

A practical geometrical comparison between free-repelled hydraulic jumps within inclined channels

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In this experimental study a practical geometrical comparison between inclined (angle φ) free and repelled hydraulic jumps (the latter in non prismatic but abruptly expanding channels) is presented, analysed and discussed. For repelled hydraulic jumps a considerable parameter is the expansion ratio r (=channels' width ratio), which here is changing from $r=0.5$ to $r=0.7$. The comparison is made with the free jump (in prismatic channel, $r=1$), in the same ranges of angles φ ($0^\circ \leq \varphi \leq 8^\circ$) and Froude numbers ($2 \leq Fr_1 \leq 8$). A practical arithmetic example is presented to show the behavioral change of conjugate depths, lengths L and depths at $0.5L$, in order to receive a comparison among all pertinent geometrical quantities. The present results may be useful for the hydraulic engineering when designing open channels.

free jumps, repelled jumps, inclined channels

1 Introduction

Two of the most important steady-turbulent hydraulic jumps are: (i) the repelled jump in two non prismatic rectangular/inclined open channels in series and (ii) the free jump, in prismatic rectangular/inclined open channels.

Figure 1 shows the flow characteristics of the repelled inclined (angle φ) jump, while Figure 2 presents the corresponding characteristics of the free inclined (angle φ) jump.

The repelled jump ($d_1, d_2, d, b_1, L, \varphi$) is created because of an abrupt channel bottom expansion from b_0 to b_1 , beyond a symmetrical separation zone (Figure 1). The discharge (per unit width, b_1) is q , $r=b_0/b_1$ is the expansion ratio, where the jump is produced from a suitable gate exit, of supercritical flow placed at α - a .

The simpler free inclined jump ($d'_1, d'_2, d', b'_1, L', \varphi, q, r=1$) is formed within a channel of constant width (Figure 2), while in both jumps the primary parameter is the

Froude number $Fr_1 = q \cdot g^{-1/2} \cdot d_1^{-3/2}$, or Fr'_1 .

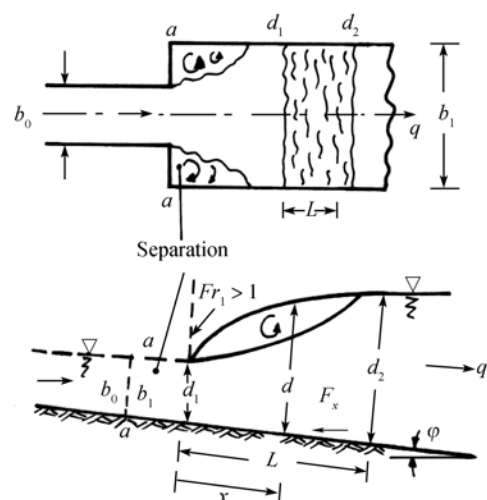


Figure 1 Flow characteristics in repelled jump.

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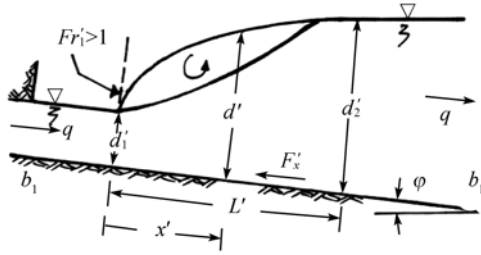


Figure 2 Flow characteristics in free jump.

The repelled jump has considerable differences from the free jump because of the preceding symmetrical separation zone, which has a serious effect on all jump quantities.

Chow^[1] presented a lot of details for both jumps, while Hager^[2] summarized almost all jumps, although, both authors only within horizontal channels. Demetriou et al.^[3-16] experimentally presented a lot of characteristics of the repelled jumps in inclined channels and also presented an extended research material on inclined free jumps. They^[17-19] also examined the corresponding inclined jumps.

This paper attempts a practical comparative presentation based on all laboratory results by the author and collaborators.

