SCIENCE CHINA
Physics, Mechanics & Astronomy

• Article •

November 2014 Vol. 57 No. 11: 2043–2048
doi: 10.1007/s11433-014-5557-3

Spin-orbit hybrid entanglement quantum key distribution scheme†

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Received June 10, 2014; accepted June 24, 2014; published online August 1, 2014

We propose a novel quantum key distribution scheme by using the SAM-OAM hybrid entangled state as the physical resource. To obtain this state, the polarization entangled photon pairs are created by the spontaneous parametric down conversion process, and then, the *q*-plate acts as a SAM-to-OAM transverter to transform the polarization entangled pairs into the hybrid entangled pattern, which opens the possibility to exploit the features of the higher-dimensional space of OAM state to encode information. In the manipulation and encoding process, Alice performs the SAM measurement by modulating the polarization state $|\theta\rangle^{\pi}$ on one photon, whereas Bob modulates the OAM sector state $|\chi\rangle^{l}$ on the other photon to encode his key elements using the designed holograms which is implemented by the computer-controlled SLM. With coincidence measurement, Alice could extract the key information. It is showed that *N*-based keys can be encoded with each pair of entangled photon, and this scheme is robust against Eve's individual attack. Also, the MUBs are not used. Alice and Bob do not need the classical communication for the key recovery.

quantum key distribution, orbital angular momentum, hybrid entangled state

PACS number(s): 03.67.Dd, 03.67.Hk, 42.79.Sz

Citation: Zhang C X, Guo B H, Cheng G M, et al. Spin-orbit hybrid entanglement quantum key distribution scheme. Sci China-Phys Mech Astron, 2014, 57: 2043–2048, doi: 10.1007/s11433-014-5557-3

1 Introduction

Quantum key distribution (QKD) provides a secret communication between the two authorized parties, namely Alice and Bob, in which Alice and Bob can exchange messages securely and prevent information from leaking to the eavesdropper Eve. Protocols for QKD can be implemented by either single photon source or entangled photon pairs, for example, BB84 [1], B92 [2] and SARG04 [3] protocols are single photon protocols, and E91 [4] and BBM [5] protocols are based on entangled photons. Because the entangled photon source has strong correlations, it serves as a better source for QKD than a single photon source [6]. So far, there already have been various protocols for QKD with the usage of entangled photon pairs [7–11]. However, most of the initial efforts have been devoted to the implementation of polarization degree of freedom.

Photons can carry both spin angular momentum (SAM) and orbital angular momentum (OAM) [12]. SAM is related to the polarization and the left- and right-handed circular polarization states are recognized as its eigenstate. OAM is related to the transverse-mode spatial structure described by $\exp(\mathrm{i}l\phi)$, where l is known as the topological charge number and ϕ is the azimuthal angle. Because the topological charge number can be any integer value, positive or negative, there are infinite OAM eigenstates with the single photon, which opens the possibility for encoding a quantum state in N dimensions (quNits) [13]. Because of its high property, OAM has attracted much attention and OAM

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states have been recognized as a new promising resource for the application for QKD [14–18]. Mhlambululi et al. [14] presented the description of (d + 1) mutually unbiased bases (MUBs) with OAM states, which results in a high-dimensional QKD. Li et al. [15] have proposed a Six-State QKD protocol using photons with OAM states. Both of the two schemes above encode information by means of OAM states to implement the high-dimensional mutually unbiased bases. They can be regarded as the frame of the BB84 protocol, which always needs a classical channel for sifting keys.

