基于部分纠缠信道的高维赤道态概率可控远程联合制备

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摘要 作为一种特殊类型的量子态, 赤道态由于在量子信息处理中的独特应用引起了广泛关注. 基于不同多粒子纠缠态信道, 研究者提出了任意双量子比特、三量子比特以及多量子比特赤道态多方联合制备方案. 本文研究了基于高维部分纠缠态的高维多粒子赤道态多方概率可控远程联合制备方案, 提出了两个基于高维部分纠缠态的高维多粒子赤道态多方可控远程联合制备方案. 一个方案以 m 个高维类GHZ态为纠缠信道, 可实现多个控制方控制下的高维赤道态多方远程联合制备, 另一个方案以一个多粒子部分纠缠态为量子纠缠信道完成单个控制方控制下的高维赤道态远程联合制备. 多个发送方共享多粒子赤道态信息, 依据已知信息对手中纠缠粒子选择执行相应正交投影测量, 控制方对他的纠缠粒子执行单粒子测量. 接收方与所有发送方和控制方合作, 仅需引入一个附加量子比特就可在他的纠缠粒子上重建需制备量子态. 方案不需要使用最大纠缠态作为量子纠缠信道实现高维赤道态远程可控联合制备, 具有可行性强的优点.

关键词 量子态可控远程联合制备, 高维多粒子赤道态, 类GHZ态, 附加粒子

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1 引言

网络中不同节点间量子态的远程传送是远程量子通信的重要任务之一^[1-12]. 基于预先共享量子纠缠信道非定域相关性, 量子隐形传态可以完成任意未知量子态的远程传送^[13-25]. 与量子隐形传态不同, 量子态远程制备中发送方已知需制备量子态信息^[26-28]. 由于发送方可以依据已知需制备量子态信息协助接收方完

成量子态远程制备, 因此量子态远程制备可以消耗较少的通信资源来完成任意量子态的远程传送^[29-35].

量子态远程传送所需的通信资源是量子信息理论的重要研究内容^[26]. 自从2000年Pati^[28]首先指出量子比特Bloch球赤道圆上量子态远程制备所消耗的资源可少于量子隐形传态以来, 任意量子比特, 特别是任意多量子比特态的多方远程联合制备引起了广泛关注^[36-46]. 例如, 2003年, Liu等人^[36]提出了基于双粒子

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Li Y H, He L M, Zhou P. Multiparty probabilistic controlled joint remote preparation of an arbitrary m -qudit equatorial state with a pure entangled quantum channel (in Chinese). Sci Sin-Phys Mech Astron, 2021, 51: 050301, doi: [10.1360/SSPMA-2020-0251](https://doi.org/10.1360/SSPMA-2020-0251)

纠缠态的双粒子高维纠缠赤道态远程制备方案, 以两个双粒子纠缠态 $|\psi\rangle = \frac{1}{\sqrt{3}}(|00\rangle + |11\rangle + |22\rangle)$ 为量子纠缠信道, 即可实现双粒子高维纠缠赤道态 $|\phi\rangle = \frac{1}{\sqrt{3}}(|00\rangle + e^{i\vartheta_1}|11\rangle + e^{i\vartheta_2}|22\rangle)$ 的确定远程制备. 2004 年, Liu 等人^[37]提出了高维多粒子赤道态远程制备方案, 以 N 个双粒子纠缠态为量子纠缠信道, 实现了 N 粒子高维赤道纠缠态 $|\phi\rangle = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} e^{i\vartheta_j} |jj \dots j\rangle$ 的远程制备. 2008 年, Wang 等人^[38]提出了单粒子高维赤道态 $|\phi\rangle = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} e^{i\vartheta_j} |j\rangle$ 的概率远程制备模型. 2010 年, Zhan 等人^[39]提出了基于部分纠缠信道的高维单粒子赤道态概率多方联合制备模型. 2011 年, Chen 等人^[40]以两个 3 粒子部分纠缠态为量子纠缠信道提出了双粒子高维赤道态多方远程联合制备方案. 2014 年, Wang 和 Liang^[41]提出了双粒子纠缠态和 3 粒子纠缠态的任意 N 量子比特赤道态可控远程制备方案. 2015 年, Choudhury 和 Dhara^[42]提出基于 3 粒子 GHZ 态的双量子比特赤道态远程联合制备方案, 方案成功率为 25%. 通过使用同样的量子纠缠信道, Li 和 Ghose^[43]将双量子比特赤道态远程联合制备成功率提高到 100%, 并提出了 N 量子比特赤道态远程联合制备方案. 2018 年, Bich 等人^[44]开展了基于高维双粒子纠缠态的杂化赤道态远程制备研究. 2020 年, Wang 等人^[45]提出了基于多粒子纠缠态的 4 量子比特赤道簇态 $|0000\rangle + e^{i\vartheta}|0011\rangle + e^{i\vartheta}|1100\rangle - e^{i\vartheta}|1111\rangle$ 多方远程联合制备模型. 现在, 已经完成了基于核磁共振技术的单量子比特赤道态远程制备实验验证^[46].