2 One-dimensional equations

The one-dimensional equations are, the continuity equation, $q = V_1 \cdot d_1 = V_2 \cdot d_2$, and the momentum equation along the flow direction x (per unit channel width b_1). For repelled jumps, $|P_x| = 0.5 \cdot \gamma \cdot |d_1^2 - d_2^2|$, γ is specific water weight, $W = K_w \cdot |P_x|$ is the water weight between d_1 and d_2 and under the free water surface profile, while $F_x = K_x \cdot |P_x|$ is the boundary resistance force.

$$P_x \cdot \cos\phi + W \cdot \sin\phi - F_x = \rho \cdot q^2 \cdot [(1/d_2) - (1/d_1)]. \quad (1)$$

If $T = \cos\phi \cdot K_w \cdot \sin\phi + K_x$, eq. (1) may theoretically be solved for d_2/d_1 ,

$$\frac{d_2}{d_1} = 0.5 \left[\left(1 + 8 \cdot \frac{Fr_1^2}{T} \right) - 1 \right]. \quad (2)$$

The above solution does not take into consideration some very important flow parameters, such as the regime of flows, the separation zone, or some other details. Thus d_2/d_1 ratio must rather be determined from the experiment. K_x (F_x) may be found from experimental water free surface profile determination (with equations) and W , d_2/d_1 ,

L , determination, after rearranging eq. (2). A similar analysis holds for the free jump (d_2'/d_1' , W' , K_w' , K_x' , F_x').

3 Experiments

The experimental results by the authors refer to $0^\circ \leq \phi \leq 8^\circ$, $r = 0.5-0.9-1.0$, $2 \leq Fr_1 \leq 6$ and $2.3 \leq (d_2/d_1) \leq 7.5$ for repelled jumps and $0^\circ \leq \phi \leq 16^\circ$, $2 \leq Fr_1' \leq 16$ for free jumps. All the pertinent experimental details may be found in the cited list of papers.

The comparison of the data mainly shows the extension of the separation effect, since the repelled inclined jump becomes exactly the same with the free jump for $r=1$ and considerably differs for $r < 1$, where this difference is increasing when r is reducing. The comparison is made under the assumption that $Fr_1 = Fr_1'$ with a small extrapolation to $Fr_1 = 8$.

4 Recent data

Demetriou (2005)^[12], experimentally presented, for the free jump, the following equations:

$$d_2'/d_1' = 0.5 \cdot \left[\left(1 + 8 \cdot Fr_1'^2 \right)^{1/2} - 1 \right] \cdot e^{3.5 \cdot J_0}, \quad (3)$$

$$L'/d_2' = (1 - 0.88 \cdot J_0^{0.5}) \cdot [8.22 - 0.12 \cdot Fr_1 - 8.1 \cdot Fr_1^{-1}], \quad (4)$$

$$\frac{d'}{d_1'} \cong 1 + \left(\frac{d_2'}{d_1'} - 1 \right) \cdot \left[(3.37 - 8.11 \cdot J_0) \cdot \left(\frac{x'}{L'} \right) - (2.37 - 8.11 \cdot J_0) \cdot \left(\frac{x'}{L'} \right)^{1.5} \right]. \quad (5)$$

The last equation (water free surface profile), may also be used to give the "half" depths, $d'_{0.5}$, at $x'/L' = 0.5$ for any angle ϕ (or J_0).

These data will be used here to further analyse the important comparison ratios between repelled and free jumps, where $d_{0.5}$ is the water depth at $x/L = 0.5$ (repelled jump). Demetriou et al. (2006)^[7], have shown that for $0.5 \leq r \leq 1$, L/d_2 may become only Fr_1 and angle ϕ dependent. The following parameters are used,

$$\begin{cases} L^* = (L/d_2)/(L_2'/d_2'), D^* = (d_2/d_1)/(d_2'/d_1'), \\ D_{0.5}^* = (d_{0.5}/d_1)/(d_{0.5}'/d_1'). \end{cases} \quad (6)$$

The practical comparison will mainly be made through arithmetic examples for $\phi = 2^\circ$ and $\phi = 6^\circ$, $r = 0.5-0.7-1.0$, $Fr_1 = 2$ to 6, and suitable diagrams.