Traditionally, it was regarded that SAM and OAM are almost separable and there is no interaction between them. However, the translation between SAM and OAM was found to be possible, and the case of so-called SAM-OAM hybrid entangled state has been reported, which could be realized using the Pancharatnam-Berry phase optical elements (q-plate) [19]. It has been pointed out that such hybrid entangled state allows the coupling of qubit-qudit entangled states, related to the different Hilbert space dimensionality of the SAM and OAM degrees of freedom [20]. In this paper, we present a novel QKD scheme using the SAM-OAM hybrid entangled state as the entangled source. In the protocol, the q-plate is the key device to implement the transformation from the polarization into the hybrid entangled state. In the manipulation and encoding process, Alice modulates the SAM by adjusting the polarization state $|\theta\rangle^{\pi}$ on one photon (signal photon), whereas Bob modulates the sector state $|\chi\rangle^{l}$ on the other photon (idler photon) for encoding his N-based keys and resends the modulated photon to Alice. With coincidence measurement, Alice could extract the key information that Bob has encoded before.

2 Description of the q-plate

q-Plate (QP) is the key device to obtain the SAM-OAM hybrid entangled state, which implements the swapping by coupling the entanglement between the SAM and OAM degrees of freedom. Generally, a QP is a planar slab of a uniaxial birefringent medium [19]. The optical axis of the QP is lying in the x-y plane with an inhomogeneous orientation and a homogeneous phase retardation of π along the z-axis. The orientation of the optical axis in a polar coordinate can be described as $\alpha(\varphi)=q\varphi+\alpha_{\mathrm{o}}$, where the parameter q and α_{o} are constant. On the circular bases, the transmitted matrix for the QP can be described by

$$T_q=\begin{pmatrix} 0 & \exp\left[\mathrm{i}2\alpha(r,\varphi)\right] \\ \exp\left[-\mathrm{i}2\alpha(r,\varphi)\right] & 0 \end{pmatrix}. \tag{1}$$

In the single photon space, the action of the QP can be associated to a quantum operator:

$$\hat{Q}(q)=\exp(\mathrm{i}2\alpha_{\mathrm{o}})|R,m+2q\rangle \times\langle L,m|+\exp(-\mathrm{i}2\alpha_{\mathrm{o}})|L,m-2q\rangle\langle R,m|, \tag{2}$$

where $|L\rangle$ and $|R\rangle$ are left- and right-handed circular polarizations and $|m\rangle$ denotes the eigenstates of the OAM operator. Some types of QP are shown in Figure 1. Here, we consider a QP with $q=1$. One can see that such QP can modify the OAM state with m of the incoming photon, imposing a variation $\Delta m=\pm 2$ whose sign depends on the input polarization. Hence, the QP can implement the following quantum transformations:

$$\begin{aligned} |L\rangle^{\pi}|m\rangle^{l} &\rightarrow |R\rangle^{\pi}|m+2\rangle^{l}, \\ |R\rangle^{\pi}|m\rangle^{l} &\rightarrow |L\rangle^{\pi}|m-2\rangle^{l}, \end{aligned} \tag{3}$$

where π denotes SAM and l denotes OAM. The QP can act as a SAM-to-OAM transferor (Figure 2). For example, the input photon which is horizontal linearly polarized $|H\rangle=\frac{|L\rangle+|R\rangle}{\sqrt{2}}\otimes|0\rangle$ is transmitted through the QP with $q=1$, and then the output from the QP will be a spin-orbit entangled state $|\mathrm{out}\rangle=\frac{|L\rangle^{\pi}|-2\rangle^{l}+|R\rangle^{\pi}|+2\rangle^{l}}{\sqrt{2}}$. As we can see, this state exhibits the spin-orbit hybrid entanglement.

3 Quantum key distribution using the hybrid entangled state

Figure 3 shows the setup of the proposed QKD system. This system consists of four units. The SAM-OAM generation

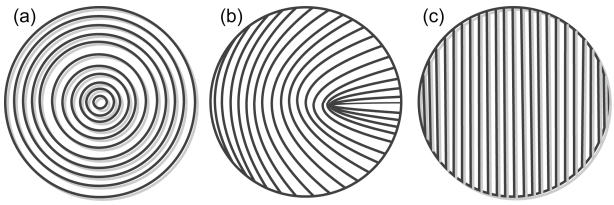

Figure 1 Illustrations of some typical q-plates. (a) q = 1, α_{o}= π/2; (b) q = 1/2, α_{o}= 0; and (c) q = 0, α_{o} = π/2.