在量子态多方可控联合制备中, 由于多个发送方共享需制备量子态信息, 接收方只有与所有控制方合作才能完成量子态制备, 因此与量子态远程制备相比量子态多方可控联合制备具有更好的安全性^[47-53]. 2009 年, An^[47]提出了基于 4 粒子 GHZ 态的任意实系数双量子比特态可控远程联合制备. 以 8 粒子簇态为量子纠缠信道, Guan 等人^[48]提出了任意双量子比特态可控联合制备方案. 2013 年, Wang 等人^[49]开展了基于多粒子纠缠态的任意单量子比特和双量子比特态多方可控远程联合制备研究. 2014 年, An 和 Bich^[50]提出了基

于 4 粒子部分纠缠信道的单量子比特态可控联合制备方案. 2017 年, Lv 等人^[51]开展了高维多粒子态可控远程联合制备研究, 通过执行半正定算子测量完成高维多粒子态远程制备. 2019 年, Jiang 等人^[52]提出了基于 $3N+1$ 粒子纠缠态的任意 N 量子比特态可控远程联合制备方案, 通过构造基于需制备量子态信息的正交投影测量完成多量子比特态可控远程联合制备. Sang 和 Yu^[53]提出了基于 7 粒子纠缠态的任意双量子比特赤道态可控联合制备方案.

虽然已有多粒子赤道态可控远程联合制备模型, 这些模型都是基于最大纠缠信道的双量子比特赤道态可控远程联合制备研究^[53]. 基于部分纠缠信道的高维多粒子赤道态可控远程联合制备方案还不多. 本文提出基于 N 个高维多粒子部分纠缠态的任意高维 N 粒子赤道态概率可控远程联合制备方案. 多个发送方共享需制备量子态信息并依据已知信息构造相应 N 粒子正交投影测量. 控制方对手中纠缠粒子执行单粒子广义 X 基测量, 接收方与所有控制方、发送方合作完成量子态远程制备. 由于基于部分纠缠信道的高维多粒子赤道态远程联合制备成功率与部分纠缠信道可提取最大纠缠态概率相等, 因而方案是最优的. 此外, 方案不需要使用最大纠缠信道来完成量子态远程制备, 具有可行性强的优点.

2 基于高维类 GHZ 的任意高维多粒子赤道态可控远程联合制备

广义 X 基可表示为^[51]

$$|r_x\rangle = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} e^{-\frac{2\pi i}{d} rj} |j\rangle, \quad (1)$$

其中任意 $r = 0, \dots, d-1$ 用于表示 d 个互相正交 d 维量子态.

与文献^[38]类似, m 粒子高维赤道态可表示为

$$|\phi\rangle = \frac{1}{\sqrt{d^m}} \sum_{j_1, \dots, j_m=0}^{d-1} e^{i\vartheta_{j_1, \dots, j_m}} |j_1 \dots j_m\rangle, \quad (2)$$

其中 $\delta_{0\dots 0} = 0, \delta_{0\dots 1}, \dots, \delta_{d-1, \dots, d-1}$ 为任意实数. k 个发送方 $Alice_n (n = 1, \dots, k)$ 共享需制备量子态信息, 即第 n 个发送方 $Alice_n$ 已知相位 $\delta_{j_1 \dots j_m}^n (j_1, \dots, j_m = 0, \dots, d-1)$ 信息^[51].

$$\delta_{j_1 \cdots j_m} = \sum_{n=1}^k \delta_{j_1 \cdots j_n}^n, \quad (3)$$

k 个发送方互相合作在多个控制方 $\text{Charlie}_s (s = 1, \dots, l)$ 的控制下协助接受方制备需制备量子态.

与相位信息 $\delta_{j_1 \cdots j_m}^n (j_1, \dots, j_m = 0, \dots, d-1)$ 相应的 m 粒子正交测量基可表示为^[51]

$$|\phi_{l_1 \cdots l_m}\rangle = \frac{1}{\sqrt{d^m}} \sum_{j_1, \dots, j_m=0}^{d-1} e^{-\frac{2\pi i}{d}(j_1 l_1 + \cdots + j_m l_m)} e^{-i\delta_{j_1 \cdots j_m}^n} |j_1 \cdots j_m\rangle, \quad (4)$$

其中 $l_1, \dots, l_m = 0, \dots, d-1$ 用于表示 d^m 个彼此正交的量子态 $|\phi_{l_1 \cdots l_m}\rangle$.