5 Comparison's results

Figure 3 presents the ratio L^* vs. Fr_1 and angles φ . The small circles came after the computations of the L^* -based experiments and gave (with a very small scatter) a family of systematic curves with a double behavior, falling (for $2 \leq Fr_1 \leq 6$) and rising again (for $Fr_1 > 6$), while most parts of the curves lie under $L^*=1$. For $Fr_1 = \text{const}$, L^* is increasing with angle φ , to a certain value, $L^* = \text{const}$ appears to correspond to two Fr_1 values, while L^* is independent of r ratios.

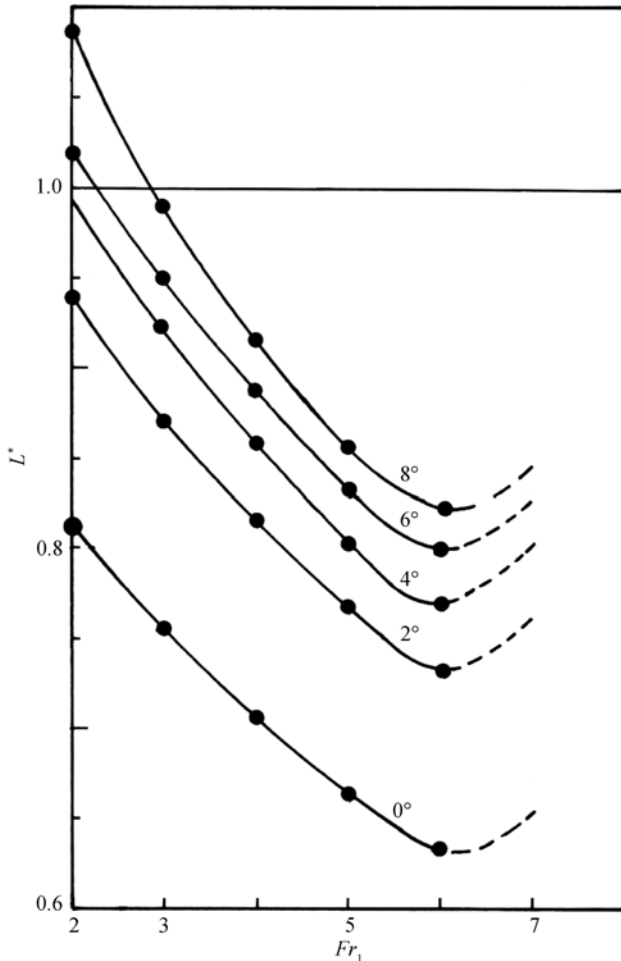


Figure 3 L^* vs. Fr_1 and angle φ .

Figure 4 shows systematic curves, representing the relationships between r and D^* for various angles φ . For any r ratio, D^* is increasing when angle φ is decreasing, while along any curve, D^* is increasing with r ratio.

Figure 5 shows $D_{0.5}^*$ vs. Fr_1 and angle φ (2° and 6°) for two r ratios, $r=0.5$ and $r=0.7$. All the corresponding curves are systematic and the couple of curves, with $\varphi=2^\circ$ — $r=0.5$ and 0.7 , lie over the respective couple of