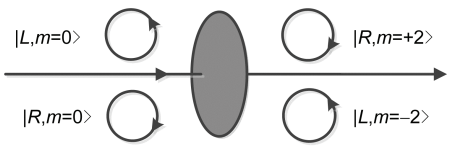

Figure 2 The transformation between the SAM and OAM with the q-plate.

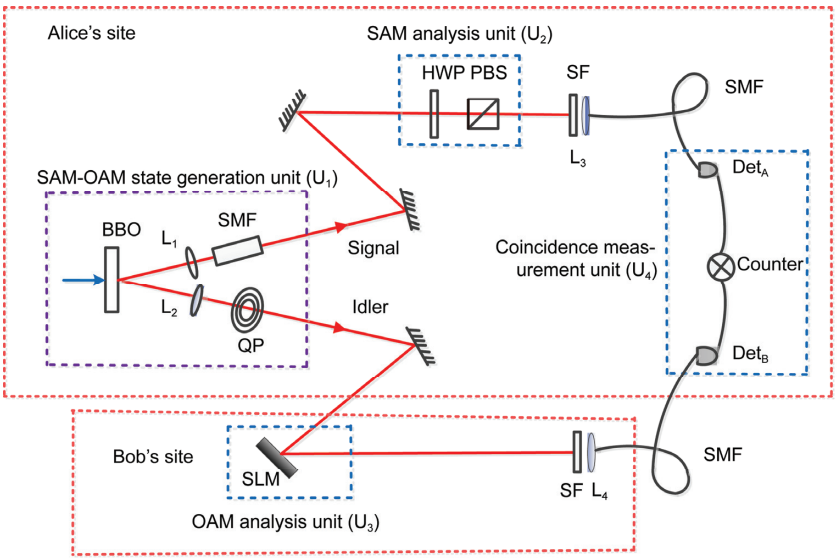

Figure 3 (Color online) Set up for the proposed QKD schematic. U_1 is used to produce the SAM-OAM hybrid entangled state, U_2 and U_3 are used to encode the key elements, and U_4 is used to retrieve the information.

Unit (U_1) is mainly composed of the key device QP, BBO crystal, and the designed single mode fiber (SMF). It produces the SAM-OAM hybrid entangled photon pairs, and each of the output pairs is split into the SAM and OAM mode, respectively. The SAM analysis Unit (U_2) and the OAM analysis Unit (U_3) implement to encode the key elements. U_2 is composed of a half-wave plate and a fixed linear polarizing beam splitter (PBS) and it can be used to modulate the SAM state. U_3 is a spatial light modulator (SLM) which is employed to manipulate the OAM state. After the manipulation, the single photon detectors (Det_A and Det_B) and the counter form the coincidence measurement Unit (U_4) to retrieve the key information. The details of this system are described in the following section.

3.1 Preparation of the hybrid entangled state

In the U_1, as seen in Figure 3, the laser beam with zero orbital angular momentum pumps the nonlinear crystal of beta barium borate BBO, which is cut for type-I phase matching. Through the parametric down-conversion process, the source generates photon pairs in the polarization entangled state $|\Psi\rangle_{\text{spin}} = \left(|H\rangle_A|H\rangle_B + |V\rangle_A|V\rangle_B\right)/\sqrt{2}$, where A and B stand for the signal (Alice's) and idler (Bob's) photons, respectively. To eliminate the potential OAM entanglement, the signal photons are coupled into the SMF, which collapse the transverse spatial mode into a pure TEM_{00}, corresponding to OAM state with $m = 0$. Thus, the possible OAM entanglement can be cancelled out [20]. To transform the polarization entangled pairs into the hybrid entangled state, the idler photons are sent through the QP with $q = 1$. Here, the QP plays the key element to implement the transformation. From eq. (2), the eigenmodes in the QP are circular polarizations, considering the substituting relation $|H\rangle = \left(|L\rangle + |R\rangle\right)/\sqrt{2}$ and $|V\rangle = \left(|L\rangle - |R\rangle\right)/\mathrm{i}\sqrt{2}$. The polarization entangled state can be rewritten as $|\Psi\rangle_{\text{spin}} = \left(|L\rangle_A|R\rangle_B + |R\rangle_A|L\rangle_B\right)/\sqrt{2}$, so that after passing through the QP, this polarization entangled state will show a hybrid pattern:

$$|\Psi\rangle_{\text{hybrid}} = \frac{1}{\sqrt{2}}\Big[|L\rangle^{\pi}_{A}\left(|L\rangle^{\pi}\otimes|-2\rangle^{l}\right)_B + |R\rangle^{\pi}_{A}\left(|R\rangle^{\pi}\otimes|+2\rangle^{l}\right)_B\Big]. \quad (4)$$

Eq. (4) shows that the SMA of photon A is simultaneously entangled with the SAM and the OAM degrees of freedom of photon B. For photon B, in the next manipulation stage we only consider employing its OAM degree of freedom to encode the key elements, On the other hand, the OAM measurement does not affect the SAM degree of freedom [21]. For simplicity, the SAM of photon B will be omitted and the hybrid state can be rewritten as:

$$|\Psi\rangle'_{\text{hybrid}} = \frac{1}{\sqrt{2}}\left(|L\rangle^{\pi}_{A}|-2\rangle^{l}_{B} + |R\rangle^{\pi}_{A}|+2\rangle^{l}_{B}\right). \quad (5)$$

As one can see, eq. (5) describes the SAM-OAM hybrid entanglement between photon A and photon B, which is the target state that we need.

3.2 Manipulation and encoding

The spin-orbit hybrid entangled state has been successfully prepared, as is above. And then, the SAM state at Alice's site and the OAM state at Bob's site should be precisely manipulated, respectively.

To analyze the SAM state, the output light from the SMF is then measured by a half-wave plate (HWP),which is oriented at a variable angle $\theta/2$ (HWP@$\theta/2$) followed by a fixed linear polarizer. This HWP-polarizer combination could act as a SAM analyzer as seen in the U_2, restoring the horizontal polarization state of photon A and filtering incoming photons having linear polarization at angle θ with respect to the horizontal direction. Hence, in the circular polarization basis, the state of the filtered photons can be written as $\left|\theta\right\rangle^{\pi}=\frac{1}{\sqrt{2}}\left(\mathrm{e}^{\mathrm{i}\theta}\left|L\right\rangle^{\pi}_{\mathrm{A}}+\mathrm{e}^{-\mathrm{i}\theta}\left|R\right\rangle^{\pi}_{\mathrm{A}}\right)$. Such state shows an equally weighted superposition of $\left|L\right\rangle$ and $\left|R\right\rangle$ with an arbitrary relative phase θ in the two-dimensional subspace. As seen in Figure 4(a), these SAM superposition states can be represented by a Pointcaré sphere, which lie on the equator of the sphere.

Equivalent to the Pointcaré sphere for polarization, in the earlier work by Leach et al. [22], the modified Pointcaré sphere was used to describe the superposition of OAM states with any modes ($m = \pm l$) when added in equal weights. In that case, the surface of the sphere maps out all possible superposition of these modes. Inspired by this, in this letter, we employ this approach to analyze the OAM superposition state with modes $m = \pm 2$. As seen in Figure 4(b), modes $\left|+2\right\rangle$ and $\left|-2\right\rangle$ are represented by the north and the south poles on the modified Pointcaré sphere, respectively. In an analogous fashion to the polarization states, an equally weighted superposition of $\left|+2\right\rangle$ and $\left|-2\right\rangle$ with an arbitrary relative phase χ is given by

$$\left|\chi\right\rangle^{l}=\frac{1}{\sqrt{2}}\left(\mathrm{e}^{\mathrm{i}2\chi}\left|+2\right\rangle^{l}_{\mathrm{B}}+\mathrm{e}^{-\mathrm{i}2\chi}\left|-2\right\rangle^{l}_{\mathrm{B}}\right). \tag{6}$$