为完成高维多粒子赤道态 $|\phi\rangle$ 的可控远程联合制备, 通信方共享 m 个高维多粒子类 GHZ 态. 与文献^[54] 类似, 高维多粒子类 GHZ 态可表示为

$$|\psi_p\rangle = \sum_{j=0}^{d-1} \alpha_j |jj \cdots j\rangle_{a_1 \cdots a_{k+l+1}}, \quad (5)$$

其中 $p = 1, \dots, m$, $\alpha_j (j = 0, \dots, d-1)$ 为任意复数, 满足归一化关系 $\sum_{j=0}^{d-1} |\alpha_j|^2 = 1$. 如图 1 所示, 接收方 Bob 将第 $p (p = 1, \dots, m)$ 个多粒子纠缠态 $|\psi_p\rangle$ 中第 $n (n = 1, \dots, k)$ 个粒子 a_{pn} 发送给第 n 个发送方 Alice_n , 第 $k+s$ ($s = 1, \dots, l$) 个粒子 $a_{p,k+s}$ 发送给第 s 个控制方 Charlie_s , 并保留第 $k+l+1$ 个粒子 $a_{p,k+l+1}$. 由粒子 $a_{11}, a_{12}, \dots, a_{m,k+l+1}$ 所组成的复合量子系统量子态可表示为

$$\begin{aligned} |\Psi\rangle &= |\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots \otimes |\psi_m\rangle \\ &= \sum_{j_1, j_2, \dots, j_m=0}^{d-1} \alpha_{j_1} \alpha_{j_2} \cdots \alpha_{j_m} |j_1 j_2 \cdots j_m\rangle_{a_{11} a_{12} \cdots a_{1,k+l+1}} \\ &\quad \times |j_2 j_2 \cdots j_2\rangle_{a_{21} a_{22} \cdots a_{2,k+l+1}} \cdots |j_m j_m \cdots j_m\rangle_{a_{m1} a_{m2} \cdots a_{m,k+l+1}}. \end{aligned} \quad (6)$$

安全建立量子纠缠信道后, 发送方 Alice_n ($n = 1, \dots, k$) 依据已知需制备量子态信息对手中纠缠粒子 $a_{1n}, a_{2n}, \dots, a_{mn}$ 执行正交投影测量. 复合系统量子态可改写为(未归一化):

$$\begin{aligned} |\Psi\rangle &= \sum_{j_1, j_2, \dots, j_m=0}^{d-1} \alpha_{j_1} \alpha_{j_2} \cdots \alpha_{j_m} e^{\frac{2\pi i}{d}(j_1 q_1 + j_2 q_2 + \cdots + j_m q_m)} \\ &\quad \times e^{i\delta_{j_1 \cdots j_m}} |\phi_{q_{11} \cdots q_{1m}}\rangle_{a_{11} \cdots a_{m1}} |\phi_{q_{21} \cdots q_{2m}}\rangle_{a_{12} \cdots a_{m2}} \cdots \\ &\quad \times |\phi_{q_{k1} \cdots q_{km}}\rangle_{a_{1k} \cdots a_{mk}} |j_1 j_1 \cdots j_1\rangle_{a_{1,k+1} \cdots a_{1,k+l+1}} \\ &\quad \times |j_2 \cdots j_2\rangle_{a_{2,k+1} \cdots a_{2,k+l+1}} \cdots \\ &\quad \times |j_m \cdots j_m\rangle_{a_{m,k+1} \cdots a_{m,k+l+1}}, \end{aligned} \quad (7)$$

其中 $q_s = q_{1s} + \cdots + q_{ks} (s = 1, \dots, m)$.

