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REFERENCES TO 'PARALLEL PHENOMENON' IN INDIAN ASTRONOMY AND INSCRIPTIONS

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Abstract: In the present paper we discuss briefly an interesting astronomical event called "Parallel phenomenon' or "Parallel aspect" in astronomical sciences. Besides explaining this phenomenon, we provide interesting historical Indian references recorded in classical astronomical texts and also in some old inscriptions.

Keywords: Parallel phenomenon, Parallel aspect, *Vyatipāta*, *Vaidhṛti*.

1 INTRODUCTION

When the Sun and the Moon in the course of their apparent motion as observed from the Earth, are placed equidistant with respect to the Celestial Equator, the phenomenon of 'Parallel Aspect' is said to occur. If both the Sun and the Moon are on the same side of the Celestial Equator (*Viṣuvadvṛtta*)–both to the North or both to the South–then the phenomenon is called *Vaidhṛti*. This means the declinations (*krānti*) of the two bodies δs and δm are equal, both in direction and magnitude. On the other hand, if the bodies are equidistant on opposite sides of the Celestial Equator they are said to be in *Vyatīpāta* phenomenon. In that case their declinations are equal in magnitude but opposite in direction. In classical Indian astronomical texts a section generally is devoted to a deep study of these two phenomena. Importance has been given to this parallel phenomenon ever since the time of the *Vedāṅga-jyatiṣa*, the earliest Indian astronomical text in Sanskrit. In the ancient Jain tradition this astronomical phenomenon is also recognized, for example in the famous *prākṛt* text *Jyotiṣkaraṇḍa*.

1.1 Definition

The combined revolutions of the Sun and the Moon added to themselves is the number of *Vyatīpāta* (in a *yuga*). The phenomenon called *Vyatīpāta* is of two types, *Lāta Vyatīpāta* and *Vaidhṛta*. The former occurs when the sum of the tropical longitudes of the Sun and the Moon accounts to 180° and the latter when that sum amounts to 360°. Let λs and λm be the longitudes of the Sun and Moon. Then if

$\delta s = \delta m$ *Vyatīpāta*
$\delta s = -\delta m$ *Vaidhṛtapāta*
$\lambda s + \lambda m = 180°$ *Vyatīpāta*
$\lambda s + \lambda m = 360°$ *Vaidhṛtapāta*

In the Jain astronomical work *Jyotiṣkaranda* (CE 514), the rule for finding the number of *Vyatīpāta* in a *yuga* of five years is formulated. The *Vedāṅgajyotiṣa,* states the number of *Vyatīpāta* in the *yuga* of five years. Following are definitions according to the different texts.

1.1.1 The *Grahalāghavam* of Ganeśa Daivajña

In Chapter 14, *Pātādhikārah śloka* 1, the method of finding *Vyatīpāta* and *Vaidhṛti pāta* is ex-

Table 1: Dates of *Vyatipāta* and *Vaidhṛti* in Indian astronomical commentaries.

Author	Commentary On	Traditional Date	CE
Pṛthūdaka	*Khaṇḍakhādyaka* (Sengupta, 1934)	*śaka* 786 *vaiśākha*, 10	21 April 864
Varuṇa	*Khaṇḍakhādyaka* (Chatterjiee, 1970)	*śaka* 962 *kanya*,16	10 September 1040
Mallikarjuna sūri	*Śiṣya-dhī-vṛddhi-da* (Chatterjee, 1981)	*śaka* 1107 *caitra śukla* 2, Tuesday	20 February 1185
Yallaya	*Makaranda sāriṇī*	*śaka* 1394 *jyeṣṭha śukla*, 5	22 May 1472
Sumatiharśa	*Karaṇakutūhala* (Dvivedi, 1991)	*śaka* 1539 *kārtika kṛṣṇa* 10 Tuesday	23/24 October 1617
Viśvanātha	*Grahalāghava* (Yugeśvaraśastri, 1946, Joshi, 1981)	*śaka* 1535 *vaiśākha kṛṣṇa* 7 Saturday	10/11 May 1613

plained as follows:

(1) Multiply *ayanāmśa* by 9. Divide the product by 60 to get the result in *ghaṭis*.
(2) Subtract the result of step (1) from 13/30. The difference is called *sāva-yava yoga*. If this *yoga* is elapsed (*gata*) *yoga* then there will be *Vyatīpāta*.
(3) Subtract the result of step (1) from 27. The difference is called *sāvayava yoga*. If this *yoga* is elapsed (*gata*) *yoga* then there will be *Vaidhṛti pāta*.
(4) Correction to be applied to yoga *ghaṭis* to get corrected *ghaṭis*.