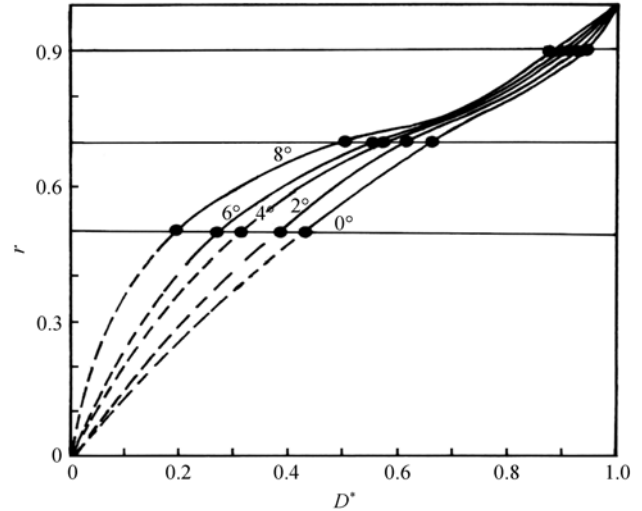


Figure 4 D^* vs. r and angle φ .

curves at $\varphi=6^\circ$. For $Fr_1 = \text{const}$, $D_{0.5}^*$ are increasing when angle φ is decreasing and r is also increasing. All the present $D_{0.5}^*$ values are less than one, while a characteristic $D_{0.5}^*$ value could correspond to $r=0.5$.

To Figures 3, 4 and 5, a number of interpolations may be made for intermediate angles, Fr_1 values or r ratios.

6 Arithmetic example

In order to better understanding the comparison between free ($r=1$) and repelled jumps ($r<1$) Table 1 presents an arithmetic example for $\varphi=2^\circ$ – 6° , Fr_1 (or Fr_1') = 2–6 and $r=0.5$ – 0.7 . As in any practical problem one must assume d_1 and d_1' depths and proceed through eqs. (3)–(5) and Figures 3–5. For the present example the depths d_1 and d_1' are considered to be equal, $d_1=d_1'=0.50$ m. These depths were chosen on purpose, since they reflect the depths of a real-practical problem, but also because they give the opportunity of some discussion. Under such circumstances $D^*=d_2/d_2'$, $D_{0.5}^*=d_{0.5}/d_{0.5}'$, while L^* is taken from eq. (6).

For the free jumps all the ratios d_2'/d_1' , $d_{0.5}'/d_1'$, L'/d_2' and d_2' , $d_{0.5}'$, L' (in m) are calculated, while for the repelled jumps D^* , $D_{0.5}^*$, L^* are taken from Figures 4, 5 and 3 correspondingly and d_2 , $d_{0.5}$, L (in m) are also calculated. In all the cases for $Fr_1 = \text{const}$, and $d_1 = \text{const}$, $q = Fr_1 \cdot g^{1/2} \cdot d_1^{3/2} = \text{const}$ (with respective arithmetic values $q=2.21$ and 6.64 m²/s).

For the free jump at $\varphi = \text{const}$, d_2'/d_1' , $d_{0.5}'/d_1'$, L'/d_2' are

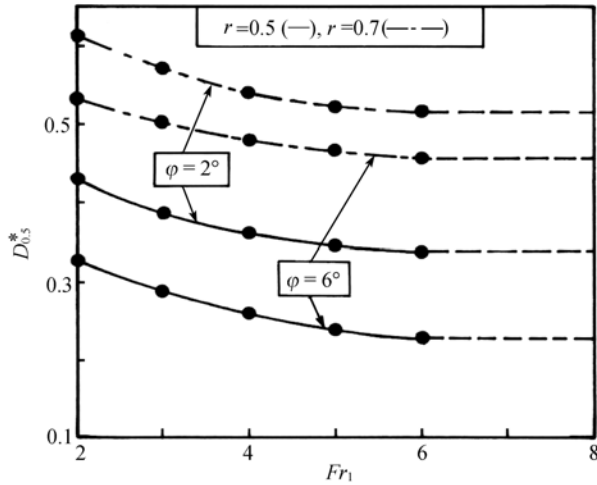


Figure 5 $D_{0.5}^*$ vs. r , Fr_1 and angle φ .

