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THE EQUATION OF THE CONJUNCTION (*ŚĪGHRAPHALA*) OF THE PLANETS IN CLASSICAL INDIAN ASTRONOMY

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Abstract: In classical Indian Astronomy the true positions of the five planets are determined by repeatedly applying two equations, viz the equation of centre (*mandaphala*) and the equation of conjunction (*śīghraphala*). In the present paper we concentrate on the equation of conjunction (*śīghraphala*). Here, conjunction refers to the conjunction of a planet with the Sun, considering the 'anomaly' of their mean positions. In this process the concepts involved are the *śīghra* anomaly (*śīghrakendra*), *śīghraparidhi* (periphery) and the *śīghrakarṇa* (hypotenuse).

Keywords: *śīghraphala, mandaphala,* equation of centre, equation of conjunction.

1 INTRODUCTION

Of the two equations applied to the mean positions of the planets, the first one is called the *mandaphala* which corresponds to the equation of centre due to the eccentricity. The second equation is called the *śīghraphala* (equation of conjunction) and corresponds, in 'Modern Astronomy' to the transformation of the true heliocentric position of a planet to the geocentric position. Of course, before the advent of Copernicus medieval Indian astronomers, as in other civilizations, were not aware of the heliocentric motion of the planets. All the same, using an epicyclic procedure and variable peripheries, Indian astronomers could predict true positions reasonably close to the actual ones (Balachandra Rao, 2005). Sustained observations of planetary phenomena over several centuries, plus intuition born out of experience and mathematical acumen, must have helped them to evolve the related equations.

In fact, the variable peripheries of the epicycles introduced by Āryabhata (born CE 476) resulted in the locus of each heavenly body approximating in ellipse. The expression for the *manda* equation in Indian astronomical texts is texts is given by

$$E = \frac{a}{R}\sin(m) \tag{1}$$

where '*a*' is the *periphery* (in degrees) of the *manda* epicycle, *R* = 360° and *m* is the *manda* anomaly, measured from the apogee of the body. The corresponding modern formula for the equation of centre is

$$E = \left(2e - \frac{1}{4}e^3\right)\sin(m) + \left[\frac{5}{4}e^2 - \frac{11}{24}e^4\right] \sin(2m) + \cdots, \tag{2}$$

Here '*e*' is the eccentricity of the body's elliptical orbit. Generally, since *e* is small, ignoring the higher powers of *e*, the equation of centre is approximated as

$$E \approx (2e)\sin(m) \tag{3}$$

From Equations (1) and (2) $a/R \approx 2e$. The periphery '*a*' (in degrees) results in a/R being close to the known modern eccentricity of the orbit. However, based on modern computational astronomy, the *manda* peripheries need to be updated for better results. In the present paper, we study the maximum *śīghraphala* for each of the planets, and their critical points.

2 CLASSICAL EXPRESSION FOR THE *ŚĪGHRAPHALA* AND THE *ŚĪGHRA* CORRECTION FOR THE *TĀRĀGRAHAS*

The *śīghra* correction corresponds to the 'elongation' in the case of *Budha* and *Śukra* from the Sun and the annual parallax in the case of *Kuja*, *Guru* and *Śani*. The *manda* correction is applied to the mean longitude of a planet to get the 'true-mean' or *manda*-corrected (*manda-sphuta graha*) position of the planet (Balachandra Rao, 2018). Now, the concept of the *śīghra* correction is explained with the help of Figure 1.

Let the circle *CDFG* with the Earth (*E*) at the centre represent the *kaksāvrtta* (or deferent circle) of a planet. Just like the *manda* epicycle, a *śīghra* epicycle is prescribed with a specified variable radius for each planet. Let

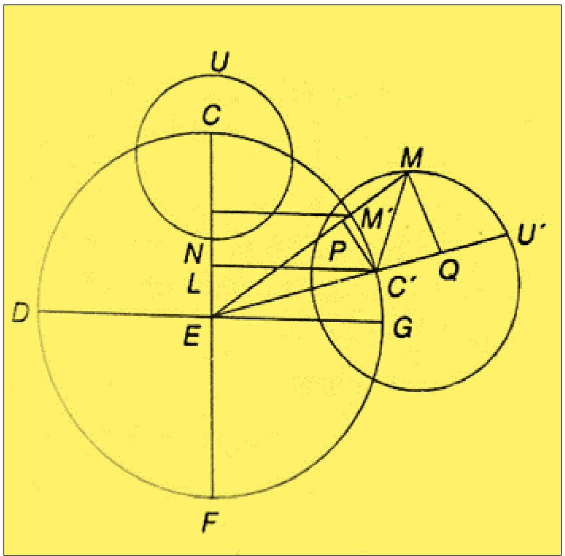


Figure 1: Epicyclic theory (diagram: P. Venugopal).

C be the centre of the *śīghra* epicycle of the planet. While *C* moves along the deferent circle, the planet moves along its epicycle. The epicycle in this case is called the *śīghra-nīcocco-vrtta*. Let *CEF* cut the epicycle at *U* and *N* which are respectively the *śīghrocco* and *śīghranīca* (*śīghra* apogee and perigee) of the *śīghra* epicycle. The centre *C* of the epicycle moves along the deferent circle with the velocity of the corrected planet (*mandasphuta graha*). Let the planet move from *U'* to *M* along the epicycle so that arc *U'M* is equal to arc *C'C*. Join *EM* cutting the deferent at *M'*. Then, *C'* is the *mandasphutagraha* and *M'* is the true planet (*sphuta graha*). Therefore, the correction is to be made to the longitude of the 'true mean' planet (i.e. the *manda*-corrected planet) the arc *C'M'*. The correction, *C'M'* in angular measure, is called the *śīghraphala.* Now in order to obtain an expression for the arc *C'M'*, draw *C'L*, *C'P* and *MQ* perpendiculars respectively to *CE*, *EM* and *U'E*.

The angle *C'EC* which is the angle between the *śīghrocco* and the *manda sphutagraha* is called the *śīghrakendra* or the anomaly of conjunction.

From Figure 1 we have *C'L* equals *R* sin (*śīghra* anomaly) and *EL* equals *R* cos (*śīghra* anomaly). Also, arc *U'M* equals arc *C'C* and the angle *U'C'M* equals the angle *C'EC*, and hence the triangles *MC'Q* and *C'EL* are similar. Therefore, *MQ*/*MC'* equals *C'L*/*C'E*, and *MQ* equals *C'L* multiplied by *MC'*/*C'E*. This equates to *R* sin (*śīghra* anomaly) multiplied by the radius of the epicycle and divided by 360°, which is known as the *dohphala.*

Again, from the same similar right-angled triangles, we have *C'Q*/*C'M* equals *EL*/*EC'*, and *C'Q* equals *EL* multiplied by *MC'*/*C'E*. This equals *R* cos (the *śīghra* anomaly) multiplied by the radius of epicycle and divided by R, or 360°. This is known as the *kotiphala.*

Now, from Figure 1, we have the *sphuta-koti EQ* equalling *EC'* plus *C'Q*, which equates to *R* plus the *kotiphala*. The *kotiphala* is positive or negative according to whether the *śīghra* anomaly is in the fourth and first quadrants (i.e. between 270° and 90°) or in the second and third quadrants (i.e. from 90° to 270°). Then we have the

$$\text{\textit{śīghrakarna}}\ EM = \sqrt{EQ^2 + MQ^2} = \sqrt{(C'E + C'Q)^2 + MQ^2}, \quad (4)$$

which is the hypotenuse of the right-angled triangle *MEQ*.

From the similar triangles *EC'P* and *EMQ*, we have *C'P*/*C'E* equals *MQ*/*EM,* and therefore *C'P* equals *MQ* multiplied by *EC'*/*EM*, which equals to the *dohphala* multiplied by *R* and divided by the *śīghrakarna.* Then the *śīghrakarna*, arc *C'M'*, is the arc corresponding to *C'P* as *R* sin (*śīghra* anomaly). It is important to note that in the *Sūrya Siddhānta* the *śīghra* anomaly equals the *śīghrocco* minus the mean planet.

In the case of the superior planets viz, *Kuja, Guru* and *Śani*, their mean *śīghrocco* is the same as the mean longitude of the Sun. In the case of *Budha* and *Śukra*, their mean longitude is taken to be that of the Sun while their *śīghroccos* are special points. In the *Siddhāntic* texts while the revolutions of the other mean planets, in a *Kalpa* or a *Mahāyuga* are given, in the case of *Budha* and *Śukra*, the revolutions of their *śīghroccos* are given.

Thus, according to the *Sūrya Siddhānta*, for the superior planets (*Kuja*, *Guru* and *Śani*) the *śīghra* anomaly equals the mean Sun minus the mean planet, while in the case of the inferior planets (*Budha* and *Śukra*) the *śīghra* anomaly equals the planet's *śīghrocco* minus the mean Sun. In both the cases, we have *R* sin (*śīghra-phala*) equals (*r*/*k*) (*R* sin *m*), where *r* is the corrected radius of the *śīghra* epicycle of the planet, *k* is the *śīghra* hypotenuse (*śīghra-karna*) and *R* sin *m* is the Indian sine of the *śīghra* anomaly *m* of the planet. It is important to note that the radius of the *śīghra* epicycle is a variable even as in the case of the *manda* epicycle. The peripheries of the *śīghra* epicycles of the five star-planets are listed in Table 1, with their maxima in Table 2.

Traditional canonical texts like the *Sūrya Siddhānta* [*SS*] are based on the expression

$$\sin(SP) = \frac{p}{SKR}[R\sin(SK)] \quad (5)$$

where *SP* is the required *śīghraphala*, *p* is the *śīghraparidhi*, the periphery of the *śīghra* epicycle, *R* is 3438′ and *SKR* is the *śīghrakarna*. The *śīghra* hypotenuse is given by SKR^2, which equals the *sphuta koti* squared plus the square of the *dohphala*. If *r* equals *p* divided by 360°, then the *dohphala* equals $r[R\ \sin(SK)]$, the *kotiphala* equals $r[R\ \cos(SK)]$, and the *sphutakoti* equals $R + r[R\cos(SK)]$, or $R\,[1+r\cos(SK)]$. The *śīghrakarna SKR* is given by (6) below. In this case, SKR^2 equals the *dohphala* squared plus the square of the *sphuta koti*, which equals

$$R^2[\{r\sin(SK)\}^2 + \{1 + r\cos(SK)\}^2]$$
$$= R^2[r^2 + 2r\cos(SK) + 1], \text{ and}$$
$$\therefore SKR = R\sqrt{r^2 + 2r\cos(SK) + 1} \quad (6)$$

Substituting (ii) in (i), we get

$$\sin(SP) = \frac{(\text{r}\ \sin SK)}{SKR}$$
$$= \frac{(\text{r}\ \sin \text{SK})}{\sqrt{r^2 + 2r\cos(SK) + 1}}$$

so that the *śīghraphala,*

$$SP = \sin^{-1}\left[\frac{(r\ \sin SK)}{\sqrt{r^2+2r\ \cos(SK)+1}}\right] \quad (7)$$

The *śīghraphala* is additive or subtractive according to whether the *śīghrakendra*, *SK* is less than or greater than 180° (Balachandra Rao, 2000). In the case of *śīghra* correction, the *śīghraparidhi* (periphery) *p* is a variable given by p_e minus ($p_e - p_o$) multiplied by the sine of *SK*. The peripheries *p*, for different planets, at the ends of even and odd quadrants according to the *Sūrya Siddhānta* are given in Table 1.

Finding the *śīghraphala*, of *Kuja* etc. is explained here:

(1) By subtracting (the already obtained) *manda* corrected planet from its *śīghrocca* we get the *śīghrakendra* of the (*manda* corrected) planet.
(2) Find the *bhuja jyā* of the *śīghrakendra*.
(3) Consider the product 2 × *kotijyā* × *parākhya.*
(4) The result of (3) is added to or subtracted from the (*parākhya*)^2 according to whether the *śīghrakendra* is in I and IV quadrants or in II and III quadrants.
(5) Add (120)^2 i.e., 14400 to the result of (4) and take its square-root. This is called the *śīghrakarna.*
(6) Then the *parākhya* × *bhuja jyā* / *śīghrakarna* gives the *jyā* of the *śīghraphala.*
(7) The *dhanu* (or *cāpa*) i.e., the inverse of *jyā* of the result of (6) gives the required *śīghraphala.*

Table 1: The peripheries of the *śīghra* epicycle.

Planet	The peripheries of *śīghra* epicycle	
	At the end of an odd quadrant (*m* = 90° or 270°)	At the end of an even quadrant (*m* = 0° or 180°)
Kuja	232°	235°
Budha	132°	133°
Guru	72°	70°
Śukra	260°	262°
Śani	40°	39°

The *śīghraphala* is added to or subtracted from the *manda sphuta* of the planet according to whether the *śīghrakendra* is in the 1st and 2nd quadrants or in the 3rd and 4th quadrants. Thus, the *śīghrakarna* equals

$$= \sqrt{(par\bar{a}khya)^2 \pm 2\,parakhya\ \times .kotij\bar{a} + 14400}$$

Jyā (*śīghraphala*) = *parākhya* × *bhuja jyā* / *śīghrakarna.* (8)

The *śīghraphala* is the *cāpa* (*dhanu*) of the above.

2.1 Sengupta's Derivation

Following Brahmagupta's *Khaṇḍakhādyaka*, Sengupta (1941) has derived the following expression for the *śīghraphala* (*SP*) of a planet:

$$SP = \frac{m}{2} - \tan^{-1}\left[k\ \tan\frac{m}{2}\right] \quad (9)$$

where *m* is the *śīghra* anomaly of the planet and *k* is a constant for a planet based on the periphery of its *śīghra* epicycle. Let us denote (m/2) by x in (9) so that we have

$$S = SP = x - \tan^{-1}(k\ \tan x) \quad (10)$$

Differentiating with respect to x, we get

Table 2: Maximum *śīghraphalas*.

