile stress.
Liu et al.[67]	$\frac{\sigma_3}{S_3^+} + \left(\frac{E_3}{2G_{13}}\right)^2\left(\frac{\sigma_5}{S_3^+}\right)^2 + \left(\frac{E_3}{2G_{23}}\right)^2\left(\frac{\sigma_4}{S_3^+}\right)^2 = 1$	NU criterion, predicting matrix failure dominated by through-thickness tensile stress.
Hou et al.[65] Patel et al.[68]	$\left(\frac{\sigma_2}{S_2^+}\right)^2 + \left(\frac{\sigma_4}{S_4}\right)^2 + \left(\frac{\sigma_6}{S_6}\right)^2 = 1,\ \sigma_2 > 0$	Characterize tensile matrix cracking along 1-3 plane, where S_4 adopts shear strength that relates to matrix cracking. Yen[64] replaced the stress components by corresponding strain ones.
Patel et al.[68]	$\left(\frac{\sigma_3}{S_2^+}\right)^2 + \left(\frac{\sigma_5}{S_5}\right)^2 + \left(\frac{\sigma_6}{S_6}\right)^2 = 1,\ \sigma_3 > 0$	Characterize tensile matrix cracking along 1-2 plane. Yen[64] replaced the stress components by corresponding strain ones.
Dávila et al.[47]	$(1-g)\frac{\sigma_2}{S_2^+} + g\left(\frac{\sigma_2}{S_2^+}\right)^2 + \left(\frac{\sigma_6}{S_6}\right)^2 = 1,\ \sigma_2 > 0$	LaRC03 criterion, where *g* is a material constant. When used to laminates, in-situ strength of lamina is adopted.
Pinho et al.[48]	$(1-g)\frac{\sigma_2}{S_2^+} + g\left(\frac{\sigma_2}{S_2^+}\right)^2 + \frac{\Lambda_{23}\sigma_4^2 + \chi(\varepsilon_6)}{\chi(\varepsilon_{6\mathrm{u}})} = 1,\ \sigma_2 > 0$	LaRC04 criterion, where *g* is a material constant. In-situ strength of lamina is adopted and shear nonlinearity is considered.
Christensen[60]	$\left(\frac{1}{S_2^+} - \frac{1}{S_2^-}\right)I_2 + \frac{I_2^2}{S_2^+ S_2^-} + \frac{I_3}{S_4^2} + \frac{I_4}{S_6^2} = 1$	Stress-invariant based failure criterion, do not distinguish tension and compression failure modes.
Camanho et al.[66]	$\alpha_1 I_3 + \alpha_2 I_4 + \alpha_3^{\mathrm{t}} I_2 + \alpha_{32}^{\mathrm{t}} I_2^2 = 1,\ I_2 > 0$	Similar to Hashin criterion, distinguish tension and compression failure modes, but owns different strength coefficients.
Daniel et al.[61]	$\frac{\sigma_3}{S_3^+} + \left(\frac{\sigma_5}{S_3^+}\right)^2\left(\frac{E_3}{2G_{13}}\right)^2 = 1$	NU criterion characterizing inter-fiber tensile failure, where E_3 and G_{13} are elastic modulus in 3-axis and shear modulus in 1-3 plane, respectively.
Sun et al.[69]	$\frac{\sigma_2}{S_2^+} + \left(\frac{\sigma_6}{S_{\mathrm{L}}}\right)^2 = 1$	Similar to NU criterion, used to characterize transverse tensile failure of matrix.

同一模式的失效, 不同研究者出于不同的研究思路, 仍然提出了形形色色的失效判别式, 包括宏观力学判据和细观力学判据, 其中细观力学判据要求进行组分材料的应力或应变计算, 而宏观判据则只需用表观应力. 从数学形式上看, 这些判据包括一次应力/应变和二次应力/应变表达形式, 极少有高阶应力/应变形式. 在用于层压板失效预测时, 有些模型强调使用单层或其组分的就位强度性能, 以表征各单层之间的约束效应或制备工艺效应. 综合全部模型及其应用情况, 不难发现, 判据的多样性主要体现在应力分量在不同材料体系和不同加载条件下的主导性及其组合模式的差异, 可以认为, 适用于同一个失效判据的不同材料体系, 其破坏机制具有相似性或等同性.

除了上述 5 种失效模式外, 复合材料层压板还有一类普遍而重要的失效模式, 即分层. 早期的分层起始判据包括最大应力判据和改进的最大应力判据[72], 但应用并不广泛. Ochoa 等[73] 考虑面外应力, 最先提出了二次应力判据. Soni 等[74] 同样假设分层失效由面外应力控制, 针对多向层压板的分层问题提出了分层起始判据, 其中引入了正应力 σ_3 的一次项, 用以表征该正应力对分层损伤的抑制或者促进作用. Brewer 等[75] 认为法向正应力对分层起始的作用并不是应力的一阶效应, 因此将正应力的一次项忽略, 给出了简化的二次分层判据. Long[76] 根据复合材料搭接板的分层实验数据, 提出了两种适用的分层起始判据; Tong[77] 进一步考虑到搭接分层可能起始于表层纤维断裂处, 因此在失效判别式中增加了沿纤

表 5 复合材料单向板压缩基体模式破坏判据

Table 5 Failure criteria for compressive matrix mode of unidirectional composites