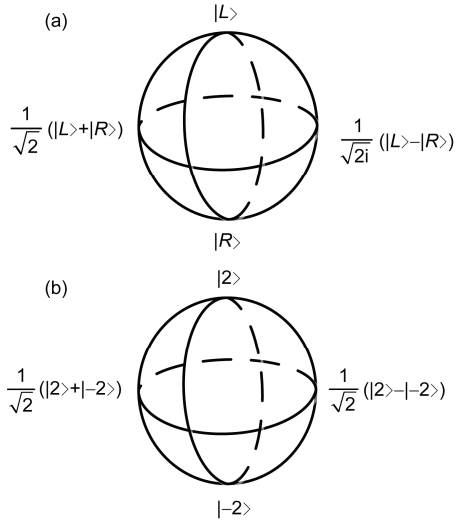

Figure 4 (a) Pointcaré sphere for polarization superposition of $\left|L\right\rangle$ and $\left|R\right\rangle$. (b) Modified Pointcaré sphere for superposition of OAM $\left(\left|\pm 2\right\rangle\right)$ states. Both $\left|\theta\right\rangle^{\pi}$ and $\left|\chi\right\rangle^{l}$ lie on the equator of the sphere.

Analogous to the linear polarization states above, these OAM superposition states described in eq. (6) are represented by points along the equator. Similar to the case in ref. [22], we may call these states as the sector states.

In the U_3, the sector states can be defined by the computer-controlled designed holograms displayed on the SLM. The SLM acts as the OAM analyzer to select these states when their relative orientation χ is changed by modifying the spatial profile. On diffraction, these holograms transform the sector states as showed in eq. (6) back into $m = 0$ state, which is then coupled into the SMF.

Hence, in the manipulation process, the overall effect is to adjust the SAM state $\left|\theta\right\rangle^{\pi}$ and the OAM state $\left|\chi\right\rangle^{l}$ at the same time by setting the two parameters θ and χ, respectively.

To quantify the hybrid entanglement of photon A and photon B, the coincidence measurement should be performed. For the hybrid entangled states described by eq. (5), the coincidence rate of photon A in the state $\left|\theta\right\rangle^{\pi}$ and photon B in the state $\left|\chi\right\rangle^{l}$ would be

$$P(\theta,\chi)=\left|\left\langle\Psi'\right|\cdot\left|\theta\right\rangle^{\pi}\left|\chi\right\rangle^{l}\right|^{2}\propto\cos^{2}(2\chi-\theta). \tag{7}$$

As shown in Figure 5, the simulation has showed that coincidence rate is a function of orientation angles χ and θ. We can use this coincidence rate to encode the key stream. If the SAM state $\left|\theta\right\rangle^{\pi}$ is fixed for photon A at Alice's site, and some discrete OAM sector states $\left|\chi\right\rangle^{l}$ are prepared with different angles χ for photon B at Bob's site, then, the coincidence rate will be different according to different angles χ. What is more, if the different angles χ here are the representations of the key for Bob's coding, Alice could judge the encoding information from the different coincidence rates.

Now, we would like to take the hybrid entangled state as the entanglement source and apply the coincidence rate to encode the key stream. Based on the scheme in Figure 3, the proposed QKD protocol is designed as follows:

(1) Through the spontaneous parametric down conversion (SPDC) process in the U_1, the polarization entangled photon pairs are created. Meanwhile the potential OAM entanglement is eliminated by the SMF. The polarization of one photon is transferred to the OAM state through the act of the QP with $q = 1$. Thus, the SAM-OAM hybrid entangled state can be obtained. And then, Alice keeps the signal photons and sends the idler photons to Bob through the free-space channel.