Multiply *gata yoga ghaṭis* by the sum of *gata* (elapsed) and *eṣya* (remaining) *ghaṭis* of the *nakṣatra* at that instant. Divide the product by 65. The result gives the corrected *gata yoga ghaṭis* (Rao and Uma, 2006).

1.1.2 The *Karnakutuhalam* of Bhāskara II

In Chapter 9 *śloka* 1 and 2, if the sum of *sāyana* Ravi and *sāyana* Chandra is equal to 6 *rāśis* (i.e., 180°), the *yoga* is called *Vyatīpāta*. If the sum is 12 *rāśis* (i.e., 360°), it is called *Vaidhṛti pāta*.

If the sum is less than 6 or 12 *rāśis*, correspondingly the *Vyatīpāta* or *Vaidhṛti pāta*is due (*eṣya*). On the other hand, if the sum is greater than 6 or 12 *rāśis* the *pāta* is over (*gata*) (Rao and Uma, 2007).

1.1.3 The *Mahā-bhāskariya* of Bhāskara I

Śloka 35-36 says, when the sum of the (true) longitudes of the Sun and the Moon amounts to half a circle (i.e., 180°), the phenomenon is called (*lāṭa*) *Vyatīpāta*; when that sum amounts to a circle (i.e., 360°), the phenomenon is called *Vaidhṛti* (*Vyatīpāta*); and when the sum extends to the end of the *nakṣatra Anurādhā* (i.e., when that sum amounts to 7 signs, 16 degrees, and 40 minutes), the phenomenon is called *sārpamastaka* (*Vyatīpāta*) (Shukla, 1960).

2 RECORDING OF THE PHENOMENON IN INDIAN ASTRONOMICAL TEXTS

The canonical texts like the *Sūryasiddhānta*, Brahmagupta's *Khaṇḍakādyaka*, the *Brahmasputa siddhānta*, *Karaṇakutūhalam* of Bhaskara II and the *Grahalāghava* of Ganeśa Daivajña (ca. CE 1520) have explained in detail the algorithmic procedure to work out the details of the parallel phenomenon. The procedure is quite elaborate in the case of eclipses. However, the actual recordings of the occurrence of the phenomena in their lifetimes are detailed by the famous astronomers who wrote elaborate commentaries on the standard treatises. We shall now consider historical records of the events mentioned and mathematically worked out by six of them, namely Pṛthūdaka (eighth century), Varuna (tenth century), Mallikarjuna sūri (eleventh century), Yallaya (fifteenth century) and Sumatiharśa and Viśhvanātha (seventeenth century). The details given by them are listed in Table1.

3 A BRIEF ANALYSIS OF THE REFERENCES

In Section 1 we mentioned that the parallel phenomena occur when the declinations of the Sun and the Moon are equal (i.e., the two bodies are equidistant from the Celestial Equator). In particular, they are called *Vyatipāta* and *Vaidhṛti* if respectively the Sun and the Moon are on the same side or on opposite sides of the Celestial Equator.

Note that in what follows, IST stands for Indian standard time, which is 5.5 hours ahead of UT.

As a first approximation, the sum of the (tropical) longitudes of the Sun and the Moon is 180° or 360° for the above phenomena.

3.1 Example 1

In Pṛthūdaka's example on the given date viz., 21 April 864 around 9^h51^m IST the sum of the longitudes of the two bodies is 180°. Now by working out the prescribed algorithm their declinations become equal (12° 58′) both in magnitude and direction (North) at 10^h IST.

3.2 Example 2

In the Kashmiri astronomer Varuṇa's example

for the *Vaidhṛti*, the sum of the longitudes is 360° on 10 September 1040 at 17^h 10^m IST, but the declinations (2° 54′) were equal in magnitude but in opposite directions and occurred earlier at 4 hours (IST) on the same day.

3.3 Example 3

Mallikārjuna sūri records and works out the occurrence of *Vyatīpāta* where the sum of the tropical longitudes is 540°. Removing the completed cycle of 360°, the sum reduces to 180°. In that case, the two declinations are equal, both in magnitude and direction (South).

3.4 Example 4

Yallaya, a famous astronomer from the Andhra region, gives an example of *Vyatīpāta* on Friday 22 May 1472, when the declination of the Sun was 21° 54′ N and the Moon 20° 46′ S, but actually the declinations become equal the next day (23 May 1472) when declinations were 22° 3′ but opposite in direction.