Contributor(s)	Expression of criterion	Model description
Jenkin[32] Stowell et al.[33]	$-\sigma_2 = S_2^-,\ \sigma_2 < 0$	Maximum stress criterion predicting compressive matrix failure dominated by stress σ_2 .
Mayes et al.[52]	$\dfrac{\sigma_{2\,\mathrm{m}}^2+\sigma_{3\,\mathrm{m}}^2+2\sigma_{4\,\mathrm{m}}^2}{(S_{2\,\mathrm{m}}^-)^2+(S_{3\,\mathrm{m}}^-)^2}+\dfrac{\sigma_{5\,\mathrm{m}}^2+\sigma_{6\,\mathrm{m}}^2}{(S_{6\mathrm{m}})^2}=1,\ \sigma_2<0$	Micromechanical criterion including compressive and shear strengths of matrix, with the subscript “m” standing for matrix.
Huang et al.[56]	$\dfrac{\sigma_{1\mathrm{m}}+\sigma_{2\mathrm{m}}}{2}-\dfrac{1}{2}\sqrt{(\sigma_{1\mathrm{m}}-\sigma_{2\mathrm{m}})^2+4\sigma_{6\mathrm{m}}^2}=S_{1\mathrm{m}}^-$	Micromechanical criterion including isotropic compressive strength of matrix, with the subscript “m” standing for matrix.
Zhao et al.[57]	$\dfrac{\sigma_\mathrm{m}^{\min}}{S_2^-}=1,\ \sigma_\mathrm{m}^{\min}=\dfrac{\sigma_2+\sigma_3}{2}-\sqrt{\left(\dfrac{\sigma_2-\sigma_3}{2}\right)^2+\sigma_4^2}\leqslant 0$	Micromechanical criterion assuming that compressive matrix cracking is determined by the minimum principal stress in transverse isotropic plane.
Hashin[39]	$\left[\left(\dfrac{S_2^-}{2S_\mathrm{T}}\right)^2-1\right]\dfrac{I_2}{S_2^-}+\dfrac{I_2^2}{4S_\mathrm{T}^2}+\dfrac{I_3}{S_\mathrm{T}^2}+\dfrac{I_4}{S_\mathrm{L}^2}=1,\ \sigma_2+\sigma_3<0$	3D failure criterion towards unidirectional laminates, with $S_\mathrm{T}=S_4$, $S_\mathrm{L}=S_5=S_6$.
Cuntze et al.[53]	$\dfrac{(b_\perp^\tau-1)I_2}{S_2^-}+\dfrac{b_\perp^\tau I_3+b_{\perp\parallel}^\tau I_4}{(S_2^-)^2}=1,\ \sigma_2<0$	This model predicting compressive matrix failure dominated by stress σ_2 .
Chang et al.[59]	$\left(\dfrac{\sigma_2}{2S_6}\right)^2+\left[\left(\dfrac{S_2^-}{2S_6}\right)^2-1\right]\dfrac{\sigma_2}{S_2^-}+\dfrac{2+3\alpha E_6\sigma_6^2}{2+3\alpha E_6 S_6^2}\left(\dfrac{\sigma_6}{S_6}\right)^2=1,$ $\sigma_2<0$	Shear nonlinearity is considered, and in-situ shear strength is adopted for S_6 .
Sun et al.[70]	$\left(\dfrac{\sigma_2}{S_2^-}\right)^2+\left(\dfrac{\sigma_6}{S_6-\eta\sigma_2}\right)^2=1,\ \sigma_2<0$	Friction-like coefficient η is introduced to characterize the negative effect of compressive σ_2 on shear strength.
Dávila et al.[47]	$\left(\dfrac{\langle\lvert\tau_\mathrm{T}\rvert+\eta_\mathrm{T}\sigma_\mathrm{n}\rangle}{S_\mathrm{T}}\right)^2+\left(\dfrac{\langle\lvert\tau_\mathrm{L}\rvert+\eta_\mathrm{L}\sigma_\mathrm{n}\rangle}{S_\mathrm{L}}\right)^2=1,\ \sigma_1\geqslant -S_2^-$ $\left(\dfrac{\left\langle\lvert\tau_\mathrm{T}^\mathrm{m}\rvert+\eta_\mathrm{T}\sigma_\mathrm{n}^\mathrm{m}\right\rangle}{S_\mathrm{T}}\right)^2+\left(\dfrac{\left\langle\lvert\tau_\mathrm{L}^\mathrm{m}\rvert+\eta_\mathrm{L}\sigma_\mathrm{n}^\mathrm{m}\right\rangle}{S_\mathrm{L}}\right)^2=1,\ \sigma_1<-S_2^-$	LaRC03 criterion based on fracture plane assumption, where the effect of normal stress on shear strength as well as fiber kinking are considered, with the superscript “m” standing for local stress of the kink band. The in-situ strength is adopted for S_L .
Pinho et al.[48]	$\left(\dfrac{\tau_\mathrm{T}}{S_\mathrm{T}-\eta_\mathrm{T}\sigma_\mathrm{n}}\right)^2+\left(\dfrac{\tau_\mathrm{L}}{S_\mathrm{L}-\eta_\mathrm{L}\sigma_\mathrm{n}}\right)^2=1,\ \sigma_1\geqslant -S_2^-$ $\left(\dfrac{\tau_\mathrm{T}^\mathrm{m}}{S_\mathrm{T}-\eta_\mathrm{T}\sigma_\mathrm{n}^\mathrm{m}}\right)^2+\left(\dfrac{\tau_\mathrm{L}^\mathrm{m}}{S_\mathrm{L}-\eta_\mathrm{L}\sigma_\mathrm{n}^\mathrm{m}}\right)^2=1,\ \sigma_1<-S_2^-$	LaRC04 criterion improved from the LaRC03, with the superscript “m” standing for local stress of the kink band. The in-situ strength is adopted for S_L .
Hou et al.[65]	$\dfrac{1}{4}\left(\dfrac{\sigma_2}{S_6}\right)^2+\left[\left(\dfrac{S_2^-}{2S_6}\right)^2-1\right]\dfrac{\sigma_2}{S_2^-}+\left(\dfrac{\sigma_6}{S_6}\right)^2=1,\ \sigma_2<0$	This model is a simplified one of Chang criterion without considering shear nonlinearity.
Christensen[60]	$\left(\dfrac{1}{S_2^+}-\dfrac{1}{S_2^-}\right)I_2+\dfrac{I_2^2}{S_2^+S_2^-}+\dfrac{I_3}{S_4^2}+\dfrac{I_4}{S_6^2}=1$	Stress-invariant based failure criterion, do not distinguish tension and compression failure modes.
Camanho et al.[66]	$\alpha_1 I_3+\alpha_2 I_4+\alpha_3^\mathrm{c} I_2+\alpha_{32}^\mathrm{c} I_2^2=1,\ I_2\leqslant 0$	Similar to Hashin criterion, distinguish tension and compression failure modes, but owns different strength coefficients.
Daniel et al.[61]	$\left(\dfrac{\sigma_3}{S_3^-}\right)^2+\left(\dfrac{\sigma_5}{S_3^-}\right)^2\left(\dfrac{E_3}{G_{13}}\right)^2=1$	NU criterion predicting inter-fiber compressive failure.
Liu et al.[67]	$\dfrac{\sigma_2}{S_2^-}+\left(\dfrac{E_2}{E_6}\right)^2\left(\dfrac{\sigma_6}{S_2^-}\right)^2+\left(\dfrac{E_2}{E_4}\right)^2\left(\dfrac{\sigma_4}{S_2^-}\right)^2=1$	NU criterion predicting matrix failure dominated by transverse compressive stress.
Liu et al.[67]	$\left(\dfrac{\sigma_3}{S_3^-}\right)^2+\left(\dfrac{E_3}{G_{13}}\right)^2\left(\dfrac{\sigma_5}{S_3^-}\right)^2+\left(\dfrac{E_3}{G_{23}}\right)^2\left(\dfrac{\sigma_4}{S_3^-}\right)^2=1$	NU criterion predicting matrix failure dominated by through-thickness compressive stress.
Hashin et al.[44]	$\dfrac{\sigma_2}{S_2^-}+\left(\dfrac{\sigma_6}{S_6}\right)^2=1$	Hashin-Rotem criterion for unidirectional laminates, predicting transverse compressive failure of matrix.
Yen[64]	$\left(\dfrac{\sigma_2}{S_2^-}\right)^2+\left(\dfrac{\sigma_3}{S_2^-}\right)^2=1,\ \sigma_2<0,\sigma_3<0$ $\left(\dfrac{\sigma_4}{S_4+\eta_2\sigma_2}\right)^2+\left(\dfrac{\sigma_6}{S_6+\eta_2\sigma_2}\right)^2=1,\ \sigma_2<0$ $\left(\dfrac{\sigma_4}{S_4+\eta_3\sigma_3}\right)^2+\left(\dfrac{\sigma_5}{S_5+\eta_3\sigma_3}\right)^2=1,\ \sigma_3<0$	Characterize bi-axial compression, transverse compression and through-thickness compression failure, assuming that only matrix compression fracture occurs, and considering the Coulomb friction mechanism.
Sun et al.[69]	$\left(\dfrac{\sigma_2}{S_2^-}\right)^2+\left(\dfrac{S_2^- -\left\lvert\sigma_{22}^\mathrm{Tran}\right\rvert}{S_\mathrm{L}}\right)^2\left(\dfrac{\sigma_6}{S_2^-}\right)^2=1$	Similar to NU criterion. Strength coefficient is modified by introducing transition point $\sigma_{22}^\mathrm{Tran}$ of failure envelop.