(2) Alice manipulates the SAM state of each signal photon using the U_2. The half-wave plate is set to orient at a variable angle $\theta/2$ to restore the horizontal polarization state. Thus, she can define the SAM state $\left|\theta\right\rangle^{\pi}$ by adjusting different orientation angles θ. For simplicity, the state $\left|\theta\right\rangle^{\pi}$

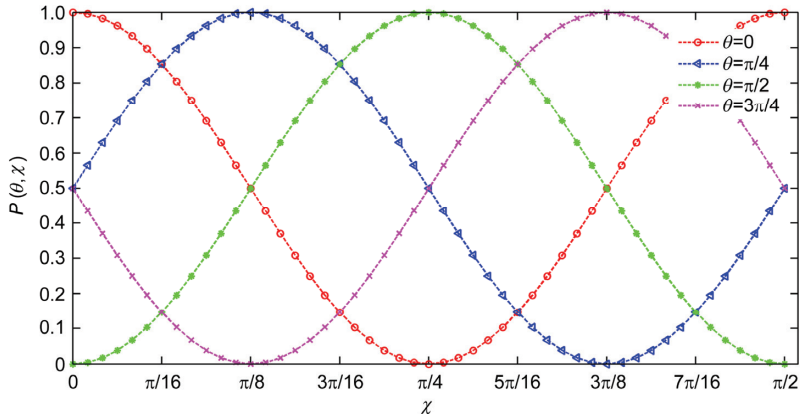

Figure 5 (Color online) Simulated coincidence rates $P(\theta, \chi)$ when varying the orientation χ.

should be selected as a fixed value.

(3) In the U_3, Bob manipulates the OAM state of each idler photon to encode his key element. He selects the sector states with different orientation angle χ using computer-controlled designed holograms with the SLM. The corresponding sector states $|\chi_1\rangle^I$, $|\chi_2\rangle^I$, …, $|\chi_N\rangle^I$ are the representations of Bob's key streams 0, 1, …, N, respectively (χ is selected in the monotone interval).

(4) Alice and Bob transmit the modulated photons to the detectors at Alice's private site through the SMF, respectively, and then, Alice performs a coincidence measurement in the U_4. According to the different coincidence rates, Alice is able to retrieve the key information encoding by Bob. Thus, they share the same keys successfully.

It should be pointed out that, in the third step, for Alice's fixed state $|\theta\rangle^\pi$, the corresponding coincidences expressed in eq. (7) are in the distinct value regions and the regions should be settled big enough to fit the requirement of the practical experiment.

4 Security analysis

We will present our argument about the security for the proposed QKD scheme and some simple attack strategies are discussed. As showed in Figure 3, Alice manipulates the SAM state of the signal photons only in her private side and keeps these photons all the time. Similar to Bob, he manipulates the OAM state of the idler photons in his private place. In these private places, Eve has no chance to access the photons. Nevertheless, Bob does not keep the idler photons all the time, it offers the chance for Eve's eavesdropping. We find that Eve still has two possible ways to intercept Bob's photons. One possible way is in the free space from Alice to Bob when Alice transmits the photons to Bob. Another is in the SMF after Bob encodes his key information and returns the photons back to Alice. We may call the two possible ways as the ATB way and the BTA way, respectively, and Eve's attack may take place on these ways.