increasing when Fr_1 is increasing, although eq. (4) for L'/d_2' shows that this ratio increases along Fr_1 values less than 8.1, while beyond this Fr_1 characteristic value L'/d_2' are decreasing. A similar behavior holds for these

ratios when angle φ is increases

For the repelled jump, at $\varphi=\text{const}-Fr_1=\text{const}$, d_2 , $d_{0.5}$ and L are increasing with $r=0.5-0.7$, while a similar comparison may also be made with $r=1$ (free jump), although d_2 , $d_{0.5}$ and L are always less than corresponding d_2' , $d_{0.5}'$ and L' quantities. A more complex comparison may be made between d_2 ($d_{0.5}$ or L) for $r=0.5$ and 0.7 and d_2' , $d_{0.5}'$, L' , for $r=1$ (free jump), where all the quantities steadily increase towards corresponding $r=1$ quantities.

More generally, when r is increasing towards $r=1$, all the quantities of the repelled jumps tend towards the free jump corresponding quantities (depths at same $x/L=x'/L'$ places, or lengths).

A local discrepancy of the above results appears (Table 1) when $\varphi=\text{large enough}$ (e.g., $\varphi=6^\circ$) while Fr_1 is small enough (e.g., $r=0.5$) in case d_1 (or d_1') is pre-designed, since for the repelled jumps (or any jump) one must be sure that this d_1 depth is a supercritical one.

Table1 Arithmetic example

Parameters		Free jump ($r=1$)			r	Repelled jump ($r<1$)			q (m ² /s)
φ	$Fr_1 (= Fr_1')$	$\frac{d_2'}{d_1'}$ eq. (3)	$\frac{d_{0.5}'}{d_1'}$ eq. (5)	$\frac{L'}{d_2'}$ eq. (4)		D^* (Figure 4)	$D_{0.5}^*$ (Figure 5)	L^* (Figure 3)	
		d_2' (m)	$d_{0.5}'$ (m)	L' (m)		d_2 (m)	$D_{0.5}$ (m)	L (m)	
2°	2	2.68	2.36	3.28	0.5	0.385	0.430	0.94	2.21
					0.7	0.52	0.51	1.57	
	6	1.34	1.18	4.39	0.5	0.625	0.615	0.94	6.64
					0.7	0.84	0.73	2.59	
6°	2	9.04	7.52	5.14	0.5	0.385	0.335	0.730	2.21
					0.7	1.74	1.26	6.53	
	6	4.52	3.76	23.23	0.5	0.625	0.520	0.730	6.64
					0.7	2.83	1.96	10.62	
	2	3.48	2.76	2.81	0.5	0.260	0.336	1.020	2.21
					0.7	< d_1	< d_1	—	
	6	1.74	1.38	4.81	0.5	0.553	0.540	1.020	6.64
					0.7	0.96	0.74	2.72	

Notes: 1) Calculations for $d_1=d_1'=0.5$; 2) approximate results; 3) $D^*=d_2/d_2'$, $D_{0.5}^*=d_{0.5}/d_{0.5}'$, $L^*=(L/d_2)/(L_1/d_2')$; 4) for $Fr_1=\text{const}$ and $d_1=\text{const}$, $q=\text{const}$; 5) For $\varphi=6^\circ-Fr_1=2-r=0.5$, the calculation fails, since critical depth is less than $d_1'=0.5$ m.

7 Conclusions

In this experimental study a practical geometrical comparison is made between depths and lengths of the repelled inclined jumps (in non prismatic channels) to corresponding free inclined jumps' quantities (in prismatic channels). The fields of parameters are common, $0^\circ \leq \varphi \leq 8^\circ$, $2 \leq Fr_1 \leq 6$ (with some extrapolation to

$Fr_1=8$) while the expansion ratios are $r=0.5-0.7$, in comparison to $r=1$ for free inclined jumps. The general conclusion is that the repelled jump for $\varphi=\text{const}$, $Fr_1=\text{const}$, tends to increase all its geometrical quantities when r is increasing, towards the value $r = 1$ (which corresponds to the free jump in a prismatic channel). The present results may be useful to the hydraulic engineering when designing open channels.

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