维方向的应力, 并同时给出了 6 个可供选择的独立判别式. 而 Christensen 等[78] 则针对准各向同性层压板在面外载荷作用下的最终分层失效问题, 基于应力不变量提出了破坏判据. 上述准则的应力交互形式较为简单, Zubillaga 等[79] 针对复合材料层压板中基体开裂引起的分层现象, 基于能量释放率提出了应力交互形式更为复杂的分层失效判据, 其中引入了正应力和剪应力之间的二次交互项. 之后, Nejad 等[80] 同样基于临界能量释放率准则提出了类似的分层起始判据. 在考虑层间摩擦机制方面, Yen[64] 针对单向板和平纹织物复合材料提出了分层失效判据, 应用较广, 并被植入了有限元软件 LS-DYNA. Xiao 等[81] 考虑到面外压缩应力很大时 Yen 准则给出的失效曲面并不光滑, 因此引入了一个过渡系数对 Yen 失效

表 6　复合材料单向板纤维-基体剪切破坏判据

Table 6　Failure criteria for fiber-matrix shear mode of unidirectional composites

Contributor(s)	Expression of criterion	Model description
Jenkin[32]	$\frac{\sigma_6}{S_6} = 1$	Maximum stress criterion predicting fiber-matrix shear failure dominated by stress σ_6 .
Hashin[39]	$\left(\frac{\sigma_6}{S_6}\right)^2 + \left(\frac{\sigma_1}{S_1^+}\right)^2 = 1$	Characterize shear stress dominated fiber-matrix interface failure.
Chang et al.[71]	$\left(\frac{\sigma_1}{S_1^-}\right)^2 + \frac{2+3\alpha E_6\sigma_6^2}{2+3\alpha E_6 S_6^2}\left(\frac{\sigma_6}{S_6}\right)^2 = 1,\ \sigma_1 < 0$	Parameter S_6 adopts in-situ shear strength of lamina, which can be measured by symmetric cross-ply laminate.
Zhao et al.[57]	$\sigma_S^{\max}/S_6 = 1,\ \sigma_S^{\max} = \sqrt{\sigma_5^2+\sigma_6^2}$	The criterion assumes that fiber-matrix shear cracking is determined by the maximum shear stress in transverse isotropic plane.
Cuntze et al.[53]	$\frac{I_4^{3/2}}{S_6^3} + 2b_{\perp\parallel}\frac{I_2 I_4 + I_5}{S_6^3} = 1$	Characterize fiber-matrix shear failure dominated by stress σ_6 in terms of stress invariants shown in Eq. (20).
Daniel et al.[61]	$\left(\frac{\sigma_5}{S_5}\right)^2 + 2\frac{\sigma_3}{S_5}\frac{G_{13}}{E_3} = 1$	NU criterion, predicting inter-fiber shear failure.
Liu et al.[67]	$\left(\frac{\sigma_6}{S_6}\right)^2 + \left(\frac{\sigma_4}{S_4}\right)^2 + \frac{2G_{12}}{E_2}\frac{\sigma_2}{S_6} = 1$	NU criterion, predicting shear failure dominated by in-plane shear stress, with E_2 and G_{12} the elastic modulus of 2-axis and shear modulus in 1-2 plane.
Liu et al.[67]	$\left(\frac{\sigma_5}{S_5}\right)^2 + \left(\frac{\sigma_4}{S_4}\right)^2 + \frac{2G_{13}}{E_3}\frac{\sigma_3}{S_5} = 1$	Modified NU criterion, predicting inter-fiber shear failure along through-thickness direction.
Sun et al.[69]	$\left(\frac{\sigma_6}{S_L}\right)^2 + \frac{\sigma_2}{\lvert\sigma_{22}^{\mathrm{Tran}}\rvert}\left[\left(\frac{\lvert\tau_{12}^{\mathrm{Tran}}\rvert}{S_L}\right)^2 - 1\right] = 1$	Similar to NU criterion. Strength coefficient is modified by introducing transition points $\sigma_{22}^{\mathrm{Tran}}$ and $\tau_{12}^{\mathrm{Tran}}$ of failure envelop.

判据进行了改进, 使得面外压应力较大时失效曲面或包线更符合实验现象、更合理. Goyal 等[82] 则突破了二次应力模型, 引入可供数据拟合的指数参数, 提出了一个唯象判据.

Daniel 等[61] 则将层间失效分为压缩控制、剪切控制和拉伸控制 3 种模式, 分别给出了失效判据, 即所谓的 NU 准则. 之后, Daniel[62] 将 NU 准则进行了改进, 用以表征应变率对失效强度的影响. Sun 等[69] 基于 Puck 理论, 引入失效包线转折点, 也对 NU 准则进行了变换, 给出了一组层间失效判别准则. 这些区分失效模式的失效判据, 在层压板的冲击损伤模拟中得到了广泛应用. 上述各分层判据见表 7.

表 7　分层破坏判据

Table 7　Failure criteria for delamination

Contributor(s)	Expression of criterion	Model description
Lee[72]	$\sigma_3 = S_3^+,\ \sigma_5^2 + \sigma_6^2 = S_4^2$	Modified maximum stress criterion for delamination initiation.
Ochoa et al.[73]	$\left(\frac{\sigma_3}{S_3^+}\right)^2 + \frac{\sigma_4^2+\sigma_5^2}{S_4^2} = 1$	Delamination initiation criterion based on out-of-plane stresses for laminates.
Soni et al.[74]	$F_{zz}(\sigma_3)^2 + F_z\sigma_3 + \left(\frac{\sigma_4}{S_4}\right)^2 + \left(\frac{\sigma_5}{S_5}\right)^2 = 1$	The model considers the effect of interlayer normal stress on the inhibition or promotion of delamination damage.
Brewer et al.[75]	$\left(\frac{\sigma_3}{S_3}\right)^2 + \left(\frac{\sigma_4}{S_4}\right)^2 + \left(\frac{\sigma_5}{S_5}\right)^2 = 1$	In the model, S_3 is interlayer tensile or compressive strength, S_4 and S_5 are interlayer shear strengths.
Long[76]	$\frac{\sigma_3}{S_3} + \left(\frac{\sigma_4}{S_4}\right)^2 = 1,\ \left(\frac{\sigma_3}{S_3}\right)^2 + \left(\frac{\sigma_4}{S_4}\right)^2 = 1$	These two interactive stress criteria are proposed based on experimental data.
Tong[77]	a: $\frac{\sigma_1^2-\sigma_1\sigma_3}{(S_1^+)^2} + \left(\frac{\sigma_3}{S_3^+}\right)^2 + \left(\frac{\sigma_4}{S_4}\right)^2 = 1$ b: $\frac{\sigma_1^2-\sigma_1\sigma_3}{(S_1^+)^2} + \left(\frac{\sigma_3}{S_3^+}\right) + \left(\frac{\sigma_4}{S_4}\right)^2 = 1$ c: $\left(\frac{\sigma_1}{S_1^+}\right)^2 + \left(\frac{\sigma_3}{S_3^+}\right)^2 + \left(\frac{\sigma_4}{S_4}\right)^2 = 1$ d: $\left(\frac{\sigma_1}{S_1^+}\right)^2 + \left(\frac{\sigma_3}{S_3^+}\right) + \left(\frac{\sigma_4}{S_4}\right)^2 = 1$ e: $\frac{\sigma_1^2-\sigma_1\sigma_3}{S_1^+S_1^-} + \left(\frac{\sigma_3}{S_3^+}\right)^2 + \left(\frac{\sigma_4}{S_4}\right)^2 = 1$ f: $\frac{\sigma_1^2-\sigma_1\sigma_3}{S_1^+S_1^-} + \left(\frac{\sigma_3}{S_3^+}\right) + \left(\frac{\sigma_4}{S_4}\right)^2 = 1$	The six independent delamination initiation criteria are proposed on the basis of Azzi-Tsai model[11] and Norris model[28], which can be selected and used according to material and structure of laminates.
Christensen et al.[78]	$\frac{\sigma_3}{S_2^+} + \frac{\sigma_4^2+\sigma_5^2}{S_4^2} = 1$	An out-of-plane delamination failure criterion based on stress invariants for quasi-isotropic laminates.