Maximum *śīghraphalas*	
Kuja (Mars)	42° 27′ 14″
Budha (Mercury)	21° 30′ 36″
Guru (Jupiter)	11° 30′ 23″
Śukra (Venus)	46° 28′ 7.8″
Śani (Saturn)	6° 13′ 9.29″

$$\frac{dS}{dx} = (1-k)(1-k\tan^2 x)/(1+k^2\tan^2 x) \quad (11)$$

For SP to be maximum, $\frac{dS}{dx} = 0$, so that

$$1 - k\tan^2 x = 0 \text{ or } \tan x = \frac{1}{\sqrt{k}}. \quad (12)$$

$$\text{Or } x = \tan^{-1}\left(\frac{1}{\sqrt{k}}\right) \text{ i.e., } m = 2\tan^{-1}\left(\frac{1}{\sqrt{k}}\right). \quad (13)$$

Substituting (13) in (9), we get

$$SP_{max} = \tan^{-1}\left(\frac{1}{\sqrt{k}}\right) - \tan^{-1}\sqrt{k}$$

$$= \tan^{-1}\left[\frac{1-k}{2\sqrt{k}}\right], \quad (14)$$

$$\text{and } m = 2\tan^{-1}\left[\frac{1-k}{2\sqrt{k}}\right]. \quad (15)$$

The values of k for the planets and corresponding maximum *śīghraphala (SP)* are listed in Table 3, where the *parākhyās* of *kuja* etc., are 81,44,23,87 and 13. These are the *śīghra-paridis* (i.e. the peripheries of the *śīghra* epicycles, with the periphery of the deferent circle as R is 120°). Usually in canonical texts like the *Sūrya Siddhānta* R is taken as 360°. For example, *kuja's śīghra* periphery ranges from 232° to 235°. Now taking R as 120°, for *kuja*, 77°.33 to 78°.33, Bhaskara II in his *Khanda Khadhyaka* (KK) takes *śīghraparidhi* r as 81°. Now *śīghrakendra* (sk) equals the *śīghrocco* minus the mean planet. For the superior planets *Kuja*, *Guru* and *Śani*, *śīghrocco* is the same as the mean position of the Sun (S) for the given time. The values of k for the planets and corresponding maximum *śīghraphala (SP)* are tabulated in Table 3.

Table 3: The values of k for the planets and corresponding maximum *śīghraphala (SP)*.

Planet	*parākhya*	*k*	SP_{max}
Kuja	81°	0.21212	40° 32′ 30″
Budha	44°	0.46341	21° 30′ 37″
Guru	23°	0.6666	11° 32′ 23″
Śukra	87°	0.16129	46° 14′ 17″
Śani	13°	0.8	6° 22′ 45.7″

In an Indian context, many astronomical tables belonging to different *pakṣas* (schools) are popular in different regions. These astronomical tables are named differently, belonging to different regions as *sārinis, padakas, koṣṭakas* and *vākyas.* The major genres of these Indian astronomical tables belonging to different *pakṣas* (schools) are namely the (1) *Saura* pakṣa, (2) *Ārya pakṣa*, (3) *Brāhma pakṣa* and (4) *Gaṇeśa pakṣa*, etc.

These compositions of tables are based on the major treatises by the great authors of the *Sūryasiddhānta* (the author of this text is unknown), the *Āryabhaṭīyam* of Āryabhaṭa (CE 499), the *Brahmasputasiddhānta* of Brahmagupta (CE 628) and the *Grahalāghavam* of Gaṇeśa Daivajña (CE 1520). We find many tables based on *saura pakṣa*, namely, the *Makaranda sāriṇī* (*MKS*) of Makaranda, the *Gaṇakānanda* (*GNK*) of Sūrya, the *Mahādevi* (*MH*) of Mahādeva, the *Pratibhāgi padakas* and *Tyagarti graha padakas* (these two manuscripts are from Karnataka). A popular astronomical table belonging to the *ārya pakṣa* is the *Vākyakaraṇa* by a legendry astronomer named Vararuci. The *Vākyakaraṇa* is followed in Kerala and Tamilnadu. The *Brahmatulya sāriṇī* and the *Karaṇakuhūla sāriṇī* belong to the *brāhma pakṣa* of Brahmagupta, but were based on Bhāskara-II's *Karaṇa Kutūhala* (e.g. see Sumatiharṣa, 1991).

In this paper we discuss the *śīghraphala* with examples as explained in the *Makaranda sāriṇī* (*MKS*), the *Vākyakaraṇa* and the *Karaṇa Kutuhūla Sāriṇī*.

3 THE *MAKARANDA SĀRIṆĪ*

The major tables in the *Makaranda sāriṇī* are for

(1) The ending moment of the *tithi, nakṣatra* and *yoga*.
(2) The mean longitudes of the Sun, the Moon and the five planets.
(3) The *mandaphala* (equation of centre) of each of the heavenly bodies.
(4) The *śīghraphala* (equation of conjunction) of the five planets.
(5) The moments of solar ingress (*saṅkramaṇa*) into zodiacal signs (*rāśis*).
(6) The Sun's declination (*krānti*).
(7) The latitude of the Moon (*śara, vikṣepa*).
(8) The angular diameters (*bimba*) of the Sun, the Moon and the Earth's shadow cone for the computations of eclipses.

According to the *Sūrya Siddhānta*, to find the true longitudes of the planets four corrections are applied to the mean planet viz, the half *śīghra*, half *manda,* full *manda* and full *śīghra* corrections. If *MP* is the mean planet then

$$P_1 = MP + \frac{SE1}{2},\ P_2 = P_1 + \frac{ME_1}{2}, \quad (16), (17)$$

$$P_3 = MP + ME_2,\ P_4 = P_3 + SE_2. \quad (18), (19)$$

But the important thing to be noted in the *Makaranda sāriṇī* is that the author has reduced the corrections only to three (instead of four *phala samskāras*) by combining the half *manda* and full *manda* correction together to the planet. Hence by discarding the 2nd step of the *Sūrya Siddhānta* the procedure followed in the *MKS* is half *śīghra*, full *manda* and full *śīghra* corrections:

$$P_1 = MP + \frac{SE1}{2},\ P_2 = MP + ME_1 \text{ and} \quad (20), (21)$$

$$P_3 = P_2 + SE_2 \quad (22)$$

According to Bhāskara-I (CE 629), the procedure to find the true planets, as explained in the *Mahābhāskarīya*, is as follows:

$P_1 = PMA$ (where *PMA* is the *mandocca* of the planet) (23)

$P_2 = P_1 + \frac{SE_1}{2}$, $P_3 = MP + ME_1$, $P_4 = P_3 + SE_2$ (24) (25), (26)

Makaranda had possibly followed Bhāskara-I by dispensing with the first step and retaining the remaining steps in his calculations. Even Gaṇeśa Daivajña in the sixteenth century had followed the same method for finding the true positions of planets in his famous composition *Grahalāghavam.*

In the original text the *dēśāntara* correction is given only for *Kāśi* and *Mitila*. Later, in 1831, Nārāyaṇa Daivajña in his commentary included the *dēśāntara* correction for all other places:

vaktrādikaṃ sthūlamidaṃ mayōktaṃ
sukhārthameveti na tad yathārthaṃ |
astōdayau spaṣṭatarau prasādhyau
siddhāntarītyā kusutādikānāṃ ||

As told by earlier astronomers, Makaranda (Uma and Balachandra Rao, 2018) also says that the tables given by him have to be corrected to get the exact positions for future periods. He had taken a rate of precession of the equinox per year as 54 *vikalās* (seconds), whereas Gaṇeśa Daivajña (Balachandra Rao and Uma, 2006) and others had taken 60 seconds or 1 minute as the rate of precession. In modern astronomy the rate of precession is 50 + 2/60sec = 50.033sec. So, the rate considered by Makaranda was close to the modern value.

Makaranda, being a poet, has used the words *valli* (creeper)*, vāṭīka* (an orchard)*, kanda* (root) etc. for the tables.

3.1 The Procedure to Find the *Kali Ahargaṇa* for 14 November 2013 According to the *Makaranda Sāriṇī*

In the calculation of the *ahargaṇa* (number of days) for a *cāndramāna* date (lunar calendar) of a year, Makaranda explains the conversion of the *śaka* year into the *kali* year, then to the *sauramāsa*, later to the *cāndramāsa*, then the number of days in *cāndramāsa* is reduced by deducting the *kṣaya dinas*. Here 703/11, i.e. 63.9090, is the improved value given by Makaranda over the value 64 in traditional texts. Calculation of the *ahargaṇa* is the same as in the case of the *Sūrya Siddhānta* to midnight of a day (Uma and Balachandra Rao, 2018).

3.1.1 Example 1: Finding the *Kali Ahargaṇa*

The date 14 November 2013 corresponds to a Thursday in *śaka* year 1935, *kārtīka, śuddha dwādaśi*, and

(1) The *kali* years = *śaka* year + 3179 = 1935 + 3179 = 5114.
(2) The *kali* years × 12 = *kalimāsas* = 5114 × 12 = 61368 *kali sauramāsas.*
(3) The number of months elapsed from *chaitra* to *kārtīka* = 7. Therefore, the total number of *saura māsas* elapsed is 61368 + 7 = 61375.
(4) 61375 ÷ 70 = 876.78 ≈ 876 *kṣaya māsas*.
(5) Adding *kṣaya māsas* to 61375, we get 62251.
6) 62551 ÷ 33 = 1886.39 ≈ 1886 (only the integer part) *adhika māsas.*
(7) Adding this to 61375, we get 63261.
(8) Multiply 63261 by 30 days = 1897830 days.
(9) The *Dwādaśi tithi* of *kārtīka* means 11 days were completed in that month. Hence the number of days elapsed = 1897830 + 11 = 1897841.
(10) Now the *ahargaṇa* for the date = 1897841 – (1897841 ÷ 703/11) = 1868145.

Here is an alternate method suggested by us to find the *ahargaṇa:*

(1) The *kali* years = Christian year + 3101 = 2013 + 3101 = 5114.
(2) Solar months (*sauramāsas*) = 5114 × 12 = 61368.
(3) The *Adhika māsas* = $\frac{1593336}{4320000} \times 5114$ = 1886.
(4) Now the number of completed lunar months (*Cāndra māsas*) = 61368 + 1886 + 7 (months elapsed from *chaitra* to *kārtīka*) = 63261
(5) Total number of *tithis* (days) = (63261 + *tithi*/30) × 29.590589 = 1868145. To ob-tain *valli* for the *ahargaṇa* 1868145, divide the *ahargaṇa* continuously by 60 then

$\frac{1868145}{60} \Rightarrow Q_1 = 31135$ and $R_1 = 45$

$\frac{31135}{60} \Rightarrow Q_2 = 518$ and $R_2 = 55$

$\frac{518}{60} \Rightarrow Q_3 = 8$ and $R_3 = 38$

$\frac{8}{60} \Rightarrow Q_4 = 0$ and $R_4 = 8$.

Here Q_1, Q_2, Q_3 and Q_4 are quotients and R_1, R_2, R_3 and R_4 are the remainders in the respective cases. Now only the remainders are considered for the computation as *valli* (i.e, *valli* is 45/55/38/8).

3.1.2 Example 2: Finding the True Sun from the *Ravi Vātika* Tables

In order to find the True Sun from *Ravi vātika* tables, the rate of motion for each component of *valli* 45/55/38/8 is considered using Figure 2 as follows:

To find the True Sun, the epochal year is

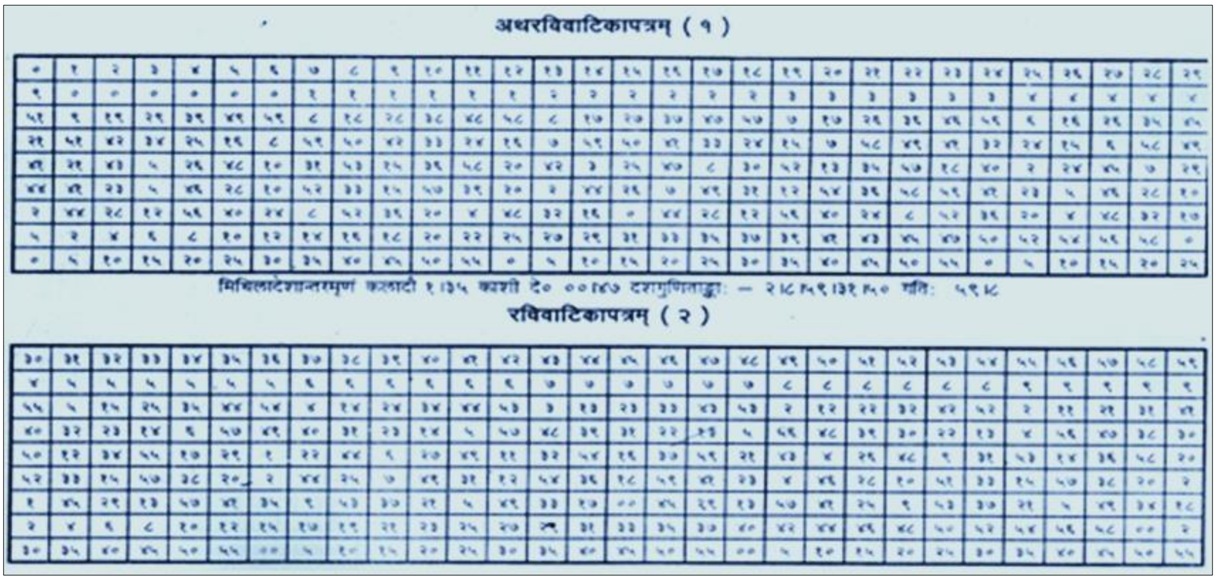


Figure 2: *The Ravi vāṭika*, a folio from a manuscript (the false colour marginally improves legibility).

śaka 1651 + 78, which is CE 1729. The epochal *mandocca* is 2^s 17° 17′ 09″ and the *dhruvāṅka* is 9. For the given date 14 November 2013, the difference in years is 2013–1729, or 284. Divide the difference in years by the *dhruvaṅka* 9 which gives 31″ 33‴. Adding this to the epochal *mandocca*, the *Ravi mandocca* for the given year is 2^s 17° 17′ 09″ plus 31″ 33‴, or 77° 17′ 40″. The *mandakendra* (*mk*) equals the *mandocca* minus the mean planet, therefore 77° 17′ 40″ minus 208° 49′ 27″ plus 360°, which equals 228° 10′ 33‴ for >180°. Therefore, the *bhuja* of *mk* is 48° 10′ 33″.