若 $\text{Alice}_n (n = 1, \dots, k)$ 对手中 m 个纠缠粒子 $a_{1n}, a_{2n}, \dots, a_{mn}$ 正交投影测量结果为 $|\phi_{q_{n1} \cdots q_{nm}}\rangle_{a_{1n} \cdots a_{mn}}$, 则剩余粒子 $a_{1,k+1}, a_{1,k+2}, \dots, a_{m,k+l+1}$ 量子态转化为

$$\begin{aligned} |\Psi_1\rangle &= \sum_{j_1, j_2, \dots, j_m=0}^{d-1} \alpha_{j_1} \alpha_{j_2} \cdots \alpha_{j_m} e^{\frac{2\pi i}{d}(j_1 q_1 + j_2 q_2 + \cdots + j_m q_m)} \\ &\quad \times e^{i\delta_{j_1 \cdots j_m}} |j_1 j_1 \cdots j_1\rangle_{a_{1,k+1} \cdots a_{1,k+l+1}} \cdots \\ &\quad |j_m \cdots j_m\rangle_{a_{m,k+1} \cdots a_{m,k+l+1}}. \end{aligned} \quad (8)$$

为完成高维多粒子赤道态可控远程联合制备, 控制方 $\text{Charlie}_s (s = 1, \dots, l)$ 对手中纠缠粒子 $a_{1,k+s}, a_{2,k+s}, \dots, a_{m,k+s}$ 执行广义 X 基测量. 与文献^[55,56] 类似, 控制方对纠缠粒子执行的广义 X 基测量可表示为

$$M = \prod_{p=1}^m M_p, \quad (9)$$

式中

$$M_p = (|0_x\rangle)^{l-t_{p1}-\cdots-t_{p,d-1}} (|1_x\rangle)^{t_{p1}} \cdots (|(d-1)_x\rangle)^{t_{p,d-1}} \quad (10)$$

表示所有控制方对第 $p (p = 1, \dots, m)$ 个多粒子纠缠态的广义 X 基测量, $t_{pj} (j = 0, \dots, d-1)$ 表示测量结果为 $|j_x\rangle$ 的控制方个数. 广义 X 基测量后, 粒子 $a_{1,k+l+1}, \dots, a_{m,k+l+1}$ 状态转化为

$$\begin{aligned} |\Psi_2\rangle &= \sum_{j_1, j_2, \dots, j_m=0}^{d-1} \alpha_{j_1} \alpha_{j_2} \cdots \alpha_{j_m} e^{\frac{2\pi i}{d}(j_1 q_1 + j_2 q_2 + \cdots + j_m q_m)} \\ &\quad \times e^{\frac{2\pi i}{d}(t_{1j_1} + t_{2j_2} + \cdots + t_{mj_m})} e^{i\delta_{j_1 \cdots j_m}} |j_1 \cdots j_m\rangle_{a_{1,k+l+1} \cdots a_{m,k+l+1}}, \end{aligned} \quad (11)$$

图 1 基于高维类GHZ态的多粒子赤道态多方可控远程联合制备原理图. 其中方框表示广义X基测量, 长框表示 m 粒子正交投影测量. Alice _{n} ($n=1, \dots, k$)为发送方, Charlie _{s} ($s=1, \dots, l$)为控制方, Bob为接收方

Figure 1 The schematic principle for controlled joint remote preparation of multiparticle equator state via high-dimensional GHZ-class state. The squares represent generalized X -basis measurements performed by controllers, and the rectangles denote m -particle projective measurements performed by senders. Alice _{n} ($n=1, \dots, k$) represents the sender, Charlie _{s} ($s=1, \dots, l$) represents the controller, Bob represents the receiver.

其中

$$t_p = 1 \cdot t_{p1} + 2 \cdot t_{p2} + \dots + (d-1)t_{p,d-1}. \quad (12)$$

为消除部分纠缠信道影响, Bob引入初态为 $|0\rangle_{b_{\text{aux}}}$ 的附加量子比特 b_{aux} , 对手中粒子 $a_{1,k+l+1}, \dots, a_{m,k+l+1}$, b_{aux} , 执行联合么正演化^[57]. 若

$$\alpha_s = \min\{\alpha_j, j = 0, 1, \dots, d-1\},$$

联合么正演化

$$U_{\max} = \sum_{j_1, \dots, j_m=0}^{d-1} \left[\frac{\alpha_s^m}{\alpha_{j_1} \dots \alpha_{j_m}} |j_1 \dots j_m\rangle |0\rangle + \sqrt{1 - \left(\frac{\alpha_s^m}{\alpha_{j_1} \dots \alpha_{j_m}} \right)^2} |j_1 \dots j_m\rangle |1\rangle \right] |j_1 \dots j_m\rangle \langle 0|, \quad (13)$$