3.5 Example 5

According to Sumatiharṣha (Jodhpur) on 23 October 1617 at 23^h 30^m IST the declinations of the Sun and Moon were respectively 11° 39′ N and 11° 42′ N.

4 RECORDINGS OF *VYATĪlPĀTA* IN KANNADA INSCRIPTIONS

In the Indian context, it is interesting to study existing records of historical stone inscriptions (Shylaja and Geetha, 2016). Here are some examples of vyatipāta and *Vaidhṛitipāta* from Kannada inscriptions—Kannada is the regional language of Karnataka State.

4.1 Example 1: South Indian Inscriptions, Volume 9—Chalukyas of Kalyani, No. 264 (A.R. No. 316 of 1925)

This is on a slab planted in front of the Temple of Hanuman at Halyam, Kudligi Taluk, Bellary District.

This is dated śaka 1085 (mistake for 1075), *Śrīmukha*, Pushya, amavasye, Somavara, Uttaryana-sankranti, Surya-grahana, *Vyatīpāta*. The date is irregular. In Srikukha, Pushya-amavasya corresponded to Friday (not Monday) CE 15 January 1154, nor was there a solar eclipse on that day. It also was not a Uttaryana day.

The record refers itself to the reign of the Chalukya King Tribhuvanamalladeva, and gives the genealogy of the dynasty from Talipa to Vikramaditya VI. It records the grant of some land for the service of the God Agastesvara at Yeleha, situated to the North of Koturu, by the King's feudatory Nachidevarasa of the Kadamba Family, which was ruling over Kogali 512 km from his capital. The teacher Hemarasi-deva and Bupajiya are said to have been placed in charge of the lands granted to the same god by Bommidevarasa and others. Soyidevavarasa, the son of Bommidefva of Koturu, Chavudisetti, Marisetti and others are also said to have granted lands for the service of the same god. Though the record itself refers to the reign of Vikramaditya VI, its date falls in the reign of Trailokyamalla Taila III. The grant was probably put on stone in the year cited.

This record cites the *Vyatīpāta.* This was a Monday on which an annular solar eclipse occurred and the instant of *Vyatīpāta* (i.e. the declinations of the Sun and the Moon were equal to 21° 32′ S at approximately 3^h 49^m IST). In the present record, the word *saṅkrānti* may be interpreted to mean equality in declinations. The date is recorded as *śaka* 1085, *Śrīmukha samvatsara* (a cycle of 60 years) and *Puṣya* (lunar calendar month).

Note that there is some confusion in fixing the date of No. 264. A.R. 316 of 1925: the epigraphists have given the date as a mistake for *śaka* 1075. However, on 26 January 1153 there was an annular solar eclipse (magnitude 0.9283692), and the declinations of the Sun and the Moon were equal to 16° 54′ at 12^h IST. Although both of the dates given above were Mondays and there were annular eclipses, the first one (CE 1163) was not '*Śrīmukha samvatsara*' of the 60-year cycle. Furthermore, there was a solar ingress (*Sānkranti*) into the sidereal sign (*rāśi*) of *kumbha* (sidereal Aquarius). The confusion is due to the fact that in the traditional Indian convention a weekday is reckoned from sunrise to sunrise whereas in modern convention it runs from midnight to midnight.

4.2 Example 2: South Indian Inscriptions, Volume XI—Bombay–Karnataka, No. 182 (B.K. No. 78 of 1927–28)

This is at Doni, Mundargi Petha in the Dharwar District. This inscription is dated Chalukya-Vikrama year 23, Bahudhanya, Pusya, amavasye, Sunday, Uttarayana-sankramana, Vyatīpāta, corresponding probably to Saturday CE 25 December 1098, which was the day of Uttarayana-sankranti—the weekday cited in the record seems to be an error.

The epigraph registers a grant of tax-free land made by the Mahajanas, who were headed by the Urodeya, and the Settis of Dronapura to the God Siddhesvaradeva. Saktisiva-Pandtia, the Acharya of the temple, received the gift.

This record cites the *Vyatīpāta.* On Saturday 25 December 1098 at $4^h\ 50^m$ the declinations of both the Sun and the Moon were 23° 13′ S, with the Moon's latitude 0° 16′. The tropical Sun was at 279° 33′ 45″, the Moon at 274° 16′ 53″, Rahu at 271° 10′ 54″ and Mercury at 274° 40′ 50″.