From the *manda* tables, for 48° ⇒ 1/37/35 and for 49° ⇒ 1/39/6. The difference between these values is 0/1/31 ⇒ 0° 1′ 31″. Multiply the difference by the remaining *bhuja* (10′ 33″ × 0° 1′ 31″), which equals 0° 0′ 16″. Adding this to the value of 48, 1/37/35 + 0° 0′ 16″, which gives 1° 37′ 51″. Thus, the *mandaphala* (equation of centre) is 1° 37′ 51″. The True Sun equals the Mean Sun ± *mandaphala.* If the *mandakendra* >180° then the *mandaphala* is subtracted from the mean Sun or if the *mandakendra* <180° then the *mandaphala* is added to the mean Sun. In this case it is >180°, so the True Sun equals the Mean Sun minus the *mandaphala.* The True Sun is 208° 49′ 27″ minus 1° 37′ 51″. Therefore, the True Sun is 207° 11′ 36″.

3.1.3 Example 3: Finding the Mean Mars from *Kuja Vātika* Tables

Now to find mean Mars (*Kuja*) from the above *Kuja vātika* tables, consider the rate of motion for each component of *valli* 45/55/38/8 for the given date 14 November 2013. From the *Kuja vātika* tables (Figure 3) we find that mean Mars is 104° 8′ 54″.

3.2 Finding True Longitude of Mars

Although the author of the *MKS* follows the text of the *SS* (*Saurapakṣa*), instead of four corrections, he applies only three *phalasam-skāra*'s to the mean planets, as shown in the following steps:

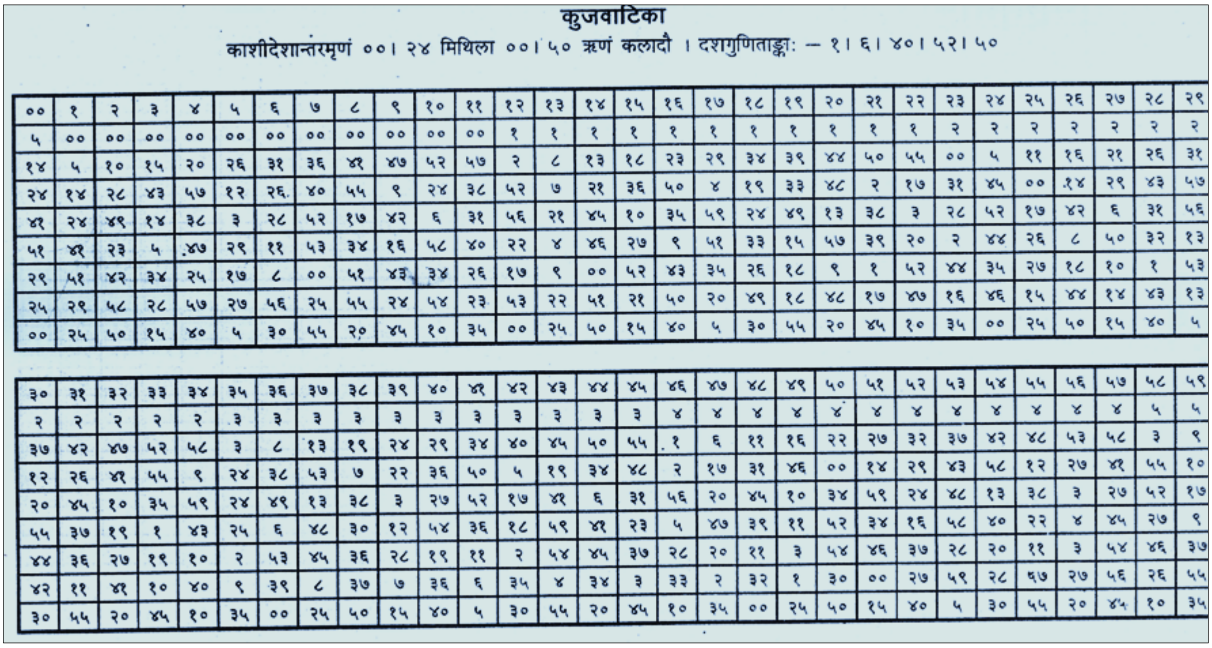
$P_1 = MP + (SE_1)/2$
$P_2 = MP + ME_1$
$P_3 = P_2 + SE_2$

Note that the mean Sun is considered as the *śīghrocca* for the superior planets, whereas for the interior planets it is *vice versa* (this means that the mean Sun is considered as the mean planet for superior planets and the mean planet is considered as the *śīghrocca* for the interior planets).

For the first correction (half-*śīghra* correction), the *śīghrakendra* (sk_1) equals the Mean Sun (*śīghrocca*) minus the Mean Planet. So sk_1 equals 208° 49′ 27″ minus 104° 08′ 54″, or 104° 40′ 52″, for <180°. The *śīghraphala* (SE_1) equals 36° 42′ 09″. Thus, the first corrected Mars equals the mean planet + $\frac{1}{2}$ (SE_1), i.e. 104° 08′ 54″ plus $\frac{1}{2}$ (36° 42′ 09″). So P_1 is 122° 29′ 58″.

For the second correction (the full-*manda* correction), the *mandakendra* (mk_1) equals the *mandocca* minus the first corrected Mars (P_1). The *mandaphala* (ME_1) equals 1° 33′ 35″. Thus, the second corrected Mars equals the mean planet plus ME_1, or 104° 08′ 54″ + 1° 33′ 35″. So P_2 is 105° 42′ 29″.

For the third correction (the full-*śīghra* correction), the *śīghrakendra* (sk_2) equals the *śīghrocca* minus P_2, or 208° 49′ 27″ minus 105° 42′ 29″, or 103° 06′ 58″, for >180°. The *śīghraphala* (SE_2) equals 36° 19′ 55″. Thus, the third



कुजवाटिका

काशीदेशान्तरमृणं ००। २४ मिथिला ००। ५० ऋणं कलादौ । दशगुणिताङ्काः — १। ६। ४०। ५२। ५०

००	१	२	३	४	५	६	७	८	९	१०	११	१२	१३	१४	१५	१६	१७	१८	१९	२०	२१	२२	२३	२४	२५	२६	२७	२८	२९
५	००	००	००	००	००	००	००	००	००	००	००	१	१	१	१	१	१	१	१	१	१	१	२	२	२	२	२	२	२
१४	५	१०	१५	२०	२६	३१	३६	४१	४७	५२	५७	२	८	१३	१८	२३	२९	३४	३९	४४	५०	५५	००	५	११	१६	२१	२६	३१
२४	१४	२८	४३	५७	१२	२६	४०	५५	९	२४	३८	५२	७	२१	३६	५०	४	१९	३३	४८	२	१७	३१	४५	००	२४	२९	४३	५७
४२	२४	४९	१४	३८	३	२८	५२	१७	४२	६	३१	५६	२१	४५	१०	३५	५९	२४	४९	१३	३८	३	२८	५२	१७	४२	६	३१	५६
५१	४१	२३	५	४७	२९	११	५३	३४	१६	५८	४०	२२	४	४६	२७	९	५१	३३	१५	५७	३९	२०	२	४४	२६	८	५०	३२	१३
२९	५१	४२	३४	२५	१७	८	००	५१	४३	३४	२६	१७	९	००	५२	४३	३५	२६	१८	९	१	५२	४४	३५	२७	१८	१०	१	५३
२५	२९	५८	२८	५७	२७	५६	२५	५५	२४	५४	२३	५३	२२	५१	२१	५०	२०	४९	१८	४८	१७	४७	१६	४६	१५	४४	१४	४३	१३
००	२५	५०	१५	४०	५	३०	५५	२०	४५	१०	३५	००	२५	५०	१५	४०	५	३०	५५	२०	४५	१०	३५	००	२५	५०	१५	४०	५

३०	३१	३२	३३	३४	३५	३६	३७	३८	३९	४०	४१	४२	४३	४४	४५	४६	४७	४८	४९	५०	५१	५२	५३	५४	५५	५६	५७	५८	५९
२	२	२	२	२	३	३	३	३	३	३	३	३	३	३	३	४	४	४	४	४	४	४	४	४	४	४	४	५	५
३७	४२	४७	५२	५८	३	८	१३	१९	२४	२९	३४	४०	४५	५०	५५	१	६	११	१६	२२	२७	३२	३७	४२	४८	५३	५८	३	९
१२	२६	४१	५५	९	२४	३८	५३	७	२२	३६	५०	५	१९	३४	४८	२	१७	३१	४६	००	१४	२९	४३	५८	१२	२७	४१	५५	१०
२०	४५	१०	३५	५९	२४	४९	१३	३८	३	२७	५२	१७	४१	६	३१	५६	२०	४५	१०	३४	५९	२४	४८	१३	३८	३	२७	५२	१७
५५	३७	१९	१	४३	२५	६	४८	३०	१२	५४	३६	१८	५९	४१	२३	५	४७	२९	११	५२	३४	१६	५८	४०	२२	४	४५	२७	९
४४	३६	२७	१९	१०	२	५३	४५	३६	२८	१९	११	२	५४	४५	३७	२८	२०	११	३	५४	४६	३७	२८	२०	११	३	५४	४६	३७
४२	११	४१	१०	४०	९	३९	८	३७	७	३६	६	३५	४	३४	३	३३	२	३२	१	३०	००	२९	५९	२८	५७	२७	५६	२६	५५
३०	५५	२०	४५	१०	३५	००	२५	५०	१५	४०	५	३०	५५	२०	४५	१०	३५	००	२५	५०	१५	४०	५	३०	५५	२०	४५	१०	३५

Figure 3: A folio of *Kuja vātika* tables from a manuscript in the *Makarandasāriṇī* (the false colour improves legibility).

Table 4: The *Mandaphala* of the planets in arc minutes according to the *MKS* and Equation (27).

mk (°)	*Kuja* (Mars)		*Budha* (Mercury)		*Guru* (Jupiter)		*Śukra* (Venus)		*Śani* (Saturn)	
	MKS	Formula	*MKS*	Formula	*MKS*	Formula	*MKS*	Formula	*MKS*	Formula
10°	111	123.53	47	49.17	52	54.44	19	19.61	76	80.97
20°	219	241.80	92	95.76	102	106.68	38	38.08	150	158.97
30°	320	351.53	134	138.50	149	155.23	54	54.91	219	231.75
40°	414	449.80	171	176.33	192	198.73	69	69.72	282	297.19
50°	498	533.97	203	208.37	209	235.98	81	82.19	338	353.46
60°	570	601.82	230	233.95	260	266.01	92	92.09	385	398.96
70°	627	651.60	250	252.56	283	288.03	99	99.26	422	432.40
80°	667	677.10	262	263.86	299	301.46	103	103.60	447	452.85
90°	689	687.55	267	267.65	305	305.98	105	105.06	459	459.74

corrected Mars equals P_2 plus SE_2, or 105° 42′ 29″ plus 36° 19′ 55″. So P_3 is 142° 02′ 24″. Therefore, the true longitude of Mars is 142° 02′ 24″.

According to modern astronomy, the true longitude of Mars (*Kuja*) is 143° 05′. This difference is due to the *ayanāmśa* (the accumulated precession of the equinox).

3.2.1 Algorithms Adopted in the *Makarandasāriṇī*

In finding the true longitudes of the Sun and the Moon only one major correction, the *mandaphala* (equation of centre) is applied (Rupa et al., 2013). But for the five planets besides the *manda* correction one more correction, the *śīghraphala* (equation of conjunction), is applied (Rupa et al, 2014). The *mandaphala* of a heavenly body is given by the classical expression sin (*MP*) equals (p ÷ r) multiplied by sin (*mk*), where *MP* is the *mandaphala*, *mk* is the *mandakendra* (anomaly from the apogee), *p* is the *mandaparidhi* (periphery of the related epicycle) of the concerned body and *r* is 360°, the periphery of the deferent circle. Since the periphery of planets is varying according to the *Sūrya Siddhānta,* it is maximum at the end of an even quadrant (i.e. for *mk* equals 0° and 180°) and minimum at the end of an odd quadrant (i.e. for *mk* equals 90° and 270°). If the peripheries at the ends of even and odd quadrants are denoted by p_e and p_o, then variable periphery is given by

$p = [p_e - (p_e - p_o)]\ |\sin(mk)|$ Thus $MP = \sin^{-1}\left[\frac{p}{r}\sin(mk)\right]$ (27)

Table 4 shows the values of the *mandaphala* of the planets according to *MKS* and Equation (27). Among all the heavenly bodies the periphery of the planet Mars is large, hence its *mandaphala* is also comparatively larger than other planets, it is shown in Figure 4, the variation of the *mandaphala* (*MP*) with the *mandakendra* (*mk*) (as per the formula) is shown graphically for the five planets. The behaviour of the graphs is sinusoidal with *MP* equals 0 for *mk* at 0° and 180° and reaching the maximum at *mk* equals 90°. But in the *MKS*

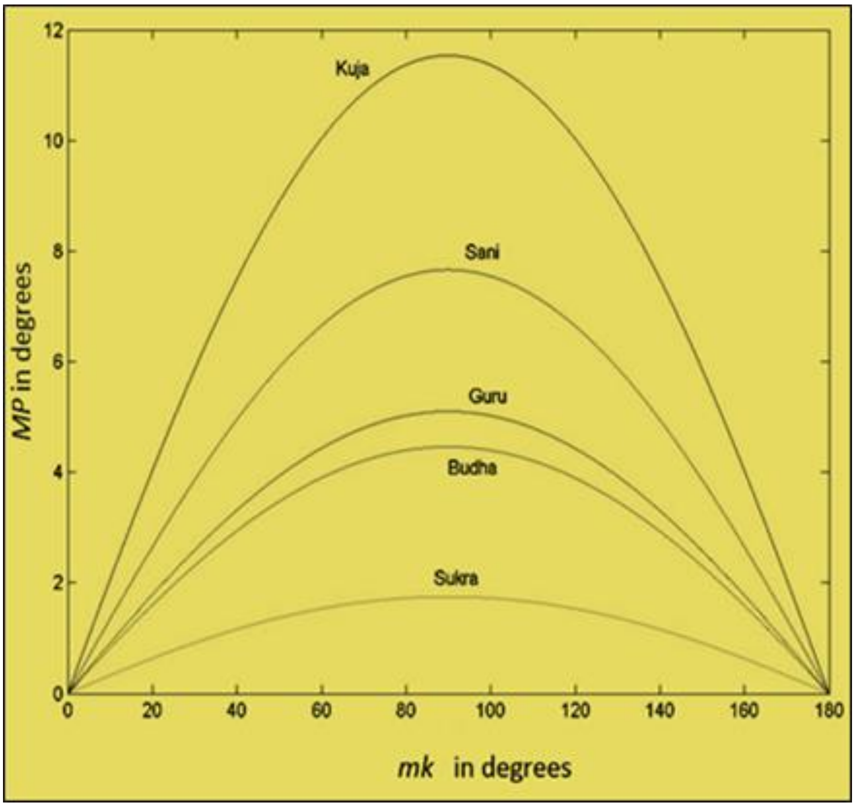


Figure 4: Variation of the *mandaphala* (equation of centre) of planets against the *mandakendra* (anomaly from the apogee) (plot: P. Venugopal).