可将粒子 $a_{1,k+l+1}, \dots, a_{m,k+l+1}, b_{\text{aux}}$ 所组成复合系统状态转化为

$$\begin{aligned} |\Psi_3\rangle &= U_{\max} |\Psi_2\rangle \otimes |0\rangle_{b_{\text{aux}}} \\ &= \sum_{j_1, j_2, \dots, j_m=0}^{d-1} \alpha_{j_1} \alpha_{j_2} \dots \alpha_{j_m} e^{\frac{2\pi i}{d}(j_1 q_1 + j_2 q_2 + \dots + j_m q_m)} \\ &\quad \times e^{\frac{2\pi i}{d}(t_1 j_1 + t_2 j_2 + \dots + t_m j_m)} e^{i\delta_{j_1 \dots j_m}} |j_1 \dots j_m\rangle_{a_{1,k+l+1} \dots a_{m,k+l+1}} \end{aligned}$$

$$\left(\frac{\alpha_s^m}{\alpha_{j_1} \dots \alpha_{j_m}} |0\rangle + \sqrt{1 - \left(\frac{\alpha_s^m}{\alpha_{j_1} \dots \alpha_{j_m}} \right)^2} |1\rangle \right)_{b_{\text{aux}}}. \quad (14)$$

执行联合么正演化后, Bob测量附加粒子 b_{aux} 状态. 若测量结果为 $|0\rangle$ 则量子态制备成功, 否则量子态制备失败. 若测量结果为 $|0\rangle$, 剩余粒子状态转化为

$$\begin{aligned} |\Phi\rangle &= \sum_{j_1, j_2, \dots, j_m=0}^{d-1} e^{\frac{2\pi i}{d}(j_1 q_1 + j_2 q_2 + \dots + j_m q_m)} \\ &\quad \times e^{\frac{2\pi i}{d}(t_1 j_1 + t_2 j_2 + \dots + t_m j_m)} e^{i\delta_{j_1 \dots j_m}} |j_1 \dots j_m\rangle_{a_{1,k+l+1} \dots a_{m,k+l+1}}. \end{aligned} \quad (15)$$

与发送方、控制方测量结果相应的单粒子操作

$$(U_{q_1 \oplus t_1})_{a_{1,k+l+1}} \otimes \dots \otimes (U_{q_m \oplus t_m})_{a_{m,k+l+1}}$$

可将粒子 $a_{1,k+l+1}, \dots, a_{m,k+l+1}$ 状态由 $|\Phi\rangle$ 转化为 $|\phi\rangle$:

$$\begin{aligned} |\phi\rangle_{a_{1,k+l+1} \dots a_{m,k+l+1}} &= (U_{q_1 \oplus t_1})_{a_{1,k+l+1}} \otimes \dots \otimes (U_{q_m \oplus t_m})_{a_{m,k+l+1}} \\ &\quad \times |\Phi\rangle_{a_{1,k+l+1} \dots a_{m,k+l+1}} \\ &= \sum_{j_1, j_2, \dots, j_m=0}^{d-1} e^{i\delta_{j_1 \dots j_m}} |j_1 \dots j_m\rangle_{a_{1,k+l+1} \dots a_{m,k+l+1}}, \end{aligned} \quad (16)$$

其中

$$\left(U_{q_s \oplus t_s}\right)_{a_{s,k+l+1}} = \sum_{j=0}^{d-1} e^{-\frac{2\pi i}{d}(q_s+t_s)j} |j\rangle\langle j|, \quad (17)$$

$s = 1, \dots, m$, $q_s \oplus t_s$ 表示 $q_s + t_s$ 模 d . 与文献[57]类似, 量子态制备成功率 $p = d^m |\alpha_s|^{2m}$. 与文献[58–60]类似, 两粒子非最大纠缠态 $|\psi\rangle_{AB} = \sum_{j=0}^{d-1} \alpha_j |jj\rangle$ 获取最大纠缠态数目 $d_{\min} \{|\alpha_1|^2, |\alpha_2|^2, \dots, |\alpha_{d-1}|^2\}$ 可用子系统约化密度矩阵的 Von Neumann 熵表示:

$$E = - \sum_{i=0}^{d-1} \alpha_i^2 \log_2 \alpha_i^2. \quad (18)$$

与两粒子非最大纠缠态 $|\psi\rangle_{AB}$ 类似^[61], 多粒子非最大纠缠态 $|\psi_p\rangle = \sum_{j=0}^{d-1} \alpha_j |jj \dots j\rangle_{a_1 \dots a_{k+l+1}}$ 提取最大纠缠态概率为 $d|\alpha_s|^2$ 与文献[57,62]类似, 基于 m 个多粒子类 GHZ 态可控联合制备方案成功率 $p = d^m |\alpha_s|^{2m}$ 与 m 个多粒子类 GHZ 态可提取的最大纠缠态概率相等, 因而方案是最优的.