续表 7

Contributor(s)	Expression of criterion	Model description
Zubillaga et al.[79]	$\left(\frac{\sigma_1}{S_{11}}\right)^2+\left(\frac{\sigma_2}{S_{22}}\right)^2+\left(\frac{\sigma_6}{S_{66}}\right)^2+\frac{\sigma_1\sigma_2}{S_{12}}+\frac{\sigma_1\sigma_6}{S_{16}}+\frac{\sigma_2\sigma_6}{S_{26}}=1$	Plane stress criterion relating to the energy release rate of matrix crack, with S_{ij} the strength coefficients.
Yen[64]	$\left(\frac{\langle\sigma_3\rangle}{S_3^+}\right)^2+\left(\frac{\sigma_4}{S_4+\eta\langle-\sigma_3\rangle}\right)^2+\left(\frac{\sigma_5}{S_5+\eta\langle-\sigma_3\rangle}\right)^2=1$	The model considers the strengthening effect of normal pressure on interlaminar shear property, and is mainly used to simulate impact delamination damage of laminates.
Xiao et al.[81]	$\left(\frac{\langle\sigma_3\rangle}{S_3^+}\right)^2+\left(\frac{\sigma_4}{S_4+\eta\langle-\sigma_3\rangle}\right)^2+\left(\frac{\sigma_5}{S_5+\eta\langle-\sigma_3\rangle}\right)^2+\left(\frac{\langle\sigma_3^-/S_3^- -\beta\rangle}{1-\beta}\right)^2=1$	The transition parameter β is introduced to improve the Yen criterion[64], and the prediction curve is more reasonable and closer to experimental data.
Goyal et al.[82]	$\left(\frac{\langle\sigma_3\rangle}{S_3^+}\right)^\alpha+\left(\frac{\sigma_4}{S_4}\right)^\alpha+\left(\frac{\sigma_5}{S_5}\right)^\alpha=1$	A phenomenological criterion with α the fitting parameter.
Daniel et al.[61]	a: $\left(\frac{\sigma_3}{S_3^-}\right)^2+\left(\frac{\sigma_5}{S_3^-}\right)^2\left(\frac{E_3}{G_{13}}\right)^2=1$ b: $\left(\frac{\sigma_5}{S_5}\right)^2+2\frac{\sigma_3}{S_5}\frac{G_{13}}{E_3}=1$ c: $\frac{\sigma_3}{S_3^+}+\left(\frac{\sigma_5}{S_3^+}\right)^2\left(\frac{E_3}{2G_{13}}\right)^2=1$	A model distinguishing failure modes: (a) failure dominated by out-of-plane compression; (b) failure dominated by interlaminar shear; (c) failure dominated by out-of-plane tension.

4 基于能量的失效判据

现有大部分失效判据, 通常都忽略了应力-应变响应历程对最终破坏的影响, 而只是关注材料失效时宏细观应力/应变所满足的近似数学条件. 因此, 一般认为不考虑应变响应过程的失效判据更适合于线弹性复合材料.

为了表征应力-应变行为对破坏的影响, 借鉴剪切非线性效应的表征方法[59], 基于应变能参数进行建模就成了必然选择. 从能量角度出发, Griffith 等[83]计算了平面应力下正交各向异性材料的歪形能 U_{d}, 并认为当歪形能达到其临界值 U_{c} 时材料发生破坏, 从而给出

$$U_{\mathrm{d}}=\frac{\sigma_1^2}{3}\left(s_{11}-\frac{s_{12}+s_{13}}{2}\right)+\frac{\sigma_2^2}{3}\left(s_{22}-\frac{s_{12}+s_{23}}{2}\right)+\frac{\sigma_1\sigma_2}{3}\left(2s_{12}-\frac{s_{11}+s_{22}+s_{13}+s_{23}}{2}\right)+s_{66}\sigma_6^2=U_{\mathrm{c}} \tag{30}$$

这是线弹性复合材料失效判别的另一途径, 式中 s_{ij} 为柔度系数. 基于同样的思路, 赵清望[84]将轴向发生拉压或剪切失效时的歪形能作为参考值, 发展了正交各向异性歪形能强度理论, 其中假定材料在复杂应力下的歪形能达到单向拉压或纯剪切破坏时的歪形能即发生破坏, 从而给出 9 个并列条件进行失效判别. 蒋国宾等[85]则指出, 由于复合材料存在拉剪和剪拉耦合, 形状与体积改变无法分离, 故歪形能理论有牵强之处; 于是将歪形能变更为弹性应变能并提出了应变能强度理论.

对于非线性正交各向异性材料, Sandhu[86]为了表征应力-应变响应历程对破坏性能的影响, 采用应变能密度提出了如下失效判据

$$\sum_{i=1}^{6}k_i\left(\int_{\varepsilon_i}\sigma_i\mathrm{d}\varepsilon_i\right)^{m_i}=1 \tag{31}$$

式中, $m_i\,(i=1,2,\cdots,6)$ 用来调控应变-能量空间中失效曲面的形状, 而参数 k_i 的表达式为

$$k_i=\left(\int_{e_i}\sigma_i\mathrm{d}\varepsilon_i\right)^{-m_i} \tag{32}$$

其中, e_i 为应变分量 ε_i 的极限值. 该判据在后续研究中得到了广泛应用, 并被改进用于判别纤维或基体失效模式[87]. 与 Sandhu 模型不同, Abu-Farsakh 等[88]明确区分拉、压两种情况, 给出的判据表达式为

$$\sum_{i=1}^{6}\left(\frac{U_i}{U_{im}}\right)_k=1 \tag{33}$$

其中, k 表示拉伸或者压缩; 而应变能密度参数为

$$\left.\begin{aligned}U_i&=\frac{1}{2}\sigma_i\varepsilon_i\ (i=1,2,\cdots,6)\\U_{im}&=\frac{1}{2}S_ie_i\ (i=1,2,\cdots,6)\end{aligned}\right\} \tag{34}$$

显然, 应变能密度值为应力-应变曲线割线下的三角形面积, 而非材料的真实应变能密度.

为了表征破坏模式, Zand 等[89]将复合材料单层应变能增量分割为纵向应变能、2-3 面平均应变能、厚度方向剪切应变能、纵向剪切应变能和耦合应变能 5 个部分, 并基于简单加载失效的能量条件, 提出了三维应力情况下考虑破坏模式的 5 个能量判据. Zubillaga 等[79]则在断裂力学的理论框架内, 通过对比基体裂纹附近分层扩展的能量释放率和界面断裂韧性, 提出了分层起始判据. 研究表明, 能量判

据能够反映不同应力状态下的能量累积效应; 而能量释放率则可能成为失效判据的另一种有效自变量并用于表征微裂纹或微裂纹群的失稳机制.

5　基于损伤的失效判据

不论是破坏模式无关判据还是破坏模式相关判据, 通常都难以表征材料的内在损伤破坏机理, 即微裂纹或微裂纹群如何扩展、连通、失稳进而导致的断裂. 这方面损伤判据可能更具优势.

复合材料层压板的最终破坏通常由局部基体开裂、界面脱黏、分层和纤维簇断裂等损伤的累积效应引起, Zhen[90] 认为当损伤区扩展至临界尺寸时材料发生断裂, 并给出了临界损伤失效判据, 即

$$d = D_c \tag{35}$$

其中, d 为表征损伤程度的有效损伤量, D_c 为材料失效时的临界损伤值. 该模型被用于带孔层压板的单向拉伸性能分析, 适用性得到了验证. Siron 等[91] 则基于各向异性有效损伤变量提出了类似于最大应力判据的损伤判据, 即

$$d_i \geqslant 1 \quad (i = 1,2,6) \tag{36}$$

并用于 C/C 复合材料的最终失效判别. 对于非线性-准脆性复合材料, Yang 等[92] 通过刚度退化的实验数据发现, 材料断裂时的临界损伤值小于 1, 从而对式 (36) 进行了变更. Luo 等[93] 借鉴最大主应力理论, 引入了最大主损伤的概念, 并给出如下判别式

$$\frac{D_1 + D_2}{2} + \sqrt{\left(\frac{D_1 - D_2}{2}\right)^2 + D_6^2} \geqslant 1 \tag{37}$$

其中 D_i 为单轴加载时的损伤量. 由于临界损伤判据通常考虑了损伤耦合效应, 即在一定程度上体现了应力之间的相互作用, 因此其强度预测值比最大应力准则要准确很多.

此外, Abu-Farsakh 等[94] 为了改进前期基于应变能的失效判据, 将式 (33) 与损伤建立联系, 提出了新的线性损伤-能量协同失效判据, 即

$$\sum_{i=1}^{6}\left(\frac{U_i}{U_{im}} d_i\right)_k = 1 \tag{38}$$

式中, 损伤量 d_i 表征材料主方向的损伤对最终失效强度的影响. 验证表明, 该判据对于线性、非线性单向板和多向层压板均具有适用性, 无论其拉压强度是否相等; 而且在任意平面应力下, 模型预测结果均与实验数据相吻合.