(a) The intercept-resend (IR) attack: For the ATB way, Eve intercepts and measures each idler photon. She prepares a new one depending on the measurement result and then resends the new photon to Bob. However, as the idler photon is entangled with the signal photon, the photon resent by Eve will not satisfy eq. (7). On the other hand, Alice keeps the signal photon all the time, thus, she could detect the action of Eve's eavesdropping by adopting the Clauser-Horne-Shimony-Holt (CHSH) inequality, given by [22,23]

$$S = \left|E(\theta,\chi) - E(\theta,\chi') + E(\theta',\chi) + E(\theta',\chi')\right| \leqslant 2, \qquad (8)$$

where $E(\theta, \chi)$ is calculated from the coincidence counts $P(\theta, \chi)$ according to

$$E(\theta,\chi) = \frac{P(\theta,\chi) + P\left(\theta+\frac{\pi}{2},\chi+\frac{\pi}{4}\right) - P\left(\theta+\frac{\pi}{2},\chi\right) - P\left(\theta,\chi+\frac{\pi}{4}\right)}{P(\theta,\chi) + P\left(\theta+\frac{\pi}{2},\chi+\frac{\pi}{4}\right) + P\left(\theta+\frac{\pi}{2},\chi\right) + P\left(\theta,\chi+\frac{\pi}{4}\right)}. \qquad (9)$$

If there is a violation of the inequality, the entanglement is preserved and Alice can make sure that there is no eavesdropping in the channel. For the BTA way, Eve may do as in the ATB way. As discussed previously Eve's eavesdropping may be detected. Also, in the SMF, the OAM states carried by the idler photons have turned to be $|m=0\rangle$, and even if Eve accesses these photons, she cannot steal any of Bob's useful information. Thus, for the IR attack pattern Eve fails to obtain Bob's key information.

(b) The man-in-the-middle attack: Of course, Eve can intercept both in the ATB way and in the BTA way at the same time. She pretends to be Bob to intercept each idler photon that Alice transmits and put them in her setup, and then, she resends her prepared photon to Bob. Simultaneously, Eve pretends to be Alice to steal the photon modu-

lated by Bob. Because Bob does not know Eve's hacking, he encodes his key information and resends the photon to Alice. Now, Eve obtains Bob's resending photon and the stolen photon. However, she still cannot perform the coincidence measurement on the two photons because the two photons are not entangled.

(c) The photon-number-splitting (PNS) attack: Another simple eavesdropping strategy we consider is the PNS attack. Weak coherent light are employed in most QKD protocols and each weak pulse may contain more than one photon. Eve can hold some of the photons modulated by Bob while letting go the others when Bob resends the photon to Alice. Because of the use of the entangled photon pairs and Alice keeps the signal photon all the time, Eve still cannot obtain the key information because Eve cannot perform the coincidence measurements.

5 Conclusions and discussion

In this paper, we describe the SAM-OAM hybrid entangled state and use this state as the physical resource to perform the QKD scheme. We manage to transform the polarization entangled states into the SAM-OAM pattern, which enables us to employ the feature of OAM state and provides the opportunity to use the coincidence rate to encode the key streams. With the designed holograms, Bob's corresponding sector states with different orientation angles χ can be the representations of his N-based keys, and Alice is able to retrieve the key information conveniently by performing the coincidence measurement. In the ideal case, N can be selected as an arbitrary number, so that, there is no limit to how many bits can be encoded in each hybrid entangled photon pairs. In the practical system, the number of N is not necessarily selected too high to cut down the noise effect on coincidence counting. What is more, because the MUBs are not used, this protocol is out of the frame of the BB84 protocol. Thus, Alice and Bob do not need the classical communication for the key recovery. The security of this scheme is also analyzed. Because the coincidence rates of the hybrid entangled state are employed to retrieve the keys, whereas the signal photons are always kept in Alice's private space, Eve has no way to perform the coincidence measurement. Thus, she cannot obtain the useful information even if she adopts any individual attacks. Hence, our SAM-OAM protocol is feasible and highly efficient.

This work was supported by the National Cryptography Development Foundation of China (Grant No. MMJJ201401011) and the Science and Technology Program of Guangzhou, China (Grant Nos. 2013J4500095 and 2014J4100050).

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