3 基于多粒子纠缠态的高维赤道态远程可控联合制备

与基于部分纠缠高维类 GHZ 态任意赤道态可控远程联合制备类似, 基于多粒子纠缠态的高维赤道态远程可控联合制备以高维多粒子纠缠态为量子纠缠信道, 完成在 1 个控制方控制下的任意高维赤道态远程联合制备. 为实现基于多粒子纠缠态任意赤道态 $|\varphi\rangle$ 的多方远程可控联合制备, 发送方 Alice_{*n*} ($n = 1, \dots, k$)、控制方 Charlie 和接收方 Bob 共享多粒子纠缠态 (未归一化):

$$|\Psi'\rangle = \sum_{t, j_1, \dots, j_m=0}^{d-1} \alpha_{j_1} \dots \alpha_{j_m} |j_1, \dots, j_1 \oplus t\rangle_{a_{11} \dots a_{1,k+1}} \dots |j_m, \dots, j_m \oplus t\rangle_{a_{m1} \dots a_{m,k+1}} |t\rangle_c. \quad (19)$$

如图 2 所示, 接收方 Bob 将纠缠粒子 $a_{1n}, \dots, a_{mn}$ ($n = 1, \dots, k$) 发送给第 n 个发送方 Alice_{*n*}, 纠缠粒子 c 发送给控制方 Charlie, 保留纠缠粒子 $a_{1,k+1}, \dots, a_{m,k+1}$.

安全建立量子纠缠信道后, 发送方 Alice_{*n*} ($n = 1, \dots, k$) 依据已知需制备量子态信息 $\delta_{j_1 \dots j_m}^n$ ($j_1, \dots, j_m = 0, \dots, d-1$)

对手中纠缠粒子 $a_{1n}, \dots, a_{mn}$ 执行测量基 $\left\{ \left| \phi_{q_{n1} \dots q_{nm}} \right\rangle \right\}$ ($q_{n1}, \dots, q_{nm} = 0, \dots, d-1$) 下的正交投影测量, 控制方 Charlie 对手中粒子 c 执行 Z 基测量, 接收方 Bob 依据发送方和控制方测量结果对手中粒子执行相应局域么正操作, 可在他的纠缠粒子 $a_{1,k+1}, \dots, a_{m,k+1}$ 上重建需制备量子态 $|\varphi\rangle$ ^[51].

发送方 Alice_{*n*} ($n = 1, \dots, k$) 对手中粒子执行 $a_{1n}, \dots, a_{mn}$ 执行测量基 $\left\{ \left| \phi_{q_{n1} \dots q_{nm}} \right\rangle \right\}$ 下的正交投影测量后, 若 Alice_{*n*} 测量结果为 $\left| \phi_{q_{n1} \dots q_{nm}} \right\rangle_{a_{1n} \dots a_{mn}}$, 则剩余粒子 $a_{1,k+1}, a_{2,k+1}, \dots, a_{m,k+1}, c$ 状态转化为

$$|\Psi_1'\rangle = \sum_{j_1, j_2, \dots, j_m, t=0}^{d-1} \alpha_{j_1} \alpha_{j_2} \dots \alpha_{j_m} e^{\frac{2\pi i}{d}(j_1 q_1 + \dots + j_m q_m)} \times e^{i\delta_{j_1 \dots j_m}} |j_1 \oplus t\rangle_{a_{1,k+1}} \dots |j_m \oplus t\rangle_{a_{m,k+1}} |t\rangle_c, \quad (20)$$

其中 $q_s = q_{1s} + \dots + q_{ks}$ ($s = 1, \dots, m$).

与文献[63]类似, 为完成高维赤道态可控联合制备, 控制方 Charlie 对手中纠缠粒子 c 执行 Z 基测量. 若测量结果为 $|t\rangle_c$ ($t = 0, \dots, d-1$), 则粒子 $a_{1,k+1}, a_{2,k+1}, \dots, a_{m,k+1}$ 状态转化为

$$|\Psi_2'\rangle = \sum_{j_1, j_2, \dots, j_m=0}^{d-1} \alpha_{j_1} \alpha_{j_2} \dots \alpha_{j_m} e^{\frac{2\pi i}{d}(j_1 q_1 + \dots + j_m q_m)} \times e^{i\delta_{j_1 \dots j_m}} |j_1 \oplus t\rangle_{a_{1,k+1}} \dots |j_m \oplus t\rangle_{a_{m,k+1}}. \quad (21)$$