除了临界损伤准则和线性损伤准则外, 研究者还提出了二次损伤准则. Key 等[95] 给出了横观各向同性损伤不变量, 即

$$\left.\begin{aligned} I_1'' &= D_1 \\ I_2'' &= D_2 + D_3 \\ I_3'' &= D_2^2 + D_3^2 + 2D_4^2 \\ I_4'' &= D_5^2 + D_6^2 \end{aligned}\right\} \tag{39}$$

并类比应力不变量判据[52], 提出了纤维和基体的损伤-失效判据, 表达式为

$$A_2'' I_2'' + A_3'' I_3'' + A_4'' I_4'' = 1 \tag{40}$$

其中, A_2'', A_3'' 和A_4'' 可通过简单加载实验确定. Yang 等[92,96] 考虑各向异性损伤耦合效应, 提出了更具一般性的损伤-失效判据, 即

$$\begin{aligned} &a_1 D_1 + a_2 D_2 + a_6 D_6 + a_{11} D_1^2 + a_{22} D_2^2 + a_{66} D_6^2 + \\ &\quad a_{12} D_1 D_2 + a_{16} D_1 D_6 + a_{26} D_2 D_6 = 1 \end{aligned} \tag{41}$$

并在特殊工况下确定了失效判据的具体表达式, 同时进行了实验验证. Jain 等[97] 在平面应力下给出了与式 (41) 相似的失效判别式.

Camus[98] 则区分 3 种加载模式, 在轴向拉伸载荷下采用有效应力进行失效判别, 以考虑损伤导致的纤维应力重分布; 在面内剪切载荷下采用的是临界损伤判据; 在轴向压缩载荷下采用的是最大压缩能判据; 而在多轴载荷作用下, 3 种模式的判据满足其一就认为材料破坏. 杨成鹏等[99] 将单轴拉伸有效应力判据进行拓展, 进一步提出了更具一般性的最大有效应力判据和二次有效应力判据, 即

$$|\bar{\sigma}_i| = \left|\frac{\sigma_i}{1 - d_i}\right| = \left|\frac{S_i}{1 - D_{ci}}\right| \quad (i = 1,2,\cdots,6) \tag{42}$$

$$\begin{aligned} &F(\bar{\sigma}_2 - \bar{\sigma}_3)^2 + G(\bar{\sigma}_3 - \bar{\sigma}_1)^2 + H(\bar{\sigma}_1 - \bar{\sigma}_2)^2 + \\ &\quad L\bar{\sigma}_4^2 + M\bar{\sigma}_5^2 + N\bar{\sigma}_6^2 = 1 \end{aligned} \tag{43}$$

由于式 (43) 考虑了有效应力分量之间的二次交互影响效应, 所以其中损伤参量取 D_i 即可.

在连续介质损伤力学理论框架内还有一类重要的损伤参量, 即热力学力. Chaboche 等[100] 针对各向异性-脆性复合材料, 基于面内的 3 个热力学力分量提出了最大热力学力准则 (多准则) 和耦合热力学力准则 (复合准则). Obert 等[101] 则针对复合材料单层

引入裂纹密度 ρ, 并给出了相对裂纹密度的热力学力分量, 进而根据断裂力学理论构建了如下损伤-断裂协同判据

$$\left(\frac{y_2}{G_{\mathrm{IC}}}\frac{\partial D_2}{\partial \rho}\right)^{\alpha}+\left(\frac{y_6}{G_{\mathrm{IIC}}}\frac{\partial D_6}{\partial \rho}\right)^{\alpha}+\left(\frac{y_4}{G_{\mathrm{IIIC}}}\frac{\partial D_4}{\partial \rho}\right)^{\alpha}=1 \tag{44}$$

式中, y_i 为热力学力分量, G_{IC} , G_{IIC} 和 G_{IIIC} 分别为细观裂纹的 I, II 和 III 型临界能量释放率. 为了形成比较完备的理论体系, Shahabi 等[102] 针对树脂基复合材料, 同时采用热力学力分量和应力分量, 给出了混合变量强度理论, 即

$$h_1\sigma_1+h_2\sigma_2+h_3\sigma_3+h_4|\sigma_4|+h_5|\sigma_5|+h_6|\sigma_6|+h_{11}y_1^2+ \\ h_{22}y_2^2+h_{33}y_3^2+2h_{12}y_1y_2+2h_{13}y_1y_3+2h_{23}y_2y_3=1 \tag{45}$$

式中, 正应力一次项用来表征拉压强度不等对失效曲面的影响, 而剪应力一次项则用来协调面内剪切强度. Yang 等[103] 则针对非线性-准脆性复合材料的失效分析和预测, 基于正交各向异性损伤能量释放率 (DERR) 分量, 即

$$y_i=\frac{\partial\left(\sigma_i\varepsilon_i-\int\varepsilon_i\mathrm{d}\sigma_i\right)}{\partial D_i}\quad (i=1,2,6) \tag{46}$$

同时提出了最大 DERR 判据和交互 DERR 判据, 表达式分别为

$$y_i=Y_i=y_i|_{\sigma_i=S_i}\quad (i=1,2,6) \tag{47}$$

$$c_iy_i+c_{ij}y_iy_j=1\quad (i,j=1,2,6) \tag{48}$$

式 (47) 和式 (48) 通过平纹织物叠层复合材料进行了初步验证, 显示了一定的合理性和准确性.

从上面的总结论述可以看出, 失效判据分门别类、数量众多, 如何进行失效判据的选用, 就成为了一个很重要的问题. 复合材料的破坏机理非常复杂, 失效模拟和强度预测需要根据应力的耦合影响程度、交互模式与作用机制, 从现有强度理论中选择合适判据以揭示材料的破坏模式和机理, 但是, 即使针对相似的破坏问题, 不同研究者也可能出于不同研究思路而选取不同的失效判据. 根据破坏准则所依据的原理, Huang 等[104-105] 给出了一种失效判据的遴选原则, 即: 首选基于物理原理的判据, 其次是基于数学原理的判据, 最后是基于唯象原理的判据. 这种方法具有合理性, 因为基于物理原理的失效判据可能具有更坚实的物理基础, 而基于现象学的唯象判据则只是一种数据拟合模型.

6 总论与展望

失效判据是复杂应力下复合材料及其结构渐进损伤分析以及极限强度预测的重要基础. 复合材料失效判据的发展已近百年, 数量不下百种, 有应力判据、应变判据、能量判据和损伤判据等. 早期的唯象判据通常不考虑具体破坏模式, 以数据拟合为主要手段和目标, 只关注材料发生整体失效时的临界应力或应变条件; 但后期考虑破坏模式的判据又通常以低阶宏观判据 (如最大应力判据和二次应力判据) 为基础和依据, 以致于低阶判据的拓展和应用贯穿了失效判据发展的整个进程, 以 Hashin 破坏面模型的应用、改进与发展最为典型. 应该指出, 破坏模式相关判据将纤维、基体、纤维-基体界面以及层间的失效进行独立判别, 这对于渐进损伤模拟意义重大, 研究者以此实现了复合材料损伤-破坏过程的有效仿真, 并揭示了材料在不同载荷下的损伤-破坏形式和机理, 例如层压板的冲击损伤模拟. 实际上, 这种多尺度的分析方法涉及了两种失效判据, 即细观力学判据和宏观力学判据. 细观力学判据要求计算组分材料的真实应力场, 进而根据组分材料的性能预测复合材料的整体性能, 如黄争鸣等发展的真应力理论; 而宏观力学判据则使用单层的宏观性能, 进而模拟层压板的损伤扩展并计算极限承载能力. 由于宏观力学判据使用了单层材料的实测数据, 因此其预测精度往往高于细观力学判据; 但是与细观力学判据相比, 宏观力学判据不能表征热残余应力等内应力的影响.