Table 5: The *śīghraphalas* of the planets according to the *MKS* and Equation (28).

Sk (°)	*Kuja* (Mars)		*Budha* (Mercury)		*Guru* (Jupiter)		*Śukra* (Venus)		*Śani* (Saturn)	
	MKS	Formula	*MKS*	Formula	*MKS*	Formula	*MKS*	Formula	*MKS*	Formula
15°	05° 54′	05° 54′ 13″	04° 02′	04° 01′ 30″	02° 26′	02° 26′ 28″	06° 18′	06° 18′ 15″	01° 29′	1° 27′ 45″
30°	11° 44′	11° 43′ 52″	07° 47′	07° 56′ 39″	04° 49′	04° 48′ 51″	12° 33′	12° 33′ 14″	02° 52′	2° 52′ 05″
45°	17° 26′	17° 25′ 28″	11° 40′	11° 39′ 14″	07° 00′	07° 00′ 46″	18° 43′	18° 42′ 13″	04° 07′	4° 08′ 17″
60°	22° 54′	22° 54′ 40″	15° 02′	15° 02′ 00″	08° 55′	08° 55′ 06″	24° 43′	24° 41′ 46″	05° 12′	5° 11′ 32″
75°	28° 05′	28° 05′ 20″	17° 55′	17° 55′ 44″	10° 24′	10° 23′ 55″	30° 27′	30° 27′ 01″	05° 57′	5° 57′ 03″
90°	32° 49′	32° 47′ 58″	20° 05′	20° 08′ 10″	11° 18′	11° 18′ 35″	35° 52′	35° 50′ 15″	06° 20′	6° 20′ 24″

the *mandaphala* (*MP*) is at a maximum for the values of *mk* slightly greater than 90°. As discussed earlier for finding the true planets, apart from the *manda* correction, the *śīghra* correction is also applied. The classical procedure to find the *śīghraphala* is based on the expression sin (*SP*) equals [*p* ÷ (360 multiplied by SKR)] and multiplied by R sin(sk), where, *SP* is the required *śīghraphala, P* is the *śīghraparidhi* (the periphery of *śīghra* epicycle), *R* is 3438′ and *SKR* is the *śīghrakaraṇa* (the *śīghra* hypotenuse) given by the equation

$$SKR = R\sqrt{r^2 + 2r\cos(sk) + 1}\text{, where } r = \frac{P}{360°}.$$

$$\text{Therefore } SP = \sin^{-1}\left[\frac{r \sin(sk)}{\sqrt{r^2+2r\cos(sk)+1}}\right] \qquad (28)$$

The *śīghraparidhis P* for Mars, Mercury and Venus are greater at the end of even quadrants (*sk* = 0°, 180°) than at the odd quadrants (*sk* = 90°, 270°). But it is *vice versa* for Jupiter and Saturn. In Table 5 the values of the *śīghraphala* of the planets are listed according to the *MKS* and Equation (28).

Among the five planets, Venus has the maximum *śīghraparidhi* and hence its *śīghraphala* is large compared to the other planets. The variation of the *śīghraphala* (equation of conjunction) of the planets against the *śīghra* anomaly (*sk*) is shown graphically for five planets in Figure 5.

4 *VĀKYA* TABLES

The *Vākyakaraṇa* is an astronomical text composed by Vararuci in early thirteenth century. It is the most popularly used text to construct almanacs in the southern parts of India, with the commentary by Sundararāja. The *vākya* tables belong to the *āryapakṣa,* based on the parameters and procedures in the *Āryabhaṭīyam* and mainly on the works of Bhāskara's '*Mahābhaskarīya*'. The *vākyas* are given in the form of *kaṭapayādi* notations, a system of letter numer-

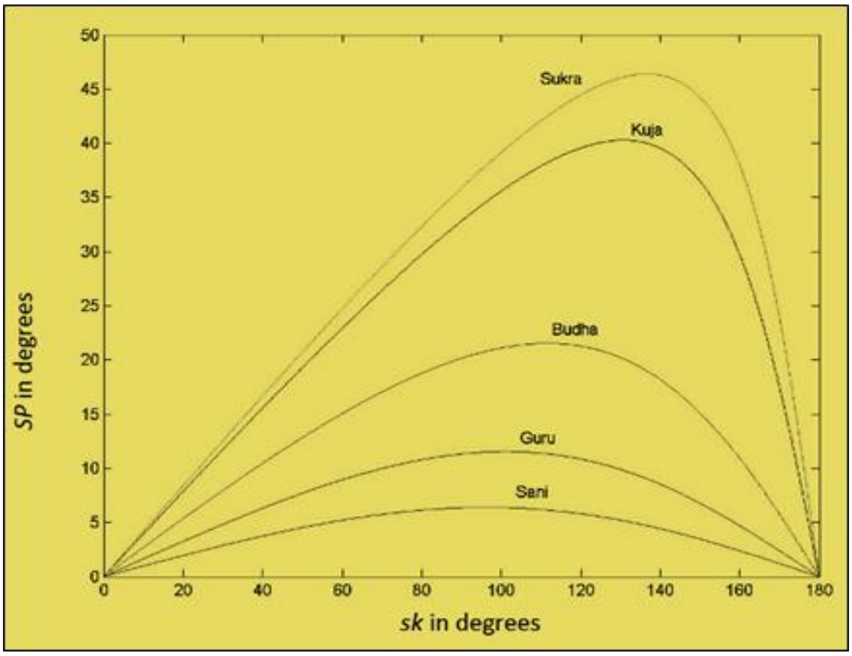


Figure 5: Variation of the *śīghraphala* (equation of conjunction) of the planets against the *śīghra* anomaly (diagram: P. Venugopal).

als. This text consists of five astronomical chapters which are very useful to the *pañcāṅga*-makers when they need to find the positions of the heavenly bodies in order to perform rituals. The five chapters are

(1) The True Positions of the Sun, the Moon and the *Rāhu*
(2) The True Positions of five Planets
(3) Problems Involving Time, Position and Direction
(4) The Eclipses
(5) Heliacal Rising and Setting and Parallel Aspects (*Mahāpatas*).

The *vākya* tables from the commentary in the *Laghuprakāśikā* by Sundararāja are studied and examined for the true positions of heavenly bodies given the Christian date, and are approximately close to those values obtained by modern methods (see Table 6). The salient feature of this text is expressing the anomalistic revolutions in terms of integral numbers (Table 7).

4.1 The Procedure for Computing the True Positions of the Sun, the Moon and the Planets

Chapter (1) deals with positions of the Sun and the Moon having only one correction known as the *manda* correction (equation of centre). To calculate the positions of the Sun and the Moon, *kali* days are required which are nothing but the number of days elapsed from the beginning of the *Kaliyuga* (3102 BC). The *Kaliyuga* is said to have begun on Friday.

4.1.1 The Procedure for Finding the *Kali Ahargaṇa* According to the *Vākyakaraṇa*

Multiply the *kali* years elapsed by 365 and add 1/4th of the years (with *nāḍikās* i.e. 60 *nāḍi* = 1 day) then multiply the sum by 5, and later deduct 1237 from it. Afterwards dividing it by 576 and adding it to the sum obtained, it gives the *kali* days of the true *sankramaṇa* (the beginning of true solar year) from the mean sunrise (Kuppanna Sastri and Sarma, 1962).

Here is an example of how to find the *kali* days for the beginning of true solar year in 2013, and determination of the *Meṣa Saṅkramaṇa*:

(1) *Kali* years elapsed = 2013 + 3101 = 5114
(2) 5114 × 365 = 1866610
(3) 5114 ÷ 4 = 1278.5
(4) Adding the values of steps (2) and (3) = 1866610 + 1278.5 = 1867888.5
(5) 5114 × 5 = 25570
(6) Deducting 1237 from the value of step (5) which is equal to 24333
(7) 24333 / 576 = 42.24479167 = 42 days 14 *nāḍis* 41 *vināḍis*
(8) Adding the values of steps (4) and (7) = 1867930.745 = 1867930 days 44 *nāḍis* 41 *vināḍis* = 1867930 days 17h 52m 24s

Step (8) gives the true *Meṣa Saṅkramaṇa* from the mean sunrise. To find the week day of this true *Meṣa Saṅkramaṇa,* add 1 to the *Kali* days obtained. i.e. 1867930 + 1 = 1867931 days and then dividing this by 7 the remainder obtained is 2, which means that the second day after the

Table 6: Sidereal periods in days.

Bodies	*Āryabhaṭīyam*	*Vākya Karaṇa*	Modern
Ravi (Sun)	365.258681	365.258681	365.256360
Chandra (Moon)	27.3216685	27.321679	27.3216604
Chandra's mandocca	3231.98708	3232.62522	3232.58853
Rahu	6794.74951	6792.3600	6793.45994
Kuja (Mars)	686.99974	686.98699	686.97985
Budha (Mercury)	87.969880	87.968084	87.969254
Guru (Jupiter)	4332.2722	4332.7788	4332.5889
Śukra (Venus)	224.69814	224.70249	224.70080
Śani (Saturn)	10766.065	10764.748	10759.227

beginning of *kali* week day (i.e. Friday). Therefore, the True *Meṣa Saṅkranthi* occurred on Sunday i.e. the new true solar year commenced on 14 April 2013 at 44 *nāḍis* 41 *vināḍis* (17 h 52m 24s) after mean sunrise. Mean sunrise at Ujjain was at 6h 27m a.m (IST). Therefore, the True *Meṣa Sankramaṇa* occurred at 6h 27m + 17h 52m 24s = 24h 19m 24s on 14 April 2013.

4.2 Finding the True Longitude of the Sun for 27 November 2013

Now from the *kali ahargaṇa* tables, the *kali* days for the date 27 November 2013 is 1868158. The Approximate Sun equals the *kali* days of the given date minus the *Meṣa Saṅkramaṇa*, which equals 1868158 minus 1867930.745, or 227° 15′ 19″. Now, to get the True Sun, apply the correction to the above Approximate Sun in the form of mnemonics (*vākyas*), given in minutes which are listed below: (1) 14, (2) 32, (3) 54, (4) 78, (5) 105, (6) 133, (7) 163, (8) 194, (9) 224, (10) 254, (11) 284, (12) 311, (13) 335, (14) 358, (15) 376, (16) 391, (17) 403, (18) 411, (19) 415, (20) 416, (21) 412, (22) 406, (23) 398, (24) 386, (25) 374, (26) 361, (27) 347, (28) 334, (29) 322, (30) 311, (31) 303, (32) 297, (33) 295, 295, (34) 296, (35) 301, (36) 309 and (37) 322.

To find the mnemonics for the Approximate Sun at 227° 15′ 19″, first consider the degree part and divide it by 10, which gives the completed mnemonics. For example; 227 ÷ 10 = 22.7, which means that 22 full mnemonics are completed. 230° means the 23rd mnemonic = 398′. 220° means the 22nd mnemonic = 406′. The difference = –8′ for 10° i.e. (230° – 220°).

Table 7: Number of revolutions in a *Mahāyuga* (civil days = 157,79,17,500).

Bodies	*Āryabhaṭīyam*	*Vākya Karaṇa*
Ravi	43,20,000	43,20,000
Chandra	5,77,53,336	5,77,53,315
Chandra's Mandocca	4,88,219	4,88,122.6
Rahu	2,32,226	2,32,307.7
Kuja	22,96,824	22,96,866
Budha	1,79,37,020	1,79,37,378
Guru	3,64,224	3,64,181
Śukra	70,22,388	70,22,252
Śani	1,46,564	1,46,582

Then the following correction is applied:

$406' - \left(\frac{-8' \times 7^0\, 15'\, 19''}{10}\right)$, which equals 6° 40′ 12″.

So, the True Sun equals the Approximate Sun minus the correction, i.e. 227° 15′ 19″ minus 6° 40′ 12″, or 220° 35′ 07″.

4.3 Finding the True Longitude of the Moon

The maximum equation of centre employed in the table for the Moon is 301′, which is same as that of the *ārya pakṣa*, and almost the same as that of the Hindu *Siddhāntas*. The algorithm for the computation of the true Moon is as follows:

(1) Find the *kali* days for the given day.

(2) Subtract the *śodhya* 16, 00, 984 from the *kali* days, and the remainder is called the *śeṣa*. There are three divisors: d_1 is 12372, d_2 is 3031 and d_3 is 248. When the *śeṣa* is divided by d_1, the quotient is q_1 and the remainder is r_1. After this r_1 is divided by d_2 and q_2 is the quotient and r_2 the remainder, and finally r_2 is divided by d_3 to get the quotient q_3 and the remainder r_3. This last remainder r_3 is always less than or equal to 248. This represents the '*vākya* number of the Moon' (which ranges from 1–248 in the *Candra Vākyas*).

(3) Multiply the quotients q_1, q_2 and q_3 respectively by 9^s 27° 48′ 10″, 11^s 7° 31′ 01″ and 0^s 27° 44′ 06″. The sum of these products is added to 7^s 02° 00′ 07″, which is the Moon's *Dhruva.* Add to this sum the value of the mnemonics (*vākyas*) from the *Candra Vākya*s, which is the uncorrected true position of the Moon.