与基于高维类 GHZ 态任意赤道态可控联合制备类似, 接收方引入初态为 $|0\rangle_{b_{\text{aux}}}$ 的附加量子比特 b_{aux} , 对手中粒子 $a_{1,k+1}, a_{2,k+1}, \dots, a_{m,k+1}, b_{\text{aux}}$ 执行联合么正演化 U'_{max} . 与文献[57]类似, 么正演化

$$U'_{\text{max}} = \sum_{j_1, \dots, j_m=0}^{d-1} \left[\frac{\alpha_s^m}{\alpha_{j_1} \dots \alpha_{j_m}} |j_1 \oplus t, \dots, j_m \oplus t\rangle |0\rangle + \sqrt{1 - \left(\frac{\alpha_s^m}{\alpha_{j_1} \dots \alpha_{j_m}} \right)^2} |j_1 \oplus t, \dots, j_m \oplus t\rangle |1\rangle \right] |j_1 \oplus t, \dots, j_m \oplus t\rangle |0\rangle. \quad (22)$$

图 2 基于高维部分纠缠态的多粒子赤道态多方可控远程联合制备原理图. 其中方框表示广义 Z 基测量, 长框表示 m 粒子正交投影测量. Alice _{n} ($n=1, \dots, k$) 为发送方, Charlie 为控制方, Bob 为接收方

Figure 2 The schematic principle for controlled joint remote preparation of multiparticle equator state via a multiparticle high-dimensional partially entangled state. The squares represent generalized Z-basis measurements performed by controller, and the rectangles denote m -particle projective measurements performed by senders. Alice _{n} ($n=1, \dots, k$) represents the sender, Charlie represents the controller, Bob represents the receiver.

将粒子 $a_{1,k+1}, a_{2,k+1}, \dots, a_{m,k+1}, b_{\text{aux}}$ 状态转化为

$$\begin{aligned} |\Psi_3\rangle &= U'_{\max} |\Psi_2\rangle_{a_{1,k+1}, \dots, a_{m,k+1}} \otimes |0\rangle_{b_{\text{aux}}} \\ &= \sum_{j_1, j_2, \dots, j_m=0}^{d-1} \alpha_{j_1} \alpha_{j_2} \dots \alpha_{j_m} e^{\frac{2\pi i}{d}(j_1 q_1 + \dots + j_m q_m)} \\ &\quad \times e^{i\delta_{j_1 \dots j_m} |j_1 \oplus t, \dots, j_m \oplus t\rangle_{a_{1,k+1} \dots a_{m,k+1}}} \\ &\quad \times \left(\frac{\alpha_s^m}{\alpha_{j_1} \dots \alpha_{j_m}} |0\rangle + \sqrt{1 - \left(\frac{\alpha_s^m}{\alpha_{j_1} \dots \alpha_{j_m}} \right)^2} |1\rangle \right)_{b_{\text{aux}}}. \end{aligned} \quad (23)$$

联合幺正演化后, 接收方 Bob 对附加粒子 b_{aux} 执行单粒子测量, 若测量结果为 $|0\rangle$, 则量子态制备成功. 若附加粒子测量结果为 $|1\rangle$, 则剩余粒子状态转化为^[57]

$$\begin{aligned} |\Phi'\rangle &= \sum_{j_1, j_2, \dots, j_m=0}^{d-1} e^{\frac{2\pi i}{d}(j_1 q_1 + \dots + j_m q_m)} \\ &\quad \times e^{i\delta_{j_1 \dots j_m} |j_1 \oplus t, \dots, j_m \oplus t\rangle_{a_{1,k+1} \dots a_{m,k+1}}}. \end{aligned} \quad (24)$$

与发送方测量结果 $q_1, q_2, \dots, q_k$ 和控制方测量结果 t 相应的单粒子操作 $(U_{q_1, t})_{a_{1,k+1} \dots a_{m,k+1}} \otimes \dots \otimes (U_{q_m, t})_{a_{m,k+1} \dots a_{m,k+1}}$ 可将粒子 $a_{1,k+1}, a_{2,k+1}, \dots, a_{m,k+1}$ 状态转化为需制备量子态

$$\begin{aligned} |\phi\rangle_{a_{1,k+1}, \dots, a_{m,k+1}} &= (U_{q_1, t_1})_{a_{1,k+1} \dots a_{1,k+1}} \otimes \dots \otimes (U_{q_m, t_m})_{a_{m,k+1} \dots a_{m,k+1}} |\Phi'\rangle_{a_{1,k+1}, \dots, a_{m,k+1}} \\ &= \sum_{j_1, j_2, \dots, j_m=0}^{d-1} e^{i\delta_{j_1 \dots j_m} |j_1 \dots j_m\rangle_{a_{1,k+1} \dots a_{m,k+1}}}, \end{aligned} \quad (25)$$

其中

$$U_{q_s, t} = \sum_{l=0}^{d-1} e^{-\frac{2\pi i}{d} l q_s} |l\rangle \langle l \oplus t|, \quad (26)$$

$s = 1, \dots, m$. 与基于类 GHZ 态可控联合制备类似, 量子态制备成功概率为 $p = d^m |\alpha_s|^{2m}$ ^[57].