从理论上讲, 所有的应力判据都可以转换为应变判据, 但是应力判据和应变判据哪个更合理、更适用尚无定论. 与应力相比, 应变具有直接可测和易测性, 所以应变判据对于工程结构的适用性更容易得到试验验证; 同时, 由于泊松效应, 应变判据还可以表征多应力对于变形和破坏的耦合影响. 总之, 应力判据和应变判据是当前使用最为广泛的两类判据. 虽然基于能量和损伤的失效判据在复合材料破坏分析中并未大量使用, 但其对于非线性-易损性复合材料同样具有适用性和合理性. 其中, 应变能判据可表征非线性应力-应变响应过程对失效行为的影响, 如 Sandhu 准则和 Chang-Chang 准则. 而能量释放率判据可用于描述细观裂纹的扩展与失稳行为.

当然, 对于易损复合材料, 连续介质损伤本构理论及损伤失效判据似乎更具应用合理性, 其中临界损伤判据、有效应力判据和损伤能量释放率判据均可以表征损伤的耦合强化或钝化效应, 对复合材料的强度理论体系也是一种必要的补充和拓展; 但是该类模型不具有广泛适用性, 且模型复杂, 发展不充分, 不便于工程应用.

目前, 应力判据、应变判据和能量判据的发展都已经相对成熟, 但仍然不足以表征复合材料的破坏特性, 其强度预测精度仍然有限, 这体现了现有判据的固有缺陷, 表明其不具有非常坚实的物理基础, 并不能完全合理地描述多应力的复杂耦合作用机制. 虽然基于破坏面假设的相关判据相比于宏观唯象判据大幅度提高了强度预测精度, 但其仍然没有从根本上突破多应力的"二次多项式"交互耦合范式, 见表 2 ~ 表 5 相关判据; 而且在三维复杂应力情形下破坏面的方位通常难以预测, 甚至预测值与真实破坏路径相悖, 即裂纹非自相似扩展. 同时, 能量判据也有缺陷, 原因是该类判据通常以单层轴向简单加载时的应变能临界值作为参考, 但是复杂应力下各轴向的能量耗散方式通常会发生改变, 导致判据应用的物理基础出现偏差, 综合而言, 不同结构形式的复合材料在不同加载方式下具有不同的能量耗散机制, 不同的能量耗散形式和路径对于极限强度并非总是单一效应. 损伤判据也是如此, 其很难准确描述多样化的开裂与破坏机制, 在内应力松弛和应力集中得以缓减的情况下损伤也并非总是导致强度性能的劣化, 例如强界面材料在多应力作用下的界面弱化可能有利于提高极限强度, 这与判据的预测往往相悖.

鉴于现有失效判据的不足, 今后强度理论的发展可以着重考虑以下几个方面. 其一, 借助先进检测表征技术进一步揭示不同载荷下不同复合材料的裂纹扩展路径及其失稳特征, 确定细观损伤的关键性参量, 通过多尺度的模型和方法, 实现跨尺度损伤-破坏机制的关联, 进而构建基于细观损伤参量的宏观性能表征理论与模型. 其二, 针对易损性-非线性-准脆性复合材料, 基于损伤力学的方法, 通过引入损伤变量进行模型构建, 分析刚度演变对材料承载性能的影响, 进而表征细观损伤引起的内应力强化对破坏行为的作用, 进一步拓展和完善基于损伤变量的失效判据. 其三, 在模型架构层面, 可进一步寻求多应力作用下不同复合材料细观裂纹失稳扩展或裂纹群连通失稳的表观应变能释放率条件, 发展更为合理、表征能力更强的损伤能量释放率判据. 特别指出, 基于损伤能量释放率的强度理论在一定程度上能够体现断裂力学中关于复合型裂纹失稳扩展的基本原理, 可对微裂纹控制的材料断裂行为进行模拟预测, 这一类强度判据可称之为损伤-断裂协同理论. 其四, 应用传统强度理论模拟预测织物增强脆性基体复合材料的损伤-破坏过程, 往往要付出极大的计算代价, 甚至是不可能完成的任务, 因此有必要发展针对编织结构复合材料的强度失效判据. 其五, 虽然基于损伤变量、有效应力和损伤能量释放率的 3 种强度理论均可用来预测准脆性复合材料的平面应力强度, 但其局限在宏观尺度, 如何将其应用在多尺度框架下, 使其能够表征基体开裂、界面脱黏滑移和纤维桥连断裂等损伤形式的影响, 同时区分基体、界面和纤维的不同破坏模式, 进而描述宏观断裂模式与细观影响机制之间的关联规律, 尚需深入分析探讨. 最后, 对于防热-承载-功能一体化复合材料而言, 其服役环境涉及高温、力-电-磁以及氧化-腐蚀介质, 在多场-多轴载荷耦合作用下, 材料的宏-细观损伤、破坏过程将变得更加复杂, 这就要求对多场-多轴耦合损伤演化行为进行多尺度模型表征, 并将基于损伤和能量的强度理论进行拓展, 使其适用于模拟预测多场耦合性能.

参 考 文 献

1 黄争鸣, 张华山. 纤维增强复合材料强度理论的研究现状与发展趋势——"破坏分析奥运会"评估综述. 力学进展, 2007, 37(1): 80-98 (Huang Zhengming, Zhang Huashan. Current status and future trend of researches on the strength of fiber-reinforced composites—A summary of the results from a "failure Olympics". *Advances in Mechanics*, 2007, 37(1): 80-98 (in Chinese))

2 Bower MV, Koedam DH. Tensor polynomial failure criterion: Coefficient limits based on convexity requirements. *Journal of Reinforced Plastics and Composites*, 1997, 16(5): 435-477

3 Goldenblat II, Kopnov VA. Strength of glass reinforced plastics in the complex stress state. *Polymer Mechanics*, 1965, 1(2): 54-59

4 Ashkenazi EK. Problems of the anisotropy of strength. *Polymer Mechanics*, 1965, 1(2): 60-70

5 Malmeister A. Geometry of theories of strength. *Polymer Mechanics*, 1966, 2(4): 324-331

6 Huang CL, Kirmser PG. A criterion of strength for orthotropic materials. *Fiber Science and Technology*, 1975, 8(2): 103-112

7 Osswald PV, Osswald TA. A strength tensor based failure criterion with stress interactions. *Polymer Composites*, 2018, 39(8): 2826-2834