(4) Multiply q_2 by 8 and deduct this from (q_3 × 32) – (8 × q_2). This must be taken as the *vināḍis*. From the true daily motion of the Moon in degrees deduct the mean motion 13° 11′ and multiply this difference by the above *vināḍis*. The result will be in seconds of arc (*vikalās*). This should be applied to the above obtained uncorrected true Moon. The final result gives the true Moon at the mean sunrise at Ujjain.

For an example from the *Candra Vākya*s see Figure 6.

3.1 Example: Finding the Longitude of the True Moon for 27 November 2013

The algorithm for determining the Mooṅ's longitude according to the *Vākyakaraṇa* (VK) is as follows:

The *kali* day for 27 November 2013 is 1868158. Deduct 1600984 (called the *śodhya*, i.e. the number to be subtracted) from the *kali* days (elapsed for the given date). This value is called the *śesa* (the remainder). There are three divisors: d1 = 12372, d2 = 3031 and d3 = 248. Dividing 267174 by d1 we get quotient q1 as 21, and the remainder, r1, is 7362. Dividing r1 (7362) by d2 (3031), we get quotient q2 as 2, and the remainder, r2, is 1300. Dividing r2 (1300) by d3 (248) we get quotient q3 as 5, and the remainder is r3 (60). This last remainder (r3) is always less than or equal to 248. This represents the '*vākya* number of the Moon' (and ranges between 1 and 248 in the *Candra Vākya*). Multiplying the quotients q1 (21), q2 (2) and q3 (5) respectively by 9^s 27° 48′ 10″, 11^s 07° 31′ 01″ and 0^s 27° 44′ 06″ we get the following:

21 × 9^s 27° 48′ 10″ = 4^s 13° 51′ 30″
2 × 11^s 07° 31′ 01″ = 10^s 15° 02′ 22″
5 × 0^s 27° 44′ 06″ = 4^s 18° 40′ 30″

The *dhruva* = 7^s 02° 00′ 07″
(Epochal position) total: 26^s 19° 34′ 29″

Note: 1^s = 1 sign = 1 *rāśi* = 30°

The above is mathematically represented as:
kali days for 27 November 2013 = 1868158
1868158
–1600984
12372) 267174 (21 × 9^s 27° 48′ 10″
–259812
3010) 7362 (2 × 11^s 07° 31′ 01″
–6062
248) 1300 (5 × 0^s 27° 44′ 06″
–1240
vākya Number 60 value for this number from the tables is 2^s 06° 05′
Note: *kali* days are the numbers of days elapsed since the epoch of the *Kaliyuga*. The epoch is taken as 17/18 February 3102 BCE.
Adding the total *dhruva* to the *vākya* (sentence) number we get:
26^s 19° 34′ 29″ + 2^s 06° 05′ = 28^s 25° 39′ 29″
Removing the multiples of 12 *raśis*:
24^s
4^s 25° 39′ 29″

Note: 12 *rāśis* = 360°
The Uncorrected True Moon is 4^s 25° 39′ 29″.
Now, the correction to get the True Moon is

$(32 \times q_3) - (8 \times q_2) = (32 \times 5) - (8 \times 2)$
= 144′ = 2° 24′.

The daily motion for the *kali* day ending 1868158 is the difference between the values of 61[st] and 60[th] *vākya* numbers respectively, i.e. 2^s 18° 52′– 2^s 06° 05′ = 12° 47′.
Deducting the mean daily motion of the Moon (13° 11′) from the difference yields –0° 24′. Meanwhile, the correction is
[2° 24′ × –0° 24′] ÷ 60 = –0° 24′ 58″.
So, the True Moon equals the Uncorrected True Moon minus the Correction:
4^s 25° 39′ 29″ – 0° 0′ 58″.
Therefore, the True Moon is
4^s 25° 38′ 31″ = 145° 38′ 31″


24-51 APPENDIX ii १२६

Days	Vakya	r	o	,
	द्रुमा धन्या नये	10	19	52
25	इष्टं राज्ञः कुर्यात्	11	2	10
	धन्या विधेयं स्यात्	11	14	19
	त्वं रक्षा राज्यस्य	11	26	24
	क्षेत्रज्ञः	0	8	26
	नीले नेत्रे	0	20	30
30	जलं प्राज्ञाय	1	2	38
	शशी वन्द्यः स्यात्	1	14	55
	गोरसप्रियः	1	27	23
	वनानि यत्र	2	10	4
	अन्नं गोत्रश्रीः	2	23	0
35	रुष्टास्ते नागाः	3	6	12
	धिगन्धः किल	3	19	39
	पुरोगा अभीः	4	3	21
	मान्यः स कविः	4	17	15
	अरिष्टनाशम्	5	1	20
40	बालो मे केशः	5	15	33
	कुशधारिणः	5	29	51
	इष्टिर्विधते	6	14	10
	स राजा प्रीतः	6	28	27
	सुगुप्तायोऽसौ	7	12	37
45	धिगस्तु ह्रासः	7	26	39
	अङ्गानि यदा	8	10	30
	सेनावान् राजा	8	24	7
	धीराः सन्नद्धाः	9	7	29
	शालीनं प्रधानम्	9	20	35
50	क्षीरं गोर्नो नयेत्	10	3	26
	रत्नचयो नृपः	10	16	2

Figure 6: *Candra vākya* folio from the *Vākyakaraṇa* (the false colour improves legibility).

4.4 The True Positions of the Five Planets

In the *vākya* tables true positions of the five planets are constructed based on the synodic periods. The synodic periods of these planets are listed in Table 8.

In the case of these five planets two equations have to be applied to make them true, the

Table 8: The synodic periods of the planets.

Planet	*Vākyakaraṇa*	Modern
Kuja	779.93745	779.936102
Budha	115.87517	115.877478
Guru	398.88521	398.884048
Śukra	583.92687	583.921367
Śani	378.08757	378.091902

Table 9: *Mandoccas* of the Sun and planets.

Body	*Vākyakaraṇa* and *Āryabhaṭīya*	Modern
Ravi	78°	77° 15′
Kuja	118°	128° 28′
Budha	210°	234° 11′
Guru	180°	170° 22′
Śukra	90°	290° 04′
Śani	236°	243° 40′

equation of centre (*mandaphala*) and the equation of conjunction (*śīghraphala*). Of these the first corresponds to finding the heliocentric longitude of the planet in modern astronomy and depends upon the anomalistic revolution. The second corresponds to converting the heliocentric into geocentric longitude, and depends on the synodic revolution. The Āryabhaṭan school considered that the apses of the planets were fixed (the higher apses *mandoccas*), and even in the *Vākyakaraṇa* the same reasoning was followed, as listed in Table 9.

If a time is chosen when a synodic revolution starts from this higher apsis (*mandocca*), then values can be tabulated for fixed days successively, until the time the synodic period begins at the higher apsis. This table can be used over and over again to find the true planet. But since this is a tedious procedure, a table for a period containing an integral number of a few synodic revolutions is provided such that at the end of the period the planet is sufficiently near the higher apsis. To make a correction in the tabular values, and to get the true planet, a correction factor called *the dhruva* is defined as the new position minus the longitude of the higher apsis.

As the new position is the starting point of the table, the *dhruva* must be added to the tabular value, which itself has to be corrected for the change in the equation of centre corresponding to the change in the mean anomaly caused by the *dhruva*. The period, for which the values are tabulated, is the *maṇḍala*. Using larger *maṇḍalas*, the *dhruva* can be made smaller and smaller. To get the true positions of the heavenly bodies the peripheries of the *manda* and *śīghra* epicycles are required which are listed in Table 10.

According to the *Āryabhaṭīyam*, the corrections to get a true planet are as follows: for the exterior planets, (1) the half-*manda,* (2) the half-*śīghra*, (3) the *manda*, and (4) the *śīghra* corrections, while for the interior planets they are (1) the half-*śīghra,* (2) and the *manda* and (3) *śīghra* corrections.

The algorithm to find the true position of the planets for a given date involves

(1) Finding the *kali* days for a given day.
(2) Deducing the *śodhya* from the *kali* days: divide the remainder by the respective *maṇḍalas* (any *maṇḍala* or *maṇḍalas* may be used for any number of times), and note the quotients. Divide the remainder by the respective synodic cycle of days. The quotients are cycles (*parivṛtta*) completed, and the remainder is the number of days in the next cycle.
(3) Finding the total *dhruva* (see Tables 11–15), which are multiplied by the respective quotients.
(4) Using the *vākya* of that cycle, taking values for the maximum number of days that are provided in *vākya* tables for an interval.
(5) Finding the motion for the remaining days by interpolation.
(6) Finding the difference of the interval, and divide by the number of interval days, which gives the daily motion of the planet. It is retrograde if the next value is less than the previous one. Then the daily motion is multiplied by the remaining days.
(7) Then adding (3), (4), (5) and (6) to get the true planet.

Note that the *vākya* of the last day of a cycle is that of the 0 day of the next cycle. The *vākya* of the 0 (zero) day of the first cycle is the respective *mandocca*. In the case of planet Mars, the last, *maṇḍala* is 11,699 minus 04 days and

Table 10: *Manda* and *śīghra* peripheries according to the *Āryabhaṭīyam*.

Body	*Mandaparidhi*		*Śīghraparidhi*	
	End of odd quadrant	End of even quadrant	End of odd quadrant	End of even quadrant
Ravi	13° 30′	13° 30′	—	—
Chandra	31° 30′	31° 30′	—	—
Kuja	81°	63°	229° 30′	238° 30′
Budha	22° 30′	31° 30′	85° 30′	130° 30′
Guru	36°	31° 30′	67° 30′	72°
Śukra	9°	18°	256° 30′	265° 30′
Śani	58° 30′	40° 30′	36°	40° 30′

the *dhruva* is +638′. The 4th *maṇḍala* is 17,158 minus 37 days with a *dhruva* of –504′. By adding these two values a third *maṇḍala* is obtained. Similarly, this can be continued until the *maṇḍala* (i.e. 6,43,089 minus 09 days with *dhruva* +4) is obtained. Then comes the *śodhya* which is the *kali* days at which the planet must be sufficiently close to the higher apsis, and a synodic revolution must begin, as in the case of a *maṇḍala* (see Table 16). The *śodhya* need not contain an integral number of synodic revolutions, for example the *śodhya* for Mars is 15,52,827 minus 37 days with a *dhruva* of –402′. If after one *maṇḍala* the *dhruva* moves x′ away from the apsis then after two *maṇḍala* it will move 2x′ away, and so on.

4.4.1 Example 1: Finding the Longitude of the True Mars for 27 November 2013.

In Classical Indian Astronomy the concept of a *yuga* is involved which meant a period of 5 years in the *Vedāṅga Jyotiṣa* (the earliest Indian astronomical text), but in later works, a *yuga* meant a large period of time.

Table 11: Days and *dhruva* for *Kuja* (Mars).

	Days (*nādis*)	*Dhruva* (min)
Śodhya	15,52,827-35	–402′
Maṇḍala 1	6,34,089-09	+4′
Maṇḍala 2	1,32,589-21	+27′
Maṇḍala 3	28,857-41	+133′
Maṇḍala 4	17,158-37	–504′
Maṇḍala 5	11,699-04	+638′
780		

Table 12: Days and *dhruva* for *Budha* (Mercury).

	Days (*nādis*)	*Dhruva* (min)
Śodhya	15,92,740-22	–32′
Maṇḍala 1	16,801-54	–01′
Maṇḍala 2	4,750-53	+149′
Maṇḍala 3	2,549-15	–447′
116		

In the traditional reckoning one *Mahāyuga* of 43,20,000 years comprises four *Yugas* viz. *Kṛta, Tretā* , *Dvāpara* and *Kali.* But Āryabhaṭa (499 CE) took them all to be of equal duration, namely 10,80,000 years, and he called them *Yugapādas*. Most the Indian astronomers take the epochal date of a *Kaliyuga* as beginning at midnight on the evening of 17/18 February 3120 BCE (by Julian reckoning). The number of days elapsed since the *Kaliyuga* began up until a chosen day is referred to as *kali* days.

The algorithm to find the true position of planets for a given date involves seven steps:

(1) Find the *kali* days for a given day.

(2) Deduct the *śodhya* (the value to be subtracted) from the *kali days*. Divide the remainder by the respective *maṇḍalas* (a number represented in days and *nādis*—an Indian unit of time) and note the quotients. Divide the remainder by the respective synodic cycle of days. The quotient is the cycles (*parivṛtta*) completed, and the remainder is the number of days in the next cycles.

(3) Find the total *dhruva* (a correction factor, which is multiplied by the respective quotients).

(4) Using the *vākya* of that cycle, take values for the maximum number of days provided in *vākya* tables for an interval.

(5) Find the motion for the remaining days by interpolation.

(6) Find the difference of the interval, and divide by the number of interval days, which gives the daily motion of the planet. It is retrograde if the next value is less than the previous one. Then the daily motion is multiplied by the remaining days.

(7) For the True Planet add (3), (4), (5) and (6).

Table 13: Days and *dhruva* for *Guru* (Jupiter).

	Days (*nādis*)	*Dhruva* (min)
Śodhya	15,70,425-17	–261′
Maṇḍala 1	4,74,875-27	+0′
Maṇḍala 2	1,25,648-50	–9′
Maṇḍala 3	65,018-17	+133′
Maṇḍala 4	30,315-17	–71′
Maṇḍala 5	21,539-48	–619′
Maṇḍala 6	4,387-44	+274′
399		

Table 14: Days and *dhruva* for *Śukra* (Venus).

	Days (*nādis*)	*Dhruva* (min)
Śodhya	15,61,937-44	+17′
Maṇḍala 1	4,37,945-09	–0′
Maṇḍala 2	1,74,594-08	+29′
Maṇḍala 3	88,756-53	–58′
Maṇḍala 4	44,962-23	+2103′
Maṇḍala 5	2,919-38	–144′
584		

Table 15: Days and *dhruva* for *Śani* (Saturn).