基于多粒子部分纠缠态的高维赤道态可控联合制备方案中, 通信方仅需共享 1 个 $(mk+m+1)$ 粒子纠缠态就可完成在单个控制方控制下的任意高维多粒子赤道态多方联合制备. 与基于高维类 GHZ 态高维赤道态可控联合制备方案相比, 在单个控制方控制下的任意高维赤道态远程联合制备中具有需分发纠缠粒子数少, 通信效率高的优点.

4 结论

与已知系数部分纠缠态纠缠浓缩类似^[64,65], 在量子通信实际应用中, 通信方 Alice, Bob 可以通过对大量

同种部分纠缠态的测量来获得纠缠粒子远程分发后部分纠缠态系数信息. 当系数 $\alpha_0 = \dots = \alpha_{d-1} = \frac{1}{\sqrt{d}}$, 量子纠缠信道由多粒子最大纠缠态组成. 与多量子比特赤道态远程制备类似, 接收方与所有发送方、控制方合作可以完成高维多粒子赤道态的确定远程制备. 演化矩阵 $U_{\max}$ 为单位矩阵, 即接收方不需要对手中纠缠粒子 $a_{1,k+l+1}, \dots, a_{m,k+l+1}$ 以及附加粒子 b_{aux} 执行么正操作, 只需依据所有发送方和控制方的测量结果对纠缠粒子 $a_{1,k+l+1}, \dots, a_{m,k+l+1}$ 执行单粒子操作就可完成高维多粒子赤道态的可控联合制备.

与任意高维多粒子态可控远程联合制备相比^[51], 任意高维赤道态远程制备中第一发送方仅需依据已知需制备量子态信息对手中粒子执行正交投影测量, 不需要依据已知需制备量子态信息将纠缠信道转化为目标信道. 此外, 第一发送方仅需将手中粒子正交投影测

量发送给接收方, 不需要将纠缠信道转化结果发送给其他发送方, 大大减少了量子态远程制备中所需传送的经典信息, 提高了量子态远程制备效率.

本文提出了两个非最大纠缠信道下的任意高维多粒子赤道态多方可控远程联合制备方案. 一个方案以 m 个 $(k+l+1)$ 粒子高维类GHZ态为量子纠缠信道, 完成任意 m 粒子高维赤道态的多方可控远程联合制备, 另一个方案以一个多粒子非最大纠缠态为量子纠缠信道, 实现在单个控制方控制下的任意高维多粒子赤道态可控远程联合制备. 发送方依据已知需制备量子态信息执行 m 粒子正交投影测量, 控制方执行广义 X 基测量, 接收方对手中纠缠粒子执行与所有发送方和控制方测量结果相应的单粒子测量即可重建需制备量子态. 为消除部分纠缠信道影响, 接收方引入附加量子比特并执行联合么正操作消除部分纠缠信道影响, 方案成功率为 $d^m |\alpha_s|^{2m}$.

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Multiparty probabilistic controlled joint remote preparation of an arbitrary m -qudit equatorial state with a pure entangled quantum channel

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The quantum equatorial state, a special kind of quantum state, has attracted much attention because of its potential for applications in quantum information processing. The remote preparations of arbitrary two-qudit, three-qudit equatorial and m -qudit equatorial states have been studied via different entangle quantum channels. In this paper, we propose two protocols for the controlled joint remote preparation of an arbitrary m -qudit equatorial state with a pure entangled quantum channel. The first is for the controlled joint remote preparation of the m -qudit equatorial state by using m multiparticle GHZ-class states as the quantum channel, and the second is for the controlled joint remote preparation of the m -qudit equatorial state via a multiparticle entangled state. All senders share information about the prepared state and perform m -particle projective measurements on their entangled particles according to their information about the prepared state. All controllers perform generalized X-basis measurements on their entangled particles. If a receiver cooperates with all the senders and controllers, he can prepare the original state probabilistically by performing unitary operations on his entangled particles and an auxiliary qubit. This protocol is more convenient for applications because it only requires nonmaximal entanglement for the controlled joint remote preparation of an arbitrary m -qudit equatorial state.

controlled joint remote state preparation, high-dimensional equatorial state, GHZ-class state, auxiliary qubit

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