8 Tennyson RC, Macdonald D, Nanyaro AP. Evaluation of the tensor polynomial failure criterion for composite materials. *Journal of Composite Materials*, 1978, 12(1): 63-75
9 Jiang ZQ, Tennyson RC. Closure of the cubic tensor polynomial failure surface. *Journal of Composite Materials*, 1989, 23(3): 208-231
10 Hill R. A theory of the yielding and plastic flow of anisotropic metals. *Proceedings of the Royal Society of London: Series A, Mathematical and Physical Sciences*, 1948, 193(1033): 281-297
11 Azzi VD, Tsai SW. Anisotropic strength of composites. *Experimental Mechanics*, 1965, 5(9): 283-288
12 Narayanaswami R, Adelman HM. Evaluation of the tensor polynomial and Hoffman strength theories for composite materials. *Journal of Composite Materials*, 1977, 11(4): 366-377
13 Fischer L. Optimization of orthotropic laminates. *Journal of Engineering for Industry*, 1967, 89(3): 399-402
14 Chamis CC. Failure criteria for filamentary composites//Composite Materials: Testing and Design. STP460. American Society for Testing and Materials, Philadelphia, 1969: 336-351
15 Marin J. Theories of strength for combined stresses and nonisotropic materials. *Journal of the Aeronautical Sciences*, 1957, 24(4): 265-274
16 Hoffman O. The brittle strength of orthotropic materials. *Journal of Composite Materials*, 1967, 1(2): 200-206
17 Franklin HG. Classic theories of failure of anisotropic materials. *Fiber Science and Technology*, 1968, 1(2): 137-150
18 Theocaris PS. Tensor failure criteria for composites: Properties and comparison of the ellipsoid failure surfaces with experiments. *Advances in Polymer Technology*, 1991, 11(1): 27-40
19 Tsai SW, Wu EM. A general theory of strength for anisotropic materials. *Journal of Composite Materials*, 1971, 5(1): 58-80
20 Cowin SC. On the strength anisotropy of bone and wood. *Journal of Applied Mechanics*, 1979, 46(4): 832-838
21 Liu C, Huang Y, Stout MG. On the asymmetric yield surface of plastically orthotropic materials: A phenomenological study. *Acta Materialia*, 1997, 45(6): 2397-2406
22 Tsai SW. A survey of macroscopic failure criteria for composite materials. *Journal of Reinforced Plastics and Composites*, 1984, 3(1): 40-62
23 DeTeresa SJ, Larsen GJ. Reduction in the number of independent parameters for the Tsai-Wu tensor polynomial theory of strength for composite materials. *Journal of Composite Materials*, 2003, 37(19): 1769-1785
24 Li S, Sitnikova E, Liang Y, et al. The Tsai-Wu failure criterion rationalised in the context of UD composites. *Composites: Part A*, 2017, 102: 207-217
25 Daniel IM, Daniel SM, Fenner JS. A new yield and failure theory for composite materials under static and dynamic loading. *International Journal of Solids and Structures*, 2018, 148-149: 79-93
26 Tan SC. A new approach of three-dimensional strength theory for anisotropic materials. *International Journal of Fracture*, 1990, 45: 35-50
27 Puppo AH, Evensen HA. Strength of anisotropic materials under combined stresses. *AIAA Journal*, 1972, 10(4): 468-474
28 Norris CB. Strength of orthotropic materials subjected to combined stresses. Report 1816. Forest Product Laboratory, 1962
29 Yeh HY, Kim CH. The Yeh-stratton criterion for composite materials. *Journal of Composite Materials*, 1994, 28(10): 926-939
30 Yeh HL. Quadratic surface criterion for composite materials. *Journal of Reinforced Plastics and Composites*, 2003, 22(6): 517-532
31 Chowdhury NT, Wang J, Chiu WK, et al. Predicting matrix failure in composite structures using a hybrid failure criterion. *Composite Structures*, 2016, 137: 148-158
32 Jenkin CF. Report on materials of construction used in aircraft and aircraft engines//Great Britain Aeronautical Research Committee, 1920
33 Stowell EZ, Liu TS. On the mechanical behavior of fiber reinforced crystalline materials. *International Journal of Mechanics and Physical of Solids*, 1961, 9(4): 242-260
34 Cabrero JM, Blanco C, Gebremedhin KG, et al. Assessment of phenomenological failure criteria for wood. *European Journal of Wood and Wood Products*, 2012, 70: 871-882
35 刘方龙, 寇长河. 复合材料复合型二次强度准则. 复合材料学报, 1985, 2(2): 52-60 (Liu Fanglong, Kou Changhe. A new quadratic strength criterion for fiber-reinforced composites. *Acta Materiae Compositae Sinica*, 1985, 2(2): 52-60 (in Chinese))
36 Labossiere P, Neale KW. A general strength theory for orthotropic fiber-reinforced composite laminae. *Polymer Composites*, 1988, 9(5): 306-317
37 Makinde A, Neale KW, Sacharuk Z. A strain-based parametric biaxial failure criterion for fiber-reinforced composites. *Polymer Composites*, 1992, 13(4): 263-272
38 Echaabi J, Trochu F. A methodology to derive the implicit equation of failure criteria for fibrous composite laminates. *Journal of Composite Materials*, 1996, 30(10): 1088-1114
39 Hashin Z. Failure criteria for unidirectional fiber composites. *Journal of Applied Mechanics*, 1980, 47(2): 329-334
40 Vogler M, Rolfes R, Camanho PP. Modeling the inelastic deformation and fracture of polymer composites—Part I: Plasticity model. *Mechanics of Materials*, 2013, 59: 50-64
41 Roetsch K, Horst T. A novel approach to consider triaxial tensile stresses within the framework of a failure criterion. *Journal of Applied Mathematics and Mechanics*, 2022, 102: e202100232
42 Gu JF, Chen PH. A failure criterion for homogeneous and isotropic materials distinguishing the different effects of hydrostatic tension and compression. *European Journal of Mechanics-A/Solids*, 2018, 70: 15-22
43 Rolfes R, Vogler M, Czichon S, et al. Exploiting the structural reserve of textile composite structures by progressive failure analysis using a new orthotropic failure criterion. *Computers and Structures*, 2011, 89: 1214-1223
44 Hashin Z, Rotem A. A fatigue failure criterion for fiber reinforced materials. *Journal of Composite Materials*, 1974, 7(4): 448-464
45 Puck A, Schürmann H. Failure analysis of FRP laminates by means of physically based phenomenological models. *Composites Science and Technology*, 1998, 58: 1045-1067
46 Dávila CG, Jaunky N, Goswami S. Failure criteria for FRP laminates in plane stress. NASA TM 2003, 613: 212663
47 Dávila CG, Camanho PP, Rose CA. Failure criteria for FRP laminates. *Journal of Composite Materials*, 2005, 39(4): 323-345
48 Pinho ST, Dávila CG, Camanho PP, et al. failure models and criteria for FRP under in-plane or three-dimensional stress states including shear non-linearity. NASA/TM-2005- 213530

49 Catalanotti G, Camanho PP, Marques AT. Three dimensional failure criteria for fiber-reinforced laminates. *Composite Structures*, 2013, 95: 63-79

50 Li N, Gu JF, Chen PH. Fracture plane based failure criteria for fibre-reinforced composites under three-dimensional stress state. *Composite Structures*, 2018, 204: 466-474

51 Guo J, Zhang Y, Zhou G, et al. A transverse failure criterion for unidirectional composites based on the puck failure surface theory. *Composites Science and Technology*, 2023, 242: 110192

52 Mayes JS, Hansen AC. Composite laminate failure analysis using multicontinuum theory. *Composites Science and Technology*, 2004, 64: 379-394

53 Cuntze RG, Freund A. The predictive capability of failure mode concept-based strength criteria for multidirectional laminates. *Composites Science and Technology*, 2004, 64: 343-377

54 Li X, Guan ZD, Li ZS, et al. A new stress-based multi-scale failure criterion of composites and its validation in open hole tension tests. *Chinese Journal of Aeronautics*, 2014, 27(6): 1430-1441

55 Chen X, Sun X, Chen P, et al. Rationalized improvement of Tsai-Wu failure criterion considering different failure modes of composite materials. *Composite Structures*, 2021, 256: 113120

56 黄争鸣, 张若京. 复合材料结构受横向载荷作用的强度问题. 复合材料学报, 2005, 22(2): 148-159 (Huang Zhengming, Zhang Ruojing. On the ultimate strength of a fiber reinforced composite laminate subjected to lateral loads. *Acta Materiae Compositae Sinica*, 2005, 22(2): 148-159 (in Chinese))

57 Zhao LB, Qin TL, Zhang JY, et al. Modified maximum stress failure criterion for composite π joints. *Journal of Composite Materials*, 2012, 47(23): 2995-3008

58 Wang X, Guan Z, Liu N, et al. A 3D micromechanics based failure criterion for fiber reinforced composites under longitudinal compression. *Composites: Part A*, 2022, 161: 107076

59 Chang FK, Chang KY. A progressive damage model for laminated composites containing stress concentrations. *Journal of Composite Materials*, 1987, 21(9): 834-855

60 Christensen RM. 2013 Timoshenko medal award paper-completion and closure on failure criteria for unidirectional fiber composite materials. *Journal of Applied Mechanics*, 2014, 81(1): 011011

61 Daniel IM, Luo JJ, Schubel PM, et al. Interfiber/interlaminar failure of composites under multi-axial states of stress. *Composites Science and Technology*, 2009, 69: 764-771

62 Daniel IM. Constitutive behavior and failure criteria for composites under static and dynamic loading. *Meccanica*, 2015, 50: 429-442

63 Waddoups ME. Advanced composite material mechanics for the design and stress analyst. General Dynamics, Fort Worth Division, Report FZM-4763, 1967

64 Yen CF. A ballistic material model for continuous-fiber reinforced composites. *International Journal of Impact Engineering*, 2012, 46: 11-22

65 Hou JP, Petrinic N, Ruiz C, et al. Prediction of impact damage in composite plates. *Composites Science and Technology*, 2000, 60: 273-281