On Saturday 25 December 1098 there was a solar eclipse with Saros No. 117, and a magnitude of 0.9179346. The circumstances were:

Beginning of the eclipse: $18^h\ 07^m$ IST
Beginning of annularity:$19^h\ 07^m$
Mid-eclipse: $20^h\ 55^m$
End of annularity: $22^h\ 44^m$
End of the eclipse: $23^h\ 44^m$

4.3 Example 3: South Indian Inscriptions, Volume XI—Bombay–Karnataka, No. 186 (B.K. No. 238 of 1928–29)

This is another example recorded in the same district, Dharwar, at Morab, Navalgund taluk. It is on a slab set up in the Hanumantadeva Temple.

The inscription, whose beginning is worn out, is dated to Chalukya-Vikrama year 28, Subhanu, Dipavali-amavase, Sunday, *Vyatīpāta*. In Subhanu, Dipavali-amavasya (i.e. Asvina-amavasya) ended on Saturday AD 3 October 1103. Probably the gift was recorded on the next day. A grant of land appears to have been made. Mallikarjuna-Pandita is mentioned.

According to our calculations for this date the declinations of the Sun and the Moon were 6° 17′ around $18^h\ 30^m$, with the Moon's latitude 1° 55′. The tropical Sun was at 195° 52′ 30″, the Moon at 200° 35′ 40″, Rahu at 179° 21′ 33″, Mercury at 203° 34′, and Venus at 217° 15′. However, there was no solar eclipse on this day.

4.4 Example 4: South Indian Inscriptions, Volume XI—No. 173 (A.R. No. 95 of 1904)

This is on the eleventh slab set up on the South side of the Kallesvara Temple at Bagali, Harapanahalli taluk. This is dated to the Chalukya-Vikrama year 32, Sarvajit Margasira, amavasye, Somavara, Suryagrahana, *Vyatīpāta*, which corresponds to Monday CE 16 December 1107, when there was a solar eclipse. It refers to the reign of the Chalukya King Tribhuvanamalladeva, whose Dandanayaka Barmmarasa is said to have granted one gadyana per month out of the pannaya of the village Balguli to the Mahajanas for the repair of the tank Hiriyakere, in the presence of the God Kalideva.

We found the declinations of the Sun and the Moon to both be 23° 33′ S around $23^h\ 45^m$ with the Moon's latitude 0° 1′ for the above-mentioned date. We also verified for the solar eclipse, with the tropical Sun at 271° 0′ 0″ and the Moon at 276° 29′ 52″.

The magnitude of the eclipse was 0.9310542, with the following circumstances:

Beginning of the eclipse: $8^h\ 54^m$ IST
Beginning of annularity: $10^h\ 11^m$
Mid-eclipse: $11^h\ 41^m$
End of annularity: $13^h\ 11^m$
End of the eclipse: $14^h\ 29^m$

5 CONCLUDING REMARKS

In the preceding sections we discussed the procedure of finding *Vyatīpāta* according to different texts. We also considered a few historical recordings of these events by commentators of classical Indian astronomical texts, and also those recorded in inscriptions in the Karnataka region of India. Examples were worked out according to improved Siddhantic procedures, and the results were verified.

6 ACKNOWLEDGEMENT

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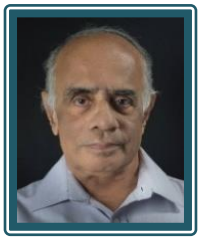

Professor Rao has been researching in the field of classical Indian astronomy since 1993 under successive research projects from INSA. He has authored, singly and jointly, quite a few papers in reputed journals and books on Indian mathematics and astronomy. The books published so far are about 30, half in English and the remainder in Kannada. The more popular ones among them are: (1) *Indian Mathematics and Astronomy—Some Landmarks*; (2) *Indian Astronomy—Concepts and Procedures*; (3) *Eclipses in Indian Astronomy*; (4) *Transits and Occultations in Indian Astronomy* [titles (3) and (4) were co-authored by Dr Padmaja Venugopal]; (5) *Grahalaghavam of Ganesha Daivajna, English Translation and Notes*; (6) *Karanakutuhalam of Bhaskara II, English Translation and Notes* [titles (5) and (6) were co-authored by Dr S.K. Uma]; (7) *Astrology—Believe it or Not?*; (8) *Traditions, Science and Society*, etc. While title (7) was translated into the Kannada and Marathi languages, title (8) was rendered into Kannada, Telugu and Malayalam. The Kannada versions of books (7) and (8) have won awards as 'The Best Works of Rational Literature' from the Kannada Sahitya Parishat (Kannada Literary Authority).