	Days (*nādis*)	*Dhruva* (min)
Śodhya	15,89,474-28	–326′
Maṇḍala 1	5,70,534-08	+5′
Maṇḍala 2	1,82,994-23	–13′
Maṇḍala 3	21,551-00	+43′
Maṇḍala 4	10,964-32	+401′
378		

Table 16: The *śodhya*, *maṇḍalas* and *dhruvas* of the Sun, Moon and five planets.

Body	*Vākya*	Indian ephemeris
Ravi	$7^s 10° 35′ 07″$	$7^s 10° 50′ 29″$
Chandra	$4^s 25° 38′ 31″$	$4^s 24° 19′ 29″$
Kuja	$4^s 28° 18′ 56″$	$5^s 00° 13′ 00″$
Budha	$6^s 27° 34′ 51″$	$6^s 23° 50′ 00″$
Guru	$2^s 26° 55′ 47″$	$2^s 25° 49′ 00″$
Śukra	$8^s 24° 53′ 35″$	$8^s 24° 39′ 00″$
Śani	$6^s 25° 03′ 34′$	$6^s 22° 37′ 00″$

Note that the *vākya* of the last day of a cycle is that of the 0 day of the next cycle. The *vākya* of the 0(zero) day of the first cycle is the respective *mandocca* (or aphelion, in modern terminology).

In the case of Mars, the last *maṇḍala* is 11,699-04 days and the *dhruva* is +638′. The 4^{th} *maṇḍala* is 17,158-37 days with a *dhruva* of –504′. By adding these two values the 3^{rd} *maṇḍala* is obtained. Similarly, this can be continued until the 1^{st} *maṇḍala*, i.e. 6,43,089-09 days with a *dhruva* of +4 is obtained. Then comes the *śodhya* (value to be subtracted) which is the *kali* days at which the planet must be sufficiently close to the higher apsis, and a synodic revolution must begin, as in the case of the *maṇḍala*. The *śodhya* need not contain an integral nuber of synodic revolutions, for example the *śodhya* for Mars is 15,52,827-37 days with the *dhruva* –402′. If after one *maṇḍala*, the *dhruva* moves x′ away from the apsis then after two *maṇḍalas* it would move 2x′ away, and so on.

Finding the true longitude of Mars for the date 27 November 2013.

Kali days: 1868158-00 *Dhruva*
Śodhya : –1552827-35 –402′
132589-21) 315220-25 (2 × 27′) = 54″
265178-42
28857-41) 50151-43 (1 ×133') = –133′
28857-41
17158-37) 21294-02 (1 × 504′) = –504′
17158-37
Total *Dhruva* –719′
= –11° 59′ 00″
780) 4135-25(5
3900-00
Vākya No: 235-25

This means that 5 cycles (*parivṛttis*) are completed and in the 6^{th} cycle find the motion for 235-25 days. Now from the 6^{th} *parivṛtti*, for 230 days the motion of Mars is 5^s 5° 40′ and the correction (–9′). The next value given is for 250 days the motion is 5^s 15° 56′ and the correction (–10′):

The correction for 230 days = 9′ × 11′ 59″ = 1° 47′ 51″

The correction for 250 days = 10′ × 11′ 59″ = 1° 59′ 50″

Motion for 230 days: 5^s 5° 40′ + 1° 47′ 51″ = 5^s 07° 27′ 51″

Motion for 250 days: 5^s 15° 56′ + 1° 59′ 50″ = 5^s 17° 55′ 50″

The motion for 20 (i.e., 250–230) days = (5^s 07° 27′ 51″) – (5^s 17° 55′ 50″) = 10° 27′ 59″

The motion per day, i.e. daily motion = 10° 27′ 59″/20 = 0° 31′ 24"

The motion for 5 days 25 *nādis* = 5° 25′ × 0° 31′ 24″ = 2° 50′ 5″

True Mars = Total *Dhruva* + Motion for 230 days + Motion for 5 days 25 *nādis*
= –11° 59′ 00″ 5^s + 07° 27′ 51″ + 2° 50′ 5″
= 4^s 28° 18′ 56″. The longitude of True Mars is therefore 148° 18′ 56″. (Note that 60 *nādis* = 24 hours.)

According to modern astronomy the longitude is 150° 14′. The difference between this figure and the *vākya* value is due to the accumulated precession of equinox.

Similarly, the true longitudes of the other four planets can be computed for the date 27 November 2013., which are listed in Table 3. According to the v*ākya* values, those the true planets were computed for the mean sunrise at Ujjain and are comparable to those in Indian ephemeris that are given for 5–30 am (IST).

It is truly a remarkable accomplishment of the *Vākya* System that the true position of each planet is given in simple sentences, so that a student of Sanskrit language can easily memorize these verses or sentences. The user of this *Vākya* System is saved time because the elaborate procedure of determining the *manda* and *śīghraphalas* repeatedly to obtain the true position of a planet is dispensed with. Especially for the *pañcāṅga*-makers the *Vākya* System is a boon because the herculean task of finding the true position for each day of the succeeding year is made simpler. Significantly, the *Vākya* System scores over other Indian astronomical tables. These *vākya* tables need to be reconstructed by updating the parameters. So, there is a necessity of revising these *vākyas* and also the Sanskrit sentences.

5 THE *KARAṆA KUTŪHALA SĀRIṆĪ*

The tables in the *Karaṇa kutūhala sāriṇī* (*KKS*) are based on the twelfth-century *Karaṇa kutūhala* (*KK*) tables of Bhāskara-II. These tables are also known as the *Brahmatulya sāriṇī*. The epoch and the parameters used to construct the tables are the same as in the *Karaṇa kutūhala.* The author and period of construction are not known, but the manuscripts of these tables are available in some of the libraries of the Oriental Research Institutes in India (Figures 7 and 8).

The name *Brahmatulya* means "… equal or corresponding to the *Brāhma …*" i.e., the *Brāhmapakṣa* School of Astronomy adhered to by Bhāskara II, which follows the parameters of the *Brahma sphuṭa sinddhānta* of Brahmagupta (CE 628). The *Brahmatulya sāriṇī* tables record *Brāhmapakṣa*-derived values of planetary mean motions with orbital and geographical corrections for computing their true motions for a given terrestrial location, topics which are addressed in Chapters 1 and 2 in the *Karaṇa kutūhala*

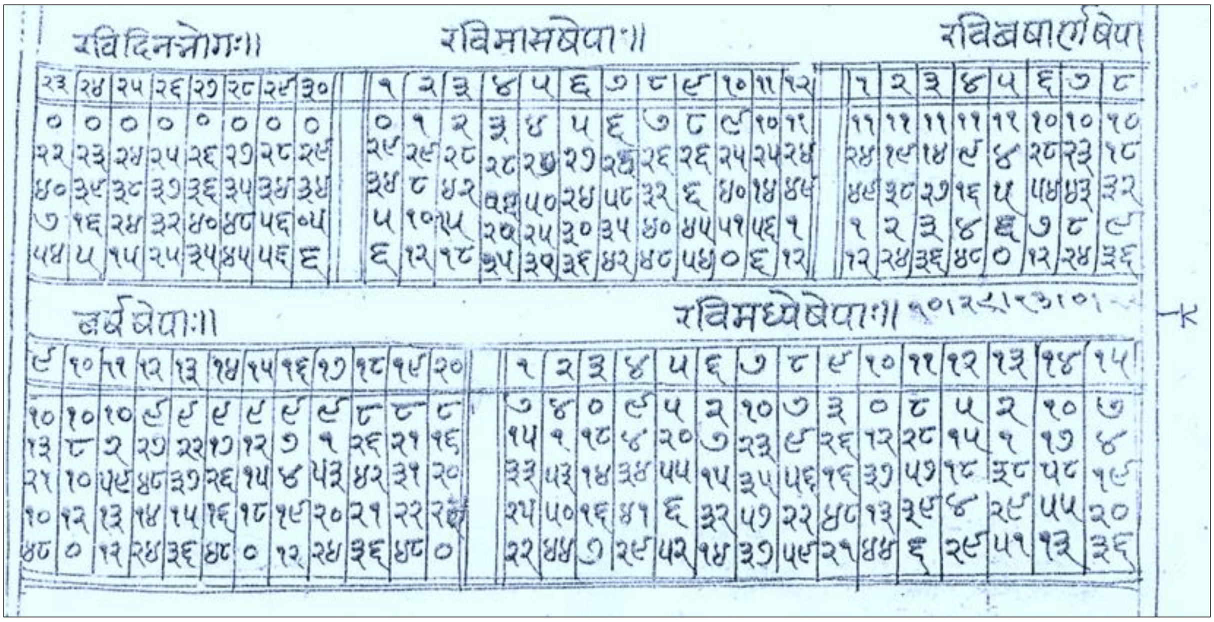


Figure 7: Mean motion table of the Sun, a folio of *Karaṇa kutūhala sāriṇī* (the false colour improves legibility).

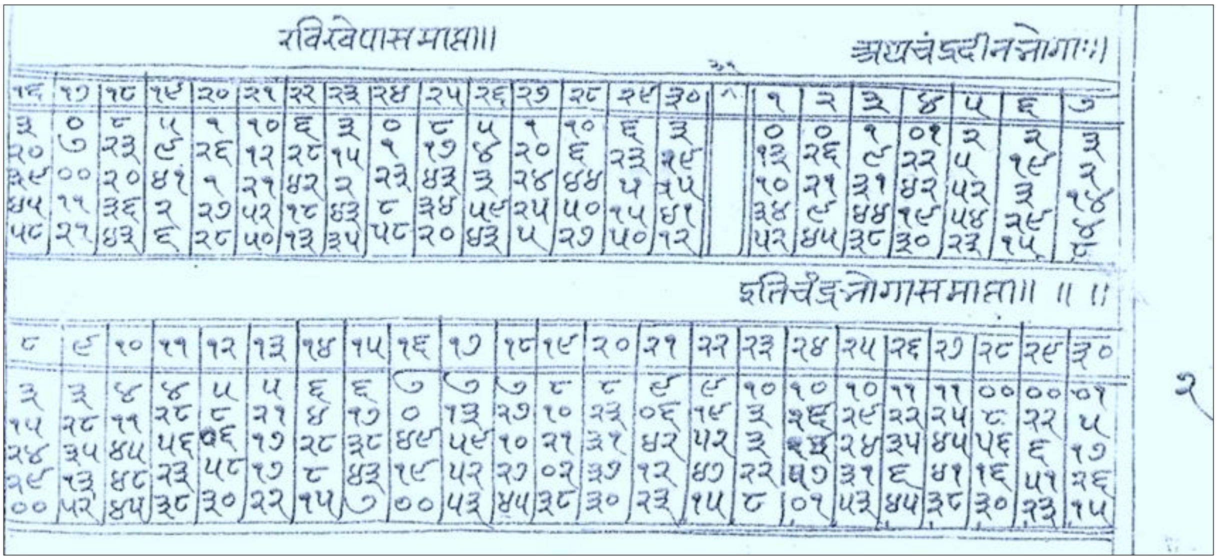


Figure 8: Daily motion table of the Moon for 30 days, a folio of *Karaṇa kutūhala sāriṇī* (false colour improves legibility).

hala (see Balachandra Rao and Uma, 2007–2008).

There are at least five extant manuscripts of the tables of the *Karaṇa kutūhala sāriṇī,* some with expository details in table headers and marginal notes. To study tables in the *Karaṇa kutūhala sāriṇī,* the manuscript 501/1895-1902 from the Bombay Oriental Research Institute was referred to.

The tables consist of

(1) The mean motion tables.
(2) The *mandaphala* tables or table of the equation of centre of the Sun for 0° to 90°.
(3) The table of solar declination and of lunar latitude for 0° to 90°.
(4) The *mandaphala* tables for the planets (the table of the equation of centre for 0° to 90°).
(5) The *śīghraphala* tables for the planets (the table of the equation of the conjunction for 0° to 180°).

The mean motion tables are given for 1 to 30 days, then for 1 to 12 months, then for 1 to 20 years and later the table is extended for 1 to 30 periods of 20 years each. This method of giving the motion for the period of 20 years is unique in the *Karaṇa kutūhala sāriṇī.* Although it is mentioned that the *Karaṇa kutūhala sāriṇī* is also known as the *Brahmatulya sāriṇī,* there is one more *Brahmatulya sāriṇī* in which the mean motion tables are given for 1 to 60 periods of 20

Table 17: Mean daily motions according to the *KKS.*

Bodies	Mean daily motion
Ravi	0° 59′ 8″ 10‴12 ^{iv}40^{v}
Chandra	13° 10′ 34″ 52‴ 31 ^{iv} 50^{v}
Kuja	0° 31′ 26″ 28″ ′09 ^{iv}50^{v}
Budha's conjunction	4° 5′ 32″ 21‴ 1 ^{iv}0^{v}
Guru	0° 4′ 59″ 8‴ 54 ^{iv}0^{v}
Śukra's conjunction	1° 36′ 7″ 43‴ 49 ^{iv}50^{v}
Śani	0° 2′ 0″ 23‴ 3 ^{iv}30^{v}
Chandra's mandocca	0° 6′ 40″ 53‴ 50 ^{iv} 10^{v}
Rahu	–0° 3′ 10″ 48‴25 ^{iv}30^{v}

Table 18: Epochal values according to the *KKS.*

Bodies	Epochal values according to the *KKS*
Ravi	329° 13' 00"
Chandra	329° 05' 50"
Kuja	231° 24' 21"
Budha's conjunction	81° 14' 30"
Guru	64° 00' 51"
Śukra's conjunction	258° 05' 55"
Śani	123° 43' 17"
Chandra's mandocca	135° 12' 59"
Rahu	287° 25' 09"

years each instead for 1 to 30 periods of 20 years each as in *Karaṇa kutūhala sāriṇī* (that means it is given over 1200 years). According to the *Karaṇa kutūhala sāriṇī*, the mean daily motions are as in Table 17.

In Table 17, the motions are given for sub seconds, sub-sub seconds, so that a mean daily motion of a body can be obtained for any day correct to a second. The epochal positions given in the *Sāriṇī* (Table 18) indicate the date 24 February 1183; it is the same epoch of the *Karaṇa kutūhala.* In the *Karaṇa kutūhala,* Bhāskara-II has adopted the mean sunrise on 24 February 1183 CE as the epoch for computation.