66 Camanho PP, Arteiro A, Melro AR, et al. Three dimensional invariant based failure criteria for fibre reinforced composites. *International Journal of Solids and Structures*, 2015, 55: 92-107

67 Liu HB, Liu J, Ding YZ, et al. A three-dimensional elastic-plastic damage model for predicting the impact behaviour of fibre-reinforced polymer-matrix composites. *Composites: Part B*, 2020, 201: 108389

68 Patel DK, Waas AM, Yen CF. Direct numerical simulation of 3D woven textile composites subjected to tensile loading: An experimentally validated multiscale approach. *Composites: Part B*, 2018, 152: 102-115

69 Sun QP, Zhou GW, Meng ZX, et al. Failure criteria of unidirectional carbon fiber reinforced polymer composites informed by a computational micromechanics model. *Composites Science and Technology*, 2019, 172: 81-95

70 Sun CT, Quinn BJ, Tao J, et al. Comparative evaluation of failure analysis methods for composite laminates. DOT/FAA/AR-95/109, 1996

71 Chang FK, Lessard LB. Damage tolerance of laminated composites containing an open hole and subjected to compressive loadings: Part I—Analysis. *Journal of Composite Materials*, 1991, 25(1): 2-43

72 Lee JD. Three dimensional finite element analysis of damage accumulation in composite laminate. *Computers and Structures*, 1982, 15(3): 335-350

73 Ochoa OO, Engblom JJ. Analysis of progressive failure in composites. *Composites Science and Technology*, 1987, 28: 87-102

74 Soni SR, Kim RY. Delamination of composite laminates stimulated by interlaminar shear. *ASTM STP*, 1986, 893: 286-307

75 Brewer JC, Lagace PA. Quadratic stress criterion for initiation of delamination. *Journal of Composite Materials*, 1988, 22(12): 1141-1155

76 Long RS. Static strength of adhesively bonded ARALL-1 joints. *Journal of Composite Materials*, 1991, 25(24): 391-415

77 Tong L. An assessment of failure criteria to predict the strength of adhesively bonded composite double lap joints. *Journal of Reinforced Plastics and Composites*, 1997, 16(8): 698-713

78 Christensen RM, Deteresa SJ. Delamination failure investigation for out-of-plane loading in laminates. *Journal of Composite Materials*, 2004, 38(24): 2231-2238

79 Zubillaga L, Turon A, Maimí P, et al. An energy based failure criterion for matrix crack induced delamination in laminated composite structures. *Composite Structures*, 2014, 112: 339-344

80 Nejad AD, Farrokhabadi A. A failure criterion to predict the onset of matrix cracking induced delamination in general composite laminates. *Composite Structures*, 2020, 235: 111564

81 Xiao JR, Gillespie Jr JW. A phenomenological Mohr-Coulomb failure criterion for composite laminates under interlaminar shear and compression. *Journal of Composite Materials*, 2007, 41(11): 1295-1309

82 Goyal VK, Johnson ER, Dávila CG. Irreversible constitutive law for modeling the delamination process using interfacial surface discontinuities. *Composite Structures*, 2004, 65: 289-305

83 Griffith JE, Baldwin WM. Failure theories for generally orthotropic materials. *Developments in Theoretical and Applied Mechanics*, 1962, 1: 410-420

84 赵清望. 正交各向异性复合材料的歪形能强度理论. 力学与实践, 1987, 9(S1): 106-111 (Zhao Qingwang. Distortion energy strength theory for orthotropic composites. *Mechanics in Engineering*, 1987, 9(S1): 106-111 (in Chinese))

85 蒋国宾, 张卫国, 蒋玉川. 复合材料的应变能强度理论. 玻璃钢/

复合材料, 1993, 2(2): 21-23 (Jiang Guobin, Zhang Weiguo, Jiang Yuchuan. Strength theory of composite strain energy. *Composites Science and Engineering*, 1993, 2(2): 21-23 (in Chinese))
86 Sandhu RS. Nonlinear behavior of unidirectional and angle ply laminates. *Journal of Aircraft*, 1976, 13(2): 104-111
87 Butalia TS, Wolfe WE. A strain-energy based nonlinear failure criterion: Comparison of numerical predictions and experimental observations for symmetric composite laminates. *Composites Science and Technology*, 2002, 62: 1697-1710
88 Abu-Farsakh GA, Abdel-Jawad YA. A new failure criterion for nonlinear composite materials. *Journal of Composites Technology and Research*, 1994, 16(2): 138-145
89 Zand B, Butalia TS, Wolfe WE, et al. A strain energy based failure criterion for nonlinear analysis of composite laminates subjected to triaxial loading. *Journal of Composite Materials*, 2012, 46(19-20): 2515-2537
90 Zhen S. The D criterion theory in notched composite materials. *Journal of Reinforced Plastics and Composites*, 1983, 2(2): 98-110
91 Siron O, Pailhes J, Lamon J. Modelling of the stress-strain behaviour of a carbon-carbon composite with a 2.5 dimensional fibre architecture under tensile and shear loads at room temperature. *Composites Science and Technology*, 1999, 59: 1-12
92 Yang CP, Jiao GQ, Wang B, et al. Damage based failure theory and its application to 2D-C/SiC composites. *Composites: Part A*, 2015, 77: 181-187
93 Luo D, Takezono S, Tao K, et al. The mechanical behavior analysis of CFCC with overall anisotropic damage by the micro-macro scale method. *International Journal of Damage Mechanics*, 2003, 12(4): 141-162
94 Abu-Farsakh GA, Odeh IN. A new damage-based failure criterion for nonlinear behavior of fibrous composite materials. *International Journal of Damage Mechanics*, 2023, 32(7): 940-961
95 Key CT, Schumacher SC, Hansen AC. Progressive failure modeling of woven fabric composite materials using multicontinuum theory. *Composites: Part B*, 2007, 38: 247-257
96 杨成鹏, 矫桂琼, 郭洪宝. 纤维增强陶瓷基复合材料的 D 失效判据. 固体力学学报, 2015, 36(6): 524-529 (Yang Chengpeng, Jiao Guiqiong, Guo Hongbao. D failure criterion for fiber reinforced ceramic matrix composites. *Chinese Journal of Solid Mechanics*, 2015, 36(6): 524-529 (in Chinese))
97 Jain N, Koch D. Prediction of failure in ceramic matrix composites using damage-based failure criterion. *Journal of Composite Materials*, 2020, 4: 183
98 Camus G. Modelling of the mechanical behavior and damage processes of fibrous ceramic matrix composites- application to a 2-D SiC/SiC. *International Journal of Solids and Structures*, 2000, 37: 919-942
99 杨成鹏, 贾斐, 矫桂琼. 陶瓷基复合材料三维有效应力强度失效模型. 复合材料学报, 2019, 36(12): 2912-2919 (Yang Chengpeng, Jia Fei, Jiao Guiqiong. Effective stress based three-dimensional strength failure model for ceramic matrix composites. *Acta Materiae Compositae Sinica*, 2019, 36(12): 2912-2919 (in Chinese))
100 Chaboche JL, Lesne PM, Maire JF. Continuum damage mechanics, anisotropy and damage deactivation for brittle materials like concrete and ceramic composites. *International Journal of Damage Mechanics*, 1995, 4(1): 5-22
101 Obert E, Daghia F, Ladevèze P, et al. Micro and meso modeling of woven composites: Transverse cracking kinetics and homogenization. *Composite Structures*, 2014, 117: 212-221
102 Shahabi E, Forouzan MR. A damage mechanics based failure criterion for fiber reinforced polymers. *Composites Science and Technology*, 2017, 140: 23-29
103 Yang CP, Jia F, Wang B, et al. Constitutive model and failure criterion for orthotropic ceramic matrix composites under macroscopic plane stress. *Journal of the American Ceramic Society*, 2021, 104(2): 1002-1013
104 Huang ZM. Mechanics theories for anisotropic or composite materials. *Advances in Applied Mechanics*, 2023, 56: 1-137
105 Wang LS, Huang ZM. On strength prediction of laminated composites. *Composites Science and Technology*, 2022, 219: 109206