5.1 The True Positions of the Sun, the Moon and the Five Planets

In the *Karaṇa kutūhala,* Bhāskara-II elaborately explains the method of obtaining the true positions of planets. Ś*loka* 3 in the *Spaṣṭādhikāra* gives the *manda* and *śīghra* anomalies of planets and positivity and negativity of *manda* and *śīghraphalas:*

grahonamuccaṃmṛducañcalaṃca
kendrebhavetāṃmṛducañcalākhye|
tribhistribhirbhaiḥpadamatrakalpyaṃ
svarṇaṃphalaṃmeṣatulādikendre ||3||
(Daivajña, 1913).

The planet subtracted from the *ucca* of the *manda* and the *śīghra* are respectively the *manda* and *śīghrakendras*. Considering each group of three *rāśis* as a quadrant, the result is positive or negative according to whether the quadrant is from *Meṣa* or *Tulā* (0° to 180°).

This *śloka* tells about the *mandakendra* and the *śīgrakendras* of the planets and positivity and negativity of the *manda* and *śīgraphalas*:

(1) *Mandakendra* (*m*) = *mandocca* – Mean planet *śīghrakendra* (*m*) = *śīghrocca* – Mean Planet.
(2) If 0° < *kendra* < 180° then the *phala* is positive and if 180° < *kendra* < 360° the *phala* is negative.
(3) *śloka* 4 gives the *bhuja* and *koṭi* of *kendra* as follows:
(4) *bhuja* = *kendra* if *kendra* < 90°
(5) *bhuja* = 180° – *kendra* if 90° < *kendra* < 180°
(6) *bhuja* = *kendra* – 180° if 180° < *kendra* < 270°
(7) *bhuja* = 360° – *kendra* if 270° < *kendra* < 360°
(8) *koṭi* = 90° – *bhuja* (in all cases)

From the tables of *mandaphala* we can notice that the equation of centre for the Sun, the Moon and for the five planets are given from 0° to 90°. The maximum equation of centre (*mandaphala)* for the bodies is listed in Table 19.

According to the *KKS*, the maximum *mandaphala* for Mercury is at 88°. This is very important to note because in the main text, the *KK*, the maximum *mandaphala* of Mercury is listed as 90°.

From the tables of *śīghraphala* (the equation of the conjunction) of the five planets, the maximum equation of the conjunction is listed in Table 20. In the *Karaṇa kutūhala* Bhāskara-II has explained the method to compute true planets with *manda* and *śīghra samskāras,* whereas to find the true Sun and the true Moon only the *manda samskāra* is discussed, as explained below:

(1) If P is the mean planet then $P_1 = P + ME_1$, where ME_1 = *manda* equation for P.
(2) The first *śīghra* corrected planet $P_2 = P_1 + SE_1$, where *śīghra* equation for P_1.
(3) The second *manda* corrected planet $P_3 = P + ME_2$, where ME_2= *manda* equation for P_2.

Table 19: Maximum equation of centre of the bodies.

Bodies	Maximum *mandaphala* according to *the KKS*	Maximum *mandaphala* according to *the KK* is at 90°
Ravi	02° 10′ 54″ at 90°	02° 10′ 30″
Chandra	05° 02′ 31″ at 90°	05° 01′ 45″
Kuga	11° 12′ 53″ at 90°	11° 08′ 30″
Budha	06° 25′ 25″ at 88°	06° 02′ 52″
Guru	05° 15′ 47″ at 90°	05° 15′ 30″
Śukra	01° 31′ 50″ at 90°	01° 45′ 02″
Śani	07° 38′ 35″ at 90°	07° 57′ 27″

(4) The second *śīghra* corrected planet $P_4 = P_3 + SE_2$, where SE_2 = *śīghra* equation for P_3 .
Now P_4 is the true planet.

Further, in the *KK,* a still more accurate correction is given for finding true Mars.

(1) If P is the mean planet then $P_1 = P + \left(\frac{1}{2}\right)ME_1$

(2) The first *śīghra* corrected planet $P_2 = P_1 + \left(\frac{1}{2}\right)SE_1$

(3) The second *manda* corrected planet $P_3 = P + ME_2$

(4) The second *śīghra* corrected planet $P_4 = P_3 + SE_2$

Now P_4 is the True Mars.

Finding the true positions of the Sun, the Moon and the Mars by using the *Karaṇa kutūhala* and the tables in the *Karaṇa kutūhala sāriṇī* (Figure 9) for the date 10 January 2016 are discussed in following section.

5.2 Finding the True Positions of the Sun, the Moon and the Mars According to the *Karaṇa Kutūhala* for 10 January 2016

The *kali ahargaṇa* for 10 January 2016 is 1868932, the *kali ahargaṇa* for the epoch 24 February 1183 is 1564737 and the difference in days is 304195. Therefore, the *KK ahargaṇa* is 304195.

To find the Mean Sun and the True Sun according to the *KK*, the Mean Sun equals $\left(1 - \frac{13}{903}\right) A + K$, where *A* is the *KK ahargaṇa. K equals Kṣepaka,* or 10^R^ 29° 13′ for the Sun, which equals = $\left(\frac{890}{903}\right)$ 304195 +10^R^ 29° 13′, or 264° 53′ 12″. The *mandakendra* (*mk*) equals the *mandocca* minus the Mean Sun, or 78° minus 264° 53′ 12″ plus 360°. This equals 173° 06′

Figure 20: Maximum equation of conjunction of the bodies.

Bodies	Maximum *śīghraphala* according to the *KKS*	Maximum *śīghraphala* according to the *KK*
Kuja	41° 18′ 16″ at 130°	42° 27′ 14″
Budha	21° 37′ 11″ at 110°	21° 30′ 36″
Guru	10° 59′ 01″ at 100°	11° 03′ 00″
Śukra	46° 18″ 41″ at 130°	46° 28′ 08″
Śani	06° 10′ 24″ at 100°	06° 13′ 10″

48″, and since this is <180° the *MP* is positive. The *bhuja* equals 180°minus the *mandakendra* if 90° < *m* < 180°, which equals 6° 53′ 12″. The *jyā* equals R sin (*mk),* which equals 14.38869. The *mandaphala* (*MP*) is 14.38869 multiplied by 10 and divided by 550, or 0° 15′ 42″. The True Sun equals the Mean Sun plus MP, or 264° 53′ 12″ plus 0° 15′ 42″. Therefore, the True Sun is 265° 08′ 54″.

To find the Mean Moon and the True Moon according to the *KK*, the Mean Moon equals $\left(14 - \frac{14}{17} - \frac{1}{8600}\right) A + K$, where *K* is 10^R^ 29° 05′ 50″*.* This equals 13.17635431 multiplied by 304195 plus 10^R^ 29° 05′ 50″, which equals 270° 11′ 46″. The *mandocca* of the Moon is $\left(\frac{1}{9} + \frac{1}{4012}\right) A + K$, where *K* equals 4^R^ 15° 12′ 59″, or 0.111360363 multiplied by 304195 plus 4^R^ 15°12′ 59″, which equals 170° 28′ 56″. The *mandakendra* (*mk*) equals the *mandocca* minus the Mean Moon, which equals170° 28′ 56″ minus 270° 11′ 46″ plus 360°. This is 260° 17′ 10″, and since this is >180° the MP is negative. The *bhuja* equals the *kendra* minus 180° if 180° < *kendra* < 270°, which equals 80° 17′ 10″. *Jyā* equals R sin (*mk*)*,* or 120 sin (80° 17′ 10″), which equals 118.2795117. The *mandaphala* (*MP*) is 118.2795117 multiplied by 10 and

Figure 9: The *śīghraphala* table of Mars, a folio of *Karaṇa kutūhala sāriṇī* (the false colour improves legibility).

divided by 238, or –4° 28′ 11″. The True Moon equals the Mean Moon plus MP, or 270° 11′ 46″ minus 4° 28′ 11″. Therefore, the True Moon is 265° 13′ 35″.

To find the Mean Mars and the True Mars according to the *KK*, the Mean Mars (*P*) equals $\left(\frac{11}{21}+\frac{11}{52444}\right)A+K$, where *K* is 7^R 21° 24′ 21″ and *P* is 155° 26′ 53″. In the first operation the *mandocca* of Mars = 128° 30′, the *mandakendra* (mk_1) equals the *mandocca* minus the Mean Mars, which equals 333° 03′ 07″, which is >180°. The *bhuja* equals 360° minus mk_1 if 270° < *kendra* < 360°, which equals 26° 56′ 53″. The *jya* (*bhuja*) equals *R* sin (*bhuja*), or 120° multiplied by sin (26° 56′ 53″), and ME_1 equals the *jya (bhuja)* multiplied by 10 and divided by 107. This equals 5° 04′ 56″. Since P_1 equals *P* minus $\left(\frac{1}{2}\right)$ ME_1, P_1 is 152° 54′ 24″.

In the second operation, the *śīghrakendra* (sk_1) equals the *śīghrocca* minus P_1 (where the Mean Sun is the *śīghrocca* for the exterior planets). sk_1 equals 264° 53′ 12′′ minus 152° 54′ 24″, or 111° 58′ 47″, which is <180°. The *bhuja* equals 180° minus the *śīghrakendra* if 90° < *m* < 180°, which equals 68° 01′ 13″. The *jya* (*bhuja*) equals *R* sin (*bhuja*), or 111° 16′ 40″. The *śīghrakaraṇa* equals $\sqrt{p^2-2pR\cos(bhuja)+120^2}$, or 116° 58′ 59″, where *p* is 81 and *R* is 120. The *śīghraphala* (SE_1) equals $\sin^{-1}\left(\frac{p\ Jy\bar{a}(Bhuja)}{R\ \acute{s}\bar{\imath}ghrakara\d{n}a}\right)$, and is 39° 56′ 49″. Since P_2 equals P_1 plus $\left(\frac{1}{2}\right)SE_1$, or 152° 54′ 24″ plus $\left(\frac{1}{2}\right)$ (39° 56′ 49″), P_2 equals 172° 52′ 48″.

In the third operation the *mandakendra* (mk_2) equals the *mandocca* minus P_2, or 128° 30′ minus 172° 52′ 48″ plus 360°. This equals 315° 37′ 12″, which is >180°. The *bhuja* equals 360° minus mk_2, if 270° < *kendra* < 360°, or 44° 22′ 48″. The *jya* (*bhuja*) equals *R* sin (*bhuja*) and is 120° sin (44° 22′ 48″). ME_2 equals the *jya (bhuja)* multiplied by 10 and divided by 107, which is 7° 50′ 38″. P_3 equals *P* minus ME_2, which equals 155° 26′ 53″ minus 7° 50′ 38″, or 147° 36′ 15″.

In the fourth operation, the *śīghrakendra* (sk_2) equals the *śīghrocca minus* P_3, and sk_2 equals 264° 53′ 12″ minus 147° 36′ 15″, or 117° 16′ 57″ (which is <180°). The *bhuja* equals 180° minus the *śīghrakendra* if 90° < *m* < 180°, and equals 62° 43′ 03″. The *jyā* (*bhuja*) equals *R* sin (*bhuja*), which is 106° 39′ 03″. The *śīghrakaraṇa* = $(\sqrt{p^2-2pR\cos(bhuja)+120^2})$, and equals 109° 46′ 23″. The *śīghraphala* (SE_2) equals $\sin^{-1}\left(\frac{p\ Jy\bar{a}(Bhuja)}{R\ \acute{s}\bar{\imath}ghrakara\d{n}a}\right)$, and is 40° 58′ 49″. P_4 equals P_3 plus SE_2, or 147° 36′ 15″ plus 40° 58′ 49″. Thus, the position of True Mars is 188° 35′ 04″.

5.3 The True Positions of the Sun, the Moon and the Mars According to the *Karaṇa Kutūhala Sāriṇī* for 10 January 2016

For 10 January 2016, now converting the *KK ahargaṇa* or 304195 days into periods of 20 years then to years, months and days, we get 42 periods, 4 years, 11 months, 25 days. Note that in the *Karaṇa kutūhala sāriṇī,* the *kṣepaka* (*K*) value is added to the values of the periods of 20 years. So, adding *K* again is not necessary, as it directly gives the mean position.

To find the Mean and the True Sun from the *Karaṇa kutūhala sāriṇī* tables the mean Sun is as follows. The motion for 30 periods is 3^R 19° 25′ 41″ 12‴, where 1^R = 30°. The motion for 12 periods is 5^R 15°18′ 04″ 29‴. The motion for 4 years is 1^R 09° 13′ 04′ ′48‴. The motion for 11 months is 10^R 25° 14′ 56″ 06‴. The motion for 25 days is 0^R 24° 38′ 24″ 15‴ by adding all of these and removing the cycles of 12 *rāśīs*. The Mean Sun equals 8^R 23° 50' 10" 50''' or 263° 50′ 10″ 50‴. The *mandocca* of the Sun is 78°, and the *mandakendra* (*m*) equals the *mandocca* minus the Mean Sun, which equals 78° minus 263° 50′ 10″ 50‴ plus 360°, or 174° 09′ 50″ (which is <180°). The *bhuj*a equals 180° minus the *mandakendra,* if 90° < *m* < 180°, and is 5° 50′ 10″. From the *Ravi manda* tables, (the *mandaphala* (*MP*) is given for every degree up to 90°) the *mandaphala* (*MP*) equals 0° 11′ 27″ plus 0° 50′ 10″ multiplied by 0° 2′ 20″, and is 0° 13′ 24″. The True Sun equals the Mean Sun plus the *MP*, or 263° 50′ 10″ plus 0° 13′ 24″. Therefore, the position of the True Sun is 264° 03′ 34″.

To find the Mean and the True Moon from the tables the mean Moon for 10 January 2016 is as follows. The motion for 30 periods is 8^R 21° 30′ 41″ 22‴. The motion for 12 periods is 2^R 26° 06′ 34″ 19‴. The motion for 4 years is 8^R 13° 57′ 00″ 44‴. The motion for 11 months is 0^R 28° 11′ 48″ 55‴. The motion for 25 days is 11^R 22° 35′ 06″ 45‴ by adding all these and removing the cycles of 12 *rāśīs*. Therefore, the Mean Moon is 8^R 12° 21′ 12″ 05‴, or 262° 21′ 12″ 05‴.

To find the *mandocca* of the Moon, the *mandaphala* (*MP*) equals 0° 11′ 27″ plus 0° 50′ 10″ multiplied by 0° 2′ 20″, or 0° 13′ 24″. The True Sun equals the Mean Sun plus the *MP*, or 263° 50′ 10″ plus 0° 13′ 24″. Thus, the True Sun = 264° 03′ 34″.

Table 21: Comparison of computed values with modern values.

Bodies	*Karaṇa kutūhala*	*Karaṇa kutūhala sāriṇī*	Modern Ephemeris
Ravi	265° 08′ 54″	264° 03′ 34″	265° 04′ 36″
Chandra	265° 13′ 35″	266° 08′ 53″	264° 17′ 06″
Kuja	188° 35′ 04″	189° 25′ 57″	189° 25′

To find the Mean and the True Moon from the tables and for 10 January 2016 is as follows. The motion for 30 periods is 8^R 21° 30′ 41″ 22‴. The motion for 12 periods is 2^R 26° 06′ 34″ 19‴. The motion for 4 years is 8^R 13° 57′ 00″ 44‴. The motion for 11 months is 0^R 28° 11′ 48″ 55‴. The motion for 25 days is 11^R 22° 35′ 06″ 45‴. By adding all these and removing the cycles of 12 *rāśīs*, the Mean Moon is 8^R 12° 21′ 12″ 05‴, or 262° 21′ 12″ 05‴. To find the *mandocca* of the Moon from the tables in the *Candrocca* is as follows. The motion for 30 periods is 2^R 09° 03′ 17″ 49‴. The motion for 12 periods is 1^R 06° 45′ 07″ 13‴. The motion for 4 years is 5^R 10° 21′ 32″ 07‴. The motion for 11 months is 1^R 06° 44′ 53″ 06‴. The motion for 25 days is 0^R 02° 53′ 43″ 58‴ by adding all these and removing the cycles of 12 *rāśīs*. Therefore, the *mandocca* of the Moon is 305° 48′ 34″ 13′″. The *mandakendra* (*m*) equals the *mandocca* minus the Mean Moon, or 305° 48′ 34″ 13‴ minus 262° 21′ 12″ 05‴, which is 43° 27′ 22″ < 180°. The *bhuja* is 43° 27′ 22″. From the *Candra manda* tables, the *mandaphala* (*MP*) equals 3° 25′ 27″ plus 0° 27′ 22″ multiplied by 0° 48′ 45″ = 3° 47′ 41". The True Moon equals the Mean Moon plus the *MP*, or 262° 21′ 12″ plus 3° 47′ 41″. Thus, the position of the True Moon is 266° 08' 53".

To find the Mean and True Mars from the tables and for 10 October 2016 is as follows. The motion for 30 periods is 0^R 19° 34′ 06″ 09‴. The motion for 12 periods is 4^R 26°40′ 15″ 01‴. The motion for 4 years is 1^R 04°35′ 15″ 54‴. The motion for 11 months is 5^R 22° 55′ 34″ 54‴. The motion for 25 days is 0^R 13° 06′ 01″ 40‴ by adding all these and removing the cycles of 12 rāśīs. The Mean Mars *(P)* is 5^R 05° 58′ 19″ or 155° 58′ 19″.

To determine the first *manda* correction for the mean planet, the *mandocca* of Mars is 128° 30′, and the *mandakendra* (mk_1) equals the *mandocca* minus the Mean Mars, or 332° 31′ 41″ (which is >180°). The *bhuja* is 360° minus *kendra* if 270° < *kendra* < 360°, and equals 27° 28′ 19″. From the *Bhauma* (Mars) *manda* tables, the *mandaphala* (ME_1) equals 5° 04′ 29″ plus 0° 05′ 25″ and multiplied by 0° 28′ 19″, or 5° 07′ 02″. P_1 equals P minus $\left(\frac{1}{2}\right)$ ME_1, or 153° 24′ 48″.

To determine the first *śīghra* correction for the planet, the *śīghrakendra* (sk_1) equals the *śīghrocca* minus P_1= Mean Sun – P_1. sk_1 equals 263° 50′ 10″ minus 153° 24′ 48″, or 110° 25′ 22″ (which is <180°). From the *Bhauma* (Mars) *śīghra* tables, the *śīghraphala* (SE_1) equals 39° 57′ 53″ plus 0° 25′ 22″ multiplied by 0°119′ 15″, or 40° 48′ 18″. P_2 equals P_1 plus $\left(\frac{1}{2}\right)SE_1$, or 153° 24′ 48″ plus $\left(\frac{1}{2}\right)$ (40° 48′ 18′), and equals 173° 48′ 57″.

For the second *manda* corrected planet, the *mandakendra* (mk_2) equals the *mandocca* minus P_2, or 128°30′ minus 173°48′ 57″ plus 360°, which is 314°41′ 03″ (and >180°). The *bhuja* equals 360° minus the *kendra,* if 270° < *kendra* < 360°, and is 45° 18′ 57′′. The *mandaphala* (ME_2) equals 7° 56′ 57′ plus 0°18′ 57′ multiplied by 0°04′ 17′, which is 7° 58′ 18″. P_3 equals P minus ME_2, or 155° 58′ 19″ minus 7° 58′ 18″, and is 148° 00′ 01″.

For the second *śīghra* corrected planet, the *śīghrakendra*(sk_2) equals the *śīghrocca* minus P_3. sk_2 equals 263° 50′ 10″ minus 148° 00′ 01″, or 115° 50′ 09″ (which is <180°). The *śīghra phala* (SE_2) equals 39° 51′ 44″ plus 0° 50′ 09″ multiplied by 0° 112′ 43″, or 41° 25′ 57″. P_4 equals P_3 plus SE_2, or 148° 00′ 01″ plus 41° 25′ 57″, and is 189° 25' 57".

From Table 21, we observe that although the methods are the same for the *Karaṇa kutūhala* and the *Karaṇa kutūhala sāriṇī*, the procedure is very simple in the *Karaṇa kutūhala sāriṇī* because of the *manda* and *śīghra* tables. The true positions of the Sun and the Moon according to the *KKS* are different, whereas that of Mars is the same as the modern value. This indicates that the author of the *Karaṇa kutūhala sāriṇī* considered better values for the rate of motion of the planets.

6 Critical *Śīghrakendra* and Maximum *Śīghraphala* by Modern Expressions

The modern expression for finding the *śīghra* correction is tan *p* equals $\frac{r\sin(H-S)\cos b}{R+r\cos b\cos(H-S)}$, where *H-S* equals the *śīghrakendra* (the anomaly of the conjunction), *r* is radius vector of the planet (the heliocentric distance), *R* is the radius vector of the Sun (the Sun-Earth distance) and b is the heliocentric latitude. Interchanging *R* and *r*, for example with Jupiter, we have *R* equals 5.0280548 a u, *r* is 1 a.u, *b* is –0° 57′ and *H-S* is 137° 28′. Tan *P* is

Table 22: Maximum *śīghraphala* of planets according to the *Karaṇa kutūhalam.*

Planet	$K=\frac{R-P}{R\times P}$	Cr.*sk* (°)	Max. *sp* (°)	Hel. Dist. (*r*)	Cr.*sk* (mod)	Inc.i (°)	Cr. sp *B* = I (°)
Kuja	0.1940299	132.4543	42.4518	1.52369	131.0184	1.84972	130.9924
Guru	0.6783217	101.0502	11.05007	5.2028	101.0814	1.294167	101.0785
Budha	0.4634147	111.5103	21.5102	0.3871	112.7741	6.991667	112.5954

$\frac{\cos(-0°57')\sin(137°28')}{5.0280548+\cos(-0°57')\cos(137°28')}$, which equals 0.6759261 divided by 4.2912719, or 0.1575118. Therefore, *P* is 8.9512201, or 8° 57′, which equals 4.39 (the *śīghraphala*). There is a negative *manda* correction for Jupiter of 326° 29′ (after applying the equation of centre). The Sun is 77° 24′, R ≈ 1, and therefore the True Jupiter is 326° 29′ minus 8° 57′ 4.39″, or 317° 31′. Note that above *SK* equals *H-S* (the True Heliocentric Longitude of the planet, or *H*, and that *S* is the True Tropical Longitude of the Sun). (i) (ii)

Note that if *SK* <180°, then the *śīghraphala* (*SP*) is negative, and if *SK* >180°, then the *śīghraphala* is positive (Table 22). Here, the *śīghrakendra* equals the planet minus the *śīghrocco.* Note that in the *Sūrya Siddhānta* it is the other way round: i.e. the *śīghra-kendra* equals the *śīghrocco* minus the planet). In that case where *SP* >0, if *SK* >180 then in deriving the maximum *śīghraphala* (from the Modern formula) we have tan *P* equals $\frac{r\sin(H-S)\cos b}{R+r\cos b\cos(H-S)}$. Let A equal *r* cos(*b*), which tentatively is assumed to be constant, and *m* equals *H-S*, *śīghrakendra*, then tan *P* equals $\frac{A\sin m}{R+A\cos m}$ and *SP* equals *P*, or $\tan^{-1}\left[\frac{A\sin m}{R+A\cos m}\right]$. Now differentiating with respect to '*m*', we have

$$P' = \frac{1}{1+\left(\frac{A\sin m}{R+A\cos m}\right)^2} \times \frac{(R+A\cos m)A\cos m+A^2\sin^2 m}{(R+A\cos m)^2}$$

$$= \frac{(R+A\cos m)^2}{(R+A\cos m)^2+(A\sin m)^2} \times \frac{(RA\cos m)+A^2\cos^2 m+A^2\sin^2 m}{(R+A\cos m)^2}$$

$$= \frac{RA\cos m+A^2}{R^2+A^2\cos^2 m+2AR\cos m+A^2\sin^2 m}$$

$$= \frac{A(R\cos m+A)}{R^2+A^2+2AR\cos m}$$

$$= \frac{A(R\cos m+A)}{R^2+A(A+2R\cos m)}$$

Now, P' is 0 then $A(R\cos m+A)$ is also 0. $A \neq 0$, $R\cos m+A=0$. cos *m* equals $\frac{-A}{R}$, cos *m* is –*r cos b* divided by *R* and $m_{(crit)}$ is $\cos^{-1}$ multiplied by –*r cos b* divided by *R*. Substituting the value of '*m*' we get the *śīghraphala P*.

6.1 Example 2: For the *Kuja* of Mars at 5:30 A.M on 9 June 1926 at Madras

r is 1.52369, *R* is 1, and *b* is 1° 50′ 59″ ≈ 1° 51′ (maximum latitude). $m_{(crit)}$ is $\cos^{-1}\left[\frac{-R\cos b}{r}\right]$, which is $\cos^{-1}\left[\frac{-\cos(1°50'59'')}{1.52369}\right]$, or $\cos^{-1}$ (–0.6559594965), and equals 49.00755584.

6.2 Example 3: For Mercury (*Budha*) on 9 March 1926

Rajan (1933) has shown that where *r* is the mean (heliocentric) distance, or 0.3871, and *b* is the heliocentric latitude of the planet, *r* equals 0.3871, *R* is 1, and b_{max} (orbital circulation with the ecliptic) is 7° 0′ 2″. We know that *m* equals $\cos^{-1}$ multiplied by –*r cosb* divided by *R*, or $\cos^{-1}$ multiplied by $\left[\frac{-0.3871 X\cos(7°0'2'')}{1}\right]$, so m_{crit} equals 112.59496, or 112° 35′ 41.8″. Since SP_{crit} equals $\tan^{-1}\left[\frac{r\cos b\sin m}{R+r\cos b\cos m}\right]$ or $\tan^{-1}\left[\frac{0.35472388535}{0.8523794193}\right]$, it is 22° 35′ 41.92″. When *b* is 0, cos *b* is 1, and *m* equals $\cos^{-1}\left(\frac{-r}{R}\right)$, or $\cos^{-1}$ (–0.3871) and is 112° 46′ 27.02″. Now, SP_{crit} equals $\tan^{-1}\left[\frac{r\sin m}{R+r\cos b}\right]$, or $\tan^{-1}\left[\frac{0.35692081}{0.8501535939}\right]$, or 22° 46′ 27.02″.

6.3 Maximum *Śīghraphala* and Critical *Śīghrakendra*

Based on the Sanskrit astronomical tables, e.g. the *Thyagarthi* manuscript, Professor Balachandra Rao (2014) has shown the variation of the *Śīghraphala* for each of the five planets graphically. Unlike in the case of the *manda-phala*, here the maximum *śīghraphala* is not attained at *śīghra* anomaly 90°. In fact, these values are higher than 90°, and different for different planets.

7 CONCLUDING REMARKS

In the current paper, we have discussed the behaviour of the *śīghraphala* (equation of conjunction) and its critical values and points for each of the traditionally known five planets.

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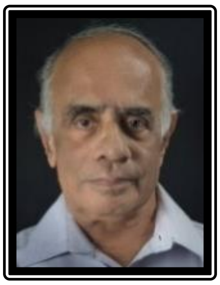

Professor Rao has been researching in the field of classical Indian astronomy since 1993 under successive research projects from INSA. He has authored, singly and jointly, quite a few papers on Indian mathematics and astronomy that have been published in Indian and international journals (including *JAHH*). He has also written about 30, half in English and the remainder in Kannada. The more popular ones among them are: (1) *Indian Mathematics and Astronomy—Some Landmarks*; (2) *Indian Astronomy—Concepts and Procedures*; (3) *Eclipses in Indian Astronomy*; (4) *Transits and Occultations in Indian Astronomy* [titles (3) and (4) were co-authored by Professor Padmaja Venugopal]; (5) *Grahalaghavam of Ganesha Daivajna, English Translation and Notes*; (6) *Karanakutuhalam of Bhaskara II, English Translation and Notes* [titles (5) and (6) were co-authored by Professor S.K. Uma]; (7) *Astrology—Believe it or Not?*; (8) *Traditions, Science and Society*, etc. While title (7) was translated into the Kannada and Marathi languages, title (8) was rendered into Kannada, Telugu and Malayalam versions. The Kannada versions of books (7) and (8) have won awards as “The Best Works of Rational Literature” from the Kannada Sahitya Parishat (Kannada Literary Authority).